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Application of Diffractions by Convex Surfaces to Irregular Terrain Situations

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Previous solutions by Rice and by Wait and Conda are combined and extended to provide more readily evaluated formulas for the diffraction of radio waves by the "rounded obstacles" encountered in irregular terrain situations. A comparison with experimental data is also provided.

1. Introduction

Schelleng, Burrows, and Ferrell [1933] were the first to successfully evaluate the diffraction of radio waves by isolated irregulated terrain features. They approximated a mountain ridge by a semi-infinite knife edge and applied the Fresnel-Kirchhoff diffraction formula. Since then, application of this formula has frequently provided usefully close approximations to the diffraction of radiowaves by hills, mountains, and mountain ridges [Selvidge, 1941; Bullington, 1947; Norton, Schulkin, and Kirby, 1949; Matsuo, 1950; Dickson, Egli, Herbstreit, and Wickizer, 1953; Kono, Uesugi, Hirai, Niwa, and Irie, 1954; Kirby, Dougherty, and McQuate, 1955].

In some applications, however, it was demonstrated that the simple knife edge is not an adequate model. Some provision must be incorporated into the model for the effect of the broad crests often encountered in hills and mountain ridges. Solutions for just such an improved model have been given by Rice [1954], Neugebauer and Bachynski [1958], and by Wait and Conda [1959]. The application of these solutions to irregular terrain situations have not, however, been too numerous to date [Crysdale, 1958; Barsis and Kirby, 1961]. This is due perhaps to certain practical difficulties in their application. The purpose of this report is to extend the work of Rice and of Wait and Conda so as to provide engineering formulas for their application to irregular terrain situations. A comparison with experimental data is also given.

2. Diffraction Formula

For the application to radio propagation over irregular terrain, the effect of the terrain may be estimated from the theoretical solutions of the wave equation. The theoretical solution chosen is usually that for some combination of simple geometrical models which approximate a modified terrain profile for the great circle plane containing the transmitting and receiving points. The modified terrain profile is that required to provide for the effects of atmospheric refraction [Norton, Rice, and Vogler, 1955]. Some simple geometrical models for irregular terrain features are illustrated in figure 1. The effect of ground reflection is included by determining equivalent reflecting planes, which then provide additive solutions for the models of figure 1 [Schelleng, Burrows, and Ferrell, 1933].

In this section, the diffraction formula is presented, deferring its basis to the following section and the appendix. It is convenient to express the diffraction effects of irregular terrain in terms of a diffraction loss, the ratio (in decibels) of the magnitude of the free space field, E_0, to that of the diffracted field, E:

$$A(v, \rho)=20 \log_{10} |E_0/E|=-20 \log_{10} a(v, \rho) \tag{1}$$

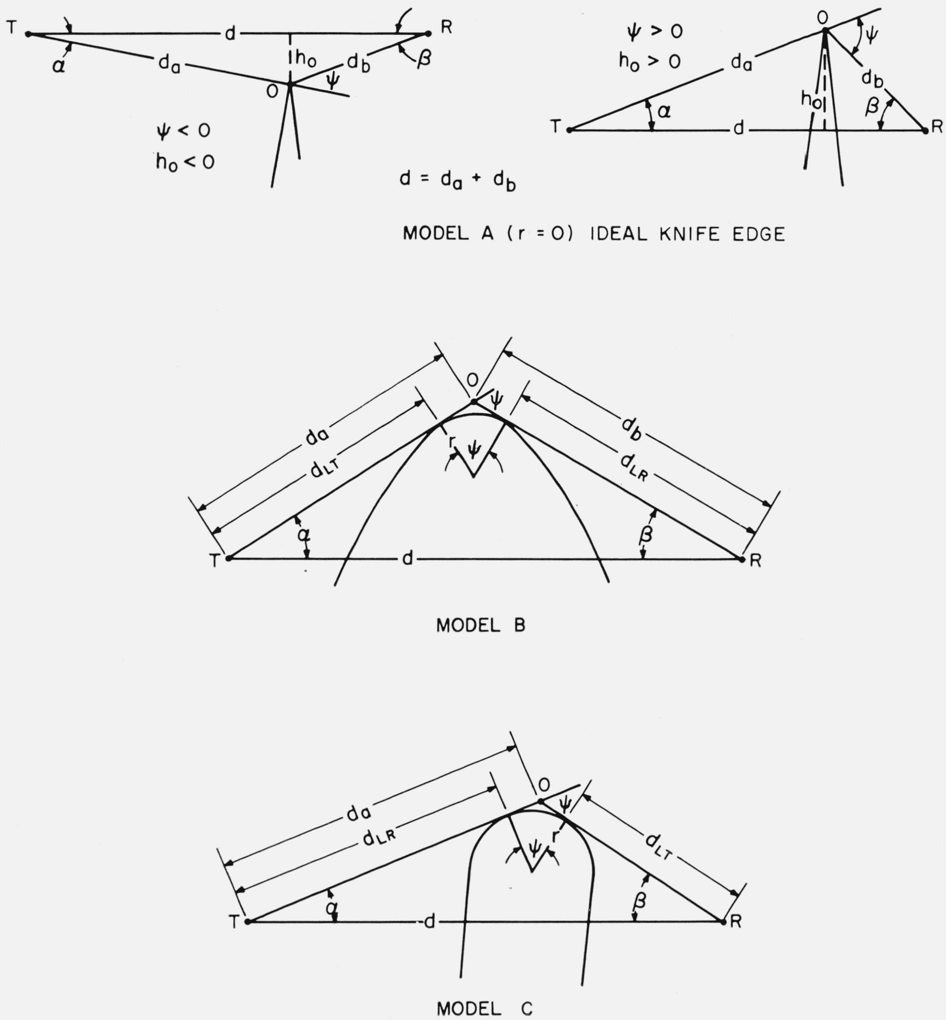

FIGURE 1. *Models for irregular terrain features.*

where

$$E/E_0 = a(v, \rho) \exp\,[-i\Phi(v, \rho)]. \tag{2}$$

The $a(v, \rho)$ is the magnitude of the ratio E/E_0, and $\Phi(v, \rho)$ is the phase by which the diffracted field lags the free space field. The v is the usual dimensionless parameter of the Fresnel-Kirchhoff diffraction formula, and ρ is a mathematically convenient dimensionless index of curvature for the crest radius, r, of the rounded knife edge. For the geometry of figure 1, for all heights, distances, etc., in the same units of length and for the diffraction angle, ψ, in radians,

