Impedance of a Monopole Antenna With a Radial-Wire Ground System on an Imperfectly Conducting Half-Space, Part II

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The impedance of a vertical monopole located over the surface of an imperfectly conducting earth and having a radial-wire ground system has been studied theoretically by J. R. Wait. An experimental investigation discussed in part I of this paper gave results in good agreement with Wait's theory. This part of the paper is a presentation of calculations based on Wait's theory showing the behavior of antenna impedance as a function of antenna height, top-loading and number, size, and length of radial wires.

1. Introduction

A vertical monopole antenna over an imperfectly conducting ground excites currents in the ground near the base of the antenna which results in a substantial loss of radiated power. To reduce this power loss and thereby increase the efficiency of the radiating system, a highly conducting concentric circular disk may be placed at the ground surface. For economic reasons, the conducting disk is usually replaced by a system of radial wires. Such a system is not as effective as the disk but can be made nearly so by use of a sufficient number of wires.

The effectiveness of the radial wire system in reducing the ground losses [1] was investigated theoretically and experimentally by observing its effects on the antenna base impedance. Part I of this paper [Maley and King, 1962] gave the results of this investigation as a function of the number of radial wires using the ground system diameter as a parameter. The investigation indicated reasonable agreement between the measured values of impedance and those calculated from formulas given by J. R. Wait [1954]. Measurements and calculations were made for frequencies of 4.20 and 9.60 G/s and for a ground plane of water. The success of Wait's formulas in predicting the effect of the radial wire ground system upon the monopole base impedance for these cases was considered sufficient justification to extend the calculations to other frequencies and ground parameters.

In this paper an analysis is made of the effects of monopole height, top-loading and number, length and size of the radial wires on the antenna impedance. Calculated values of impedance are plotted as functions of the various ground system and antenna parameters. These calculations extend those published by Wait and Pope [1955] who employed both graphical and analytical methods. The effect of ground parameters and frequency will be considered in part III of this paper.

2. Theory

The impedance of a monopole antenna over an imperfectly conducting ground has been given by Wait [1954] as [2]

$$Z = Z^\infty - \frac{2\pi}{I_0^2} \int_0^\infty E_r(r,0) H_\varphi^\infty(r,0) r\, dr. \tag{1}$$

Z^∞ is the monopole impedance which would exist if the ground were perfectly conducting, $H_\varphi^\infty(r,0)$ is the magnetic field at the surface of the ground for the same condition and with an antenna base current of I_0. $E_r(r,0)$ is the radial component of the electric field at the surface of the imperfectly conducting ground with the radial wire system in place for an antenna base current of I_0. The time dependence is $\exp(i\omega t)$ where ω is angular frequency, t is time and $i = \sqrt{-1}$. The coordinate system is shown in figure 1.

In most practical antenna systems the boundary conditions at the surface of the ground may be approximated to a good order of accuracy by the relation

$$E_r(r,0) \simeq -\eta H_\varphi(r,0) \simeq -\eta H_\varphi^\infty(r,0), \tag{2}$$

where $H_\varphi(r,0)$ is the magnetic field at the surface of the imperfectly conducting ground, and η is the surface impedance which, in general, is a function
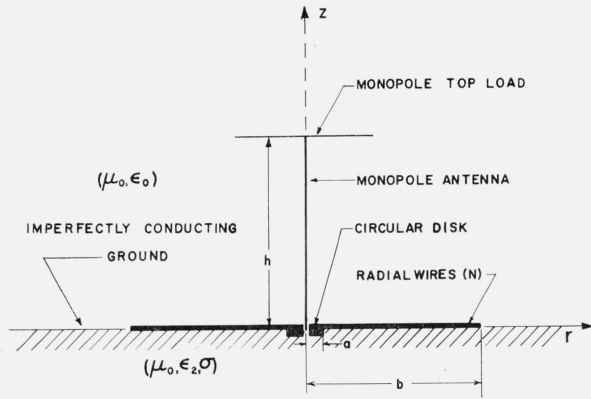

