RADIO SCIENCE Journal of Research NBS/USNC-URSI
Vol. 68D, No. 10, October 1964

# A Discussion of the Theory of Ionospheric Cross Modulation

Robert F. Benson [1]

Contribution From the Geophysical Institute, University of Alaska, College, Alaska

(Received November 18, 1963; revised April 25, 1964)

The basic equations in the theory of ionospheric cross modulation are reviewed. The suggestion by Rumi that the variations in the electron density, caused by perturbations of the attachment coefficient, can contribute to the total cross modulation is considered. It is found that the cross modulation resulting from these variations is negligible compared to the cross modulation resulting from the variations in the electron collision frequency in the region above about 40 km. In the 30 km region, however, the two components are approximately equal. The fractional change in electron energy as predicted by the original theory of cross modulation, introduced by Bailey and Martyn in 1934, is compared with the same quantity as predicted by the alternate theory of cross modulation proposed by Huxley in 1953.

Cross-modulation profiles are presented for these two theories, corresponding to various model ionospheres, and discussed in light of previously published cross-modulation observations from College, Alaska. It is concluded that neither of the theories hold over the entire *D* region and that a new theory of ionospheric cross modulation is necessary.

The requirements of such a theory, in order to be consistent with observations, are the following: for given ionospheric conditions the cross modulation should change sign in approximately the same height region as is predicted by the original theory of cross modulation but the absolute magnitude of the cross modulation, at least in the lower *D* region, should be equal to or greater than the value predicted by the alternate theory of cross modulation (which, in turn, is greater than the value predicted by the original theory). A recent theoretical investigation by Altshuler, which indicates an energy dependence for the fractional change in electron energy similar to the predictions of the original theory, seems to satisfy the first requirement. The second requirement remains to be investigated.

## 1. Introduction

The phenomenon of ionospheric cross modulation is commonly associated with Tellegen's [1933] observation that the reception of a Beromünster, Switzerland radio program (on 650 kc/s) was, at times, marred by the presence of a Luxembourg radio program (on 252 kc/s) in the background. After eliminating all possible local causes, Tellegen suggested that the interference effect was due to the interaction of the two radio waves in the ionosphere.

This phenomenon has been known under various titles: "Luxembourg effect," "radio wave interaction," and "ionospheric cross modulation." Huxley and Ratcliffe [1949], in their review article on ionospheric cross modulation, suggested that the latter name be adopted and that the interacting radio waves be identified as the "wanted" and "disturbing" waves.

Bailey and Martyn [1934] explained the cross-modulation effect in terms of the absorption of the radio waves in the ionosphere. The absorption of a radio wave as it travels in an ionized medium is dependent on the electron density and on the electron collision frequency. The radio wave from a high powered disturbing transmitter (200 kW in the case of the Luxembourg transmitter) literally heats up the free electrons in the ionosphere, which results in an increased electron velocity and electron collision frequency. The increase in the electron collision frequency alters the absorption of the wanted wave as it passes through the disturbed region. If the disturbing wave is amplitude modulated, the energy transferred to the electrons, and hence the absorption of the wanted wave, will vary in accordance with the degree of amplitude modulation on the disturbing wave. Thus, a fraction of the amplitude modulation present on the disturbing wave is transferred to the wanted wave giving rise to ionospheric cross modulation.

The original theory of Bailey and Martyn [1934] was revised by Bailey [1937] and was later put into a form more familiar to the ionospheric physicist [Shaw 1951, and references therein]. Huxley [1953] proposed an alternate development of the theory of ionospheric cross modulation in an attempt to reconcile inconsistencies between the apparent behavior of electrons in the laboratory and in the ionosphere. His alternate theory was rejected [Huxley, 1955; Fejer, 1955] on the basis that it was inconsistent with the magneto-ionic theory. Also, the cross-modulation observations conducted in Norway [Landmark and Lied, 1961; Holt, Landmark, and Lied, 1961; Barrington and Thrane, 1962] and in Alaska [Rumi, 1961; Flock and Benson, 1961; Rumi, 1962a, 1962b] seem to be in agreement with the original theory.

Some of the recent observations in Alaska, however, do not agree with the predictions of the original theory of cross modulation [Benson, 1963]. Pre-

[1] Present address: NASA—Goddard Space Flight Center, Greenbelt, Md.

liminary calculations indicated that the cross modulation observed below about 50 km were in better agreement with the alternate theory proposed by Huxley [1953] than with the original theory of Bailey and Martyn [1934]. The above work [Benson, 1963] considered only the original theory of cross modulation in detail, and it is the purpose of this paper to critically analyze and compare the above two theories of ionospheric cross modulation. The validity of the suggestion given by Rumi [1962a], indicating that the contribution to the total cross modulation caused by variations in the electron density can be comparable to the contribution caused by variations in the electron collision frequency, will also be examined.

## 2. Basic Equations

Consider the situation where the wanted wave is traveling vertically downward and the disturbing wave is traveling vertically upward. The absorption of the wanted wave as it passes through an infinitesimal homogeneous layer of thickness $dh$ is given by

$$E=E_0e^{-K_w dh} \tag{1}$$

where, referring to the wanted wave,

$E$=amplitude of the electric field of the emergent wave
$E_0$=amplitude of the electric field of the incident wave
$K_w$=absorption coefficient.

The absorption coe $K$ is given by

$$K=(5.31\times10^{-6})\frac{N}{\nu_m}\left\{\frac{5}{2}\,\mathfrak{C}_{5/2}(\alpha)\right\} \tag{2}$$

when the absorption is of the nondeviative type, i.e., the real component of the refractive index is unity, and the radio wave propagation is parallel to the direction of the earth's magnetic field [Sen and Wyller, 1960, combining their equations 39, 41, and 43]. In (2), the symbols have the following meaning:

$N$=electron density ($m^{-3}$)
$\nu_m$=the mean electron collision frequency associated with the most probable electron speed (sec$^{-1}$)
$\mathfrak{C}_p(\alpha)=\dfrac{1}{p!}\displaystyle\int_0^\infty \frac{\epsilon^p e^{-\epsilon}}{\epsilon^2+\alpha^2}\,d\epsilon$
$\alpha=\dfrac{\omega+s}{\nu_m}$ for the ordinary wave and $\dfrac{\omega-s}{\nu_m}$ for the extraordinary wave (radians).
$\omega$=angular radiofrequency (radians/sec), and
$s$=angular gyrofrequency of the electrons due to the earth's total magnetic field in the region of interest (radians/sec).

The script C integrals $\mathfrak{C}_p(\alpha)$ have been tabulated, for $\alpha$ ranging from zero to 20, by Dingle, Arnt, and Roy [1957]. Rationalized mks units were used in (2).

Assume that the downward traveling wanted wave passes through this infinitesimal layer after the passage of the upward traveling disturbing wave. The energy absorbed from the disturbing wave increases the electron collision frequency $\nu_m$ which causes the amplitude of the wanted wave to change by an amount

$$\Delta E=\left(\frac{\partial E}{\partial \nu_m}\right)_{h'}(\Delta\nu_m)_{h'}$$

$$=-E\left(\frac{\partial K_w}{\partial \nu_m}\right)_{h'}(\Delta\nu_m)_{h'}dh' \tag{3}$$

where $h'$ is the height of the disturbed layer of thickness $dh'$.

Rumi [1962a] also considered the contribution to the cross modulation caused by variations in the electron density $N$. These variations in $N$ are attributed to perturbations of the attachment processes rather than to direct ionization. His analysis is based on recent laboratory measurements of the attachment of slow electrons in oxygen which indicate that the electron attachment is a three-body process with an attachment coefficient that increases with increasing electron energy in the energy range appropriate to the ionospheric $D$ region [Chanin, Phelps, and Biondi, 1959]. Thus, an increase in the electron energy, caused by the absorption of energy from the disturbing wave, should produce a decrease in the electron density.

When the above effect is considered, (3) becomes

$$\Delta E=-E\left\{\frac{\partial K_w}{\partial \nu_m}\,\Delta\nu_m+\frac{\partial K_w}{\partial N}\,\Delta N\right\}_{h'}dh'. \tag{4}$$

If the wanted and disturbing waves are both pulse modulated, and if the pulse repetition rate of the disturbing transmitter is one half that of the wanted transmitter, then only every other wanted pulse will be altered in amplitude (provided that the repetition period of the wanted pulses is chosen to be much longer than the decay time of the excess electron temperature caused by the disturbing pulse). In this case the cross modulation is defined as the fractional change in echo amplitude, i.e., $\Delta E/E$ where $E$ is the amplitude of one received echo pulse and $E-\Delta E$ is the amplitude of the adjacent pulse. Thus the cross modulation resulting from one infinitesimal disturbed layer at the height $h'$ is given by

$$\Delta T=\frac{\Delta E}{E}=-\left\{\frac{\partial K_w}{\partial \nu_m}\,\Delta\nu_m+\frac{\partial K_w}{\partial N}\,\Delta N\right\}_{h'}dh'. \tag{5}$$

If the wanted wave is a continuous wave (such as a satellite signal or cosmic noise) the same comments apply provided that the wanted wave is sampled at a repetition rate that is twice that of the disturbing pulse-modulated transmitter.

