

The 1962 He³ Scale of Temperatures. I. New Vapor Pressure Comparisons¹

S. G. Sydorik and R. H. Sherman

Los Alamos Scientific Laboratory, University of California
Los Alamos, N. Mex.

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A comparison of He³ and He⁴ vapor pressures, (P_3 , P_4), has been made in an apparatus designed to reduce the number and magnitude of corrections associated with the refluxing film in the He⁴ pressure sensing tube and the attached bulb. The critical pressure of He³ has been redetermined to be at 873.0 ± 1.5 mm Hg at 0° C and standard gravity; the corresponding temperature as measured by a He⁴ thermometer is 3.3240 ± 0.0018° K on the 1958 He⁴ scale. These (P_3 , P_4) comparisons and the 1958 He⁴ scale are the basis of the 1962 He³ scale of temperatures derived and evaluated in the papers following this one.

Empirical interpolation equations containing only P_3 and P_4 are described by means of which existing P_4 measurements may be converted to an equivalent P_3 . A comparison has been made between this interpolation and a conversion in which the 1958 He⁴ and 1962 He³ scales are used as parameters. Deviations between the two procedures are within the estimated errors of the (P_3 , P_4) measurements.

In subsidiary experiments on techniques for He⁴ thermometry a typical vapor pressure bulb arrangement was tested. It is shown that the refluxing film introduces a heat flux, \dot{Q}_f , and a resulting Kapitza temperature drop, ΔT_k , between the He II and its container, which may amount to many millidegrees.

The feasibility of calculating ΔT_k for a particular He⁴ vapor pressure bulb was studied. The necessity of measuring $\Delta T_k/\dot{Q}_f$ at least once *in situ* is pointed out. In addition, it is necessary to redetermine the film flow rate periodically at the same time that He⁴ vapor pressure measurements are being made.

1. Introduction

At the VIIth International Conference of Low Temperature Physics a proposal was made [1]² to calculate a new He³ vapor pressure-temperature scale based on the 1958 He⁴ Temperature scale [2], the newly available He³ specific heat data [3, 4], and the He³-He⁴ vapor pressure comparisons of Abraham, Osborne, and Weinstock (AOW) [5].

This is the first of four papers describing: I. New (P_3 , P_4) data; II. The derivation of a new He³ temperature scale [6]; and III. Its evaluation [7]. Paper IV [8] provides detailed tables of the scale.

The 1962 He³ scale is intended to replace the T_E and T_L He³ scales of reference [9].

A Thermodynamic Inconsistency. In the course of the attempt to determine a new He³ vapor pressure-temperature scale, it became apparent [10] that there was a thermodynamic inconsistency equivalent to several millidegrees in the temperature interval between 2 and 1 °K. It seemed reasonable to consider that either the 1958 He⁴ scale or the AOW comparison data was in error, and that the error

increased as one approached 1 °K, because the gas thermometer data of Keller [11, 12] on which the 1958 He⁴ scale is partially based are in reasonable agreement with the 1958 He⁴ scale near 2 °K and are nonexistent below 1.5 °K; and because unexplained experimental difficulties were encountered by AOW below about 1.7 °K. If the 1958 He⁴ scale and the AOW data are assumed to be correct at 2 °K, then the thermodynamic inconsistency was such that P_4 appeared to be too high (or P_3 too low) at 1 °K. A detailed discussion of the inconsistency and of possible errors in the 1958 He⁴ scale will be given in III [7]. In this paper we shall examine possibilities for error in vapor pressure measurements.

Techniques of He⁴ Vapor Pressure Measurement. In the AOW measurements He³ and He⁴ were condensed in identical adjacent bulbs drilled in a single block of copper and connected to unjacketed pressure sensing tubes, all being immersed in a liquid He⁴ bath. The use of a He⁴ bulb pressure rather than bath pressure has been thoroughly substantiated in subsequent years, for measurements above T_λ . Extensive studies of hydrostatic head correction to a He I bath pressure measurement of the temperature of an immersed body [13, 14, 15] have shown these

¹ Work performed under the auspices of the U.S. Atomic Energy Commission.

² Figures in brackets indicate the literature references at the end of this paper.

corrections to be unreliable. The temperature gradient in the bath liquid is generally less than that calculated from the measured surface temperature and liquid depth. In the thesis of Durieux [15] it is recommended that the need for hydrostatic head corrections be avoided by use of a copper vapor pressure bulb in good thermal contact with the object whose temperature is desired to be known.

Unfortunately, below 1.7 °K, where ΔOW expected the cryostat and He⁴ bulb pressures to agree because of the high conductivity of HeII, they did not do so. This was attributed by AOW "chiefly to the impracticably long time required for pressure equilibrium in the He⁴ vapor pressure bulb system. The cryostat pressure was therefore used to determine the temperature of the He³" for $T < T_\lambda$. For their last three data points, all below 1.3 °K, they used a He⁴ pressure sensing tube ten times larger in diameter and found agreement with cryostat pressures to within 1 millidegree. The agreement may, however, have been fortuitous.

Concern about the qualitative design details of pressure-sensing tubes has been the subject of a great deal of discussion in recent years. A summary has been given by Brickwedde [16]. Some experimenters have favored use of vacuum-jacketed metal tubes, some unjacketed glass tubes, and some jacketed tubes with helium vapor in the jacket. It has been contended by some that, until the problem is better understood, it might be desirable to provide a different temperature scale with the particular arrangement one prefers of the above design details.

In the present paper we propose to show that due to two phenomena associated with film reflux a He⁴ vapor pressure bulb is not well suited, below T_λ , for accurately indicating the temperature of the wall of the bulb or of the liquid in which it is immersed, regardless of the type of pressure-sensing tube used. One of these effects is the pressure drop, ΔP_f , between that part of the pressure-sensing tube where the film vaporizes and the parent liquid where it recondenses. The other effect is the temperature drop, ΔT_k , due to heat flow between the HeII liquid and the refrigerated wall of the bulb.

A bath pressure measurement provides a more reliable thermometer for the contents of a HeII bath because there is no ΔT_k to consider. However, depending on design details, one may need to make significant ΔP_f or other types of corrections to a measurement of bath pressure below P_λ . To make these corrections, it may be necessary to carry out complex subsidiary experiments.

Our principal objective is, of course, to obtain a reliable correlation between He³ and He⁴ vapor pressure over a broad temperature range to the critical point.

2. Calculation of Film Reflux and Recondensation Corrections

Film Reflux Correction. The existence of a "driving" pressure, ΔP_f , between the level where the mobile He⁴ film vaporizes, and the (lower) pressure

in the bulb, where the film vapor recondenses, is a well-known correction [17] to HeII thermometry based on Poiseuille's Law [17, 18]. If vaporization of the film takes place a distance, L_f , above the bottom of a pressure-sensing tube of radius a , then, for viscous flow of the refluxing vapor, ΔP_f (microns) = $6\eta L_f \dot{V}_f / \pi a^4$, where η is the viscosity, and \dot{V}_f the volume flow rate of the refluxing vapor. For η we can use an empirical equation, $\eta = (1.3T + 2.1) \times 10^{-6}$, which we fitted to the theoretical calculations of deBoer and Cohen [19] in the temperature range 0.8 to 2.2 °K. (Their theory is in excellent agreement with experiments [20, 21] done between 1.3 and 4.2 °K.)