FIGURE 1. *Sketch of antenna system.*

of r. Equation (2) assumes $rH_\varphi^\infty(r,0)$ varies slowly in a distance $\left|\dfrac{1}{\gamma}\right|$ where $\gamma = [i\mu\omega(\sigma+i\omega\epsilon_2)]^{1/2}$ is the propagation constant of the ground. Using (2), (1) may be written

$$Z \cong Z^\infty + \frac{2\pi}{I_0^2} \int_0^\infty \eta[H_\varphi^\infty(r,0)]^2 r\,dr. \qquad (3)$$

In the case of a circular conducting disk ground system, $\eta=0$ for $0<r<a$ where "a" is the radius of the disk. Then (3) becomes

$$Z \cong Z^\infty + \frac{2\pi}{I_0^2} \int_a^\infty \eta_g[H_\varphi^\infty(r,0)]^2 r\,dr, \qquad (4)$$

where η_g is the effective surface impedance of the ground.

The wires of a radial wire ground system are normally connected to a small metal disk at the base of the antenna as shown in figure 2. Then (3) becomes

$$Z \cong Z^\infty + \frac{2\pi}{I_0^2} \int_a^\infty \eta_e(r)[H_\varphi^\infty(r,0)]^2 r\,dr, \qquad (5)$$

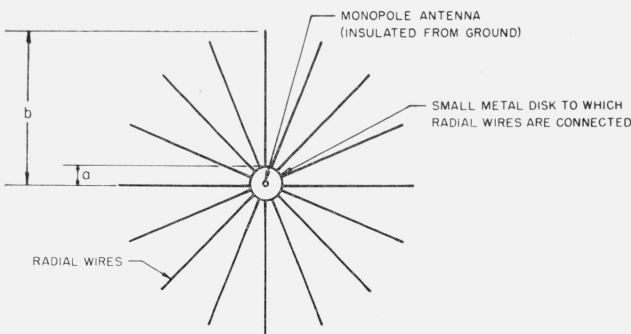where $\eta_e(r)$ is the surface impedance of the ground with the system of radial wires in place.

FIGURE 2. *Top view of radial wire ground system.*

It is interesting to compare the behavior of the impedance for the circular conducting disk ground system with that for the radial wire system. If η_g in (4) is taken as the surface impedance of the imperfectly conducting ground, $\Delta Z = Z - Z^\infty$ may be calculated as a function of disk radius. Typical results as plotted on a rectangular impedance chart are shown in figure 3.

Now consider a radial wire ground system in which the radials are connected to a small disk of radius "a" and extend to radius "b". The base impedance as a function of the number of radial wires and given value of "b" for such a system is shown typically as the dashed curve AB on figure 3. When no radial wires are present the impedance is the same as that for a conducting disk of radius "a". As the number of radials increases without limit for a constant value of "b", the impedance should approach that for a disk of radius "b". Thus, the curve of impedance as a function of the number of radials N should start at point A and end at point B in figure 3. Quantitative data on this variation and a comparison between calculated and measured values has been given in part I of this paper.

The approximate variation of the impedance between these two points may be found from (5) and from the relation [Wait, 1954; Maley and King, 1962]

$$\eta_e = \frac{\eta_g \eta_w}{\eta_g + \eta_w}, \qquad a<r<b, \qquad (6)$$

where

$$\eta_w = \frac{i2\pi\eta_0 r}{\lambda N} \ln\left(\frac{r}{Nc}\right), \qquad (7)$$

$$\eta_g = \left[\frac{i\mu\omega}{\sigma + i\omega\epsilon_2}\right]^{1/2} \qquad (8)$$

η_0 is the intrinsic impedance of free space and c is the radius of the wire.

The parallel impedance assumption (6) ignores mutual effects between ground and grid and assumes radial symmetry as well as perfect electrical contact between the wires and the earth. Experimental evidence [Smith and Devaney, 1958; 1959] indicates that mutual coupling and a potential difference can exist between the wires and earth, but may be neglected in (6) for a large number of wires and high soil conductivity. Gustafson, Devaney, and Smith [1959] give an approximate evaluation of these losses, as well as copper losses.

