JOURNAL OF RESEARCH of the National Bureau of Standards—C. Engineering and Instrumentation
Vol. 68C, No. 4, October–December 1964

Standards for the Calibration of Q-Meters 50 kHz to 45 MHz

R. N. Jones*

(May 27, 1964)

The National Bureau of Standards is now equipped to provide improved calibration services for Q-standards in the frequency range extending from 50 kHz to 45 MHz. As a result of recent development work, calibration uncertainties have been reduced to magnitudes which are comparable to the resolution of Q-meters in both resonating capacitance and Q. The uncertainties in the values of the NBS standards are consistent with the best two-terminal impedance measurements currently obtainable, but beyond this the values of the NBS standards have been statistically adjusted to provide a higher degree of standardization. Included in the paper is a discussion of the differences between the Q-meter indicated values for a standard and the effective values of the standard as given by NBS. These differences are largely due to residual immittances in the Q-meter circuit and methods for evaluating these residuals are presented.

1. Introduction

For grounded, two-terminal immittance measurements at radio frequencies, rf bridges are probably the most commonly used type of instrumentation. However, in the measurement of high Q devices, bridge techniques become impractical because of insufficient resolution for the resistive or conductive component. For such measurements, instrumentation based upon resonance principles is frequently used, with the most notable example being the Q-meter. As a result of this wide application of Q-meters, many laboratories have been required to show calibration traceability to NBS.

Most Q-meters covering the 50 kHz to 50 MHz frequency range are a combination of five more basic instruments including an oscillator, an insertion impedance, a thermoelement, an adjustable capacitor, and a vacuum tube voltmeter. A complete calibration of the Q-meter would require that each of these instruments be calibrated individually. This is impractical because they are not readily accessible, and also because of the large requirements in time and equipment.

The most satisfactory alternative is to employ transfer standards which may be used as spot checks for various points over a selected range of impedance and frequency. It is this method which is being employed, and the following discussion will be devoted to the NBS standards: What they are, how their values were derived, how to use them to best advantage, and the accuracies involved.

2. Q Defined

The mathematical definition for the Q of a circuit is 2Π times the ratio of the maximum instantaneous energy stored to the energy dissipated over a unit of time corresponding to one cycle. In equation form it may be written as:

$$Q=2\Pi\,\frac{\text{energy stored}}{\text{energy dissipated per cycle}}.$$

At the instant when maximum energy is stored in the inductance of a resonant circuit, the amount of energy stored may be expressed as $\frac{1}{2}L(I_{\max})^2$, where I denotes current and L is the inductance. The amount of energy dissipated per cycle in the circuit resistance is the product of the resistance and the square of the rms current times the period, t, of one cycle, or

$$R\left(\frac{I_{\max}}{\sqrt{2}}\right)^2\Big/f \text{ since } t=1/f.$$

Using these values the expression for Q becomes:

$$Q=2\Pi\,\frac{\frac{1}{2}L(I_{\max})^2}{R\left(\frac{I_{\max}}{\sqrt{2}}\right)^2\Big/f}=\frac{\omega L}{R},$$

and thus we have Q in terms of impedance.

3. Problems Associated With the Standardization of Q

Standards of measurement are ordinarily very fundamental, so that their values are directly related to basic physical dimensions or some invariant natural constant. This practice has been followed in the development of rf impedance standards [1, 2],[1] and prototype standards have been constructed which have uncertainties of the order of 0.1 percent relative to the true values. Actually Q-standards are only a special type of impedance

*Radio Standards Engineering Laboratories, NBS Boulder Laboratories, Boulder, Colo.

