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Oblique Propagation of Groundwaves Across a Coastline—Part II [1]

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The amplitude and phase are calculated for oblique propagation across a coastline with a sloping beach. In this case, the land and sea are taken to be plane surfaces and the beach slope is constant. It is shown that the reflected wave may be quite significant and it has a fundamentally different character from the reflected wave in the case of a flat-lying coastline.

1. Introduction

In part I [Wait, 1963] of this series, the propagation of radiowaves across a flat-lying coastline was considered in some detail. It is the purpose of this paper to extend the analysis to include the effect of a gradual elevation change between land and sea. Numerical results are given for a beach which has a plane or constant slope. The same problem has been treated by Feinberg [1946] and more recently by Kalinin [1958]. Our results, derived in a different manner, appear to be in agreement with the Soviet work.

2. Formulation

The general situation is illustrated in figure 1. With respect to a Cartesian coordinate system, the xy plane is taken to be the plane surface of the sea to the right of the waterline at $x=d_0$. The elevation of the land is then defined by

$$z=z_0 \text{ for } x<0,$$

and

$$z=z_0(d_0-x)/d_0 \text{ for } 0<x<d_0.$$

To simplify the discussion, it is assumed that the electrical characteristics of the land may be described in terms of surface impedance Z right up to the waterline at $x=d_0$. The medium to the right (i.e., $x>d_0$), which is the sea, is described by a surface impedance Z'. As in part I, the transmitter at A with coordinates $(-x_0, y_0)$ is regarded as a vertical electric dipole, of effective height h_a, on the surface of the land. The receiving antenna, of effective height h_b, is located at B with coordinates $(d_1, 0)$ where d_1 may be positive or negative.

It is apparent that a rigorous solution of the problem would be extremely difficult. Furthermore, even if it were available, the idealization of the model would limit its usefulness. Therefore, an approximate approach is adopted which leads to a relatively simple formula for computation.

Provided the slope of the beach is small, it can be expected, on the basis of physical consideration, that the main influence of the elevation change is to modify the tilt of the electric field. For example, if the slope in the x direction is defined by γ_x, then the *modification* of the horizontal electric field E_x is approximately equal to $-\gamma_x E_z$. To within a first order, the horizontal electric field E_y is not changed if the slope γ_y is zero.

In order to achieve a further simplification, the source dipole is assumed to be sufficiently removed (to the left in fig. 1) that the incident wave is nearly plane in the vicinity of the coastline. Thus, locally the incident field is proportional to exp $[ikS_1y-ikC_1x]$ where C_1 and S_1 are the cosine and sine of the angle

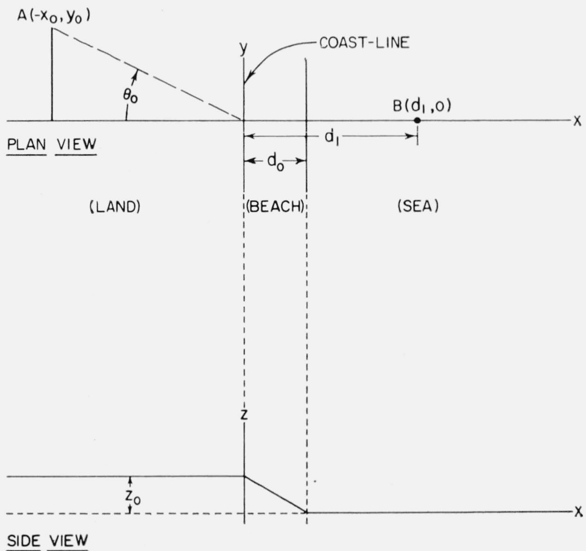

FIGURE 1. *Plan and side view of the mixed path showing the location of dipoles* A *and* B.

The scales are exaggerated

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of incidence θ_0 at the coastline. It is then a simple matter to describe the tilting of the electric field in terms of the modification of the effective surface impedance. Thus, over the plane $z=z_0$ and in the interval $0<x<d_0$, it is found that the impedance boundary conditions may be written

$$E_x=-Z_xH_y \text{ and } E_y=Z_yH_x,$$

where

$$Z_x=Z-\gamma_x\eta_0/C_1,$$

and

$$Z_y=Z.$$

Consequently, we have, in effect, replaced the sloping beach by a horizontal plane surface with a modified surface impedance. Boundary conditions of this type can be deduced directly from the results of Feinberg [1946] after a long and involved derivation. According to Feinberg, the restrictions on the slope are equivalent to requiring that $\gamma_x<<1$ and $z_0<<\sqrt{d_0}\lambda/2\pi$.