Equation (7) assumes $|\gamma_e d| \ll 1$ where $d = 2\pi r/N$ is the spacing between wires and γ_e is the propagation constant for a thin wire in the interface and is given by [Wait, 1954]

$$\gamma_e = \left[\frac{\gamma_0^2 + \gamma^2}{2}\right]^{1/2}. \qquad (9)$$

If calculations are made for the impedance as a function of the number of radials N for several different values of wire lengths, the results may be
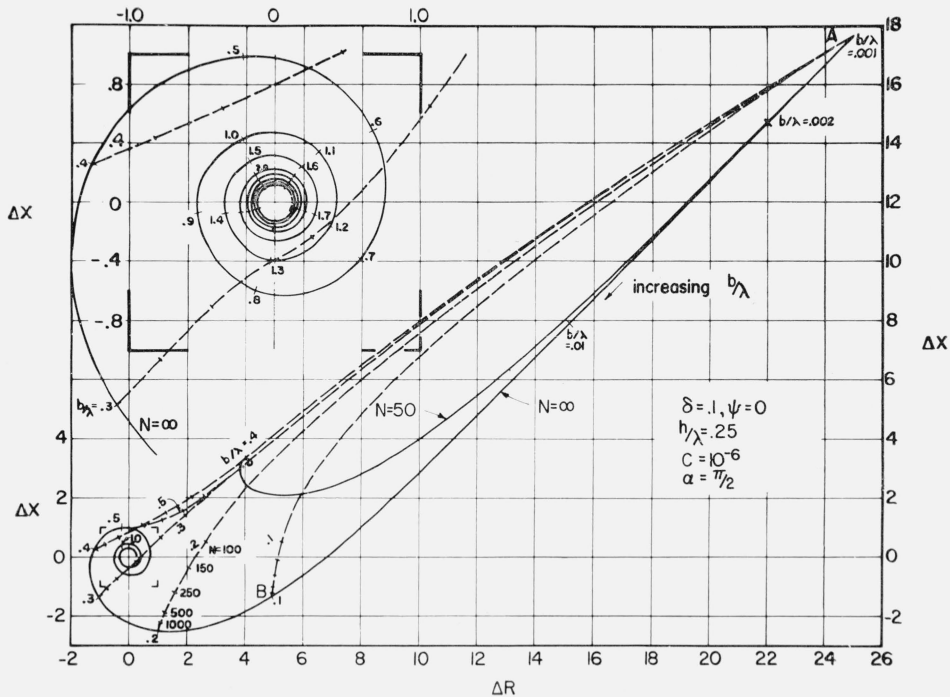
FIGURE 3. *Monopole base impedance loci showing the effect of varying* N *and system radius.*

The solid curve corresponds to a perfectly conducting disk ground system and the dashed curve shows the loci of impedance for varying N for a given value of b/λ. The inset shows an expansion of the region of convergence of the solid spiral.

shown as the family of dashed curves on figure 3. Each starts at point A corresponding to a small disk of radius "a", and terminates at a point corresponding to the given value of "b". From this figure the variation of impedance ΔZ as a function of "b" is apparent. A curve connecting the points representing the same number of radials but of varying length has the shape of a spiral similar to that for the conducting disk ground system of varying diameters. As N decreases, the spirals become smaller and closer to point A, and as N increases, the spirals become nearly coincident with the spiral corresponding to a solid disk.

The change of impedance with respect to an infinite perfectly conducting plane (the limit of the spiral as $b \rightarrow \infty$) may be obtained from (5), (6), (7), and (8). The result is

$$\Delta Z \simeq \frac{2\pi}{I_0^2} \int_a^b \eta_e(r)[H_\varphi^\infty(r,0)]^2 r dr$$

$$+ \frac{2\pi}{I_0^2} \int_b^\infty \eta_g[H_\varphi^\infty(r,0)]^2 r dr. \quad (10)$$

3. Calculations

The magnetic field, $H_\varphi^\infty(r, 0)$ for a perfectly conducting ground can be found if the current distribution on the monopole is known. It can usually be assumed that this distribution is sinusoidal and is given by

$$I(z) = I_0 \frac{\sin (\alpha - \beta z)}{\sin \alpha}, \quad (11)$$

where $\beta = 2\pi/\lambda$, $\alpha = \beta(h + h')$, λ is the free space wavelength, h is the actual monopole height, and h' is a parameter specifying the top-loading of the monopole. For this choice $H_\varphi^\infty(r, 0)$ is given by Wait (1954) as

$$H_\varphi^\infty(r,0) = \frac{iI_0}{2\pi \sin \alpha} \left[\frac{e^{-i\beta\rho}}{r} \cos (\beta h - \alpha) \right.$$

$$\left. - \frac{e^{-i\beta r}}{r} \cos \alpha + \frac{ihe^{-i\beta\rho}}{r\rho} \sin (\beta h - \alpha) \right] \quad (12)$$

where $\rho = (r^2 + h^2)^{1/2}$. Equation (12) applies sufficiently well near a monopole antenna with symmetrical top loading.