For \dot{V}_f we can substitute $2\pi a \dot{n}_f v_g$ where v_g is the molar volume of the vapor, for which the ideal gas value is adequate, and \dot{n}_f is the molar flow rate of film per centimeter circumference [22]. With these substitutions we get

$$P\Delta P_f = \frac{(970T + 1570)TL_f \dot{n}_f}{a^3} \quad (1)$$

in units of (microns)² if L_f and a are in centimeters. As an example, for clean glass tubing at 1 °K, $\dot{n}_f \approx 3.5 \times 10^{-6}$ moles $\text{cm}^{-1} \text{sec}^{-1}$ and eq (1) gives $P\Delta P = 0.0089L_f a^3$. Allowing for the higher values of η we used, this is in excellent agreement with the equation of Bleaney and Hull [18].

Figure 1 shows the temperature correction equivalent to ΔP_f calculated for "clean" tubing of various typical sizes assuming $L_f = 10$ cm. The correction is seen to be 1 mdeg at a temperature of the order

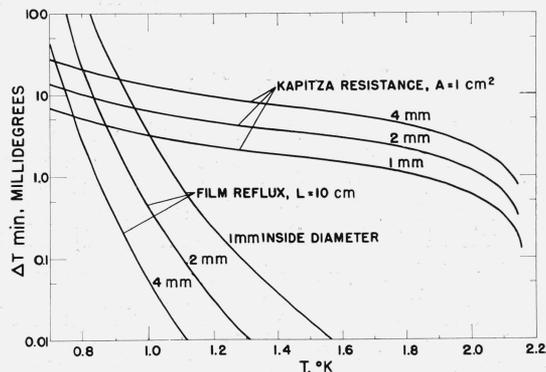


FIGURE 1. Examples of minimum temperature corrections for a He⁴ vapor pressure thermometer bulb based on the minimum superfluid film flux rate for pressure-sensing tubes of the indicated inside diameters.

For the curves marked film reflux, ΔT_{min} is the temperature inferred from the pressure minus the liquid temperature. In calculating the film reflux pressure drop, the mobile film is assumed to evaporate a distance L from the cold end of the pressure-sensing tube and the vapor to return to the bulb by laminar flow. For the curves marked Kapitza resistance, ΔT_{min} is the temperature difference between the thermometric fluid and the bulb wall assuming the area of thermal contact to be A . If the bulb is immersed in liquid the temperature of which is being measured, a second ΔT_k correction will be needed to account for the Kapitza resistance between the bulb wall and the external liquid.

Minute deposits of air on the tubing can increase the film flow and thus cause the required correction to be greater than ΔT_{min} by as much as a factor of twenty and extraordinary precautions are needed to come within a factor of two of the plotted corrections. Because it is so nonreproducible, the film flow rate should be concurrently determined, for example by a measurement of the recondensation heat leak.

of 1 °K if the tubing is scrupulously clean. If impurities on the walls cause the film flow to be an order of magnitude higher a 1 mdeg correction will be needed at about 1.2 °K.

Unfortunately (1) is difficult to apply to an actual measurement of vapor pressure for many reasons. The transfer rate depends markedly on the condition of the surface which is generally not as smooth as glass. What is worse, insofar as a calculation of ΔP_f is concerned, is the fact that \dot{n}_f has been found [23] to depend very markedly on traces of impurities, such as solid air, on the wall over which the film passes. If $\dot{n}_{f,\min}$ is the value of \dot{n}_f for perfectly clean tubing and if precautions are taken to clean the tubing wall or to prevent entry of condensable gases into the assembled system, \dot{n}_f may be expected to exceed $\dot{n}_{f,\min}$, and a factor of 2 or more is not unreasonable [24]. A layer of solid air so thin as to be invisible can cause \dot{n}_f to exceed the value measured for clean tubing [22] by as much as a factor of 10 or more [23]. It has been our experience that even though no leaks could be detected in our plumbing by a sensitive mass spectrometer leak detector, still \dot{n}_f gradually increased, over a period of several days, by almost an order of magnitude. We presume the increase to have been due to an undetectable air leak or to migration of adsorbed gases from warm parts of the system to cold parts. In the light of the above observations, it is clearly imperative, for a proper evaluation of ΔP_f , that an experimental determination of \dot{n}_f be made *in situ*. This we have done, in effect, by measuring the heat of recondensation of the refluxing film, \dot{Q}_f , since $\dot{n}_f = \dot{Q}_f / (2\pi a \lambda_f)$, where λ_f is the heat of transport of the film per mole. Since the mobile part of the film does not carry any entropy, $\lambda_f = TS_G$; and since the heat of vaporization of bulk liquid is $\lambda_{II} = T(S_G - S_l)$, therefore

$$\lambda_f = \lambda_{II} + TS_l \quad (2)$$

For λ_{II} and the entropy of the bulk liquid, S_l , we refer the reader to table VI of ref. [25].

Another difficulty in applying eq (1) is that the appropriate value of L_f , the height of rise of the mobile film, is very much dependent on the detailed design of the pressure sensing tube, especially on whether or not it is vacuum jacketed. We base the calculation on the assumption that the film will rise to a level at which sufficient heat flux, \dot{Q}_f , is available to vaporize the film. For a tube in a vacuum, having total length L_s , and if w is the tube wall thickness then

$$L_s - L_f = \frac{2\pi aw}{\dot{Q}_f} \int_{T_f}^{T_s} k dT = \frac{w}{\dot{n}_f \lambda_f} \int_{T_f}^{T_s} k dT \quad (3)$$

where k is the thermal conductivity of the tubing. (It is easy to show that for the practical thermometry case where $\Delta P_f \ll P$, the temperature rise, ΔT_f , of the mobile (saturated) part of the film will also be small; since \dot{n}_f varies but slowly with T , it follows that the flow rate will be essentially constant up to the level where vaporization of the entire film is

energetically possible. Hence practically all of the vaporization will take place within a narrow band of the tubing, as has been assumed in writing down eq (3).)

Table 1 shows what one might expect L_f to be for a typical "clean" and "dirty" Inconel tube, with $w=0.01$ cm, $L_s=30$ cm, $T=1.5$ °K, and various values of T_s . We note that when the tubing is lagged (thermally shorted) at helium temperatures the film will rise almost to the level of the heat source, especially if the tubing wall is "dirty." If the tubing is lagged at liquid nitrogen temperature it is unlikely that the film will rise appreciably in a clean sensing tube, since there will be more than enough heat approaching the liquid to completely vaporize the film. If this happens, the ΔP_f correction will, of course, vanish, but the additional heat reaching the cell will give rise to temperature gradients throughout the experimental cell whose disposition will need to be considered carefully. More will be said later about the temperature excess of the walls of an immersed bulb transmitting heat to its bath liquid.

TABLE 1.—Height of rise of film, L_f , in a typical pressure-sensing tube held at temperature T_s a distance 30 cm above the bulk liquid.

The tube is made of inconel having 0.01 cm wall thickness and a mean thermal conductivity, k . For "clean" tubing the volume flow rate of film is 7.5×10^{-5} cm (cm perimeter) $^{-1}$ sec $^{-1}$ and the assumed rate for "dirty" tubing is 10 times greater. Negative L_f means more than enough heat reaches the bulb to vaporize the film before it can enter the pressure sensing tube, therefore $L_f=0$.