Equation (5) gives the cross modulation resulting from only one infinitesimal disturbed layer. The downgoing wanted wave will encounter a succession of such disturbed layers below the height $h_0$ where it first encounters the upgoing disturbing pulse. The total cross modulation observed on the received wanted wave is given by

$$T=\sum\Delta T=-\int_0^{h_0}\left\{\frac{\partial K_w}{\partial \nu_m}\,\Delta\nu_m+\frac{\partial K_w}{\partial N}\,\Delta N\right\}_{h'}dh'. \qquad (6)$$

The evaluation of the partial derivatives that enter in (6) is straight forward when the longitudinal expression for $K_w$, as given in (2), is used. This compact expression can be retained, rather than using the more involved expression for arbitrary direction of propagation, even when the direction of propagation deviates from the direction of the earth's magnetic field if $s$ is replaced by $\omega_L=|s\cos\phi|$, where $\phi$ is the angle between the propagation vector and the earth's lines of magnetic force. Just as in the classical magneto-ionic theory, the above "quasi-longitudinal" approximation in the generalized magneto-ionic theory must satisfy certain conditions before it can be used [Benson, 1964a]. In the 1–4 Mc/s frequency range, common to many experiments in ionospheric cross modulation, greater accuracy is obtained by using the unmodified longitudinal equation (2), even when $\phi\neq 0$, rather than by introducing $\omega_L$. In the cross modulation experiment in Alaska, the lowest frequency used is 4.865 Mc/s and $\phi=13°$; under these conditions the quasi-longitudinal expression for $K_w$, i.e., using $\omega_L$ in place of $s$ in (2), can be used with an accuracy of better than 1 percent (with respect to the more general expression for arbitrary value of $\phi$). The derivatives of $K_w$, required in (6), are then given by

$$\frac{\partial K_w}{\partial \nu_m}=-(5.31\times 10^{-6})\,\frac{N}{\nu_m^2}\left\{\frac{5}{2}\,Y(\alpha)\right\} \qquad (7)$$

$$\frac{\partial K_w}{\partial N}=(5.31\times 10^{-6})\,\frac{1}{\nu_m}\left\{\frac{5}{2}\,\mathfrak{C}_{5/2}(\alpha)\right\} \qquad (8)$$

where

$$Y(\alpha)=\frac{5}{2}\,\mathfrak{C}_{5/2}(\alpha)-\frac{7}{2}\,\mathfrak{C}_{7/2}(\alpha).$$

In deriving (7), the dummy variable $\epsilon$ (the normalized electron energy) in the script C integral was written in its complete form as $\epsilon=Q/k\theta$, where $Q$ is the energy of any given electron (which is the true dummy variable) and $k\theta$ is the thermal energy associated with the electron having the most probable speed of all the electrons at the temperature $\theta$(°K). The assumption $\theta=b\nu_m$, where $b$ is a constant, was then used in order to carry out the differentiation. This assumption, that the electron collision frequency is directly proportional to the electron energy, was also used by Sen and Wyller [1960] to obtain their generalized magneto-ionic equations, and it is based on recent laboratory studies with $N_2$ by Phelps and Pack [1959]. Using the above assumption, the $\Delta\nu_m$ term can be expressed as

$$\Delta\nu_m=\nu_m(\Delta\theta/\theta) \qquad (9)$$

where $(\Delta\theta/\theta)$ is the fractional increase in the electron temperature caused by the energy absorbed from the disturbing wave. The derivation of a similar expression for the $\Delta N$ term, and an evaluation of the importance of the $\Delta N$ contribution, will be considered next.

## 3. Variations in the Electron Density

In order to obtain an expression relating the term $\Delta N$ to $\Delta\theta$ the following equations, which describe the rate of change of charge density in the ionosphere, will be considered:

$$\frac{dN}{dt}=q+\gamma_p N^-+\gamma_c nN^- -\alpha_d NN^+ -\beta[n(O_2)]N-\kappa[n(O_2)]^2N \qquad (10)$$

$$\frac{dN^-}{dt}=-\gamma_p N^- -\gamma\, nN^- -\alpha_i N^-N^+ +\beta[n(O_2)]N+\kappa[n(O_2)]^2N \qquad (11)$$

$$N^+=N+N^- \qquad (12)$$

where

$N$ = electron density
$N^-$ = negative ion density
$N^+$ = positive ion density
$[n(O_2)]$ = number density of molecular oxygen
$n$ = number density of neutral particles
$q$ = electron production rate
$\alpha_d$ = electron-ion dissociative recombination coefficient (effective value)
$\alpha_i$ = ion-ion recombination coefficient
$\beta$ = 2 body attachment coefficient
$\kappa$ = 3 body attachment coefficient
$\gamma_c$ = collisional detachment coefficient
$\gamma_p$ = photodetachment coefficient.

Rumi [1962a] considered two cases of the equilibrium equation $\left(\text{which follows from (10), (11), and (12) when } \frac{dN}{dt}=\frac{dN^-}{dt}=0\right)$

$$q=(1+\lambda)(\alpha_d+\lambda\alpha_i)N^2 \qquad (13)$$

namely,

$$N=\frac{1}{\lambda}\left(\frac{q}{\alpha_i}\right)^{1/2} \text{ when } \lambda>>1 \text{ and } \lambda\alpha_i>>\alpha_d \qquad (14)$$

and

$$N=\left(\frac{q}{\alpha_d\lambda}\right)^{1/2} \text{ when } \lambda>>1 \text{ and } \lambda\alpha_i<<\alpha_d \qquad (15)$$

where $\lambda=N^-/N$. From (14), which applies to the lower $D$ region, he derives the following expression for $\Delta N$:

$$\Delta N=-N\left[\theta\frac{\partial(\ln\kappa)}{\partial\theta}\right]\frac{\Delta\theta}{\theta}$$

and presents a curve of $\theta\frac{\partial(\ln\kappa)}{\partial\theta}$ versus $\theta$ which is based on the data of Chanin, Phelps, and Biondi [1959].

In the above analysis, however, it is inherently assumed that the time constant for the variations in the electron density is the same as the time constant for the variations in the electron collision frequency. A recent report by Molmud, Altshuler, and Gardner [1962] indicates that this is not the case in the ionospheric $D$ region. The above authors discuss the time constant for electron density changes resulting from electron energy changes in their study of a method to reduce the electron density in the ionospheric $D$ region by means of high-powered ground-based transmitters. Their study is based on the fact that the time constant for electron density variations is long compared to the time constant for average electron energy variations.

Molmud, Altshuler, and Gardner [1962] combined (10), (11), and (12) by eliminating $N^-$ while retaining the time parameter $t$. They then consider the solution of the resulting differential equation under the following initial conditions: for $t<0$, $\theta=\theta_0$ and steady state conditions prevail; for $t\geq 0$, $\theta=\theta_1$. The rise in electron temperature, however, is generally not instantaneous, even if a pulse modulated disturbing wave is used. (This subject is discussed in detail in the next section.) Thus the solution of the differential equation for $N$, under the above initial conditions, will be useful only in determing an upper limit for the variations in $N$ due to the true variations in $\theta$ caused by the disturbing pulse. Combining (10), (11), and (12), using the method of Molmud, Altshuler, and Gardner [1962], gives

$$2\frac{dN}{dt}=A-2BN \tag{16}$$

where

$$A=q+2(\gamma_p+\gamma_c n)N^+ +\alpha_i(N^+)^2$$

$$B=\gamma_p+\gamma_c n+\left(\frac{\alpha_i+\alpha_d}{2}\right)N^+ +\kappa[n(0_2)]^2.$$

In deriving (16) it has been assumed that the changes in $N^+$ are slow compared to the changes in $N$. Molmud, Altshuler, and Gardner [1962] show that this is a reasonable assumption.

The rate coefficients, that appear in (16), will be assigned the following values [Crain 1961]:

$\alpha_d=1.0\times10^{-7}$ cm$^3$/sec
$\alpha_i=6[p\text{ (atm)}]\theta^{-5/2}+10^{-9}$ cm$^3$/sec
$\gamma_p=0$ during the night
$=0.4$ sec$^{-1}$ during the day
$\gamma_c n\leq 100\, p$ (atm) sec$^{-1}$
$q=(\alpha_d+\lambda\alpha_i)NN^+$ (the value for equilibrium conditions).