3. Integral Representation

The mutual impedance between the dipoles A and B is again denoted by z_m if the whole ground plane were flat and had a constant surface impedance Z. The change of the mutual impedance which results in the differing electrical characteristics of the sea is denoted Δz_m. In part I, the calculation of Δz_m was carried out under the assumption of a sharp boundary or sudden change from Z to Z' at $x=0$. The change of the mutual impedance resulting from the elevation change is denoted δz_m. In this case, δz_m is expressed as a surface integral over the strip $0\leqq x\leqq d_0$. Thus eq (1) of part I is now replaced by

$$\delta z_m=\frac{1}{I^2}\iint_{\text{strip}}(-\gamma_x\eta_0/C_1)H_{ay}H'_{by}dx\,dy, \tag{2}$$

where H_{ay} is the y component of the tangential magnetic field over the strip if the surface were unperturbed and H'_{by} is the tangential magnetic field over the strip under perturbed conditions. In formulating this integral, the current at the terminals of dipoles A and B is I.

The simplification and reduction of the above representation for δz_m is carried out in the manner described in part I. There is an essential difference here in that only the y components of the magnetic field are involved. Thus, omitting numerous details, it is found that

$$\frac{\delta z_m}{z_m}\simeq\frac{i}{2}\int_0^{d_0}\left[\frac{\partial}{\partial x}\gamma_x-ikC_1\gamma_x\right]e^{ikC_1d_1}e^{-ikC_1x}\times H_0^{(2)}(k|d_1-x|C_1)dx. \tag{3}$$

Actually, this result is valid even when γ_x is some smooth function of x. However, it is necessary that γ_x have a magnitude that is small compared with

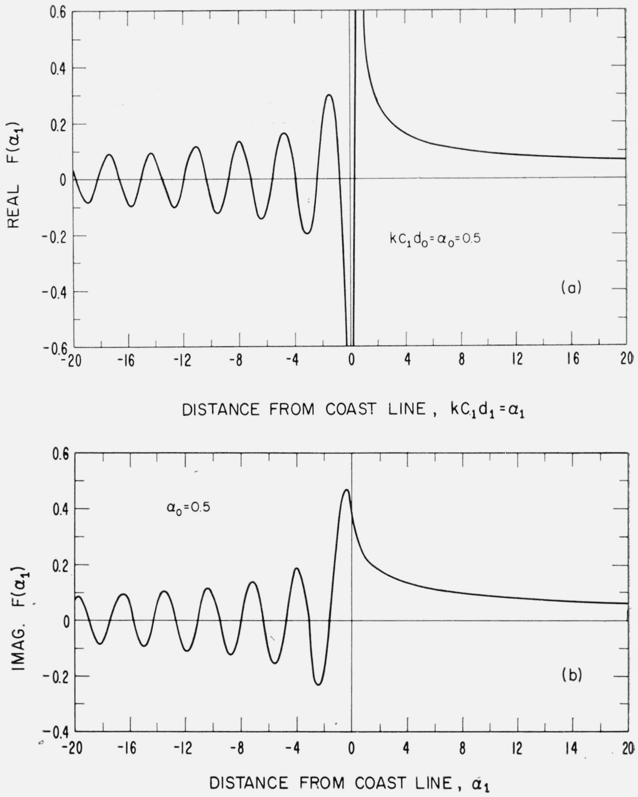

FIGURE 2. *Real and imaginary parts of the function* $F(\alpha_1)$ *as a function of* α_1 *for* $\alpha_0=0.5$.

unity corresponding to a very gentle slope. For the problem described here,

$$\gamma_x=-\frac{z_0}{d_0}[u(x)-u(x-d_0)], \tag{4a}$$

and

$$\frac{\partial\gamma_x}{\partial x}=-\frac{z_0}{d_0}[\delta(x)-\delta(x-d_0)], \tag{4b}$$

where $u(x)$ and $\delta(x)$ are the unit step and unit impulse functions, respectively. The integrations, with respect to x, may now be carried out readily if the identity,

$$\frac{\partial}{\partial\alpha}[\alpha e^{\pm i\alpha}(H_0^{(2)}(\alpha)\mp iH_1^{(2)}(\alpha))]=e^{\pm i\alpha}H_0^{(2)}(\alpha), \tag{5}$$

introduced in part I, is again utilized. Thus

$$\frac{\delta z_m}{z_m}=-\left(\frac{z_0}{2d_0}\right)F(\alpha_1), \tag{6}$$

where

$$F(\alpha_1)=f(\alpha_1)-f(\alpha_1-\alpha_0) \text{ for } \alpha_1>\alpha_0, \tag{7a}$$

$$=f(\alpha_1)+g(\alpha_0-\alpha_1) \text{ for } 0<\alpha_1<\alpha_0, \tag{7b}$$

$$=g(\alpha_0-\alpha_1)-g(-\alpha_1) \text{ for } \alpha_1<0, \tag{7c}$$

where $\alpha_0=kC_1d_0$ and $\alpha_1=kC_1d_1$.