$$v = h_0[2d/(\lambda d_a d_b)]^{\frac{1}{2}} = \psi[2d_a d_b/(\lambda d)]^{\frac{1}{2}}, \tag{3}$$

$$\rho = \left[\frac{\pi r}{\lambda}\right]^{\frac{1}{3}} \frac{\psi}{v} \left[\frac{2}{\pi}\right]^{\frac{1}{2}} \tag{4}$$

where λ is the transmission wavelength. The diffraction loss and phase lag for diffraction by a rounded knife edge may be expressed for irregular terrain applications as:

$$A(v, \rho) = A(v, 0) + A(0, \rho) + U(v\rho), \tag{5}$$

$$\Phi(v, \rho) = 90v^2 + \phi(v, 0) + \phi(0, \rho) + \phi(v\rho). \tag{6}$$

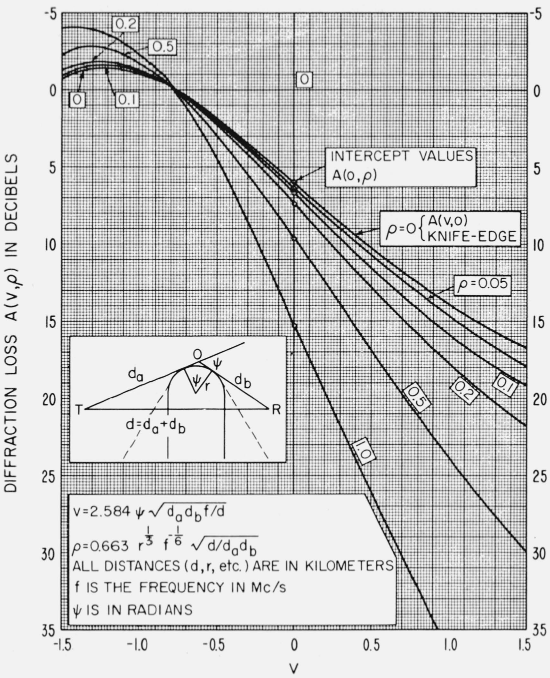

FIGURE 2. *Diffraction Loss,* A(v,ρ), *for a rounded obstacle.*

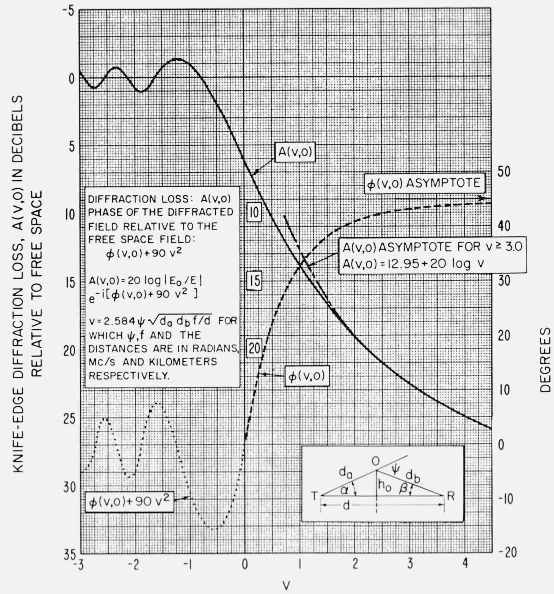

FIGURE 3. *Knife-edge diffraction loss,* A(v,O).

Equation (5) yields curves such as in figure 2, where expressions for v and ρ are given, which are convenient for calculations. The leading terms, $A(v, 0)$ and $90v^2+\phi(v, 0)$, of (5) and (6) are the diffraction loss and phase lag for the ideal knife-edge ($r=0$) model. Their values are presented in figure 3 versus v. Note that only the $\phi(v, 0)$ part of the phase lag is plotted for $v>0$. This form of the phase angle is that suggested by J. H. Crysdale [1955]. The terms $A(0, \rho)$ and $\phi(0, \rho)$ of (5) and (6) are the magnitude and phase of the intercepts (see fig. 2) which are presented in figure 4 as a function of ρ. The final terms of (5) and (6), dependent only upon the *product* of v and ρ, are presented in figure 5. The expressions for v, ρ, and $v\rho$ [(3) and (4) above] are also presented in figures 3, 4, and 5 for the transmission frequency in megacycles per second, all distances in kilometers, and the diffraction angle in radians.

The expressions for diffraction loss and phase lag given above may be associated with models A, B, or C of figure 1 for either horizontal or vertical polarization when applied to irregular terrain and provided the following conditions are met:

(1) the distances d, d_a, r, etc., are all large relative to the wavelength;

(2) the extent of the rounded knife edge, transverse to the propagation path, is of the order of the radius for the first Fresnel zone width,

$$[\lambda d/4]^{1/2}\left[1-\left(\frac{2d_1}{d}-1\right)^2\right]^{1/2} \tag{7a}$$

where d_1 is the shorter of d_a or d_b;

(3) the components, α and β, of the diffraction angle ψ are small (10° or less);

(4) the radius of curvature is sufficiently large so that

$$[\pi r/\lambda]^{1/3}>>1.0. \tag{7b}$$

3. Sources of the Diffraction Formula

Examination of (5) and (6) above shows that the diffraction loss for the rounded knife edge is given by corrections to the results for an ideal ($r=0$) knife edge. This is inherent in the form of the solutions given by Rice [1954] and Wait and Conda [1959].