[1] Figures in brackets indicate the literature references on page 247.

standard because their values are derived from impedance measurements and, therefore, the same accuracy limitations apply. The complications arise in attempting to qualify the percentage uncertainty of Q. Any practical alternating current circuit contains both reactance and resistance, and at the present state of the art of impedance measurement there is no way to evaluate either component so that its value is free of errors contributed by the other. This is true, regardless of the method of measurement employed, be it bridge, slotted line, or a resonance technique. All of these methods merely compare impedances rather than measuring resistance and reactance individually, regardless of the readout of the instrument. This places serious limitations on the absolute accuracy to which Q can be determined. As large values of Q are encountered, the magnitude of the reactive component approaches the magnitude of the complex impedance, and it becomes increasingly difficult to achieve accurate resolution of the resistance. Q, then, is limited in accuracy by the accuracy of the impedance measurement from which it is derived. Figure 1 is a plot of the percentage uncertainty of Q as a function of Q for impedance uncertainties of 0.1 and 1.0 percent. The uncertainties shown are pessimistic because they are based upon the assumption that all of the error in the impedance is attributable to the resistance and none to the reactive component. A second modification is necessary for the upper percentage limit, because by definition Q cannot be negative in a passive network. Therefore the maximum limits of uncertainty in any value for Q would be plus infinity and minus 100 percent. In individual circumstances these limits might be reduced by

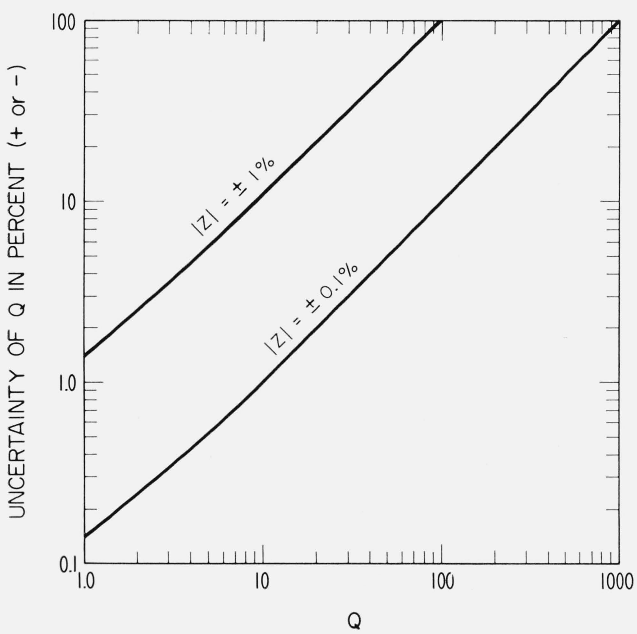


FIGURE 1. *Uncertainty of* Q *as a function of* Q *for an impedance uncertainty of 0.1 and 1.0 percent.*

the weight of experimental evidence, but it would be difficult to impose other limits categorically.

A second area of difficulty associated with the establishment of Q-standards, and one common to all types of immittance measurement, is associated with connectors. Precision coaxial connectors with well-defined reference planes are a distinct advantage in immittance measurement because they establish measurement conditions which are conducive to higher accuracy and better repeatability. Such conditions are important at all frequencies, but become increasingly so at the higher frequencies where wavelengths become short, or where measured immittances approach connector error immittances. Ideally, standards for immittance measurement of any type should utilize such laboratory precision connectors to permit the most direct evaluation in terms of national standards.

4. NBS Q-Standards

The complications arising from connector uncertainties, and the inability to make absolute determinations of Q to high percentage accuracies, have made it necessary to compromise ideal conditions in order to provide a practical system of reference standards and calibration services.

Experience gained from a large number of calibration requests over a period of several years has provided a clear indication of the type of Q-standards required. Most Q-meters provide a readout in terms of resonating capacitance and Q, and the NBS standards provide an arbitrary reference for these quantities. The established group of reference standards provides for calibrations at fifteen frequencies in the interval from 50 kHz to 45 MHz. The standards are wire-wound, air-core inductors, hermetically sealed, and equipped with banana plug connectors at a spacing of 1 in. on centers. At each frequency there are three standards, all having nearly the same value of resonating capacitance and Q. This permits periodic intercomparison to insure that no instabilities exist, and also permits three essentially independent determinations to be made on any standard received for calibration.