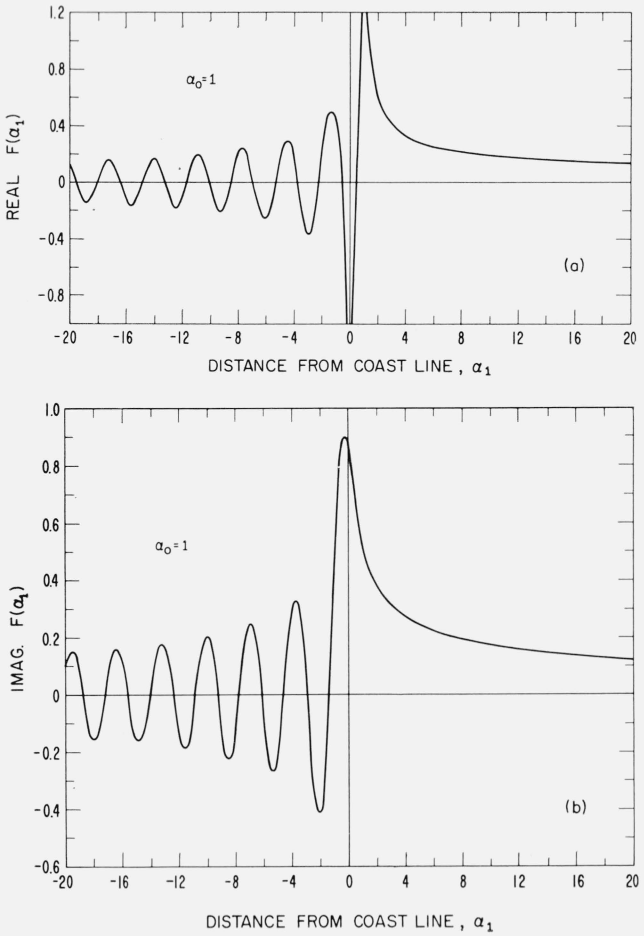

FIGURE 3. *Real and imaginary parts of the function* F(α_1) *as a function of* α_1 *for* $\alpha_0=1.0$.

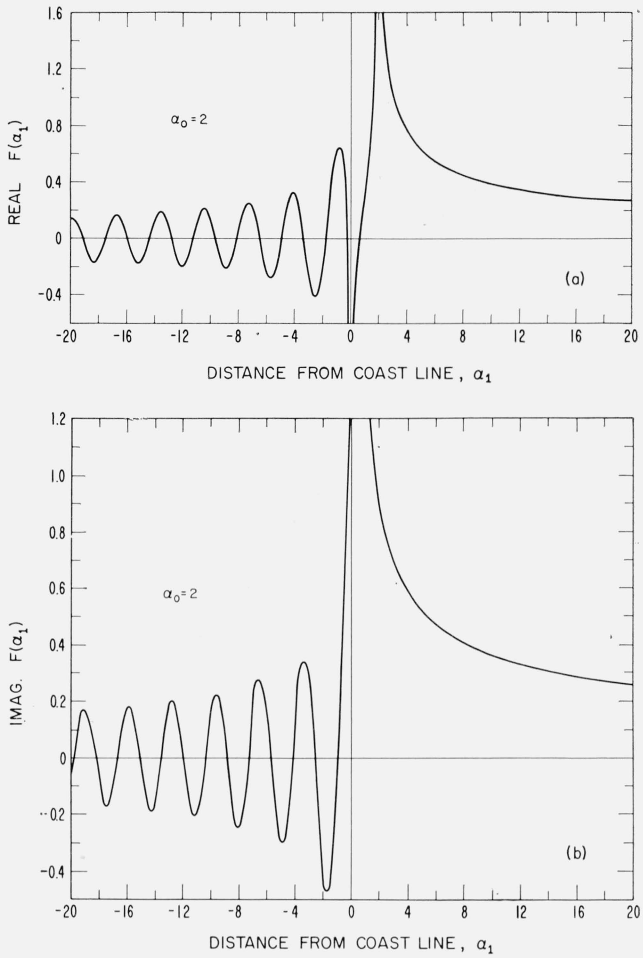

FIGURE 4. *Real and imaginary parts of the function* F(α_1) *as a function of* α_1 *for* $\alpha_0=2.0$.