Calculations were made from (10) for a variety of parameters. In most applications, the normalized radius of the wire $C = c/\lambda$ varies between $0.05 \times 10^{-6} \leq C \leq 20 \times 10^{-6}$. The theoretical calculations do not account for copper loss. These may be estimated by other means. The monopole height varies between $0.003 \leq h/\lambda \leq .25$ and the number of radials lies in the range $50 \leq N < \infty$. The top-loading parameter, α, lies between $2\pi h/\lambda \leq \alpha \leq \pi/2$, the lower and upper limits corresponding to $h' = 0$ and full top-loading, respectively. The permittivity ϵ_2 and conduc-

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tivity σ of the imperfectly conducting earth are accounted for in the parameter δ which may be defined in terms of η_g;

$$\eta_g = \left[\frac{i\mu\omega}{\sigma + i\omega\epsilon_2}\right]^{\frac{1}{2}} = \sqrt{i}\,\eta_0 \left[\frac{(\delta')^2}{1 + i(\delta')^2\epsilon}\right]^{\frac{1}{2}} = \sqrt{i}\,\eta_0\delta \quad (13)$$

where $\epsilon = \epsilon_2/\epsilon_0$ is the relative dielectric constant, and

$$\delta' = \left(\frac{\omega\epsilon_0}{\sigma}\right)^{\frac{1}{2}}. \quad (14)$$

Then

$$\delta = \delta'\left[\frac{1}{1 + i(\delta')^2\epsilon}\right] = |\delta|e^{-i\psi} \quad (15)$$

and

$$\psi = \frac{1}{2}\tan^{-1}[(\delta')^2\epsilon].$$

If $(\delta')^2\epsilon \ll 1$ then $\psi \cong 0$ which corresponds to highly conducting earth and negligible displacement currents. The ranges of δ and ψ were chosen to be $0.01 \leq \delta \leq 0.30$ and $0 \leq \psi \leq \pi/4$. Part II of this paper is mainly concerned with the variation of N, C, h/λ, and α; δ and ψ will be considered in part III.

4. Conclusions

Figures 4 through 10 reveal nearly the same conclusions as those made by Wait. Essentially the same formulation was used here except that Wait assumed $\alpha = \pi/2$. This corresponds to $h/\lambda = 0.25$ [Wait, 1954] or a uniform current distribution for short monopoles where $h/\lambda \leq 0.1$ [Wait, 1955]. Where the results can be compared good agreement exists with the curves in the present papers.

Figure 4 shows the effect of varying the number of radials N and antenna height h/λ, for $\delta = 0.10$ and $\psi = 0$, and wire radius to wavelength ratio $C = c/\lambda = 10^{-6}$. These curves show that, in general, there is little to be gained by making N greater than approximately 250. ΔZ is evidently a rapidly varying function for small b/λ, whatever the values of the parameters although ΔX generally varies less rapidly with b/λ than ΔR does. Figure 4c shows that ΔX varies over a much larger range with N than ΔR does for small h/λ. This is to be expected since the imaginary part of Z^∞ is very large for short monopoles. The rate of change of both ΔR and ΔX for small b/λ is extremely rapid for short antennas and small b/λ, although the curves are not considered accurate when b/λ is less than the earth skin depth due to reflections from the ends of the radials. Also, the boundary condition (2) assumes $rH_\varphi^\infty(r, 0)$ varies slowly in a distance $\left|\frac{1}{\gamma}\right|$.