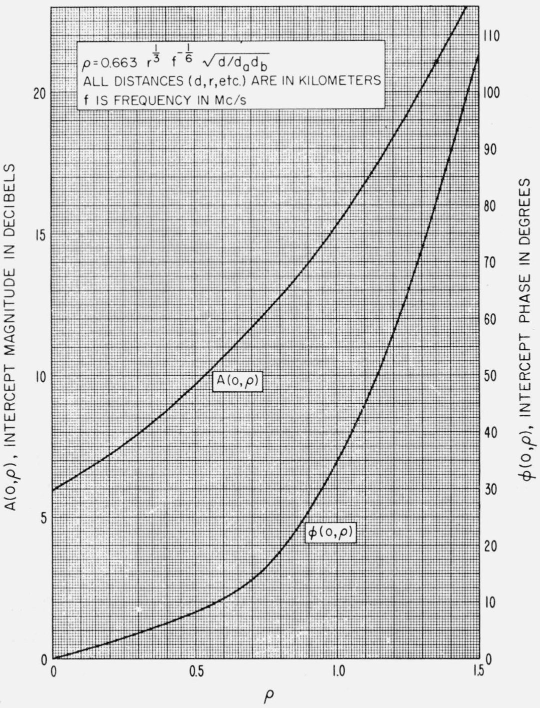

FIGURE 4. *Intercept magnitude and phase for diffraction over a rounded obstacle.*

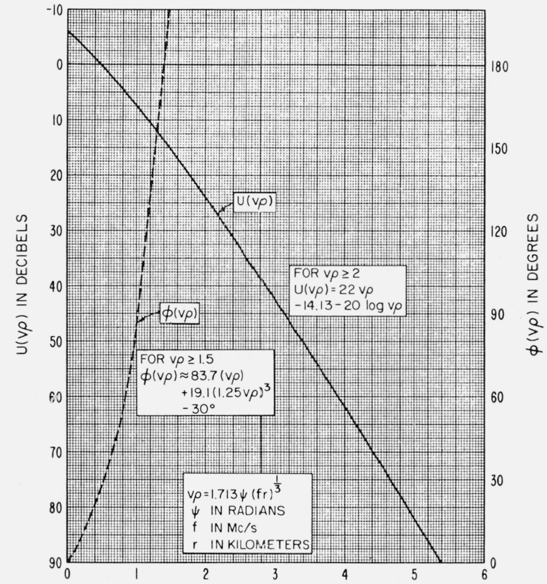

FIGURE 5. *Universal diffraction curve for a rounded knife-edge.*

The solution provided by Rice [1954] is that for a perfectly conducting parabolic cylinder (model B in fig. 1), an infinitely remote source (plane wave incidence), small diffraction angles, and a point of observation sufficiently remote to define a field proportional to a Sommerfeld crest wave [Sommerfeld, 1896; Baker and Copson, 1950]. The Sommerfeld crest wave corresponds to the asymptotic form of the ideal knife-edge diffraction loss of figure 3. Rice's solution may therefore be extended to values of v less than 2.0 when the Sommerfeld crest wave is replaced by the more general form of figure 3. If in addition, Rice's evaluation of the magnitude of the "Artmann shift" of the shadow boundary [Artmann, 1950] is corrected to include the appropriate phase (60°), subsequently given by Rubinow and Keller [1960], then Rice's solution for horizontal polarization reduces to (5).

A more general and rigorous solution was given by Wait and Conda [1959]. Starting with the wave equation and a spherical wave incident upon a circular cylinder of arbitrary dielectric constant and conductivity, they obtained the solutions for small diffraction angles with either horizontal or vertical polarization for model C of figure 1. By numerical integration they were able to reduce their solution to a set of curves which permitted plots of diffraction loss such as in figure 2. Due to the slight degree of approximation which permitted this numerical integration, for $\rho \leq 0.5$ and $v\rho \leq 0.55$, the evaluation of diffraction loss breaks down for larger values of ρ or of $v\rho$. The quantities ρ and $v\rho$ are equivalents of Wait and Conda's $1/u$ and $0.8X$. As described in the appendix, the present authors have eased the restriction on the value of ρ by a partial evaluation of the approximation error for $v=0$. For the condition of horizontal polarization and a highly conducting rounded knife edge, it may be readily shown that both of the above solutions provide identical corrections to the simple knife-edge expressions. As a consequence the asymptotic forms of Rice's solution are applicable, and the above mentioned limitation on the values of $v\rho$ in Wait and Conda's solution is removed.

In further support of the validity of (5), it should be noted that Wait and Conda's solution has agreed within a fraction of a decibel with the experimental (laboratory) measurements of Neugebauer and Bachynski [1958] and smooth earth diffraction theory.

4. Application to Irregular Terrain

For the ground constants generally encountered for irregular terrain and for transmission frequencies of VHF or higher, the theoretical solution for highly conducting rounded knife edges and horizontal polarization is directly applicable to irregular terrain. Hence, for the conditions itemized in section 2, (5), and (6) will closely approximate the diffraction effect of hills, mountains, and ridges, independent of polarization. There is, however, one major difficulty in the application. Neugebauer and Bachynski [1958] have shown that the radius of curvature of interest for the rounded knife edge is that in the great circle plane of the transmitting and receiving points. The difficulty arises because the radius of curvature is rarely constant for hills, mountains, etc.