The positive ion density is given by $N^+=(1+\lambda)N$, and $\lambda=N^-/N$ can be approximated by

$$\lambda=[n(0_2)]^2\kappa/(\gamma_p+\gamma_c n)\ \text{[Rumi, eq (13), 1962a]}.$$

The following *approximate* expression for the three body attachment coefficient $\kappa$ can be derived by extrapolating the $0_2$ curve in figure 3 of the paper by Chanin, Phelps, and Biondi [1959]:

$$\kappa\ (\text{cm}^6/\text{sec})\cong -2.20\times10^{-30}+(1.63\times10^{-32})\theta.$$

Sample calculations using the above expressions indicate that the parameter $A$, in (16), is dominated by the term $2(\gamma_p+\gamma_c n)N^+$, which is independent of the electron energy. Thus, to first approximation, $A(\theta_0)=A(\theta_1)=A$. The parameter $B$, on the other hand, is dominated by the energy dependent term $\kappa[n(0_2)]^2$. The initial conditions appropriate to (16) are then given by

$$\left.\begin{array}{l}\theta=\theta_0\\ B=B_0\\ N=N_0\end{array}\right\}t<0\qquad \left.\begin{array}{l}\theta=\theta_1\\ B=B_1\\ N=N_0\end{array}\right\}t=0\qquad \left.\begin{array}{l}\theta=\theta_1\\ B=B_1\\ N=N\end{array}\right\}t>0$$

where $N_0$ is the equilibrium value of $N$ in the absence of the disturbing pulse ($t<0$) which is found from (16), with $dN/dt=0$, to be

$$N_0=A/2B_0. \tag{17}$$

The integration of (16) then proceeds as follows:

$$\int_{N_0}^{N}\frac{2dN}{A-2B_1N}=\int_{t=0}^{t}dt$$

$$\ln\left\{\frac{A-2B_1N}{A-2B_1N_0}\right\}=-B_1t$$

$$N=\frac{A}{2B_1}(1-e^{-B_1t})+N_0e^{-B_1t}. \tag{18}$$

Combining (17) and (18) gives

$$\Delta N=N-N_0=\frac{A}{2}\left(\frac{1}{B_1}-\frac{1}{B_0}\right)(1-e^{-B_1t})$$

$$=\frac{A}{2B^2}(B_0-B_1)(1-e^{-Bt}) \tag{19}$$

where the distinction between $B_0$ and $B_1$ is made only in the difference term $(B_0-B_1)$, since the difference between the two quantities is slight (the maximum value of $\Delta\theta/\theta$ is of the order of a few percent or less). Consider the term $(B_0-B_1)$. From (16), and the values for the rate coefficients given below (16), $(B_0-B_1)$ can be written as

$$B_0-B_1=1/2[(\alpha_i)_0-(\alpha_i)_1]N^+ + (\kappa_0-\kappa_1)[n(0_2)]^2$$

$$=3[p(\text{atm})]\left[\frac{\theta_1^{5/2}-\theta_0^{5/2}}{\theta^5}\right]N^+$$

$$+(1.63\times10^{-32})(\theta_0-\theta_1)[n(0_2)]^2.$$

Let $\theta_1=\theta_0+\Delta\theta$, then,

$$\theta_1^{5/2}=\theta_0^{5/2}\left(1+\frac{\Delta\theta}{\theta_0}\right)^{5/2}\simeq\theta_0^{5/2}\left(1+5/2\,\frac{\Delta\theta}{\theta_0}\right)$$

and

$$\theta_1^{5/2}-\theta_0^{5/2}\simeq 5/2\left(\frac{\Delta\theta}{\theta}\right)\theta^{5/2}.$$

Thus

$$B_0-B_1\simeq\left\{\frac{15}{2}\,\frac{[p(\text{atm})]N^+}{\theta^{5/2}}-(1.63\times10^{-32})\theta[n(0_2)]^2\right\}\frac{\Delta\theta}{\theta}$$

and (19) becomes

$$\Delta N\simeq H(h')\,\frac{\Delta\theta}{\theta} \tag{20}$$

where

$$H(h')=\frac{A}{2B^2}\left\{\frac{15}{2}\,\frac{[p(\text{atm})]N^+}{\theta^{5/2}}-(1.63\times10^{-32})\theta[n(0_2)]^2\right\}(1-e^{-Bt})\ \text{cm}^{-3}$$

and $A$ and $B$ are given in (16). Note: cgs units were used exclusively in the above equations because the rate coefficients are most commonly expressed in these units. In the following discussion, $H(m^{-3})=10^6\,H\ (\text{cm}^{-3})$ will be used.

Combining (6), (7), (8), (9), and (20) gives the following expression for the total cross modulation observed on the received wanted wave:

$$T=\frac{5}{2}(5.31\times10^{-6})\int_0^{h_0}\{N(m^{-3})Y(\alpha)-H(m^{-3})\mathfrak{C}_{5/2}(\alpha)\}\,\frac{1}{\nu_m}\,\frac{\Delta\theta}{\theta}\,dh'. \tag{21}$$

In (21), the term $NY(\alpha)$ determines the component of cross modulation due to variations in $\nu_m$ ($CM\nu$), and the term $H\mathfrak{C}_{5/2}(\alpha)$ determines the component of cross modulation due to variations in $N$ ($CMN$). From this equation, the importance of $CMN$, as compared with $CM\nu$, can be estimated. Sample calculations, using the model atmosphere given by Miller [1957] together with a model profile for the electron density [see Benson, 1963, profile $N_1$], indicate that $CMN$ is negligible compared with $CM\nu$ except in the region below about 40 km. The results of these calculations are listed in the following table.

| $h_0$(km) | $CMN/CM\nu$* | |
|---|---|---|
| | $f_w$=17.5 Mc/s | $f_w$=4.865 Mc/s |
| 30 | 1.0 | 0.9 |
| 40 | .1 | .07 |
| 50 | .07 | .009 |

*$CMN=-H(M^{-3})\mathfrak{C}_{5/2}(\alpha)$; $CM\nu=N(m^{-3})Y(\alpha)$.

The above figures are maximum figures appropriate to a disturbing pulse of 50 $\mu$sec duration; i.e., in (20), $t$ was set equal to $50\times10^{-6}$ sec. It must be kept in mind that the true contribution of $CMN$ is less than is indicated by the above figures, especially in the region above about 40 km, because the increase in electron temperature is not instantaneous. (In the region below about 40 km, the increase is nearly instantaneous when the original theory of cross modulation is used and the above figures approximate the true situation; the increase is not instantaneous, however, when the alternate theory of cross modulation is used, and in this case the true figures are less than is indicated above.) The above results are in agreement with the results of Rumi [1962a] in the lower ionosphere (30 km region) when the original theory of cross modulation is used, namely, that $CMN$ is comparable to $CM\nu$, but differ greatly from his results above this region. This situation exists because the time constant $(1/B)$, associated with $\Delta N$ in (20), increases with increasing altitude, which prevents significant changes in $N$ from occurring during the duration of the disturbing pulse. Rumi's treatment did not include the time constant effect since the time parameter was eliminated from (10) and (11).

## 4. The Term $\Delta\theta/\theta$

The terms $\Delta\nu$ and $\Delta N$, that enter in the cross modulation equation (6), have each been expressed in terms of the fractional increase in the electron temperature $\Delta\theta/\theta$ due to the disturbing pulse [see (9) and (20)]. This term, $\Delta\theta/\theta$, is equivalent to $\Delta Q/Q$, where $Q$, the thermal energy of the electron, is given by $Q=ak\theta$ ($k$ is Boltzmann's constant and $a$ is a constant which takes the value 1, $4/\pi$, or 3/2 depending on whether the electron thermal energy is considered to be associated with the most probable electron velocity $v_m$, the mean electron velocity $\bar{v}$, or the root-mean-square electron velocity $v_{\text{rms}}$). The value $a=1$ will be used throughout this paper so as to be consistent with the use of $\nu_m$ in (2). The variation of $Q$ is determined by the equation

$$\frac{dQ}{dt}=w-\nu_m\delta Q \tag{22}$$

where $w$ is the power supplied to each electron and $\delta Q$ is the mean energy lost by an electron per collision with a heavy particle. In the original theory of ionospheric cross modulation, Bailey and Martyn [1934] assumed that

$$\delta Q = G(Q - Q_0) \tag{23}$$

where $G$ is a dimensionless constant generally referred to as the energy loss coefficient, $Q$ is the thermal energy of an average electron after the passage of the disturbing wave, and $Q_0$ is its thermal energy in the absence of the disturbing wave. Equation (22) then becomes

$$\frac{dQ}{dt} = w - G\nu_m(Q - Q_0).$$

Since $Q_0$ is considered to be constant, this equation can be written as

$$\frac{d(\Delta Q)}{dt} = w - G\nu_m(\Delta Q) \tag{24}$$

where $\Delta Q = Q - Q_0$.

Consider the solution of (24) appropriate to a pulse modulated disturbing wave traveling vertically upward with a velocity $c$ (assumed to be equal to the velocity of light in vacuum). Since the wanted wave is traveling vertically downward at the same velocity, any portion of the wanted wave is only in contact with the disturbing pulse for a time $\tau/2$ where $\tau$ is the duration of the disturbing pulse in seconds. Thus, if a portion of the wanted wave first encounters the leading edge of the disturbing pulse at the height $h_0$, it will encounter the trailing edge of the disturbing pulse at the height $h_0 - c\tau/2$; accordingly, its position at any later time $t$ is given by $h_0 - ct/2$.

The value of $\Delta Q$ at a height $h'$ when $h_0 \geq h' \geq h_0 - c\tau/2$ (i.e., inside the pulse) can be obtained by integrating (24). In this integration, $\nu_m$ will be approximated by an average value $\nu_a$ appropriate to the region between $h_0$ and $h_0 - c\tau/2$. The integration proceeds as follows:

$$\int_{\Delta Q=0}^{\Delta Q=(\Delta Q)_{h'}} \frac{d(\Delta Q)}{w - G\nu_a(\Delta Q)} = \int_{t=0}^{t=2(h_0-h')/c} dt$$

$$-\frac{1}{G\nu_a} \ln\left[\frac{w - G\nu_a(\Delta Q)_{h'}}{w}\right] = \frac{2(h_0 - h')}{c}$$

$$(\Delta Q)_{h'} = \frac{w}{G\nu_a}\left[1 - e^{-2(h_0-h')\frac{G\nu_a}{c}}\right] \tag{25}$$

when $h_0 \geq h' \geq h_0 - \dfrac{c\tau}{2}$.