Tube environment	T_s	k	Clean wall L_f	Dirty wall L_f
	deg	W/cm deg	cm	cm
Vacuum.....	1.6	0.007	29.98	29.998
Vacuum.....	4.2	0.017	28.6	29.86
Vacuum.....	75	0.072	-130.0 → 0	14.0
Vacuum.....	300	0.120	-1050 → 0	-78 → 0
Effluent vapor.....	300	0.120	-6 → 0	26.4

If the sensing tube is not in a vacuum, as assumed above, but is exposed to effluent He⁴ vapor, the heat flux down the tube may be greatly diminished. Sydoriak and Sommers show in figure 2 of ref. [26] that for complete heat exchange between effluent vapor and a glass tube the heat flux reaching the bath by conduction down the tube will be reduced by a factor of at least 30 if $T_s=300$ °K. If conduction heat is not the major cause of vaporization the reduction in heat flux will be even greater. As a result $L_s - L_f$ may be so much lower than given by eq (3) that even for $T_s=300$ °K the film may "need" to rise to such high levels that ΔP_f may be a significant correction.

In view of the many and practically unpredictable factors discussed above which can influence ΔP_f and the heat conveyed by a pressure sensing tube it is not surprising that agreement has been lacking as to how a He⁴ vapor pressure measurement should be made.

Film Recondensation Correction. If a bulb temperature measurement is being made there will be an additional source of error in He⁴ thermometry. We refer to the temperature jump, ΔT_k , across the boundary between a liquid and its container when a heat flux, \dot{Q} , exists between the liquid and the

container wall. The temperature dependence of ΔT_k for the case of liquid He⁴ was first studied in some detail by Kapitza [27], whose data for heat transfer from polished copper to He II between 1.6 and 2.1° (his fig. 30) fits the relation $\Delta T_k = 5.9 \dot{Q}/AT^3$, where A is the immersed surface area of copper across which heat \dot{Q} in watts flows. More recently, Fairbank and Wilks [28] found $\Delta T_k = 45 \dot{Q}/AT^2$. Using a better defined area of contact, Challis, Dransfeld, and Wilks [29] have found a $T^{2.5}$ power law dependence. Their data yield a coefficient of 19.0 for polished copper and 9.0 for etched copper.

The most complete investigation has been done recently by Kuang Wey-Yen [30], whose measurements extended from 2 to 0.6 °K. He finds $\Delta T_k = 21 \dot{Q}/AT^{2.6}$, in excellent agreement with the polished copper data of ref. [29]. He also investigated the reversibility of the effect, and found ΔT_k to be different, but of the same order of magnitude, for heat flow from liquid to solid as for the usually studied case of flow from solid to liquid. Evidently the discrepancy was not considered significant, since neither the data nor the sense of the discrepancy are given. This is unfortunate because, we shall see, our results, which also pertain to heat flow from liquid to solid, are also in disagreement, by an order of magnitude, from published data above 1 °K for heat flow from solid to liquid. According to the phonon radiation theory of Khalatnikov [31], $\Delta T_k \propto T^{-3}$, in fair agreement with observed temperature dependences, but the predicted values of ΔT_k are 10 to 30 times greater than the highest observed values.

Some time ago it was pointed out that this so-called Kapitza resistance is a likely cause of serious error in He⁴ thermometry [32]. For a closed, vacuum-jacketed tube containing liquid He II at 1 °K to a depth of h cm it was indicated that

$$\Delta T_k \sim (10/h) \text{ mdeg}, \quad (4)$$

using ref. [28] data. Thus, for $h \sim 1$ cm, which is more or less typical, a temperature error of 1 percent was indicated at 1 °K. Actually, the above equation gives only a minimum value of ΔT_k since it is based on the same purity assumption often made in a ΔP_f estimate of error: i.e., that the surface over which the film flows is completely free of traces of impurities. With minute traces of solid air on the tubing wall the effective perimeter for film flow can increase by an order of magnitude. This would cause \dot{Q}_f and hence ΔT_k to also increase by an order of magnitude. On the other hand, if the tube is not vacuum jacketed, as in the AOW experiments, the appropriate value of h may be comparable to the immersion depth of the pressure sensing tube, since recondensation may take place throughout the immersed parts of the tubing wall. This would tend to bring the expected ΔT_k down, for a typical geometry and tubing cleanliness, from the order of several tens of millidegrees to the order of several millidegrees at 1 °K. In support of this viewpoint we refer the reader to the thesis of Durieux, ref. [15],

figure 44. For example, with a He⁴ bath temperature of 1.35 °K the temperature of a glass vapor pressure thermometer exceeded the bath temperature by 13 mdeg when vacuum jacketed, but only by 4 mdeg when the (immersed) jacket was filled with gas. He stated the "warming up was probably mainly caused by the heat input due to the film creep."

Figure 1 shows examples of minimum temperature corrections due to the Kapitza resistance, ΔT_k , for three tubing diameters. The wall of the thermometer bulb is assumed to have an area of 1 cm² and to be at temperature, T . For the selected geometries of figure 1 the ΔT_k correction is seen to be much greater than the ΔP_f correction, if $T > 1.2^\circ$.

To our knowledge, neither of these possible sources of error, ΔP_f nor ΔT_k , has been examined experimentally. In their report, AOW stated "there was an uncertain (although probably negligible) correction due to film flow." Because of this uncertainty and our observed thermodynamic inconsistency between the 1958 He⁴ scale and the AOW data, new (P_3, P_4) comparisons were undertaken in an apparatus with which ΔT_k and ΔP_f could be determined separately and compared with calculations. In view of the uncertainty of film flow rates, it was felt that a simultaneous determination should be made of the film flow rate or, alternatively, of the consequent increase in heat leak, \dot{Q}_f , due to recondensation of the vapor. The \dot{Q}_f measurement is feasible with the apparatus described below.

It should be clear by now that calculations of ΔP_f and ΔT_k corrections to a He II vapor pressure measurement of temperature may be subject to many serious uncertainties, and with many geometrical arrangements such corrections may be altogether impractical. But by giving sufficient attention to the details of the geometrical arrangements and by making subsidiary measurements it may be possible to make moderately reliable corrections down to, perhaps, 0.9 °K. This we have attempted to do, as described below.

3. Cryogenic Apparatus

To accomplish our objectives the apparatus was, in essence, a calorimeter; it is shown in figure 2, where we represent poorly conducting Inconel tubing with fine lines and thick copper walls with crosshatching. Thus the walls of cells A, B, and D are in good thermal contact with each other, but in poor thermal contact with cell C when no helium is present in D. Heat leak from above is reduced to a minimum by maintaining a temperature within a few tenths of a degree of the cell temperatures at tubing levels several centimeters above the cells.

When He II is present in D, there will be some recondensation of the refluxing film on the surface of the bulk liquid, which will therefore be at a temperature, T_4 , which is greater than that of the refrigerated wall of the cell, by an amount ΔT_k . Since the He³ cell, C, is well insulated, its temperature will be the same as the liquid in which it is immersed. For He³-He⁴ vapor pressure comparisons, which are the

principal objective of this work, we therefore use the He^3 pressure, P_3 , in cell C and the He^4 pressure, P_4 , in cell D.