As illustrated in figure 1, the transmitting and receiving antenna horizons are marked by the distances d_{LT} and d_{LR} for the simple geometrical model. Of primary interest is the crest radius of curvature between the two horizons. One method of estimating an effective radius of curvature for the model from the terrain feature is related to the definition of angular distance and is given by:

$$r=(d-d_{LT}-d_{LR})/\psi \qquad (8)$$

where r, d, etc., are defined in figure 1 and determined from the modified terrain profile [Norton, Rice, and Vogler, 1955]. The distances r, d, etc., in (8) are in the same units of length, and ψ is in radians.

5. Comparison With Experimental Data

Experimental data were obtained for two general propagation paths in the vicinity of Denver, Colo. (see fig. 6). The audio signal was recorded at the indicated sites (A, B, and C) for transmission from four VHF TV stations on Lookout Mountain. The terrain profiles, modified for standard atmospheric refraction, are indicated in figures 7 and 8. Also indicated are the transmission frequencies, transmitting antenna elevations, and the maximum receiving antenna

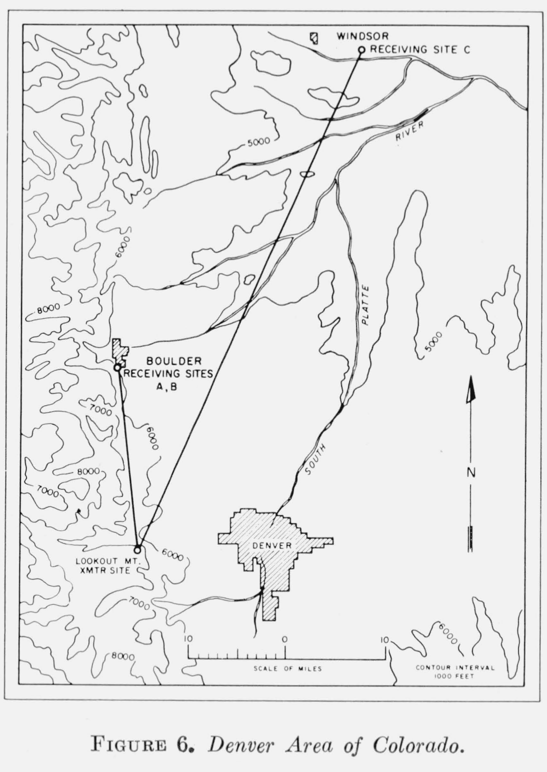

FIGURE 6. *Denver Area of Colorado.*

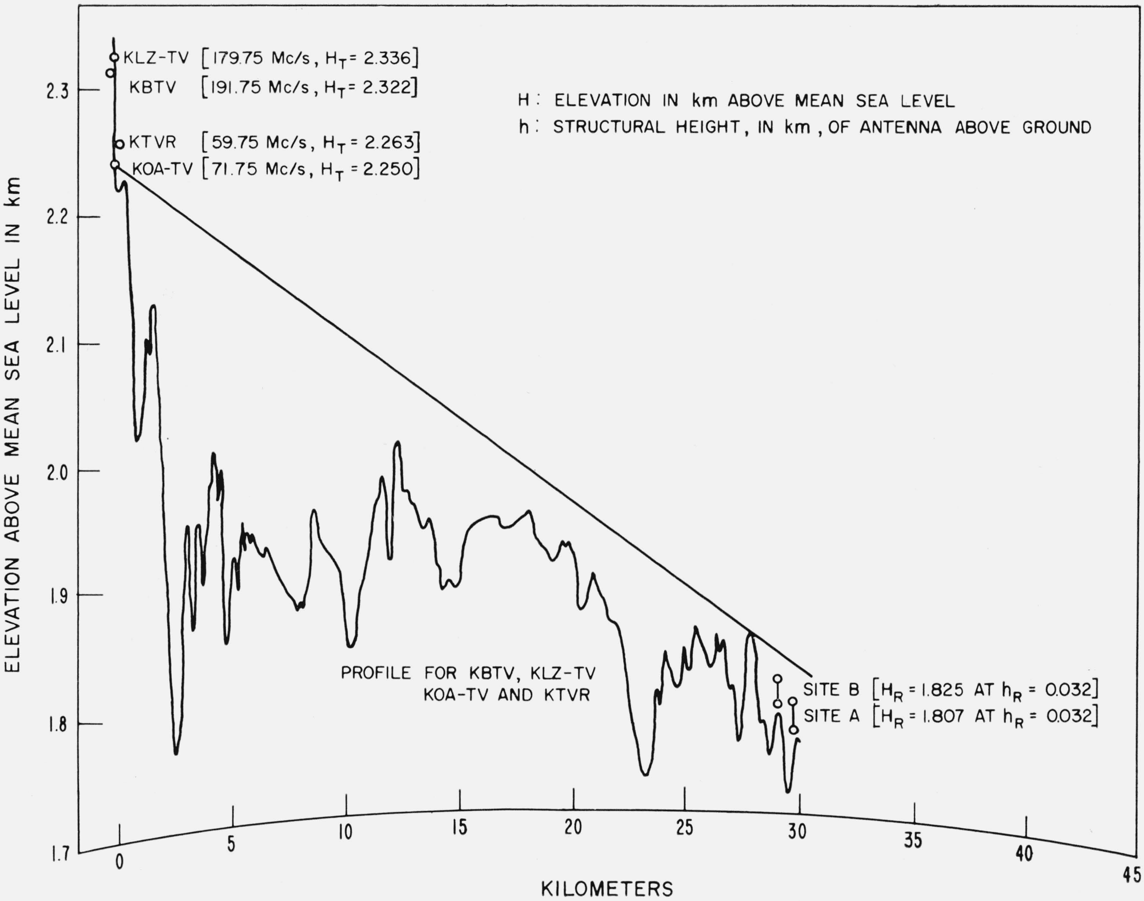

FIGURE 7. *Terrain profile from Lookout Mt. to receiving sites A and B.*

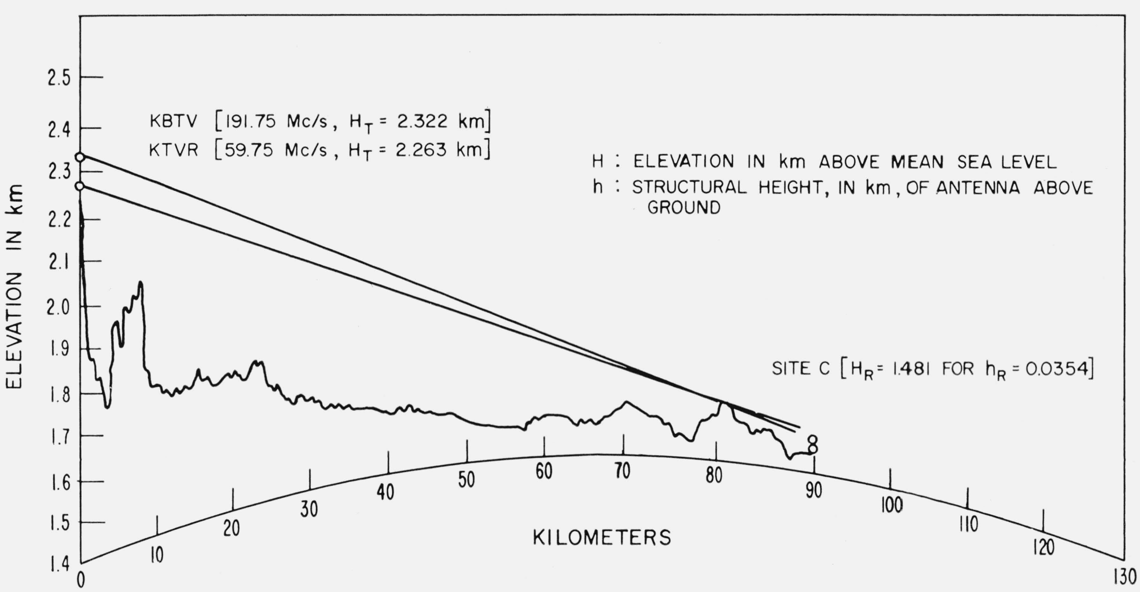

FIGURE 8. *Lookout Mountain to Windsor site C.*

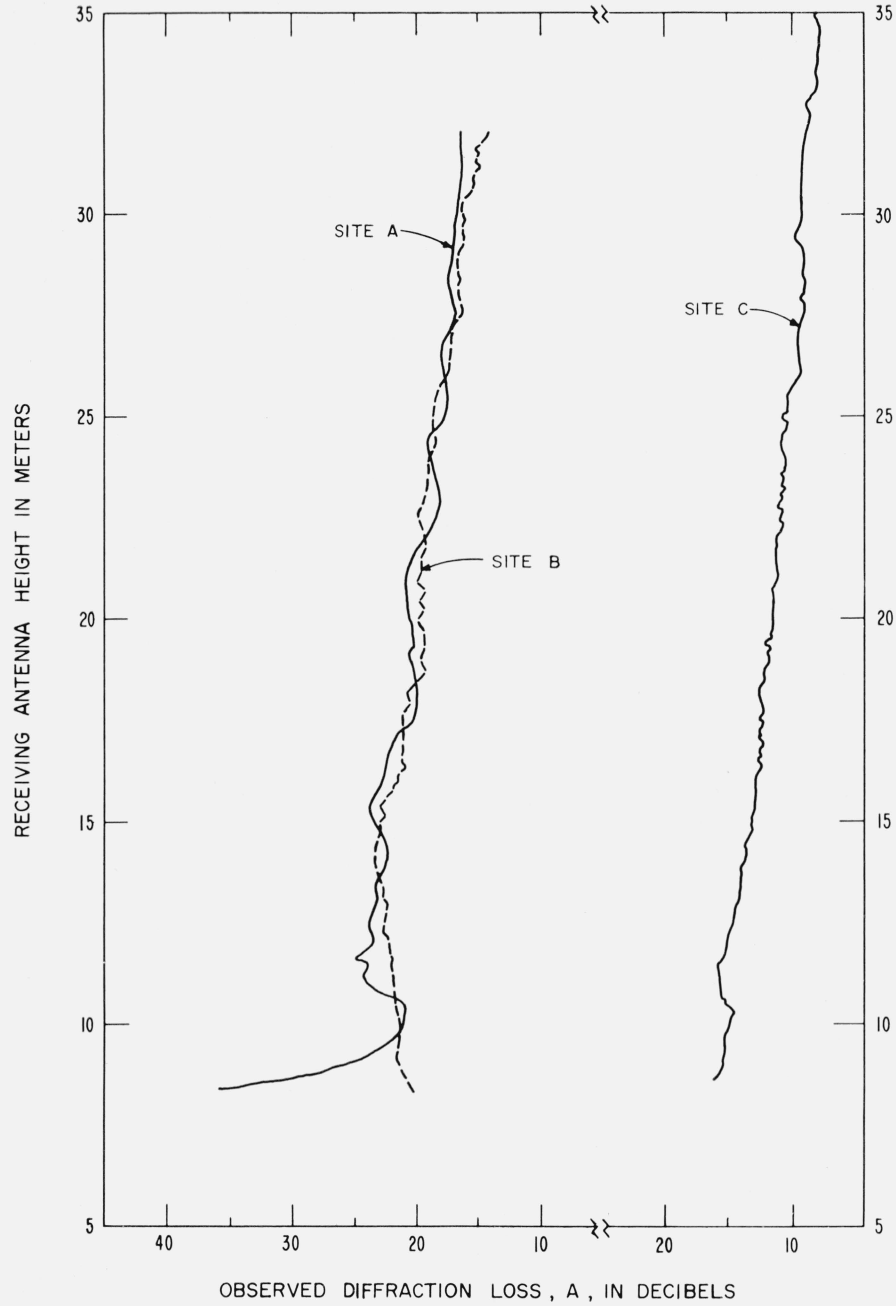

FIGURE 9. *Observed diffraction loss versus antenna height KBTV, 191.75 Mc/s.*