At this point it is important to make a distinction between indicated and effective values of a Q-standard. The indicated values of resonating capacitance and Q are those values obtained from measuring the standard on a Q-meter and may vary from one Q-meter to another. The effective values, on the other hand, are intended to be the values of the Q-standard when it is dissociated from any other circuitry or surrounding object and are, therefore, independent of the Q-meter. The effective values of the NBS standards have been assigned after extensive evaluation procedures involving both bridge and resonance techniques.

In the calibration procedure, the Q-standard to be calibrated is compared with each of the three NBS standards by observing the difference between the indicated values of resonating capacitance and Q on a Q-meter. These differences are then applied to the effective values of the NBS standards to obtain the effective values of the unknown, and the averages obtained from the three determinations provide the values given in the Report of Calibration. Thus it is desirable that the unknown and the standards be as nearly alike in value as possible to avoid the errors which can accrue from assuming that the differences in indicated value are equal to the differences in effective value.

5. Effective Versus Indicated Values

It is not unusual for the NBS effective values of the unknown to differ appreciably from the indicated values obtained from the Q-meter. These differences become increasingly pronounced at the higher frequencies and are attributable to three major causes. In order of significance these include the residual immittances of the Q-meter circuitry, the inaccuracy of the NBS effective values, and the nonrepeatability of the connectors. For purposes of checking the calibration of a Q-meter, it is most convenient to attribute the entire discrepancy to the residual Q-meter immittances and include the contribution of the latter two causes in the accuracy statement.

The following numerical example is included to assist the user in rationalizing the indicated values obtained from his Q-meter with the effective values as given by NBS for a particular Q-standard. This method may be used to evaluate the Q-meter circuit residuals at any frequency within its range, thereby achieving a definite traceability to NBS standards. The values used in the example were taken from actual measurement results to provide a general quantitative idea of the magnitude of the residuals in the Q-meter circuit, and their relative importance in any particular measurement situation.

For a particular Q-standard, the indicated values given by a Q-meter were 191 for Q and 429 pF for resonating capacitance at 15 MHz. The corresponding NBS effective values for the same Q-standard were 360 for Q and 461 pF for resonating capacitance.

Consider the approximate equivalent circuit of figure 2, which we will use to represent the measuring circuit of the Q-meter. Here C is the resonating capacitor in the Q-meter, and L_r is its residual series inductance. L_c and R_c are the inductance and resistance of the coil to be measured, and R_i is the insertion resistance. G_c and G_v are the respective conductances of the resonating capacitor and the Q-voltmeter.

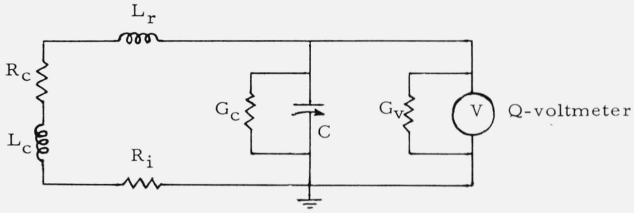


FIGURE 2

The residual inductance, L_r, may be computed from the following relationship:

$$L_r = \frac{C_e - C_i}{\omega^2 C_e C_i}, \tag{1}$$

where:

L_r=residual inductance in henries,
C_e=effective resonating capacitance in farads,
C_i=indicated resonating capacitance in farads,
ω=angular frequency in radians per second.

Here it is assumed that the Q-meter capacitor has been calibrated at low frequency and appropriate corrections applied. The expression (1) results from equating the reactance of C_e to the reactance of an equivalent circuit for the Q-meter capacitor composed of the inductance, L_r, in series with the capacitance, C_i, at the angular frequency ω. At 15 MHz this yields a value of 0.018 μH for the residual inductance, L_r. Having the value for L_r, the expression (1) may be rearranged to compute C_e, if C_i is known, or C_i, if C_e is known.