Another objective is to show that the film reflux phenomenon is an important source of error in an improperly designed He^4 thermometer system. For this we need to determine the temperatures of liquid He^4 , T_4 , and the wall of its container, T_w , and to compare $T_4 - T_w$ with a calculation of ΔT_k based on a measurement of the heat leak, \dot{Q}_f , due to the refluxing film. We get T_w from one additional measurement, namely, a He^3 vapor pressure in cell B, whose wall is in good thermal contact with the wall of the He^4 cell, D. Then:

$$T_4 - T_w = (P_C - P_B)(dT/dP), \quad (5)$$

where the derivative is the slope of any reasonably good He^3 temperature-versus-vapor pressure relationship. For \dot{Q}_f we need to determine the total heat leak to ABCD. From the measured evaporation rate of refrigerant He^3 condensed in the annulus, A, we can calculate the total heat leak. On subtracting the background heat leak measured (in a few runs) in the absence of He^3 we obtain \dot{Q}_f .

Other details of the Dewar are shown schematically in figure 3 and have been described in some detail elsewhere [33]. We merely wish to point out that the experimental cells are completely surrounded by concentric copper radiation shielding bolted to pots of liquid He^4 and liquid N_2 . Except at the various flanges to which they are lagged, the pressure-sensing tubes are everywhere in vacuum, and hence have a gas retention which is small and independent of the levels of the various refrigerants. This greatly simplifies the filling of a metal vapor pressure bulb to a desired liquid level, once a determination has been made of the gas retention volume for a particular value of T . Also, possible inverse temperature gradients in the refrigerant liquids do not introduce the danger of condensation at cold spots, as might occur if the pressure sensing tubes were not vacuum jacketed. As shown, inside diameters of the pressure sensing tubes become progressively larger at higher levels in order that thermomolecular pressure gradients be roughly the same in each section. Couplings used at these joints are designed to block the flow of radiation from higher temperatures (see insert to fig. 3). Several carbon resistors (not shown) are used to monitor temperatures of the various flanges and radiation shields.

4. Pressure Measurement

He^3 Pressures. He^3 vapor pressures were observed on oil and mercury manometers having inside diameters of 20 mm and 14.5 mm, respectively. A Wild cathetometer having a 70 cm focal length and read to 0.01 mm Hg was used to read P_3 and mercury meniscus heights. For each data point two to three readings of P_3 were taken at 10-min

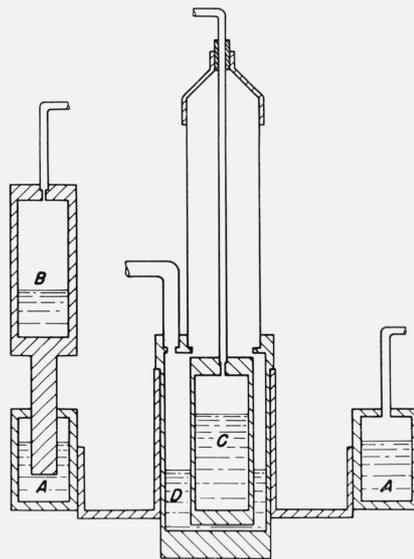


FIGURE 2. Vapor pressure cells.

Fine lines represent poorly conducting tubing, while crosshatching represents copper. He^3 refrigerant is contained in annulus A. Liquid He^3 thermometer bulb, C, measures P_3 , and is at the same temperature as the liquid He^4 in cell D, even when recondensing film warms the liquid He^4 . The wall temperature of cell D is measured by He^3 bulb, B.

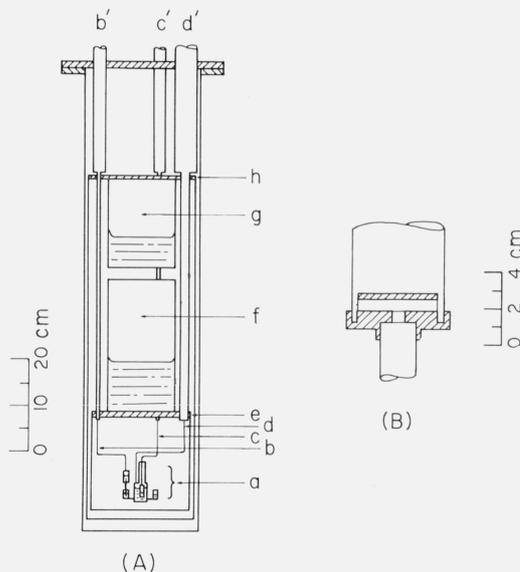


FIGURE 3A. Dewar.

The cells shown in figure 2 are shown in brackets at *a*. Stepped pressure-sensing tubes for cells B, C, and D are labeled *bb'*, *cc'*, and *dd'* respectively. The steps are in thermal contact with liquid He^4 refrigerant pot, *f*, at flange, *e*, and with liquid N_2 pot *g*, at flange, *h*. Copper radiation shields are bolted to *f* at flange, *e*, and to *g* at flange, *h*. To minimize the thermomolecular pressure gradient without unduly increasing heat leaks, tubes *bb'* and *cc'* increase in inside diameter from 0.133 cm to 0.583 cm at *e*, and to 2.49 cm at *h*. Tube *dd'* increases from 0.266 to 1.85 at *e*, and to 4.89 cm at *h*.

FIGURE 3B. A sectional view shows the details of reducing couplings which also serve as radiation shields in each of the vapor pressure-sensing tubes.

The couplings are soft soldered to flange, *h*.

intervals. The temperature was held constant to 0.1 mdeg for at least 10 min prior to each reading.

Meniscus corrections for the He³ mercury manometer were taken from Cawood and Patterson [34], who list capillary depressions which are 1.1 to 1.5 times greater than those of Kistemaker [35]. The choice was made on the basis of a number of auxiliary experiments done at several controlled pressures: at each pressure meniscus heights were forcibly altered by tapping the manometer legs. Using Kistemaker's table the average spread in the corrected pressures was reduced to 0.07 mm. With the Cawood and Patterson table the corrected pressures had an average spread of 0.02 mm. The indication was that the Cawood and Patterson table is somewhat preferable for use with our particular manometer tubing and mercury. We do not intend to imply any generality to this conclusion. The surface tension of mercury in glass tubes is known to vary widely even if purified with care [35], and it is necessary, as pointed out by Kistemaker, to adjust published tables of meniscus corrections in accordance with a calibration *in situ* of each particular combination of type of glass and sample of mercury. Should neither ref. [34] or ref. [35] provide suitable tables of the correction, one can refer to the more extensive tables given in the NBS Monograph 8 [36].

The cathetometer scale was calibrated against an Invar meter bar. The net reading, meniscus correction and scale errors were estimated to amount to 27 μ in a measurement of P_3 .

He⁴ Pressures—Brief Description of a Special Manometer. For He⁴ vapor pressure readings the mercury manometer tubing diameter used was 22 mm and menisci in the two arms were found to be sufficiently flat and equal to render meniscus corrections negligible.

At 1 °K, P_4 is so small that a cathetometer would not provide adequate accuracy. A special high-sensitivity manometer system containing oil and mercury U tubes of 25 mm bore was therefore designed to read P_4 . This is capable of an accuracy of a few microns of mercury and requires less time and eyestrain than is required in reading a cathetometer to tens of microns or an ordinary mirror-scale manometer to a few hundred microns. Because of its general usefulness a brief description will be given. A separate traveling microscope comparator is used to view each arm of the manometer. The travel of each comparator is 10 cm and a vernier gives direct readings to 1 micron. For several readings at more or less the same pressure it is therefore not necessary to reposition the frame of either comparator. Hence successive readings at more or less the same pressure can be taken rapidly. An illuminator attached to each microscope barrel projects a collimated beam of light from behind the meniscus into the telescope. With this arrangement parallax effects are virtually eliminated.