elevations. Continuous recordings were obtained with horizontal half-wave dipole antennas which were raised and lowered between a minimum of 8.5 m and a maximum of 32 m (sites A and B) or 35.4 m (site C) above ground. The recordings were obtained on winter afternoons and required approximately 20 min to complete the raise-lower cycle. A comparison of the recordings for the up and down portions of each cycle exhibited an average difference in signal level of 2 db or less and no marked variation in structure. The variation of KBTV (191.75 Mc/s) recordings for increasing antenna height is illustrated in figure 9 for sites A, B, and C. These are typical also of the recorded signals for the other three frequencies. The lobing due to foreground reflections was observed only for site A and at antenna heights of less than 20 m.

Due to the roughness of the terrain, it is assumed that the contribution due to ground reflections between the transmitting antenna and its horizon is negligible. To permit a comparison with theory and to minimize the effect of the foreground reflections at site A, a least-square fit was obtained for the variations of observed diffraction loss with height. For each frequency and site, five values of observed diffraction loss were selected at equal intervals of antenna height and compared to the calculated values. These values were selected so as to also include the least and most favorable points for comparison with theory. The results are presented in figure 10 in terms of the observed value less the calculated value. The calculated value, $A(v, \rho)$, is plotted as a reference. Also indicated are the calculated values, $A(v, o)$, for the simple ($r=0$) knife edge. Figure 10 indicates that the calculated values provide estimates which, in this situation, improve with transmission frequency. It further demonstrates the superior estimate available when allowance is made for the effect of the crest curvature upon the diffracted field. The apparent exception to this are the results for KTVR at site B, where the apparent frequency dependence is not as mentioned above, although the calculated and observed values are still comparable. This is felt to be erroneous and possibly due to a calibration error, for the following reason. Although sites A and B are not in line, they are only 1 km apart and have, for some values of antenna height, common diffraction angles. Due to their close grouping in position and frequency, the received signals for KTVR and KOA–TV at sites A and B would be expected, from calculation, to be within 3 db of one another for antenna heights above 20 m. The same holds for KLZ–TV and KBTV at sites A and B. This was also the case for the recorded signals with the exception of KTVR at site B. The received signal for KTVR at site B was approximately 5 db below the comparable observed levels. In this application, the values of v, ρ, and $v\rho$ were in the ranges 0.1 to 1.3, 0.31 to 0.78, and 0.02 to 0.44, respectively.

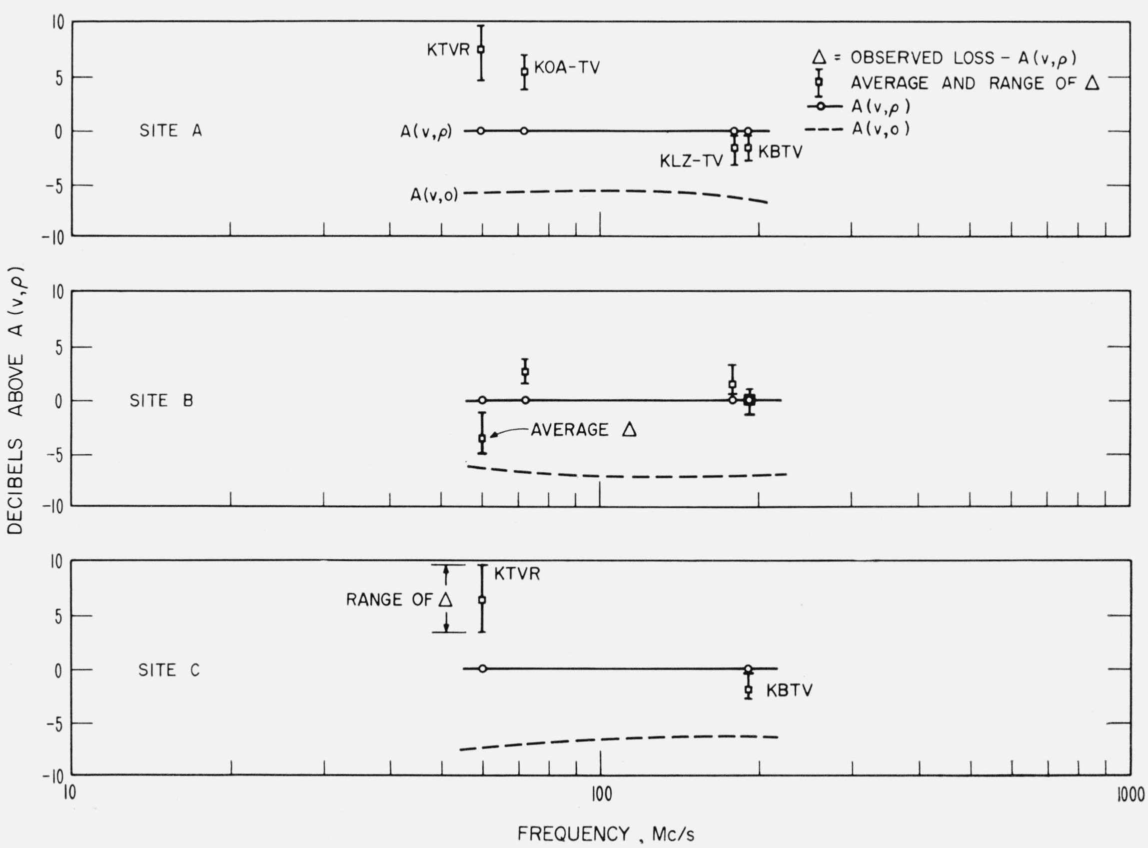

Figure 10. *Observed diffraction loss and* A(v,O) *relative to* A(v,ρ).