Larsen [1962] has investigated the ratio between the E-field and H-field losses per unit area for a monopole of various heights, current distributions, and top-loading using a radial-wire ground system. It was shown that for short antennas the E-field

losses are comparable or larger than the H-field losses especially for large N, and increase somewhat with top-loading. Heavy ground cover such as snow, vegetation, etc., increases the E-field losses even more. Gustafson, Devaney, and Smith [1959] give a method of approximating these cover losses. For higher antennas with sinusoidal current distribution, the E-field losses become negligibly small.
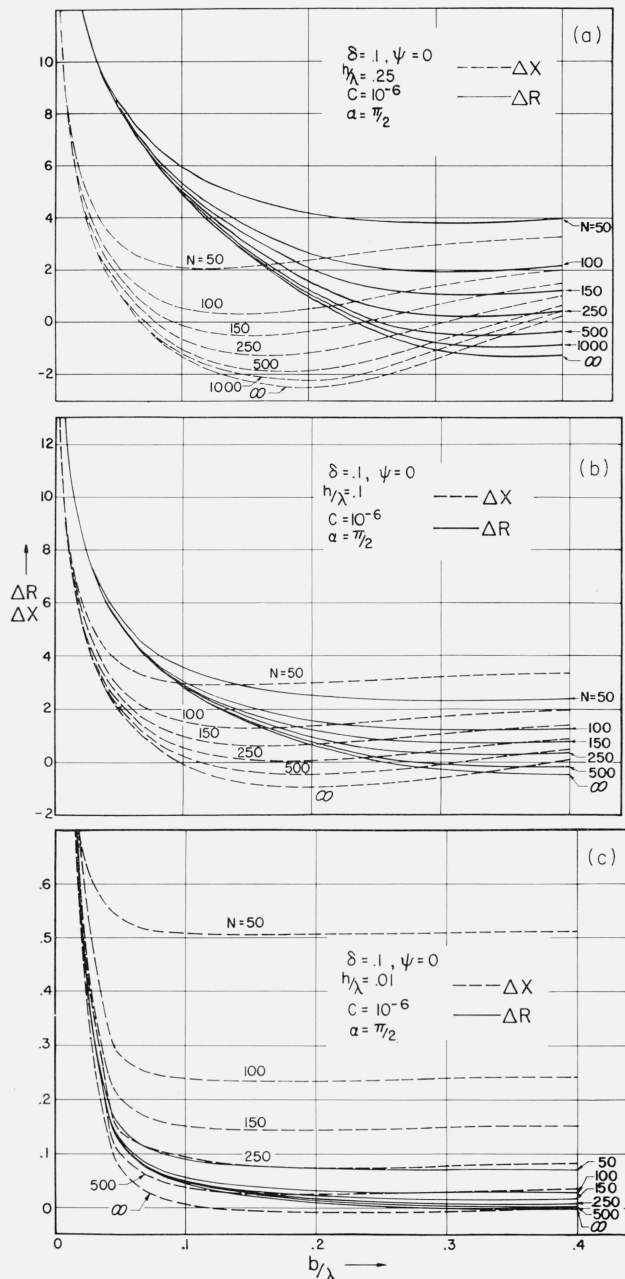

FIGURE 4. a, b, and c. ΔR and ΔX versus length of radial wires for selected values of N and h/λ and parameters as specified.
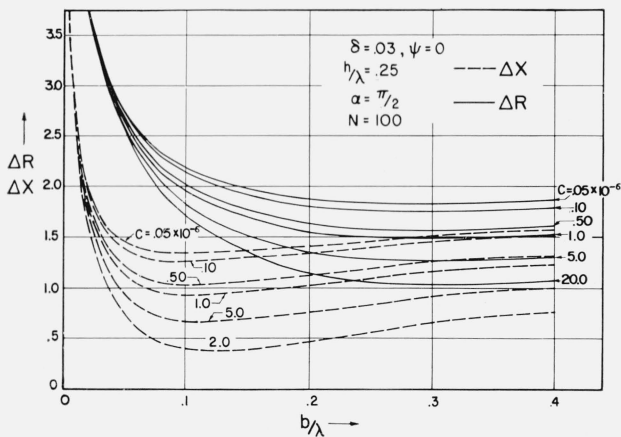
FIGURE 5. ΔR and ΔX versus length of radial wires for parameters as specified for selected values of C.
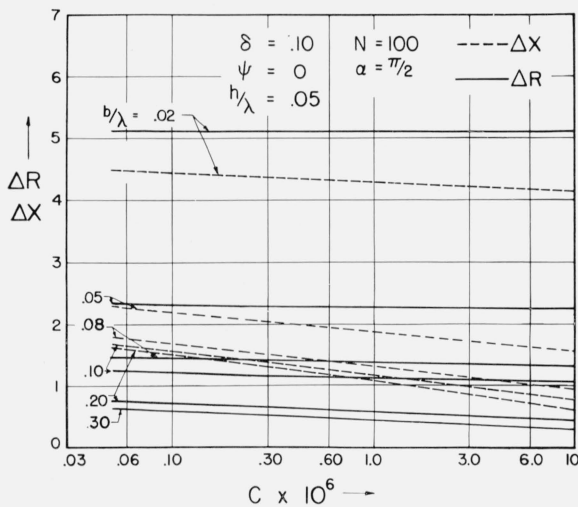