The manometer temperature was monitored by standardized mercury thermometers marked every 0.1 °C. In order that they have approximately the

same response time as the oil manometer, the monitoring thermometers were immersed in oil-filled test tubes of the same diameter as the manometer tubing to which they were fastened. Room temperature was controlled to a fraction of a degree by forced-air circulation and a sensitive thermostat. With this system and a box of heavy walled aluminum surrounding the manometers the two monitoring thermometers differed by less than 0.1 °C. Calibrations against a standard Invar meter bar were made for each comparator. They were found to be correct in the measurement of several lengths on the meter bar ranging from 1 to 10 cm with standard deviations of ± 2.4 and $\pm 1.9 \mu$. For a pressure measurement the net standard deviation due to errors in the scales was therefore $\pm 3.1 \mu$. In a series of 23 static pressure measurements with three readings of each pressure, the average deviation was found to be $\pm 1.7 \mu$. The net reading and scale errors for mercury pressure, P_4 , were therefore $\sqrt{3.1^2 + 1.7^2} = \pm 3.5 \mu$ of mercury.

In an experimental comparison of vapor pressures there is an additional source of error associated with differing equilibration times for the He³ and He⁴ systems and slow variations in cell temperature. This was minimized by controlling the temperature to a tenth of a millidegree for several minutes prior to each set of readings of P_3 and P_4 , using a suitable carbon resistor or a Consolidated Engineering Corporation Micromanometer, which is a capacity-sensing differential pressure gage having a sensitivity of 0.20 μ at any pressure.

Conversion of Millimeters Oil to Standard Millimeters Mercury. *n*-Dibutyl phthalate was used in both oil manometers. The conversion factor, F , in microns of mercury at 0 °C per millimeter of oil at t °C, was determined by two different methods: first, by means of a pycnometer, using both water and mercury in standard procedures for determining the volumes of the pycnometer bulb and capillary; and second, by a direct comparison *in situ* of the mercury and oil manometers. The oil was outgassed by refluxing under vacuum while stirring; the manometers were evacuated when not in use.

In the region of temperatures occurring in our P_3 -versus- P_4 measurements, the above oil density and direct determinations of F are in agreement to 0.012 percent or better.

McLeod Gage. The McLeod gage had an extra-long Truebore tubing capillary, about 500 mm long \times 1 mm bore. It was calibrated by means of an oil manometer read with a traveling microscope and used as a $P = h^2 K(h)$ instrument, where h is both the height of the compressed gas column and the mercury pressure head on it and $K(h)$ is the gage "constant" which may have a slight variation with h due to a tapering capillary or for a reason given in the following paragraph.

An important procedural detail when taking a group of McLeod gage readings is to isolate the Dewar from the gage prior to each reading. If this is not done, warm gas will be pushed into the vapor pressure bulb as the mercury rises in the gage, and

several minutes will be required to restore an equilibrium temperature prior to the next reading. As a consequence of closing the valve the McLeod factor, K , will deviate from a constant more than would otherwise be the case. Naturally the same valve should be closed during calibration as is closed during subsequent vapor pressure measurements. In our apparatus the variation in $K(h)$ amounted to a fraction of a percent. For the function $K(h)$ a smooth curve was drawn through the calibration data. For the average of three readings at each of 17 calibration pressures, ranging from 40 to 2600 μ , the standard deviation from $K(h)$ was ± 0.38 percent.

Thermomolecular Pressure Ratio. Despite the large tubing sizes used, it was necessary at $T \leq 1.2$ °K to make thermomolecular pressure corrections of He⁴ pressures. With stepped tubing this is a tedious calculation, even when graphs of $P_{\text{cold}}/P_{\text{warm}}$ of adequate sensitivity are available [37] for each section of tubing. For this reason we recommend to others use of a graph of the form of figure 4, in which the thermomolecular correction is expressed in terms of a correction in millidegrees to a scale temperature read from uncorrected pressures. Once the graph has been prepared for a particular geometry and isotope, no further calculations are required, beyond a simple subtraction.

For the stepped tubing used in the present measurements, the equivalent temperature correction was 4.5 mdeg at 0.90 °K.

Gravity Correction. The measured pressures have been reduced to standard gravity by multiplying by the ratio of local gravity, 979.125 cm/sec², to standard gravity, 980.665 cm/sec².

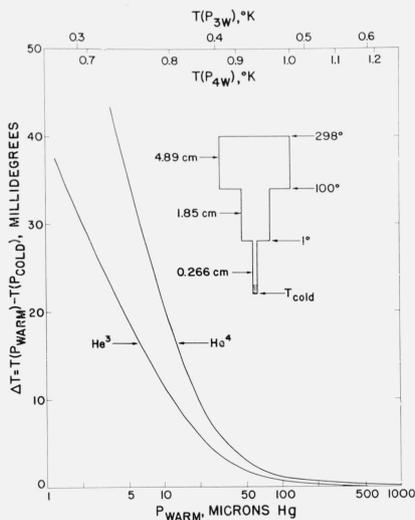


FIGURE 4. A convenient method of correcting for thermal transpiration.

The observed pressure at the warm end of the vapor pressure sensing tube, P_{warm} , plus a vapor pressure scale of temperatures, yield a first approximation to the liquid temperature, $T(P_{\text{warm}})$. The figure shows the correction, ΔT , to be subtracted from $T(P_{\text{warm}})$ to account for thermal transpiration in the pressure sensing tubes shown in the insert. Although the ratio of P_{cold} to P_{warm} is the same for He⁴ and He³, dT/dP is not the same for the two isotopes and the function $\Delta T(P_{\text{warm}})$ is therefore not the same.

Hydrostatic Pressure Correction. A small correction was made for the hydrostatic head of the gas column. This was appreciable only for the region below the flange h shown in figure 3, and most of the correction (80%) applied to the 18 cm high column between the He³ and He⁴ refrigerants. In equivalent millidegrees the total correction ranged from 0.14 to 0.67 mdeg for He³ between 1° and 3.323° respectively and from 0.08 to 0.30 mdeg for He⁴ over the same temperatures.

He⁴ Impurity Correction. The He⁴ impurity in the stored He³ was measured several times in the course of these experiments, using a mass spectrometer readable to 0.003 percent He⁴ in He³. The measured He⁴ impurity was 0.041 mole percent. The correction was calculated from a smoothed curve obtained from the data of Sydoriak and Roberts [32] and of Esel'son and Berezniak [38] for 90 percent He³ solutions. Since the liquid phase diagrams for both these sets of data show that mixture vapor pressure, P_x , varies linearly with He³ mole fraction, X , between 90 and 100 percent, we assume that $(dP_x/dX)_{T, X=1} \approx (P_3 - P_x)/(1 - X)$.

The correction raised P_3 by 0.037 and 0.030 percent at temperatures of 0.9 and 3.0 °K respectively. A more complete discussion and a table of this correction are given in paper III of this series.

ΔP_f Correction. The considerations on which ΔP_f , the film reflux pressure correction is based, have already been discussed. Using eqs (1), (2), and (3) we found ΔP_f to be equivalent to temperature corrections ranging from 3.7 mdeg at 0.9 °K, to 0.18 mdeg at 1.1 °K, and to negligible (< 0.02 mdeg) at 1.2 °K and at higher temperatures.