6. Conclusion

For application to irregular terrain, the solutions for the rounded knife edge as given by Rice [1954] and by Wait and Conda [1959] have been extended and reduced to engineering formulas which determine both magnitude and phase of the diffracted field. Although the comparison with experimental data has been promising to date, much additional investigation is indicated. This is so in the matter of determining the effective radius of curvature from terrain data. Also of interest is the effect of less smoothly rounded crests than those investigated to date. Of additional interest is the application of the curved knife-edge formulas to the problem of multiple diffraction, which has been treated by Furutsu [1963] for the ideal knife-edge model only.

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7. Appendix

A major problem of extending the previous solutions for the diffraction of radio waves by conducting cylinders or spheres is that of determining the diffraction loss for grazing conditions. The grazing condition ($v=0$) provides a diffraction loss given by the intercept value $A(0, \rho)$. Wait and Conda [1959] determined the intercept value for $u \geq 2 (\rho \leq 0.5)$. This appendix describes the evaluation of the intercept value for $0.7 \leq u \leq 1.0$ which should cover most applications to irregular terrain.

7.1. Extension for $u > 1.0$

In the case of a cylinder or rounded knife edge [Wait and Conda, 1958, 1959] as in the case of a sphere [Fock, 1951], the diffracted field may be expressed in terms of the Fresnel knife-edge diffraction field plus a correction term. The general form of the correction term $G(X)/u$ may be expressed in terms of the Airy Integrals, $W_1(t)$, $W_2(t)$, and their imaginary component $v(t)$, as defined by Wait and Conda [1959] and Spies and Wait [1961].

$$G(X)=\left[\frac{i\sqrt{y_1y_2}}{\pi}\right]^{1/2}\left\{\begin{aligned}&\frac{1}{2i}\int_{C_1} e^{-ixt}\frac{W_2'(t)-qW_2(t)}{W_1'(t)-qW_1(t)}W_1(t-y_1)W_1(t-y_2)dt\\&+\int_{C_2} e^{-ixt}\frac{v'(t)-qv(t)}{W_1'(t)-qW_1(t)}W_1(t-y_1)W_1(t-y_2)dt\end{aligned}\right. \tag{9}$$

where the contour C_1 is for t from $\infty e^{-j2\pi/3}$ to 0 and the contour C_2 is for t from 0 to ∞. The q is a function of the polarization and ground constants. For the application to irregular terrain and the conditions specified at the end of section 2, q is very large. The distance from transmitting to receiving antenna via the obstacle (fig. 1) is $\mathrm{TOR}=\lambda v^2/4+d\approx d_a+d_b=d_{\mathrm{LT}}+d_{\mathrm{LR}}+r\psi$ which when multiplied by $(kr/2)^{\frac{1}{3}}/r$ yields $x=\sqrt{y_1}+\sqrt{y_2}+X$. In this appendix it is convenient to use the $X, \left(=\sqrt{\frac{\pi}{2}}\,v\rho\right)$, and $u, (=1/\rho)$, of Wait and Conda [1959] to permit direct comparison with their expressions.

To determine the form of (9) suitable for large values of y_1 and y_2 as well as the required correction terms for lesser values of y_1 and y_2, we may expand the factors $\exp\,[-ixt]W_1(t-y_1)$ $W_1(t-y_2)$ in the integrals of (9) in terms of the asymptotic expressions for $W_1(t)$, [Spies and Wait, 1961], to obtain:

$$e^{-ixt}W_1(t-y_1)W_1(t-y_2)=-je^{-jXt}(y_1y_2)^{-1/4}e^{-j2/3}(y_1^{3/2}+y_2^{3/2})$$
$$\left\{1+\frac{1}{4}t\frac{y_1+y_2}{y_1y_2}+\frac{5}{32}t^2\left(\frac{y_1+y_2}{y_1y_2}\right)^2+\ldots\right\} \tag{10}$$

which, omitting the phase relative to the reference path T to R, reduces to:

$$-je^{-jXt}(y_1y_2)^{-1/4}\left[1+\frac{t}{4u^4}+\frac{5t^2}{32u^8}\ldots\right]. \tag{11}$$

In the above, the relationship $u^2=\sqrt{y_1}\sqrt{y_2}/(\sqrt{y_1}+\sqrt{y_2})$ has been used. For $X=0$, $q\rightarrow\infty$, (1) reduces to:

$$G(0)\simeq\frac{1}{\sqrt{\pi}}\left[\tilde{G}(X)+\frac{j}{4u^4}\tilde{G}'(X)-\frac{5}{32u^8}\tilde{G}''(X)+.\quad\right]_{X=0} \tag{12}$$

where the $\tilde{G}(X)$ is (9) evaluated with only the first series term of (11) and the primes indicate the order of the derivative with respect to the argument X. The $\tilde{G}'(X)$ and $\tilde{G}''(X)$ have been evaluated at $X=0$ by numerical integration as $0.1816e^{j15°}$ and $-0.0191e^{j45°}$. Wait and Conda have already given $\tilde{G}(0)$ as 0.3568 exp $[-i14.94°]$, and the intercept value for $u>1.0$ is given by:

$$0.5-\frac{0.3568}{u}e^{-j14.94°}+\frac{0.0454}{u^5}e^{j15°}+\frac{0.002}{u^9}e^{j45°}. \tag{13}$$

Equation (13), with $1/u$ replaced by ρ, provide the values of $A(0,\rho)$ andϕ $(0,\rho)$ for $\rho<1.0$ shown in figure 4.