The basic functions f and g are defined by

$$f(\chi)=\chi e^{i\chi}[H_0^{(2)}(\chi)-iH_1^{(2)}(\chi)]+ie^{i\chi}H_0^{(2)}(\chi), \quad (8a)$$

and

$$g(\chi)=\chi e^{-i\chi}[H_0^{(2)}(\chi)+iH_1^{(2)}(\chi)]-ie^{-i\chi}H_0^{(2)}(\chi), \quad (8b)$$

where χ is the general argument (which is positive real).[2]

The function $F(\alpha_1)$ is proportional to the fractional change of the field at B for a source dipole at A. Conversely, $F(\alpha_1)$ may be regarded as the fractional change of the field at A due to a source at B. The reciprocity is an inherent feature of a mutual impedance formulation. It is emphasized that the results are valid only when $|\delta z_m/z_m|<<1$.

The parameter α_0 is proportional to the width of the coastal strip whereas α_1 is proportional to the distance d_1 which is measured from $x=0$. The real and imaginary parts of the function $F(\alpha_1)$ are shown plotted in figures 2 to 6 for a range of values of α_0 from 0.5 to 5.0. The curves all have an oscillatory behavior for negative values of α_1. This is a manifestation of the interference between the incident wave and the wave reflected from the coastline. For positive values of α_1, corresponding to points beyond the coastline, $F(\alpha_1)$ has a monotonic character.

As may be observed in figures 2a, 3a, 4a, 5a, and 6a, the real parts of the function $F(\alpha_1)$ exhibit singularities at the edges of the coastal strip (i.e., at $\alpha_1=0$ and $\alpha_1=\alpha_0$). Actually, the results are not valid in the vicinity of these singular points since $|\delta z_m/z_m|$ is then no longer small compared with unity. Nevertheless, it can be expected that the real part of the field function $F(\alpha_1)$ should show marked changes in these regions.

The phase of the function $F(\alpha_1)$, as indicated in figures 2b, 3b, 4b, 5b, and 6b, is a continuous function right across the coastal strip. It is particularly interesting to note that the pronounced amplitude changes at the edges of the coastal strip are not accompanied by marked phase variations. Of course, this fact is compatible with the behavior of the rigorous field solution in the vicinity of the apex of a perfectly conducting wedge [Wait, 1959].

[2] It may be noted that for $\chi<<1$,

$$f(\chi)\cong-g(\chi)\cong Y_0(\chi)-\chi Y_1(\chi)\cong-\frac{2}{\pi}[\log_e(2/\chi)-1.5772].$$

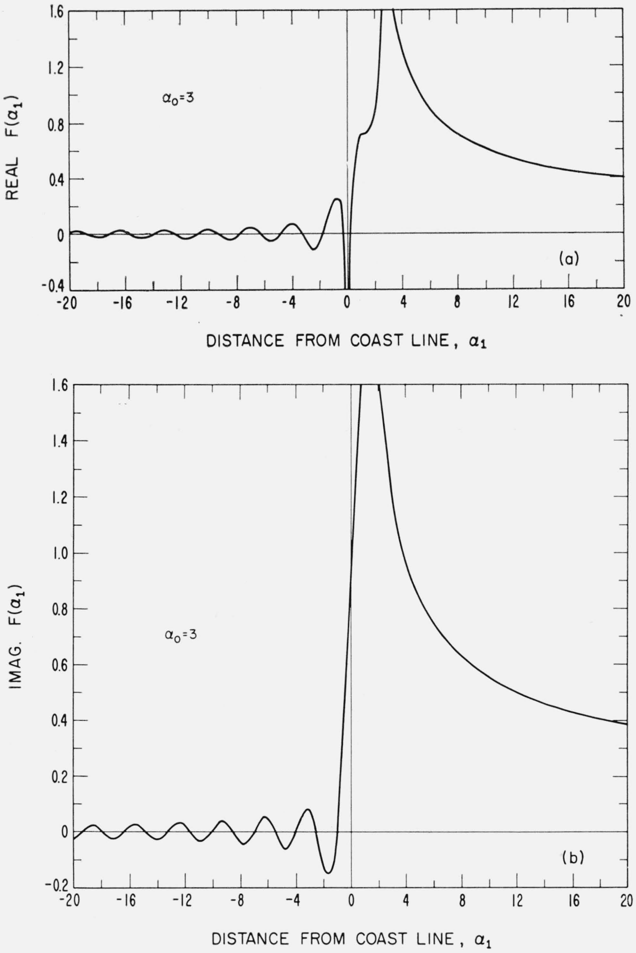

FIGURE 5. *Real and imaginary parts of the function* F(α_1) *as a function of* α_1 *for* $\alpha_0=3.0$.