FIGURE 6. ΔR and ΔX versus normalized wire radius C for selected values of b/λ and parameters as specified.

Equation (7) is not expected to be valid when $|\gamma_e d|$ is comparable to or greater than unity. If $|\gamma| >> |\gamma_0|$, then

$$|\gamma_e d| = \frac{(2\pi)^2}{N} \frac{r}{\lambda} \sqrt{\frac{1}{2} \frac{\epsilon_2}{\epsilon_0} \left| 1 - \frac{i\sigma}{\omega\epsilon_2}\right|} \qquad (16)$$

Thus, for example, if $\epsilon_2/\epsilon_0 \left|1 - \dfrac{i\sigma}{\omega\epsilon_2}\right| = 10$ and $N = 50$, the results are not expected to be correct for b/λ greater than about 0.20. However, part I of this paper shows that this restriction may be relaxed somewhat. Comparison of figures 4 a, b, and c clearly shows that the necessary length of radials decreases significantly for short antennas.

Figure 5 shows that changing the wire size by a factor of 200 for the indicated parameters changes ΔR and ΔX by less than 1 ohm and for very small values of b/λ the wire size has practically no effect. This is even more apparent in figure 6 where it is seen that the ground system radius affects ΔR and

ΔX much more significantly than C does, and that ΔX is affected much more than ΔR. For these reasons, in nearly all of the calculations, the value of C was taken to be 10^{-6} which corresponds to B and S No. 8 (0.327 cm diameter) at 183 kc/s and B and S No. 4 (0.518 cm diameter) at 115 kc/s. The value of C should be primarily determined so that the wires will be large enough to prevent breakage during installation, but not so large as to be wasteful of copper.

The effects of varying the top-loading parameter α and antenna height h/λ are shown in figures 7 and 8. As expected, these curves exhibit an increasing ΔR
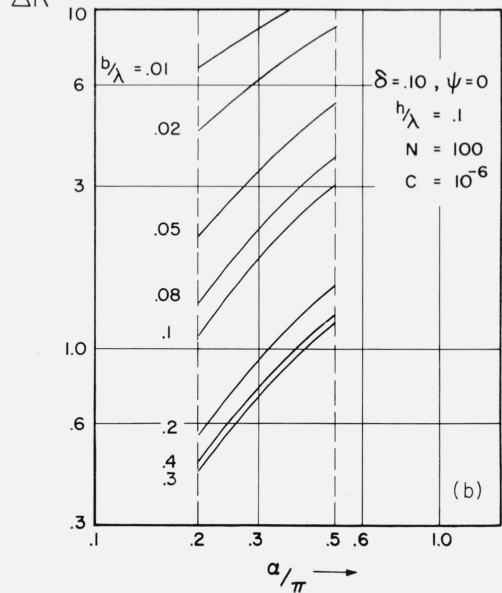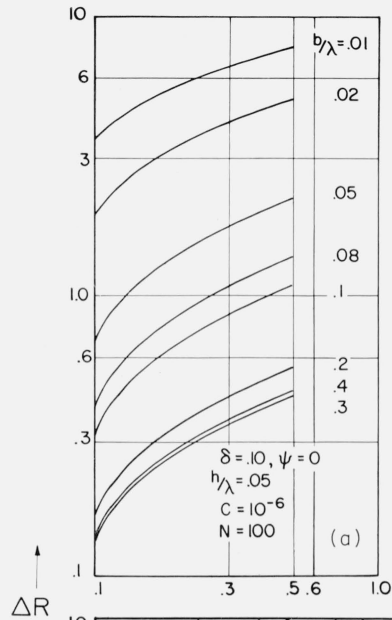