5. Results

Corrected (P_3, P_4) Data. Table 2 shows results of measurements of P_3 and P_3 , corrected as described above. Since P_3 is the pressure observed in cell C, which is immersed in a "bath" of He⁴ in cell D, no ΔT_k correction was needed with these data. The error columns, whose inverse was used as a weighting factor in subsequent calculations of a temperature scale (part II of this series), are the arithmetic sum of the estimates of probable error in the meniscus correction, manometer readings, mercury density, impurity correction, thermomolecular correction, and ΔP_f .

We defer until later a discussion of data point 44, which represents a redetermination of the critical point of He³. We only wish to point out that the indicated estimates of error for this point are large because of the inherent difficulty of any vapor pressure measurements near a critical pressure. Actually, the effort put into data point 44 was several times greater than the effort put into any other data point in the table.

In the sixth column δP_3 and δP_4 have been combined to show the effect on P_3 of errors in both P_3 and P_4 : $\delta P'_3 = \delta P_3 + \delta P_4 (dP_3/dP_4)$, the derivative being calculated from existing He³ and He⁴ temperature scale tables, T_L [9] and T_{L55} [25], respectively.

The entries in this column will be appreciably exceeded by deviations of the data from an interpolation table, calculated as described below, only if an important source of random error has been underestimated or overlooked.

Direct Interpolation Equations. For various reasons it was felt desirable to derive interpolation equations by which any He⁴ vapor pressure could be converted to an equivalent He³ vapor pressure in one step, i.e., without the intervention of any vapor pressure-*versus*-temperature scales. A table calculated from interpolation equations has been used, for example, to compare the 1962 He³ vapor pressure-temperature scale with absolute gas thermometer temperatures taken in conjunction with He⁴ vapor pressure measurements. The results of the comparison will be given in paper II of this series. Similarly, existing paramagnetic salt data taken in conjunction with He⁴ vapor pressure measurements can be compared with a He³ temperature scale by the use of such direct $P_4 \rightarrow P_3$ interpolation equations.

Another use, of more immediate interest to us, is an examination of the data of table 2 for excessive scatter or for unrealistic assignment of probable errors. Of course, a two-step comparison is always possible, using any two smooth He⁴ and He³ temperature scales. This, however, introduces an added uncertainty in that the temperature scales used as interpolation devices may not themselves be completely smooth or may have an incorrect "shape" in certain regions such as, for example, the region of the He⁴ lambda temperature.

It was not possible to obtain a single equation $f(P_3)=g(P_4)$. Therefore, the interpolation equations were obtained as follows: successive groups of eight adjacent data points in table 2 were fitted by the method of least squares to equations of the forms

$$P_3 = a + bP_4 + cP_4^2 \quad (5)$$

and

$$\ln P_3 = a + b \ln P_4 + c (\ln P_4)^2. \quad (6)$$

TABLE 2. *Results*

P_3 and P_4 are the corrected He³ and He⁴ vapor pressures and δP_3 and δP_4 the estimated errors in P_3 and P_4 respectively. If the function $P_3(P_4)$ is properly deduced from the table the estimated error in P_3 for a given P_4 is assumed to be $\delta P_3' = P_3 + \delta P_4 (dP_3/dP_4)$. The equations used are actually quadratic in P_4 or $\ln P_4$. See p. 554 (eqs 5&6). The seventh column shows the fit of a direct (one-step) interpolation table giving ${}_1P_3$ as a function of P_4 . ${}_1P_3$ was obtained from a series of quadratic equations in P_3 and P_4 fitted to successive groups of eight consecutive data points in the table. The eighth column shows the fit of a two-step "conversion" of a P_4 to a P_3 at the same temperature, using temperature as an interpolation parameter; i.e., ${}_2P_3$ was obtained from P_4 by way of the 1958 He⁴ and 1962 He³ temperature scales. The broad similarities between the last three columns give evidence of the adequacy of the error estimates and of the various curve and equation fitting procedures used to construct the above-mentioned tables.

Error estimates for point 44, which is at the critical pressure and temperature of He³, refer only to the error in the measurement of P_3 and P_4 . That is, they do not include the error in the decision that this is the critical point. Units of all columns in the table are microns of mercury at 0 °C and standard gravity.

Point	P_4	$\pm \delta P_4$	P_3	$\pm \delta P_3$	$\pm \delta P_3'$	$P_3 - {}_1P_3$	$P_3 - {}_2P_3$
1.....	40.85	0.34	5254	3	26	-1.6	-4.3
2.....	67.27	0.39	6671	3	22	-3.7	-7.6
3.....	115.56	0.45	8692	4	20	10.5	10.5
4.....	159.41	0.70	10176	4	26	8.1	13.6
5.....	188.19	0.69	11041	4	24	5.4	10.5
6.....	282.08	1.00	13487	5	29	-5.7	1.5
7.....	615.05	1.20	20001	8	28	5.5	3.4
8.....	1210.3	0.7	28395	11	19	4.0	12.9
9.....	2154.7	0.8	38512	14	22	-21	3
10.....	3521.6	1.0	50234	18	26	-26	11
11.....	5791.8	1.3	66069	27	35	-37	-48
12.....	8490.2	1.6	81978	60	69	-75	-112
13.....	8583.6	1.6	82515	28	37	-22	-85
14.....	12516	8	102750	60	108	41	-7
15.....	12706	8	103650	60	108	34	-38
16.....	12745	2	103810	60	71	17	-47
17.....	17263	3	124230	70	85	-36	-122
18.....	17647	3	125860	60	72	-29	-151
19.....	23972	8	151880	70	100	-22	-37
20.....	24361	8	153490	35	65	54	89
21.....	31791	9	181580	48	82	56	64
22.....	35937	9	196610	45	77	36	54
23.....	37807	9	203290	55	88	0	45
24.....	38498	9	205830	55	89	59	121
25.....	40975	9	214470	80	111	-48	7
26.....	44635	9	227210	59	91	3	30
27.....	51023	9	248770	80	110	-5	-25
28.....	63396	9	288860	90	119	-30	-41
29.....	77731	10	333140	90	118	-13	-6
30.....	94167	10	381810	110	140	136	173
31.....	112412	10	433390	100	127	-21	64
32.....	133810	11	491850	110	140	-110	-6
33.....	157652	11	555000	110	136	9	16
34.....	181836	12	617090	130	161	-164	-214
35.....	197613	22	657110	109	164	-43	-94
36.....	197852	19	657780	110	161	34	-25
37.....	198314	19	658950	110	161	54	-14
38.....	223380	22	721410	114	168	35	82
39.....	226755	22	729700	114	168	-21	50
40.....	230422	23	738630	116	171	29	-31
41.....	257649	19	804980	120	170	-162	-59
42.....	258223	19	806350	120	170	-198	-108
43.....	277744	21	853730	140	192	-52	253
44.....	285660	170	873000	300	710	174	448

Because of the specific heat anomaly of the liquid He^4 we excluded groups of data points which overlapped the lambda temperature. Point 23 is very close to the lambda point vapor pressure, P_λ , and was used as a terminal point for the last equation below P_λ and the first equation above P_λ .

Except for the last five equations, the fit of the logarithmic equation was much better than the fit of eq (5), by a factor of 2 to 5 times. For the five equations of highest pressures the fit of equation (5) was best, by an average factor of 3 times. In accord with these observations the most appropriate of the derived equations were retained.

The coefficients of the selected equations and the detailed $P_3(P_4)$ interpolation table are available from the authors on request. The seventh column of table 2 shows how the interpolation table fits the data.