It is of practical importance to observe that the phase perturbation resulting from the coastline is reduced to negligible proportions within several wavelengths of the coastline. This is in sharp distinction to the phase variations which result from the conductivity contrast between land and sea. As indicated in part I, the phase beyond the coastline (away from the transmitter) increases continuously with distance.

In order to summarize conveniently the field variation behind the coastline, the real and imaginary parts of $F(\alpha_1)$, for positive values of α_1, are plotted in figures 7a and 7b.

To facilitate further calculations for the influence of the sloping beach, the general functions $f(\chi)$ and $g(\chi)$ are presented in graphical form in figures 8a and 8b. These functions, defined by eqs (8a) and (8b), are now written in the form

$$f(\chi)=f_r(\chi)+if_i(\chi), \tag{9}$$

and

$$g(\chi)=g_r(\chi)+ig_i(\chi), \tag{10}$$

where f_r, f_i, g_r, and g_i are real functions of χ. As

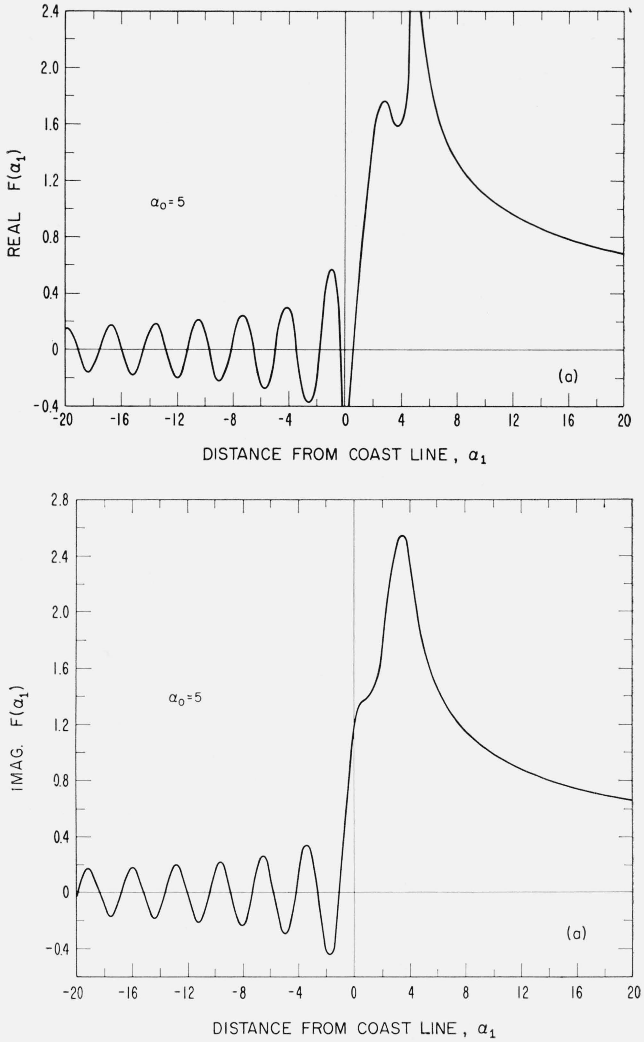

FIGURE 6. *Real and imaginary parts of the function* F(α_1) *as a function of* α_1 *for* $\alpha_0=5.0$.

indicated, the function $g(\chi)$ is generally oscillatory.