FIGURE 7. a and b. ΔR versus top-loading parameter α for selected values of b/λ and parameters as specified.
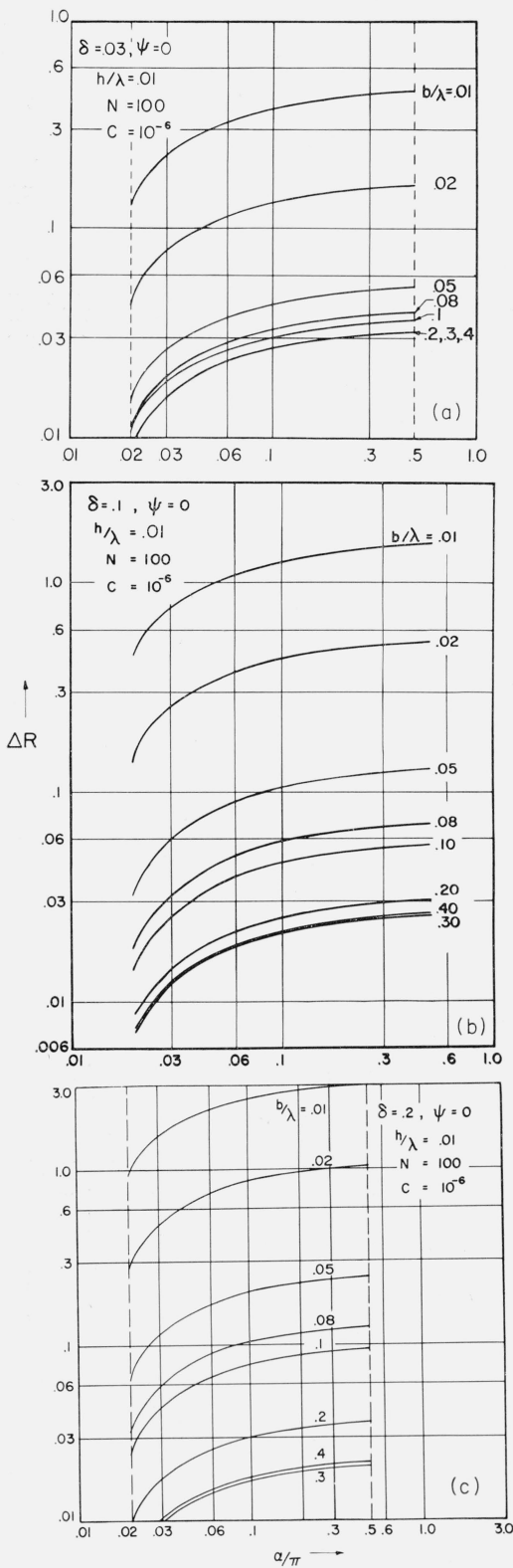(a) $h/\lambda = 0.05$, (b) $h/\lambda = 0.10$.

for increased top-loading, due to the increased radiated power. It is also obvious that increasing the ground system radius causes a considerable decrease of ΔR due to decreased ground losses; however, increasing b/λ to values greater than 0.2 has very little effect upon ΔR. Comparison of figures 7a, b, and 8b shows how the impedance increases significantly for all values of top-loading with increasing antenna height. A similar effect is achieved as δ is increased as may be seen from figures 8a, b, and c; however, this increase in ΔR as δ increases for all values of top-loading is due to increased ground losses. Decreasing h/λ and $|\delta|$ corresponds to smaller spirals on the rectangular chart of figure 3.