Having evaluated the reactive residual in the Q-meter measuring circuit, we can proceed to an analysis of the resistive residuals. Taking the Q-meter indicated values for resonating capacitance and Q, we arrive at a value of 0.129 ohm for the resistance of the entire circuit of figure 2. From the NBS effective values for the Q-standard, its resistance, R_c, is 0.064 ohm. Thus the residual resistance in the Q-meter circuit is the difference between these two values or 0.065 ohm. In this particular instance the d-c resistance of the insertion resistor, R_i, was given by the manufacturer as 0.020 ohm. Assuming this d-c value to be valid up to a frequency of 1 MHz and taking into account the rise in resistance of R_i due to skin effect, we achieve a value of $0.020\sqrt{15}=0.077$ ohm at 15 MHz.[2] This represents a disagreement of $\frac{0.077-0.065}{23.0}\times 100 = 0.05\%$ in $|Z|$.

This, of course, would be degraded somewhat if the conductances G_c and G_v had been taken into account, but the losses contributed by them are small and the agreement is well within expected tolerance limits for measurements of this type.

6. Statistical Adjustment of NBS Effective Values

Having assigned arbitrary values for effective resonating capacitance and effective Q based upon the best impedance measurements obtainable, further improvement in the absolute accuracy is extremely difficult to realize. For the purpose of establishing a set of reference standards, it is important that the values of the individual standards for each frequency be consistent with respect to one another.

[2] Above one megahertz, the resistance is assumed to be proportional to the square root of the frequency in megahertz.

A statistical analysis of all the data obtained from previous calibrations was undertaken to accomplish a threefold objective: (1) Determine whether or not the values assigned to the standards were consistent with one another, (2) to determine the corrections necessary to eliminate any inconsistencies found to exist, and (3) to utilize past experience to derive expected tolerance limits for future calibrations. The data used in the analysis were taken from calibrations performed over a period of approximately 5 years. The number of unknowns calibrated at each frequency varied from 74 to 106, thus providing an ample amount of data for statistical treatment.

As described in section 3, each unknown received for calibration is compared with each of the three NBS standards at a particular frequency. Because the statistical procedure is identical for analyzing and adjusting the values for both effective resonating capacitance and effective Q, the discussion will be confined to the Q-values for the sake of brevity. The data resulting from intercomparing the unknowns with the NBS standards at any one frequency may be arranged as follows:

$$\begin{matrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ Q_{n1} & Q_{n2} & Q_{n3} \end{matrix}$$

The first subscript denotes a particular unknown, and the second subscript denotes the NBS standard with which it was compared. Thus Q_{ij} is the effective Q of unknown number i resulting from comparing it with standard number j. From this arrangement, three arithmetic averages were computed:

$$\overline{Q}_{\cdot 1}=\frac{\sum_{i=1}^{n} Q_{i1}}{n},\ \overline{Q}_{\cdot 2}=\frac{\sum_{i=1}^{n} Q_{i2}}{n},\ \text{and}\ \overline{Q}_{\cdot 3}=\frac{\sum_{i=1}^{n} Q_{i3}}{n},$$

where n is the total number of unknowns calibrated at a particular frequency. Comparison of these averages $\overline{Q}_{\cdot 1}$, $\overline{Q}_{\cdot 2}$, and $\overline{Q}_{\cdot 3}$ provided an indication of the inconsistency of the values originally assigned to the standards. Following this, the arithmetic mean of the values $\overline{Q}_{\cdot 1}$, $\overline{Q}_{\cdot 2}$, and $\overline{Q}_{\cdot 3}$, which we will call $\overline{A}$, was used as a criterion for adjusting the original values to bring about better agreement. Standard number 1 was adjusted by an amount $\overline{D}_1=\overline{A}-\overline{Q}_{\cdot 1}$. Standard number 2 was adjusted by $\overline{D}_2=\overline{A}-\overline{Q}_{\cdot 2}$, and standard number 3 by $\overline{D}_3=\overline{A}-\overline{Q}_{\cdot 3}$. By this procedure the first two objectives were accomplished.