The value of $\Delta Q$ at a height $h'$ when $h' < h_0 - c\tau/2$, i.e., outside the pulse, can be obtained by integrating (24) with $w=0$. The equation then becomes

$$\int_{(\Delta Q)_{\max}}^{(\Delta Q)_{h'}} \frac{d(\Delta Q)}{\Delta Q} = -G\nu_m \int_0^{t'=2(h_0-c\tau/2-h')/c} dt'$$

where $(\Delta Q)_{\max}$ is the value of $\Delta Q$ at $h' = h_0 - c\tau/2$ and $t'$ is the time measured with reference to the passage of the trailing edge of the disturbing pulse. Since the decay of $\Delta Q$ at a given height is under consideration, $\nu_m$ is a constant and has been taken outside of the integral sign. Integrating the above equation gives

$$(\Delta Q)_{h'} = (\Delta Q)_{\max} e^{-2(h_0-c\tau/2-h')\frac{G\nu_m}{c}} \tag{26}$$

when $h' < h_0 - c\tau/2$.

Huxley [1953] proposed an alternate expression for the mean energy lost by an electron per collision when laboratory studies did not seem to confirm the assumption in (23). The development of his alternate expression was also encouraged by the discrepancy existing between the laboratory measurements of $G$ and the ionospheric cross modulation measurements of the quantity $G\nu$. Following Huxley [1953], the following expression can be derived:

$$\delta Q = B\,\frac{(\Delta Q)^2}{Q^2} \tag{27}$$

where $B = 3.18 \times 10^{-23} j$. The details leading to this expression are given in the appendix. An important point to notice is that (27) is based on a *steady* electric field, and thus only applies to the region where $\nu_m >> f_d$, where $f_d$ is the frequency of the disturbing transmitter. Substituting (27) into (22) and proceeding in the same manner as was used to derive (25) and (26) gives

$$(\Delta Q)_{h'} = \frac{Q}{B^{1/2}}\left(\frac{w}{\nu_a}\right)^{1/2} \tanh\left[\frac{2B^{1/2}(\nu_a w)^{1/2}(h_0 - h')}{cQ}\right] \tag{28}$$

when $h_0 \geq h' \geq h_0 - \dfrac{c\tau}{2}$

and

$$(\Delta Q)_{h'} = \frac{\Delta Q_{\max}}{1 + \Delta Q_{\max}\left[\dfrac{2B\nu_m(h_0 - c\tau/2 - h')}{cQ^2}\right]} \tag{29}$$

when $h' < h_0 - c\tau/2$

where $\Delta Q_{\max}$ is the value of $\Delta Q$ from (28) when $h' = h_0 - c\tau/2$.

Sample calculations were performed in order to compare the values of $\Delta Q/Q$ obtained from the original theory (25) and (26) with the values of $\Delta Q/Q$ obtained from the alternate theory (28) and (29). The results are presented in figure 1. In performing these calculations, the following value was used for $w$:

$$w = \frac{P_d g K_d}{2\pi N(h')^2}\, e^{-\int_0^{h'} 2K_d dh} \text{ watts} \tag{30}$$

where

$P_d$ = peak pulse power radiated by the disturbing transmitter (watts)
$g$ = disturbing antenna gain factor
$K_d$ = absorption coefficient for the disturbing wave

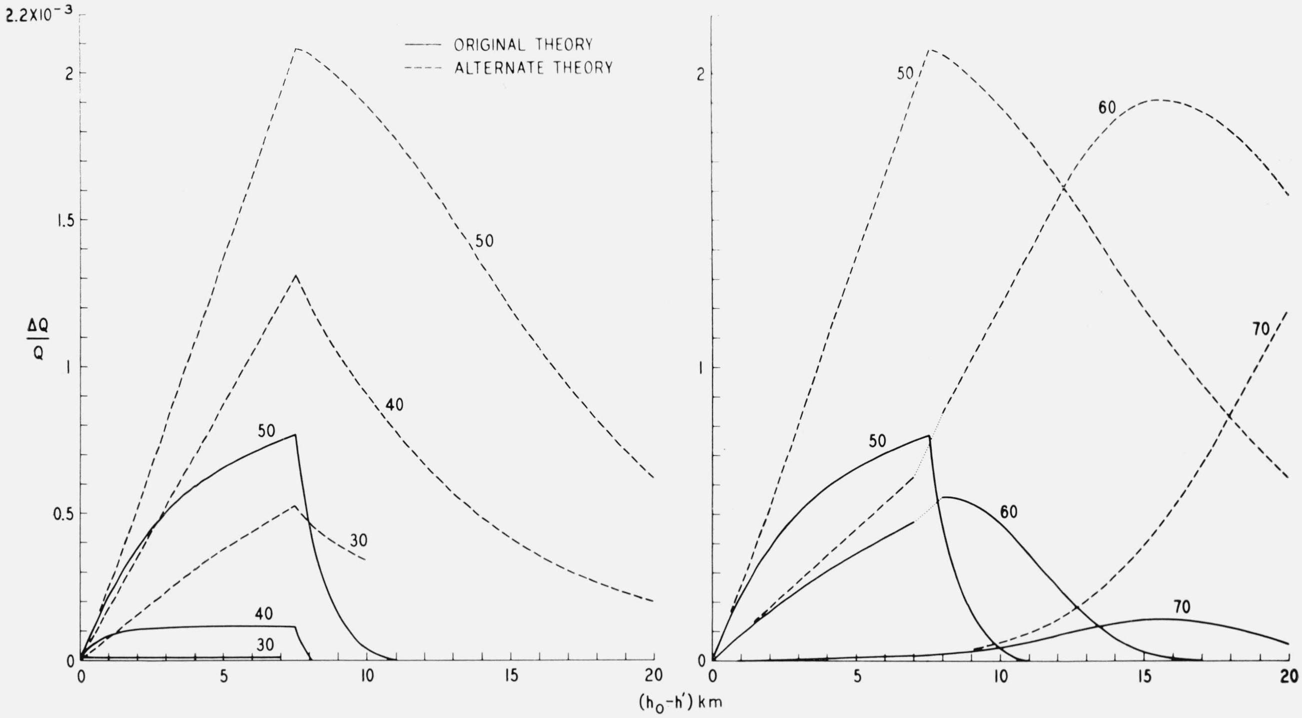


FIGURE 1. *The fractional increase in the electron energy* Q, *caused by a rectangular shaped upward traveling disturbing pulse of 50 μ sec duration, as predicted by the original theory of cross modulation* [*Bailey and Martyn, 1934*] *and the alternate theory of cross modulation proposed by Huxley* [*1953*].

Each curve corresponds to a particular value of $h_0$, the height where the leading edge of the upward traveling disturbing pulse first encounters the downward traveling wanted wave. The curves correspond to the following parameters: $P_d$=10kW, $g\simeq 5$, $f_d$=4.865 Mc/s (E), $f_w$=17.5 Mc/s.

(see appendix). Model electron density and collision frequency profiles of a previous publication were used in the calculations ($N_1$ and $\nu_1$ of Benson [1963]). The parameters were chosen so as to agree with the recent cross-modulation experiment in Alaska which uses cosmic noise at a frequency of 17.5 Mc/s as the wanted wave [Benson, 1962]; i.e., $P_d=10$ kW, $g\simeq 5$, and $f_d=4.865$ Mc/s. The calculations correspond to the extraordinary component of the disturbing wave. Each curve in figure 1 indicates the fractional variations in electron energy $\Delta Q/Q$ that are encountered by a portion of the downward traveling wanted wave as it passes the upward traveling disturbing pulse when the leading edge of the disturbing pulse is at a given height $h_0$. The curves are drawn for the case $\tau=50$ μsec; i.e., the trailing edge of the disturbing pulse is located at the height $h_0-c\tau/2=h_0-7.5$ km.

Figure 1 indicates that the expected value for $\Delta Q/Q$ is larger and that it decays much slower in the region outside (below) the disturbing pulse, for any given value of $h_0$, when the alternate theory is used rather than the original theory. The maximum value for $\Delta Q/Q$ increases with increasing $h_0$ up until $h_0=50$ km; the maximum value for $h_0>50$ km decreases with increasing $h_0$ due to the absorption of energy from the disturbing wave in the lower regions and to the slower rise of $\Delta Q/Q$ in the upper regions. The discontinuity in the curves labeled $h_0=60$ km and $h_0=70$ km at the height $h'=h_0-7.5$ km, in both theories, results from the approximation $\nu_m=\nu_a$ inside the pulse ($\nu_a$ was evaluated at the height $h'=h_0-3$ km). (The larger values of $\Delta Q/Q$ outside the pulse are due to larger values of $(\Delta Q/Q)_{\max}$ in the lower regions.) The curves for $h_0=60$ km and $h_0=70$ km do not satisfy the condition $\nu_m>>f_d$, which must be satisfied in the alternate theory; this point will be considered further in the next section where the original and alternate theories are compared by calculating the cross modulation expected for each theory for various model ionospheres.