7.2. Extension for $0.7\leq u\leq 1.0$

Values of $u\leq 1.0$ (or $\rho>1.0$) will be encountered when one antenna is appreciably closer than the other to the rounded knife edge. In the case of $y_2>>y_1$ then $u^4=y_1$ for small values of X. The asymptotic expression may then be retained for $W_1(t-y_2)$. However, the $W_1(t-y_1)$ is then better approximated by a Taylor's series expansion [Spies and Wait, 1961]. First, however, we return to (9) and substitute $t'e^{-j2\pi/3}$ for t in the first integral so as to permit common limits for both integrals. Then retaining only the first term of the asymptotic form for $W_1(t-y_2)$ and for $X=0$, $q\rightarrow\infty$, (9) reduces to:

$$G(0)\approx\frac{u}{\sqrt{\pi}}e^{+j2u^6/3}\left\{\begin{array}{l} e^{j\pi/3}\displaystyle\int_0^\infty e^{+tu^2/2(j-\sqrt{3})}W_1(te^{-j2\pi/3}-u^2)\frac{v(t)}{W_2(t)}dt \\ +\displaystyle\int_0^\infty e^{-jtu^2}W_1(t-u^2)\frac{v(t)}{W_1(t)}dt. \end{array}\right. \tag{14}$$

The expressions $W_1(te^{-j2\pi/3})=W_2(t)e^{-j\pi/3}$, etc., have been employed in reducing (9) to (14). The Taylor series expansion of $W_1(t-u^4)$ is given by:

$$W_1(t-u^4)=W_1(t)[a_0+a_1t+a_2t^2+a_3t^3+\ldots]-W_1'(t)[b_0+b_1t+b_2t^2+b_3t^3+\ldots] \tag{15}$$

where

$$\begin{array}{ll} a_0\approx 1-u^{12}/6 & b_0\approx[1-u^{12}/12]u^4 \\ a_1\approx[1-u^{12}/15]u^8/2 & b_1\approx[1-u^{12}/20]u^{12}/6 \\ a_2\approx[1-u^{12}/23.3]u^{16}/24 & b_2\approx[1-u^{12}/28]u^{20}/120 \\ a_3\approx[1-u^{12}/31.5]u^{24}/720 & b_3\approx[1-u^{12}/36]u^{28}/5040. \end{array} \tag{16}$$

For the expansion of $W_1(te^{-j2\pi/3}-u^4)$, the t's of (7) are simply replaced by $te^{-j2\pi/3}$ and the relations such as $W_1'(te^{-j2\pi/3})=e^{j\pi/3}W_2(t)$, etc., are employed. Substituting these series expansions into (14) yields

$$G(0)=\frac{u}{\sqrt{\pi}}e^{+j2u^6/3}\left\{\sum_{n=0}^{3}a_nA(n,u)-b_nB(n,u)-ib_nC(n,u)\right\} \tag{17}$$

where

$$A(n, u)=\int_0^\infty t^n v(t)\left\{e^{-ju^2t}+e^{\frac{-\sqrt{3}u^2t}{2}}e^{j\left[\frac{u^2t}{2}-\frac{2n\pi}{3}\right]}\right\} dt$$

$$B(n, u)=\int_0^\infty t^n v(t)\{e^{-ju^2t}+e^{-\sqrt{3}u^2t/2}e^{j[u^2t/2-(2n-2)\pi/3]}\}\frac{u(t)u'(t)+v(t)v'(t)dt}{|W(t)|^2}$$

$$C(n, u)=\int_0^\infty \frac{t^n v(t)}{|W(t)|^2}\{e^{-ju^2t}+e^{-\sqrt{3}u^2t/2}e^{j[u^2t/2-(1-4n)\pi/3]}\}dt. \tag{18}$$

Despite the appearance of these integrals, they are readily determined by computer programs for numerical integration, since they converge very rapidly. For any one particular value of $u\leq 1$ they may also be determined by graphical integration from tabulated values or curves of $v(t)$, $u(t)$, $|W(t)|$. The values of $A(0, \rho)$ and $\phi(0, \rho)$ for $\rho\geq 1 (u\leq 1)$ were determined in figure 4 from (17), (18), and (19):

$$a(0, \rho)=0.5-\rho G(0)$$

from which (1) and (2) determine $A(0, \rho)$ and $\phi(0, \rho)$.

7.3. Comparison of Solutions of Wait and Conda [1959] and Rice [1954]

The above function $G(X)$ from Wait and Conda's solution and the basic function of Rice's solution, $\psi(\tau)$, are both defined in terms of Airy integrals. It may be readily shown that $\psi(-\tau)=2\sqrt{i\pi}G(X)$ for horizontal polarization and an infinitely conducting rounded knife edge. For large values of v and $|\tau|=|X|$, Rice's solution for the diffracted field is proportional to Wait and Conda's solution. Furthermore, Rice's evaluation of the Artmann shift, when the phase is incorporated, closely approximates the second term of (13) in magnitude and phase. Because of the foregoing, Wait and Conda's solution for horizontal polarization and an infinitely conducting rounded knife edge may be expressed as (5) and (6) and evaluated from figures 3, 4, and 5.

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