JOURNAL OF RESEARCH of the National Bureau of Standards—D. Radio Propagation
Vol. 67D, No. 6, November–December 1963

Statistical Methods in Radar Astronomy. Determination of Surface Roughness

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(Received June 13, 1963)

An experimentally established statistical model of a rough surface is used to show that sufficient information about the roughness of such a surface in the form of its standard deviation, mean horizontal size of lumps, and average slope can be obtained from experimental data when used in conjunction with a theory based on statistical analysis.

1. Introduction

Recently some lunar theories based on the use of statistical theory have been questioned [Siegal and Senior, 1962], even though most of the theoretical work in the field of the determination of planetary surface roughness has been statistically oriented [Davies 1954; Cooper 1958; Hayre and Moore 1961; Hayre 1961 and 1962; Winters 1962; Evans 1963; Millman 1963, etc.]. Later on, Evans [1963] published some further data on lunar echoes to show that lunar theories employing statistical theory offer feasible results. This is another simple illustration of the direct use of the statistical theory in surface roughness studies by radar.

A naturally occurring rough surface may be described either by infinitesimally closely spaced contour maps or by its statistical properties. Without much ado, it is apparent that the latter is probably the most logical and compact way in the absence of any other exact description unless some other form of microscopic scale description is needed. Almost any rough surface can be shown to have a probability density function of its heights, and a height-distance autocorrelation function. It has been previously shown [Hayre, 1961] that a large number of naturally occurring rough surfaces may be said to have their heights normally distributed above and below their mean value. It has also been shown that an exponential height-distance autocovariance function seems to describe many such cases, or

$$p(h)=\frac{1}{\sqrt{2\pi\sigma^2}}\,e^{-(h-m)^2/2\sigma^2}, \tag{1}$$

$$\rho(r)=\sigma^2 e^{-(|r|/B)} \tag{2}$$

where

$p(h)$ = probability density function of heights
h = height
m = mean height
σ = standard deviation of heights
$\rho(r)$ = height-distance autocovariance
B = horizontal distance at which autocovariance drops to $1/e$th of its value at zero r.

It is now necessary to establish the physical meaning of these three statistical parameters m, σ and B. The usefulness of mean height, m, and its relationship to a physical roughness is undoubtedly self-explanatory. Now let us see what one can infer about the mean slope of the surface, mean size of lumps along the surface, and, in general, roughness in terms of vertical and horizontal roughness parameters after these constants have been determined from experimental data. The classical theories [Taylor, 1935] very clearly established that (σ/B) is a measure of the average slope of the surface, because σ is the standard deviation of vertical roughness, while B is a measure of horizontal size of surface perturbations.

The roughness parameters B and σ can specify many types of roughness. For instance, a large value of B would indicate relatively smooth variation of heights, whereas a large value of σ would represent the ruggedness of the terrain. Small values of B and σ indicate a very rough terrain with small scale (vertical) roughness, whereas a large B and large σ represent a relatively smooth surface with large scale perturbations. On the other hand, a small B and a large σ would seem to signify an extremely rough surface with large size undulations, while a large value of B and a small value of σ are characteristic of a surface which is very smooth and has small size disturbances in its contours.

A summary of the physical interpretation of the statistical roughness parameters is given in the following table.

This interpretation, when used with Pettengill's [1960] experimental data on lunar echoes and Hayre and Moore's [1961] lunar echo theory, yields an average slope of 1 in 10 [Hayre, 1963, to be published]. It is in the general range for the mean slope of 1 in 7.4 and 1 in 11 obtained by Daniels [1963] and Evans and Pettengill [1963], respectively. This seems to indicate that the lunar theories based on statistical analysis alone can yield reasonable and experimentally verifiable results. It may not be out of place to add that the use of statistical theory in radar return from a rough surface is a step in the right direction when other deterministic information is not available.

Specification of surface roughness in terms of characteristics constants B *and* σ

Horizontal size of roughness (average length of areas at one elevation)	Vertical size of roughness		
	Small	Medium	Large
Large	Very small slope Small σ Large B	Medium slopes Medium σ Large B	Large σ Large B
Medium	Small slopes Small σ Medium B	Medium σ Medium B	Medium slopes Large σ Medium B
Small	Small σ Small B	Large slopes Medium σ Small B	Very large slopes Large σ Small B

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Note: On page 430, 2d col, 11th line, (3.8) should read (3.9); in line 15. (3.9) should read (3.8)

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(Paper 67D6–305)