## 5. Model Cross-Modulation Profiles

When the term $\Delta N$ is neglected and the term $\Delta\nu_m$ is written as $\nu_m(\Delta\theta/\theta)$ from (9), then the equation for the total cross modulation (6) becomes

$$T=-\int_0^{h_0}\left(\frac{\partial k_w}{\partial \nu_m}\right)(\nu_m)\left(\frac{\Delta\theta}{\theta}\right)dh'. \tag{31}$$

The partial derivative in this equation is given by (7); the $\Delta\theta/\theta$ term is given by (25) and (26) when the original theory of cross modulation is used, and by (28) and (29) when Huxley's alternate theory of cross modulation is used.

The cross modulation was calculated from (31), with the aid of the IBM 1620 computer at the University of Alaska, for several model ionospheres. In these calculations the script C integrals were evaluated using the tabulated values, for the argument ranging from 0 to 20, as given by Dingle, Arndt, and Roy [1957]. These integrals were approximated as $1/\alpha^2$ for values of the argument $\alpha$ greater than 20. The electron temperature $\theta$ was assumed to be comparable to the gas temperature, and the Fort Churchill, Canada, winter average rocket measurements of the $D$ region gas temperature by Stroud, Nordberg, Bandeen, Bartman, and Titus [1960, see fig. 13] were used in the calculations. The electron collision frequency $\nu_m$ was evaluated from the equation $\nu_m = 8.40 \times 10^7\ p$ (mmHg) using the atmospheric pressure values for November 1956 above Fort Churchill as given by Lagow, Horowitz, and Ainsworth [1960, see fig. 5]. This equation is based on the laboratory studies in $N_2$ by Pack and Phelps [1961] and is discussed in the following paper [Benson, 1964b].

Several model profiles for the electron density $N$ were used in the calculations; these profiles are shown in figure 2. The curve labeled $N_1$ (the curve without the bumps) represents a possible electron density profile in the arctic regions during relatively quiet ionospheric conditions. Each of the curves labeled $N_2$, $N_3$, $N_4$, and $N_5$ represent particular perturbations of the main curve $N_1$. These perturbed curves should be thought of as short period electron density profiles resulting from transient irregularities in the $D$ region electron density rather than as stable electron density profiles. The nondeviative absorption of a radio wave at a frequency of 27.6 Mc/s (a standard riometer frequency at College, Alaska) for each of the above electron density profiles is given in table 1.

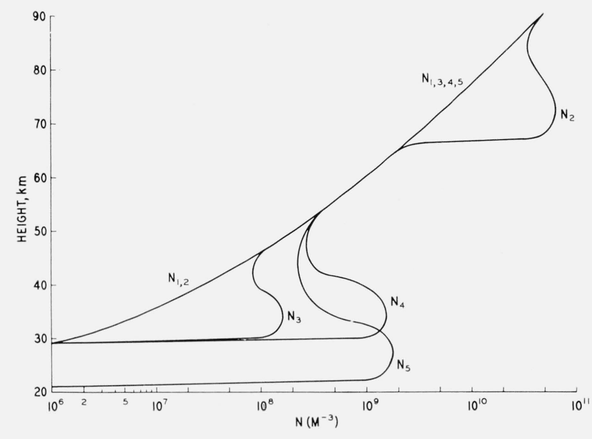


FIGURE 2. *These curves are not the result of an experimental program; they are merely model electron density profiles used in calculating the model cross-modulation profiles.*

The curves $N_2$, $N_3$, $N_4$, and $N_5$, which represent particular perturbations of the main curve $N_1$, are introduced for the sake of discussing the equations of cross modulation.

TABLE 1. *Radio wave absorption figures for the model electron density profiles*

| Electron density profile | $N_1$ | $N_2$ | $N_3$ | $N_4$ | $N_5$ |
|---|---|---|---|---|---|
| Absorption in decibels at 27.6 Mc/s | 1.1 | 5.2 | 1.2 | 2.3 | 1.9 |

In the original theory of cross modulation, it is necessary to assign a value to the energy loss coefficient $G$, which appears in (25) and (26), before model cross-modulation profiles can be computed. The value $G = 1 \times 10^{-3}$, which is consistent with laboratory measurements in air [Crompton, Huxley, and Sutton, 1953], was used in the present calculations.

In the calculations, the parameters of frequency and power were chosen so as to be consistent with the cross-modulation experiment at College, Alaska. Three separate conditions were considered: the first condition corresponds to the recent cross-modulation experiment which used cosmic noise as the wanted wave [Benson, 1962], the second condition corresponds to the original cross-modulation experiment which used a reflected pulse modulated radio wave for the wanted wave [Rumi, 1961]; and the third condition corresponds to the cross-modulation experiment that will be under way when the present construction of a new cosmic noise receiving antenna is completed. These parameters are presented in table 2.

TABLE 2. *Frequency and power parameters employed in the calculation of model cross-modulation profiles*

| | Disturbing transmitter | | | Frequency of wanted wave [a] |
|---|---|---|---|---|
| | Frequency [a] | Power | Antenna gain factor | |
| | *Mc/s* | *kW* | | *Mc/s* |
| Condition 1 | 4.865(E) | [b]20 | 5 | 17.5 |
| Condition 2 | 17.5 | 100 | 150 | 4.865(O) |
| Condition 3 | 17.5 | 100 | 150 | 18.0 |

[a] (E) designates extraordinary component; (O) designates ordinary component.
[b] In the early phases of the experiment a value of 10 kW was used; the curves in figures 3 and 4 correspond to the value of 20 kW.

The radiofrequencies listed in the above table are considerably higher than the frequencies used in other cross-modulation experiments (usually in the 1 to 3 Mc/s range). One advantage of the higher frequencies is that the cross-modulation experiment is most efficient during periods of ionospheric disturbance which are common at College, Alaska; one disadvantage of the higher frequencies is that the sensitivity of the experiment is decreased in the upper $D$ region. The latter point is illustrated in figure 3 where a large electron density perturbation in the upper $D$ region (see fig. 2, profile $N_2$) is seen to produce only an insignificant perturbation in the cross-modulation profile. The cross-modulation profiles presented in figure 3 are based on a wanted frequency of 17.5 Mc/s (condition 1 of table 2). The sensitivity of the experiment to electron density perturbations

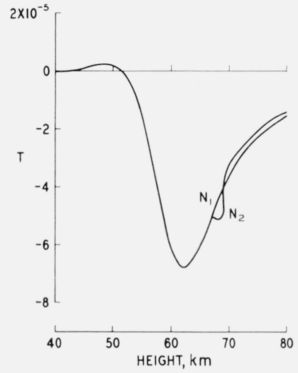


FIGURE 3. *A comparison of the expected cross modulation for* $N_1$ *and* $N_2$ *of figure 2 when the original theory of cross modulation is employed, using the parameters of condition 1.*

in the upper $D$ region increases considerably for lower frequencies. It is important to notice, however, that the lack of sensitivity indicated in figure 3 is partly due to the slow rise of $\Delta Q/Q$ in the upper $D$ region (see fig. 1).

In figures 4, 5, and 6, the ionosphierc cross-modulation profiles, corresponding to the model electron density profiles $N_1$, $N_3$, $N_4$, and $N_5$, are presented for conditions 1, 2, and 3 (see table 2), respectively. Note: Because the absolute value of $w$ in (30) was used, the curves indicate a sign reversal of $T$ was respect to the curves presented in an earlier publication [Benson, 1963]. In each figure the cross modulation predicted by the original theory is compared with the cross modulation predicted by the alternate theory up to an elevation of 55 km. The dashed curves, corresponding to the alternate theory, are terminated at 55 km because above this height (approximately) the radiofrequency of the disturbing transmitter fails to satisfy the condition