Negative ΔR does not necessarily signify that the system is more efficient than a perfectly conducting ground system. However, in general, the longer the radials the more efficient the system. Normally, ground systems are designed to operate in a region where ΔR decreases with increasing "b" and in this region it is generally said that the antenna system efficiency increases with decreasing R.
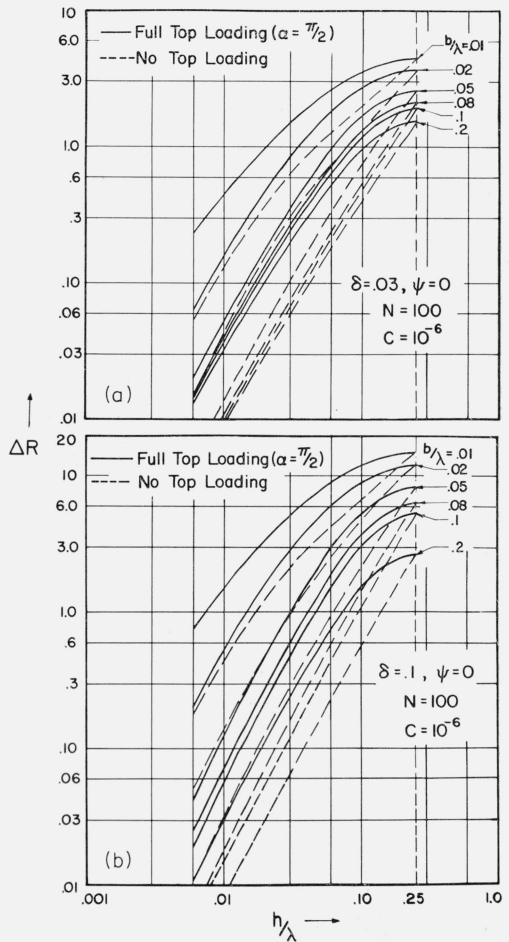

FIGURE 8. a, b, and c. ΔR *versus top-loading parameter* α *for selected values of* b/λ *and parameters as specified.*

(a) $\delta = 0.03$, (b) $\delta = 0.1$, (c) $\delta = 0.2$.

FIGURE 9. a and b. ΔR *versus antenna height* h/λ *for the two cases of full top-loading and unloaded monopoles for parameters as specified.*
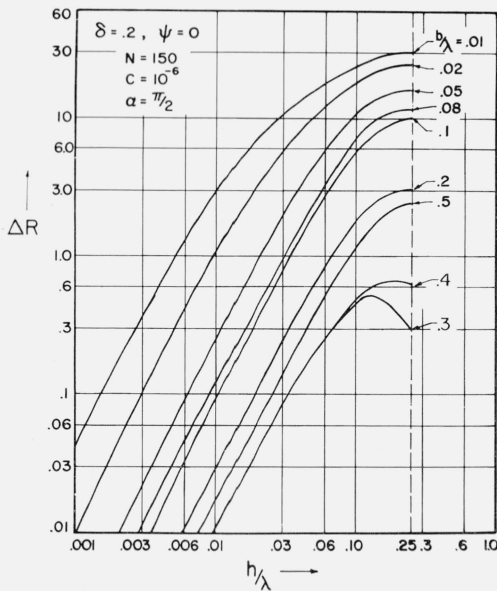
(a) $\delta = 0.03$, (b) $\delta = 0.10$.

FIGURE 10. ΔR *versus antenna height* h/λ *for full top-loading and parameters as specified.*

Figure 9 illustrates the effect of varying the antenna height for a given set of system parameters for fully loaded and unloaded monopoles. The advantage of using top-loading for h/λ less than 0.25 is quite significant as may be seen from figures 10a and b. The near linear log-log relationship makes it possible to write an empirical formula for ΔR where the curves are sufficiently linear, e.g., for $b/\lambda = 0.1$ and the parameters of figure 9b,

$$\Delta R = K\left(\frac{h}{\lambda}\right)^k = 72.4\left(\frac{h}{\lambda}\right)^{1.818}$$

is valid for h/λ less than 0.2.

Wait and Pope [1955] proposed that if the radiation resistances are equal for unloaded and fully loaded monopoles, then they are electrically equivalent in length. It was shown that for the fully loaded monopole of heights 0.025, 0.050, and 0.10 wavelength, the same ΔR is obtained for unloaded monopoles of heights 0.050, 0.095, and 0.175 wavelength, respectively. Figures 9a and b are in good agreement with this conclusion. Figure 9 shows that for loaded monopoles of height 0.025, 0.050, and 0.10 wavelength have the same ΔR as unloaded monopoles of height 0.053, 0.104, and 0.18 wavelength, respectively, except for small b/λ where the values of the latter are slightly higher. In figure 10, one could sketch the behavior of the unloaded case fairly accurately by observing the behavior of figure 9.

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5. References

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Wait, J. R., and W. A. Pope (1954), The characteristics of a vertical antenna with a radial conductor ground system, Appl. Sci. Res **B4** 177–195. Note that in equation (5), sin $(\beta h - \alpha)$ should be sin $(\alpha - \beta h)$.

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