We note that, in general, the deviations of the data points from the direct interpolation table are comparable to and usually less than the quantity $\delta P'_3$, which is representative of the combined estimated random errors in P_3 and P_4 . That is to say, each data point lies on a smooth curve with its neighbors, with a deviation comparable to the estimated random errors in the measurements. For the four entries at which $P_{3-1}P_3$ exceeds $\delta P'_3$, the average value of $(P_{3-1}P_3)/\delta P'_3$ is 1.08. We conclude that the error estimates are probably satisfactory for use as weighting factors in deriving a vapor pressure equation from these data.

The last column shows the results of a two-step interpolation using the temperature as a parameter: from P_4 and the 1958 He^4 scale we get T_{58} and from this temperature and the 1962 He^3 scale (paper II of this series) we get ${}_2P_3$. The table shows that the deviation from the observed He^3 pressure of the two-step conversion from P_4 to an equivalent P_3 are, on average, 1.5 times as great as the deviations of the one-step conversion, but they are nowhere excessively greater than the "expected" deviations $\delta P'_3$. For the 10 points which exceed $\delta P'_3$ the average ratio of $P_{3-2}P_3$ with respect to $\delta P'_3$ is 1.40. It appears from this that the procedures employed in the establishment of the 1958 He^4 scale did not introduce irregularities capable of preventing calculation of an adequate He^3 temperature scale equation.

The general similarity of the last three columns also suggests that an adequate number of floating coefficients were used to fit the data to the $P_3(P_4)$ and the $P_3(T_{62})$ equations. For information as to whether an excessive number of floating coefficients had been used it is necessary to compare ${}_1P_3$ and ${}_2P_3$ in regions between the fitted points. If, for example, N data points were fitted to equations of different form each of which contained a number of floating coefficients approaching N the fit would be excellent at the data points but very poor elsewhere. We have compared ${}_1P_3$ and ${}_2P_3$ at 63 values of P_4 selected at random and find the average deviation between them to be 0.026 percent of ${}_1P_3$ with a maximum deviation of 0.10 percent of ${}_1P_3$. (In terms of equivalent millidegrees the average de-

viation between the two conversion procedures is 0.14 mdeg, with a maximum deviation of 0.5 mdeg.) By comparison, for P_4 's at the fitted data points we find from table 2 that ${}_1P_3$ and ${}_2P_3$ differ by an average of 0.030 percent of ${}_1P_3$. We conclude that an excessive number of fitted coefficients have not been used in deriving the $P_3(P_4)$ and $P_3(T_{62})$ equations.

Redetermination of the Critical Point. Point 44 of table 2 represents a redetermination of the critical pressure and temperature by the method of pseudo-isotherms [9, 5] from data shown in figure 5. Below the critical point, the presence of a liquid level within cell A (cf. fig. 2) is recognized by the fact that for a portion of an isotherm the observed cell pressure is independent of the amount of He^3 taken out of the storage container and admitted into the Dewar and pressure gauge system. A sensitive bellows type gage marked every mm Hg was used in these measurements. It was calibrated against the same sensitive He^4 mercury manometer described above.

For isotherms approaching the critical isotherm from lower temperatures the liquid and vapor densities approach the same value and the width of the flat for an "isotherm" approaches zero. The dashed line in the figure is an estimate of the loci of points at which liquefaction within the copper cell began or ended. The top of the dashed line therefore lies on the critical isotherm and the value of the ordinate equals the critical pressure. We estimate the uncertainty in choice of the ordinate

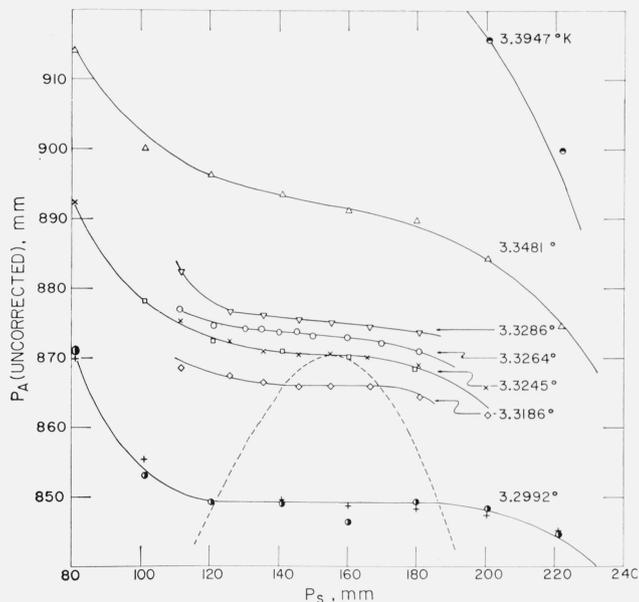


FIGURE 5. Data for critical pressure determination.

The equilibrium pressure, P_A , in cell A (fig. 2) as a function of pressure of He^3 in storage, P_s , at a number of temperatures controlled by pumping on He^3 in cell B and measured by a liquid He^4 pressure in cell D. P_A represents a vapor pressure when P_A is independent of P_s . The dashed line therefore encloses the region in which cell A is partly full of liquid and the top of the dashed line is at the pressure and temperature of the critical point. The corrected critical pressure is found to be 873.0 ± 1.5 mm Hg at 0°C and standard gravity. The error estimate includes the uncertainty in location of the peak of the coexistence curve. The critical temperature is 3.3240°K on the 1958 He^4 scale.

of the top of the dome to be ± 1.2 mm Hg. As drawn in the figure the top occurs at a corrected pressure of 873.00 ± 0.30 mm Hg. The critical pressure is therefore 873.00 ± 1.50 mm Hg. The corrected temperature of the chosen "critical isotherm" is 3.3240 ± 0.0005 °K. If we add on the uncertainty in locating the top of the dome the critical temperature is 3.3240 ± 0.0018 °K.

The first determinations of the critical pressure were by Abraham, Osborne, and Weinstock [5], who obtained 890 mm Hg by the above method and 860 mm Hg by the visual method of detecting disappearance of the liquid meniscus. Next, by the above pseudo-isotherm method, Sydoriak, Grilly, and Hammel (39) found $P_c = 875$ mm Hg. From measurements of density by an optical method, Peshkov [40] obtained $T_c = 3.38 \pm 0.03$ °K and 930 ± 20 mm Hg. Our correction for He⁴ impurity was made by direct linear extrapolation from the He⁴ critical point through the observed point (0.031% He⁴) to 0.0 percent He⁴.

Demonstration of ΔT_k Errors in a He⁴ Bulb Thermometer Below T_λ . The (P_3 , P_4) data of table 2 were taken with a He³ bulb (cell C, fig. 2) immersed in a bath of He⁴ (cell D, fig. 2). Since all of the heat of condensation of the refluxing film was taken up in the refrigerated walls of cell D, there was no need for a ΔT_k correction between the liquids in C and D.

In many experiments a He⁴ vapor pressure bulb is used to determine the temperature of an object to which it is attached. To demonstrate the need for a ΔT_k correction to such a measurement when $T < T_\lambda$, we also made some comparisons between T_C and temperatures, T_B , calculated from the He³ vapor pressure in cell B, which is in good thermal contact with the walls of the He⁴ cell. Some of the results are shown in table 3. We note that above T_λ the two He³ bulbs read the same temperature but below T_λ they are in serious disagreement, by an amount which increases rapidly as the temperature is progressively reduced below 2 °K.