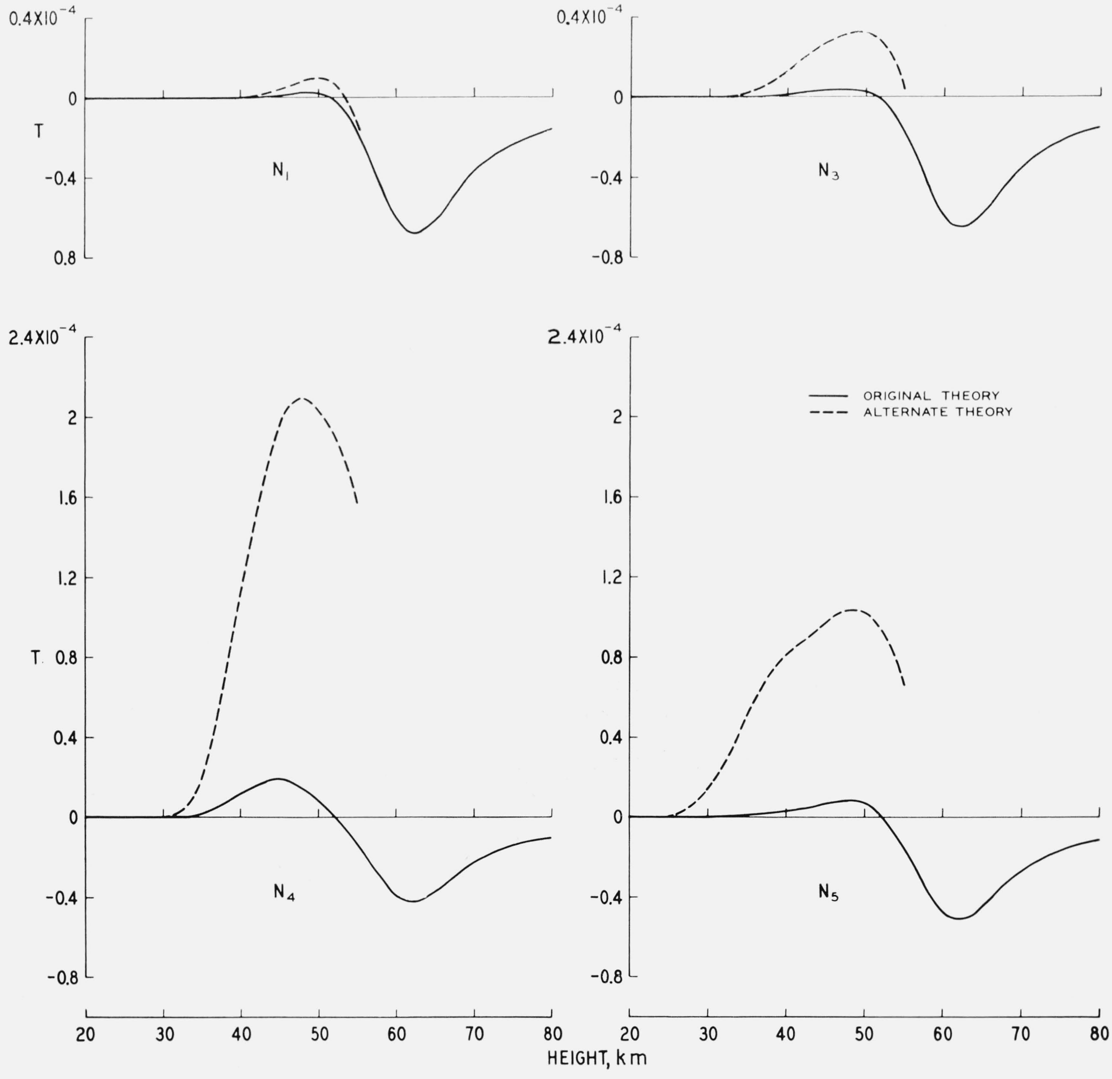


FIGURE 4. *Cross-modulation profiles based on condition 1 of table 2.*

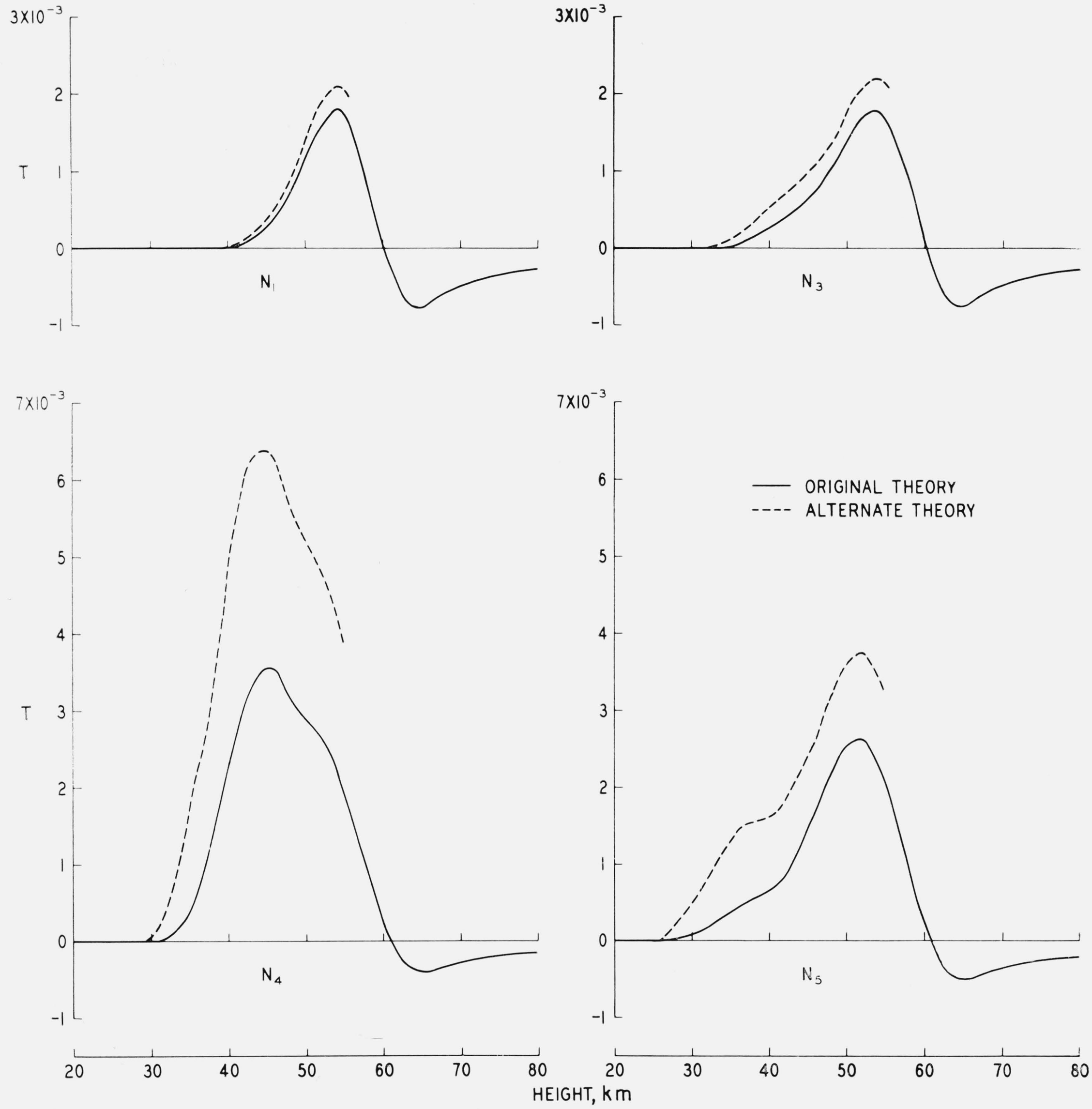


FIGURE 5. *Cross-modulation profiles based on condition 2 of table 2.*

$f_d << \nu_m$, which is implied in the derivation of the alternate theory [see discussion following (27)]. Also, calculations based on the alternate theory predict that a significant portion of the excess electron energy $\Delta Q$, caused by a given disturbing pulse, persists until the next sample of the wanted wave is taken when $h_0$ is greater than about 55 km.

At first glance it appears that the choice of frequency and power parameters as given in condition 2 is preferable to the choice of these parameters as given in conditions 1 and 3. All of the curves, however, correspond to the case when a usable wanted signal is available at the receiving antenna, and in condition 2 the wanted signal, which is a radio wave at a frequency of 4.865 Mc/s that has been reflected from the $E$ or $F$ region of the ionosphere, is often completely absorbed before reaching this antenna. For example, the total *nondeviative* absorption of the ordinary component of a reflected radio wave (two way path through the D region) at a frequency of 4.865 Mc/s is of the order of 30 dB when the electron density profiles $N_1$, $N_3$, $N_4$, or $N_5$ are used. The absorption of the cosmic noise signal, which is the wanted wave in conditions 1 and 3, is much less than the absorption of the reflected radio wave in condition 2 because the received cosmic noise is at a relatively high frequency and it travels through the $D$ region only once. For example, the total nondeviative absorption for the cosmic noise at 17.5 Mc/s (condition 1) and at 18.0 Mc/s (condition 3) is of the order of 3 dB or less when the electron density profiles $N_1$, $N_3$, $N_4$, or $N_5$ are used. It is to be empha-