To derive expected tolerance limits for future calibrations, using the new values for the NBS standards, the previous data were compensated to take into account the adjustments made on the values for the standards. The variance,

$$s_i^2=\frac{1}{3-1}\sum_{j=1}^{3}(Q'_{ij}-\overline{Q}'_{i\cdot})^2,$$

was computed for each unknown where $Q'_{ij}=Q_{ij}+\overline{D}_j$ is the compensated value obtained from comparing the ith unknown with the adjusted jth standard and $\overline{Q}'_{i\cdot}=\overline{Q}_{i\cdot}$ is the average of the three values of the unknown resulting from comparing it with each of the three adjusted standards. A best estimate of the common true variance, s^2, was then derived from the individual variances by means of a weighted average. This was done by the relationship:

$$s^2=\frac{\sum_{i=1}^{n}(3-1)s_i^2}{(n-1)(3-1)}=\frac{\sum_{i=1}^{n}s_i^2}{n-1}.$$

The fact that the denominator is $(n-1)(3-1)$ follows from the application of a general method, two-way analysis of variance. The square root of this estimate of the common true variance was then used as the standard deviation upon which the tolerance limits for future calibrations should be based. However, such tolerance limits are valid only if the data from which they were derived conform to a normal or gaussian distribution. In an attempt to verify normality of the data, histograms were plotted and the chi-square coefficient computed and compared with tabulated theoretical values. At this point some difficulty was encountered because the measurements displayed a very high degree of repeatability with respect to the resolution of the Q-meter. This resulted in many measurements deviating from the mean by the same amount and tended to distort the shape of the histogram by concentrating too many values near the center. Consequently, the chi-square coefficient failed to establish normality in a number of instances for both resonating capacitance and Q.

Under the criteria of normality, the significance of a factor, K, times the standard deviation is that the probability is γ that at least a proportion, P, of the distribution will be included between $\overline{x}\pm Ks$, where $\overline{x}$ and s are estimates of the mean and the standard deviation computed from a sample size of n [3]. In these instances, the sample size was taken as 150 because there were approximately 75 unknowns compared against each of three standards, giving two degrees of freedom per unknown. Choosing a value of 0.99 for both the probability, γ, and the proportion, P, gives a value for K of approximately 3 for a sample size of 150. Examination of the data indicated that the number of measurements falling outside the $3s$ limits conformed very well to that expected of a normal distribution, and it was therefore deemed appropriate that these limits be imposed as the tolerance for use in connection with the calibration of Q-standards. Table 1 shows the frequen-

Table 1. *Approximate effective values of NBS Q-standards and estimated calibration tolerances based upon previous calibration data.*

Frequency	Effective resonating capacitance	Effective Q
50 kHz	[a] 400 pF (±0.6)	[a] 100 (±2)
100 kHz	90 pF (±0.1)	150 (±2)
150 kHz	34 pF (±0.1)	150 (±2)
150 kHz	430 pF (±0.6)	180 (±2)
300 kHz	100 pF (±0.3)	200 (±2)
450 kHz	40 pF (±0.1)	160 (±2)
0.5 MHz	400 pF (±0.6)	190 (±2)
1.0 MHz	90 pF (±0.2)	240 (±2)
1.5 MHz	37 pF (±0.2)	210 (±2)
1.5 MHz	430 pF (±0.7)	180 (±2)
3.0 MHz	100 pF (±0.2)	240 (±2)
4.5 MHz	40 pF (±0.1)	250 (±2)
5 MHz	390 pF (±0.7)	220 (±2)
10 MHz	90 pF (±0.1)	270 (±3)
15 MHz	36 pF (±0.1)	240 (±2)
15 MHz	440 pF (±1.0)	350 (±8)
30 MHz	100 pF (±0.3)	500 (±9)
45 MHz	40 pF (±0.1)	620 (±7)

[a] Numbers in parentheses indicate $3s$ calibration tolerances.

cies and approximate effective values of the NBS standards together with the tolerance limits for calibrations.

7. Conclusion

The system of standards described has been developed to meet a specialized need for standardization and traceability for high Q impedance measurements. So far as possible their accuracies are consistent with the best impedance measurements obtainable. Beyond this, the values have been statistically adjusted to provide a still higher degree of precision so that it is practical to provide calibration data which utilize nearly all of the significant figures that the Q-meter can resolve.