From the concurrently measured boil-off of the He³ refrigerant we were able to estimate the heat of recondensation, \dot{Q}_f , of the refluxing film in cell D. From this it should be possible, in principle, to calculate ΔT_k and to decide, by comparison with $(T_C - T_B)$, whether we have a valid explanation of the origin of the discrepancy and a means of correcting from a measured He⁴ bulb pressure to the temperature of the bulb wall.

Unfortunately, as we have seen in the introduction, experimental values of $\dot{Q}/\Delta T_k$ for copper are not reproducible from one sample to another, so a quantitative check against our measured ΔT_k is not possible. Qualitatively, however, our results are similar to experimental and theoretical values appearing in the literature. Figure 6 shows our data, Khalatnikov's theoretical curve as interpreted by Challis et al. [29], and the results of various experimenters. The fact that none of the reported measurements overlaps our data does not appear to be too significant, in view of the very large differences between the various experiments. Kuang Wey-Yen

has clearly demonstrated the large effect that the condition of the surface (the Beilby layer) has on ΔT_k . Quite possibly our He⁴ cell surface, which was thoroughly annealed when being silver-soldered, was altogether different from the surfaces on which the other ΔT_k measurements were made (if these other cells or surfaces were soft soldered, as appears likely from the description of the various experiments). It is also possible that the direction of heat flow, from liquid to solid in our experiments and from solid to liquid in all the others, affects the magnitude of ΔT_k .

TABLE 3.—The Kapitza resistance, ΔT_K , due to film recondensation heat flux, \dot{Q}_f , to the He⁴ cell; $\Delta T_K = T_C - T_B$

T	ΔT_K	\dot{Q}_f
deg	mdeg	mW
0.82	7.2	0.44
0.85	9.0	0.42
0.90	2.4	0.31
0.94	2.1	0.31
0.98	4.6	0.53
0.99	9.2	1.19
0.99	9.1	1.50
1.00	1.9	0.26
1.05	1.4	0.29
1.09	3.5	0.50
1.10	1.0	0.28
1.30	1.2	0.39
1.40	1.5	0.64
1.50	2.3	0.77
1.81	0.5	0.52
2.00	0.4	0.36
2.01	0.4	0.24
2.23	0.1	None
2.60	-0.3	Do.
2.80	-0.1	Do.
2.91	-0.1	Do.
3.00	-0.1	Do.
3.00	-0.2	Do.
3.10	-0.3	Do.
3.15	0.0	Do.

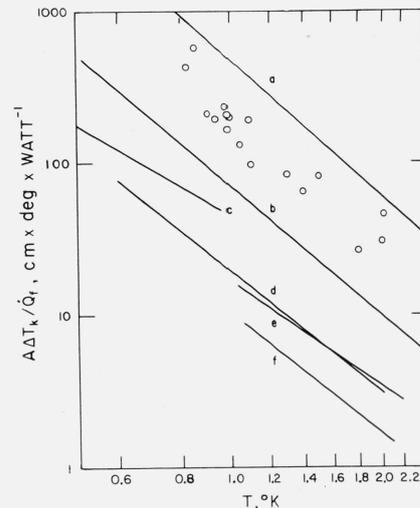


FIGURE 6. The Kapitza resistance, $A \Delta T_K / \dot{Q}_f$, where $\Delta T_K = T_C - T_B$.

\dot{Q}_f is obtained from the increase in evaporation rate due to the presence of He⁴ in cell D, and A is the total inside surface area of cell D. Curves a and b are from Khalatnikov's theoretical result [31] as calculated for copper by Challis et al. (ref. [29]) and Kuang Wey-Yen (ref. [30]).

Curves c and d are experimental results for copper, refs. [28] and [30] respectively. Curves e and f are experimental results on polished and etched copper respectively, as given in ref. [29].

As noted earlier, Kuang Wey-Yen looked for such a directional dependence and found a difference of less than an order of magnitude, but unfortunately he does not state which direction gave a higher ΔT_k , so no conclusion can be drawn. Finally, it may possibly be significant that our liquid to solid ($T_C - T_B$) values lie between ΔT_k 's based on solid to liquid measurements and ΔT_k as given by Khalatnikov's theory. Perhaps the properties of bulk material used [29] in making calculations from the theory are more applicable to our well-annealed surface than to the surfaces studied by the other experimenters.

In any case, it is clearly impossible, in the present state of knowledge of the Kapitza temperature discontinuity, to correct properly a measured He⁴ vapor pressure bulb measurement below T_λ to obtain the temperature of the bulb wall. Even if the conversion from \dot{Q} to ΔT_k were established experimentally for a particular bulb, as we have done, it would be impractical, in most cases, to meet the requirement of measuring \dot{Q}_f frequently. Nor, in our experience, can the measurement of \dot{Q}_f be justifiably avoided by using known film flow rates for clean tubing, $\dot{n}_{f,\min}$, or a predetermined multiple of $\dot{n}_{f,\min}$. As noted earlier, we have found that, although no leaks were detectable into our He⁴ cell system, using a mass spectrometer leak detector sensitive to 10^{-10} cm³/sec, yet our measured \dot{Q}_f was not reproducible from day to day. During an early series of runs \dot{Q}_f steadily increased from a value corresponding to $1.2 \dot{n}_{f,\min}$ immediately after cooling the Dewar off to $11 \dot{n}_{f,\min}$ during the ensuing 7 days of experimentation at liquid helium temperatures. It is possible that the increase was attributable to impurities present in the He⁴ storage can although this was filled from vapor drawn off the large liquid He⁴ pot. In later runs the ratio $\dot{n}_{f,\text{obs}}/\dot{n}_{f,\min}$ averaged 2.9 and ranged no higher than 4.2.

Possibility of Gas Oscillations. Resonant oscillations of a gas column between warm and cold portions of a tube are a possible cause of error in vapor pressure bulb thermometry for two reasons: The pressure measured at the warm end may differ from the vapor pressure; and heating of the liquid by the downward surges of warm gas will occur.

We believe gas oscillations were not a significant cause of error in the present measurements for two reasons: During the measurements no pressure fluctuations were detected on a Consolidated Micromanometer which was capable of detecting fluctuations of 0.2μ at any pressure and at frequencies up to about 30 c/s; and pressures above the lambda point measured simultaneously in cells B and C gave no evidence of the existence of gas oscillations. In support of the latter statement we note in table 3 measurements of P_B and P_C made at seven different pressures above P_λ . Since bulbs B and C and their pressure sensing tubes are identical, it is reasonable to assume that the heat leak to bulbs B and C due to gas oscillations would be the same. Since bulb B is in much better thermal contact with the refrigerant than is bulb C, one would expect B to be appreciably

colder than C if significant gas oscillations were occurring. Actually we found $T_C - T_B$ to average -0.14 mdeg for the seven measured temperatures above T_λ . We conclude that gas oscillations were not a significant cause of error above P_λ .

For temperatures below T_λ we have no such evidence because $T_C - T_B$ is predominantly a measure of the film reflux phenomenon. However, the absence of detectable pressure oscillations provides some assurance that gas oscillations were not occurring below T_λ .

6. Conclusion

The data of table 2 appear to be suitable for derivation of a He³ scale of temperatures. The paper which follows this one describes this derivation. A He⁴ vapor pressure bulb is found to be impractical for accurate thermometry below T_λ because of the difficulty of correcting for the pressure gradient heating effects due to the refluxing film.

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