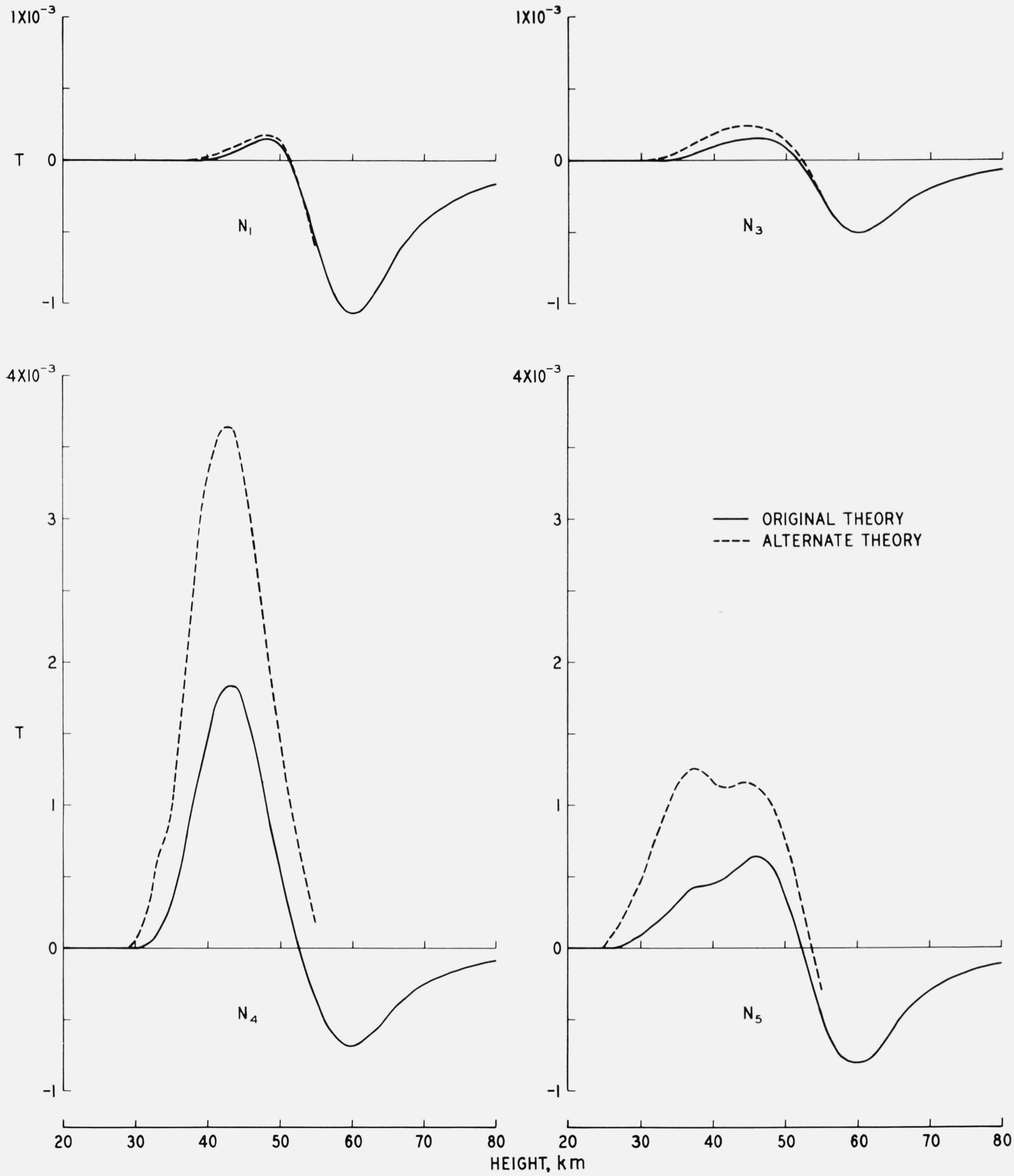


FIGURE 6. *Cross-modulation profiles based on condition 3 of table 2.*

sized that the above electron density profiles are merely model profiles used for the sake of comparing the two theories of cross modulation under discussion. During typical periods of ionospheric absorption at College, Alaska the absorption of radio waves is more severe than the absorption indicated by the above electron density profiles; under such conditions the echo technique (condition 2) becomes ineffective due to the excessive absorption of the wanted wave, and the cosmic noise cross-modulation technique is more advantageous [Benson, 1962].

The main points to observe in figures 4, 5, and 6 are the following:

(1) In all cases, $T_{\mathrm{alt}} > T_{\mathrm{orig}}$ where $T_{\mathrm{alt}}$ refers to the cross modulation predicted from the alternate theory and $T_{\mathrm{orig}}$ refers to the cross modulation predicted from the original theory.

(2) The ratio $T_{\mathrm{alt}}/T_{\mathrm{orig}}$ is greatest when excessive ionization is present in the lower regions, i.e., for the electron density profiles $N_3$, $N_4$, and $N_5$. This is to be expected since the decay of $\Delta Q$ is slower, and thus

the cross-modulation contribution of the lower regions persists longer, when the alternate theory is used.

(3) The ratio $T_{\mathrm{alt}}/T_{\mathrm{orig}}$ is greater for condition 1 than it is for conditions 2 and 3. This is to be expected since the power term $w$ for the disturbing wave, as given by (30), is different in condition 1 than it is in conditions 2 and 3 and this term enters as the first power in the original theory [see (25)] but enters as the square root in both the coefficient and the argument of the hyperbolic tangent term of the alternate theory [see (28)].

(4) Significant cross modulation in the lower regions is predicted only by the alternate theory when the disturbing power is small (condition 1).

(5) For a given value of the wanted frequency, the point where the cross modulation $T$ changes from positive to negative depends only slightly on the form of the electron density profile when the original theory of cross modulation is used. For example, this "crossover" point is always between 51 and 53 km for all of the model $N$ profiles in conditions 1 and 3 where $f_w=17.5$ Mc/s, and 18Mc/s respectively, and it is always between 60 and 61 km in condition 2, where $f_w=4.865$ Mc/s (ordinary component).

## 6. Discussion

Figures 4, 5, and 6 indicate that the cross modulation predicted by the alternate theory [Huxley, 1953] does not differ greatly from the cross modulation predicted by the original theory [Bailey and Martyn, 1934] when the $D$ region electron density corresponds to the fairly moderate profile (for the arctic regions) given by $N_1$ of figure 2. This is especially true when a disturbing transmitter of high power is used (see the cross-modulation curves corresponding to $N_1$ in figs. 5 and 6). This is in marked contrast with the criticism of the alternate theory given by Huxley [1955] and Fejer [1955]. The criticism of Huxley [1955] was based on the cross-modulation measurements with obliquely traveling radio waves where the region of interaction was approximately 90 km. In these experiments $f_d \approx \nu_m$, whereas in the present discussion, concerned with the lower $D$ region, $f_d << \nu_m$. Also, the classical magneto-ionic theory was used in the earlier experiments; the generalized magneto-ionic theory [Sen and Wyller, 1960] was used in the present calculations.

The criticism of Fejer [1955] was based on cross-modulation observations which were confined to the upper $D$ region where the condition $f_d << \nu_m$, as required by the alternate theory, was not satisfied. Also, the classical magneto-ionic theory was used in the cross-modulation equation.

From the above comments it appears that the objections to the alternate theory are not sufficient to completely neglect this theory in the lower $D$ region.

The observations of considerable cross modulation in the lower $D$ region above College, Alaska are in better agreement with the alternate theory than with the original theory. For example, in the cosmic noise cross-modulation experiment at College, with the system parameters corresponding to condition 1, cross modulation was frequently observed in the 40 to 45 km region [Benson, 1962 and 1963]; in the echo-type cross-modulation experiment at College, with the system parameters corresponding to condition 2, short duration periods of cross modulation were observed as low as 30 km [Rumi, 1961 and 1962b; Flock and Benson, 1961]. The model electron density profiles $N_3$ and $N_4$ were constructed to approximately simulate the former case and the profile $N_5$ was constructed to approximately simulate the latter case. The corresponding cross-modulation profiles are shown in figures 4 and 5, respectively. In every case the alternate theory predicts more cross modulation than the original theory. [When $h_0=45$ km and $N_4$ is used in condition 1 (see fig. 4), then $T_{\mathrm{alt}}/T_{\mathrm{orig}}=10$; when $h_0=30$ km and $N_5$ is used in condition 2 (see fig. 5), then $T_{\mathrm{alt}}/T_{\mathrm{orig}}=5.5$.] This indicates that less ionization is required in the lower regions to produce a given amount of cross modulation when the alternate theory is used in place of the original theory.

The "crossover" point, mentioned in point 5 of the last section provides, at present, the most reliable information that can be obtained from an experiment in ionospheric cross modulation. This information concerns the electron collision frequency, and the next paper is completely devoted to this subject [Benson, 1964b]. The total cross modulation $T$, as given by the integral in (31), reverses sign because the term $(\partial k_w/\partial \nu_m)$ in the integrand reverses sign. This term, which is given by (7), changes sign at the point where $\alpha=\dfrac{2\pi f_w \pm \omega_L}{\nu_m}=2.18$. Since $f_w$ and $\omega_L=|s \cos\phi|$ are known, the electron collision frequency $\nu_m$ can be determined at the height where $(\partial k_w/\partial \nu_m)=0$. The location of this height can be estimated by observing the value of $h_0$ that causes the complete integral in (31), i.e., the observed cross modulation, to change sign. As mentioned in the last section, this "crossover" point is only slightly dependent on the form of the electron density profile when the original theory of cross modulation is used. Unfortunately, the alternate theory of cross modulation is valid only when $f_d << \nu_m$ and thus cannot be used in the vicinity of, or above, the crossover region. The crossover point should not be altered by more than a few km, however, even if a more general expression for $\Delta\theta/\theta$, which holds for the entire $D$ region, is used since this point is determined mainly by the term $(\partial k_w/\partial \nu_m)$ in (31). In support of this last statement are the observations of this cross-modulation "crossover" point that yield values for the electron collision frequency that are in agreement with other independent observations [Benson, 1964b].

In summary, it appears that the general shape of the cross-modulation curve can be predicted from the original theory of cross modulation [Bailey and Martyn, 1934], but that the absolute magnitude of the cross modulation is larger than is predicted by this theory, and that it is in better agreement with the alternate theory proposed by Huxley [1953] in the lower $D$ region. It is apparent that a different expression, that will hold for the entire $D$ region, is required for the term $\Delta\theta/\theta$ that enters in (31). This implies that a different expression is required for the term $\nu_m\delta Q$ in (22). A recent theoretical investigation by Altshuler [1963] shows that the electron cooling law is determined by inelastic electron collisions and that the term $\nu_m\delta Q$ is proportional to $\Delta Q/Q^{1/2}$ rather than to $\Delta Q$ as predicted by the original theory of cross modulation or to $\nu_m(\Delta Q)^2/Q^2$ as predicted by the alternate theory of cross modulation. The energy dependence of this term in Altshuler's analysis is very similar to the energy dependence of this term in the original theory; thus, a cross-modulation profile based on Altshuler's results should have approximately the same general shape as a profile based on the original theory of cross modulation. The absolute magnitude of the cross modulation, however, will depend on the constant terms (which were not evaluated in Altshuler's analysis) in the expression for $\nu_m\delta Q$. This problem will be considered further in future publications concerning the cross-modulation experiment at College, Alaska.