The tolerance limits established by the statistical analysis serve a twofold purpose. The first is to provide reliability limits within which the data may be expected to repeat. Secondly, but of equal importance, such limits enable calibration personnel to evaluate measurement results. This further reduces the probability of large excursions due to human error or erratic performance of the instrumentation.

The major possibility for improvement in the standards is in the absolute accuracy. Should means become available to improve this accuracy through better impedance measurement, the assigned values of the standards may be altered, but the same tolerance limits for calibration repeatability would still be applicable.

The data used in the development of these standards were accumulated over a period of several years and involved the cooperation of many NBS personnel. The author would like to thank all those who participated. Particular mention is due three individuals who were especially helpful. These include R. C. Powell for his helpful technical suggestions, Carl Love for his assistance with computer programming and data analysis, and E. L. Crow for his advice in statistical procedures.

8. References

[1] R. C. Powell, R. M. Jickling, and A. E. Hess, High frequency impedance measurements at the National Bureau of Standards, IRE Transactions on Instrumentation, Vol. I–7, numbers 3 and 4, December, 1958.

[2] R. N. Jones and R. E. Nelson, The role of capacitance in the National Reference Standards of high frequency impedance, Proceedings of the 17th Annual I.S.A. Conference and Exhibit, 1962, Paper No. 18.1.62.

[3] M. G. Natrella, Experimental statistics, NBS Handbook 91, pp. T–10 to T–12.

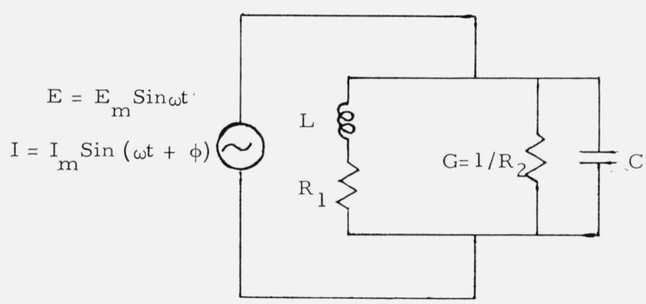
9. Appendix

In dealing with resonant circuits it is often convenient to utilize the expression:

$$\frac{1}{Q_{ckt}}=\frac{1}{Q_L}+\frac{1}{Q_C},$$

which expresses the relationship between the Q of the entire circuit and the Q of its individual reactive components. The derivation of this relationship is not commonly found in the literature and it is included here because of its usefulness in measurement applications involving the Q-meter.

Consider the following circuit:



By definition

$$Q=\frac{\omega_0\text{ (energy stored in circuit)}}{\text{average power lost}}.$$

At the instant when all of the energy is stored in the magnetic field of the inductor, the energy stored is:

$$U=1/2LI_m^2.$$

At the instant when all of the energy is in the electrostatic field of the capacitor, the energy stored is:

$$U=1/2CE_m^2.$$

The average power lost per cycle in the resistance, R_1, and the conductance, G, are $1/2R_1I_m^2$, and $1/2GE_m^2$, respectively. From the above definition, the circuit Q may be expressed as:

$$Q_{ckt}=\frac{\omega_0 \frac{LI_m^2}{2}}{1/2R_1I_m^2+1/2GE_m^2},$$

or

$$Q_{ckt}=\frac{\omega_0 \frac{CE_m^2}{2}}{1/2R_1I_m^2+1/2GE_m^2};$$

$$\frac{1}{Q_{ckt}}=\frac{R_1I_m^2+GE_m^2}{\omega_0 LI_m^2}.$$

But $E_m=\frac{I_m}{G}$, and using this substitution the expression becomes:

$$\frac{1}{Q_{ckt}}=\frac{R_1}{\omega_0 L}+\frac{1}{\omega_0 LG}.$$

Since $\omega_0 L=\frac{1}{\omega_0 C}$, we have $\frac{1}{Q_{ckt}}=\frac{1}{Q_L}+\frac{1}{Q_C}$ where $Q_c=\frac{\omega_0 C}{G}$.

Q.E.D.

(Paper 68C4–170)