I am grateful to W. L. Flock for encouraging this work and for many helpful discussions, and to G. C. Rumi for his critical correspondence.

This work was supported by the Air Force Cambridge Research Laboratories Office of Aerospace Research under Contract No. AF 19(604)-8833 and, in part, by the Defense Atomic Support Agency, Washington, D.C. under Web No. 04.0063.

## 7. Appendix

Huxley's [1953, section 3] alternate theory of cross modulation is based on laboratory measurements [Crompton, Huxley, and Sutton, 1953] of the temperatures, energy losses, and collisional frequencies of electrons drifting through air due to a uniform and constant electric field. After converting from cgs to mks units and from $\bar{\nu}$ to $\nu_m$ ($\bar{\nu}=1.128\ \nu_m$), to be consistent with the present paper, his basic equations become

$$\nu_m = ak_T p(\text{mmHg}) \tag{1}$$

where $k_T=Q/Q_0$ and $a=8.30\times10^7$,

$$k_T = 1+\frac{bZ(v/m)}{p(\text{mmHg})} \tag{2}$$

where $Z$ is the steady applied electric field and $b=0.33$, and

$$W_m = (1.49\times10^{11})\ \frac{Z(v/m)}{\nu_m}\ \text{m/sec} \tag{3}$$

where $W_m$ is the most probable drift velocity.

The average power $w$ supplied to an electron by the constant electric field $Z$ is

$$w = Z(v/m)e(\text{coulomb})\,W_m(\text{m/sec})\ j/\text{sec}. \tag{4}$$

In equilibrium conditions, $w=\nu_m\delta Q$, from (22), and the above equations can be combined to give

$$\nu_m\delta Q = B\ \frac{(Q-Q_0)^2}{Q^2}\ \nu_m$$

where

$$B=\frac{(1.49\times10^{11})\,[e\,(\text{coulomb})]}{a^2b^2}=3.18\times10^{-23}j$$

as stated in (27).

Next, $w$ (the power supplied to each electron) must be expressed in terms of $P_d$ (the peak power radiated by the antenna of the disturbing transmitter). If the antenna has a gain $g$, then the disturbing power per unit area at the height $h'$ is given by

$$\frac{P_d gF(h')}{4\pi(h')^2}$$

where

$$F(h')=e^{-\int_0^{h'}2K_d dh}$$

takes into account the absorption of power up to the height $h'$. The power per unit area absorbed in an infinitesimal layer of thickness $dh'$ is given by

$$(\Delta P_d)_{h'}=\frac{P_d g}{4\pi(h')^2}\ \frac{\partial F(h')}{\partial h'}\ dh'$$
$$=\frac{P_d g}{4\pi(h')^2}\left\{-2(K_d)_{h'}e^{-\int_0^{h'}2K_d dh}\right\}dh'.$$

Thus, the power supplied to each electron $\dfrac{(\Delta P_d)_{h'}}{Ndh'}$ is

$$w=-\frac{P_d gK_d}{2\pi N(h')^2}\ e^{-\int_0^{h'}2K_d dh}.$$

In (30) the absolute value of $w$ is given since the quantity $(w)^{1/2}$ enters in the equations for $\Delta Q$ in the alternate theory of cross modulation.

## 8. References

Altshuler, S. (1963), Effects of inelastic collisions upon electrical conductivity and electron heating in the lower ionosphere, J. Geophys. Res. **68,** 4707–4716.

Bailey, V. A., and D. F. Martyn (1934), The influence of electric waves on the ionosphere, Phil. Mag. **18,** 369–386.

Bailey, V. A. (1937), On some effects caused in the ionosphere by electric waves—Part I, Phil. Mag. **23,** 929–960.

Barrington, R. E., and E. Thrane (1962), The determination of *D*-region electron densities from observations of cross modulations, J. Atmospheric Terrest. Phys. **24,** 31–42.

Benson, R. F. (1962), Cross modulation of cosmic noise, J. Geophys. Res. **67,** 2569–2572.

Benson, R. F. (1963), The cross modulation of cosmic noise: a technique for investigating the disturbed *D*-region, Univ. of Alaska, Geophys. Inst. Rept. 137, May 1963; or Ph. D. dissertation, Univ. of Alaska.

Benson, R. F. (Feb. 1964a), The quasi-longitudinal approximation in the generalized theory of radio wave absorption, Radio Sci. J. Res. NBS/URSI **68D,** No. 2, 219–223.

Benson, R. F. (Oct. 1964b), The electron collision frequency in the ionospheric *D* region, Radio Sci. J. Res. NBS/URSI **68D,** No. 10, 1123–1126.

Chanin, L. M., A. V. Phelps, and M. A. Biondi (1959), Measurements of the attachment of slow electrons in oxygen, Phys. Rev. Letters **2,** 344–346.

Crain, C. M. (1961), Ionization loss rates below 90 km, J. Geophys. Res. **66,** 1117–1126.

Crompton, R. W., L. G. Huxley, and D. J. Sutton (1953), Experimental studies of the motions of slow electrons in air with application to the ionosphere, Proc. Roy. Soc. **218A,** 507–519.

Dingle, R. B., D. Arndt, and S. K. Roy (1957), The integrals

$$\mathfrak{C}_p(x)=(p!)^{-1}\int_0^\infty \epsilon^P(\epsilon^2+x^2)^{-1}e^{-\epsilon}d\epsilon$$

and

$$\mathfrak{D}_P(x)=(p!)^{-1}\int_0^\infty \epsilon^P(\epsilon^2+x^2)^{-2}e^{-\epsilon}d\epsilon$$

and their tabulation, Appl. Sci. Res. **6B,** 155–164 (The Hague, Netherlands).

Fejer, J. A. (1955), The interaction of pulsed radio waves in the ionosphere, J. Atmospheric Terrest. Phys. **7,** 322–332.

Flock, W. L., and R. F. Benson (Aug. 1961), Experiment Luxembourg final rept., Contract AF 19(604)–3880; Univ. of Alaska, Geophys. Inst. Rept. 121.

Holt, O., B. Landmark, and F. Lied (April 1961), A study of polar radio blackouts, experimental studies, NDRE Rept. 35, Parts I to III; Norwegian Defence Research Establishment. Kjeller-Lillestrøm, Norway.

Huxley, L. G. H., and J. A. Ratcliffe (1949), A survey of ionospheric cross-modulation, Proc. Inst. Elec. Engrs. **96,** Pt. 3, 433–440.

Huxley, L. G. H. (1953), Alternative developments of the theory of radio wave interaction, Proc. Roy. Soc. **218A,** 520–536.

Huxley, L. G. H. (1955), Comment on the theory of radio-wave interaction, Proc. Roy. Soc. **229A,** 405–407.

Lagow, H. E., R. Horowitz, and J. Ainsworth (1960), Results of IGY atmospheric density measurements above Fort Churchill, Space Research, Proceedings of the First International Space Science Symposium, Nice, Jan. 11–16, 1960. Ed. by H. Kallmann, 164–174.

Landmark, B., and F. Lied (1961), Observations of the *D*-region from a study of ionospheric cross moculation, J. Atmospheric Terrest. Phys. **23,** 92–100.

Miller, L. E. (1957), "Molecular Weight" of air at high latitudes, J. Geophys. Res. **62,** 351–365.

Molmud, P., S. Altshuler, and J. H. Gardner (Sept. 1962), The study of a method to reduce electron density in the ionospheric *D*-region by means of high-powered ground-based transmitters, Final Rept., Contract No. AF 19(604)–8844; Space Technology Lab. Inc., One Space Park, Redondo Beach, Calif.

Pack, J. L., and A. V. Phelps (1961), Drift velocities of slow electrons in helium, neon, argon, hydrogen, and nitrogen, Phys. Rev. **121,** 798–806.

Phelps, A. V., and J. L. Pack (1959), Electron collision frequencies in nitrogen and in the lower ionosphere, Phys. Rev. Letters **3,** 340–342.

Rumi, G. C. (1961), Preliminary results of experiment Luxembourg, J. Atmospheric Terrest. Phys. **23,** 101–105.

Rumi, G. C. (1962a), Experiment Luxembourg: Cross modulation at high latitude, low height. Part I: Theoretical aspects, IRE Trans. Ant. Prop. **AP–10,** 594–600.

Rumi, G. C. (1962b), Experiment Luxembourg: Cross modulation at high latitude, low height. Part II: Experimental aspects, IRE Trans. Ant. Prop. **AP–10,** 601–607.

Sen, H. K., and A. A. Wyller (1960), On the generalization of the Appleton-Hartree magnetoionic formulas, J. Geophys. Res. **65,** 3931–3950.

Shaw, I. J. (1951), Some further investigations of ionospheric cross-modulation, Proc. Phys. Soc. **64B,** No. 1, 1–20.

Stroud, W. G., W. Nordberg, W. R. Bandeen, F. L. Bartman, and P. Titus (1960), Rocket-grenade measurements of temperatures and winds in the mesosphere over Churchill, Canada, J. Geophys. Res. **65,** 2307–2323.

Tellegen, B. D. H. (1933), Interaction between radio-waves?, Nature **131,** 840.

(Paper 68D10–410