4. Refraction Effects Resulting From the Sloping Beach

It is interesting to note that the sloping beach actually causes a refraction error which is more or less additive to the error resulting from the conductivity contrast with the sea. As in part I, it is assumed that the field incident on the boundary at $x=0$, has the form

$$E_0=e^{-ikC_1x}e^{ikS_1y},$$

which is appropriate if the transmitter at A is sufficiently far removed to the left in figure 1. Under the further assumption that the total perturbation of the field E is small, it follows readily that

$$E\cong E_0\left(1+\frac{\Delta z_m}{z_m}+\frac{\delta z_m}{z_m}\right), \tag{11}$$

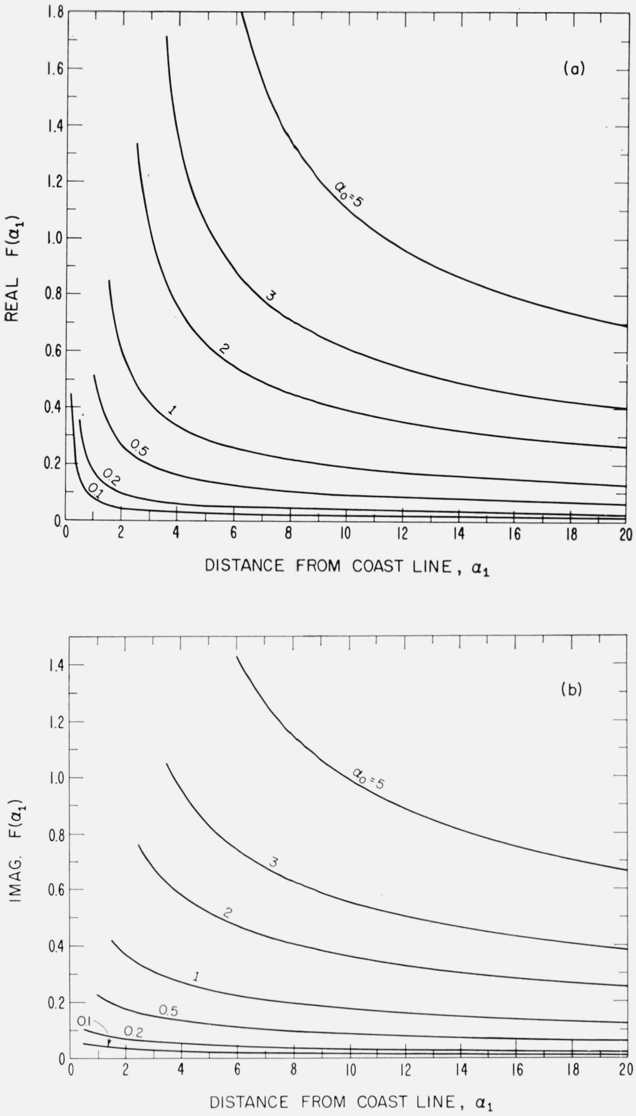

FIGURE 7. *Real and imaginary parts of* F(α_1) *for positive values of* $(\alpha_1-\alpha_0)$ *for various values of* α_0.

where $\Delta z_m/z_m$ is the relative perturbation resulting from the conductivity contrast and $\delta z_m/z_m$ is the relative perturbation resulting from the sloping beach.

Following the line of reasoning given in part I, the total refraction or bearing error $\delta\theta_T$ may be expressed in the form

$$\delta\theta_T = \delta\theta + \delta\theta', \tag{12}$$

where $\delta\theta$ is given explicitly by eq (36) of part I, while

$$\delta\theta' \cong S_1 C_1 \frac{\partial}{\partial \alpha_1} \operatorname{Im}\left(\frac{\delta z_m}{z_m}\right). \tag{13}$$

Restricting attention to the region beyond the coastline (away from the transmitter) and employing eq (6) it follows, without difficulty, that

$$\delta\theta' \cong \frac{S_1 C_1}{2}\left(\frac{z_0}{d_0}\right)[\sin \alpha_1 Y_1(\alpha_1) + \cos \alpha_1 J_1(\alpha_1)$$
$$-\sin(\alpha_1-\alpha_0) Y_1(\alpha_1-\alpha_0) - \cos(\alpha_1-\alpha_0) J_1(\alpha_1-\alpha_0)], \tag{14}$$

where the symbols have their usual meaning. In particular, J_1 and Y_1 are the Bessel and Neumann functions, respectively, of order one. Provided $(\alpha_1-\alpha_0)>>1$, the above result may be simplified to

$$\delta\theta' \cong \frac{S_1 C_1}{2}\left(\frac{z_0}{d_0}\right)\left[\frac{1}{[\pi(\alpha_1-\alpha_0)]^{\frac{1}{2}}} - \frac{1}{[\pi\alpha_1]^{\frac{1}{2}}}\right]$$
$$= \frac{\sin\theta_0(\cos\theta_0)^{\frac{1}{2}}}{2\pi^{\frac{1}{2}}}\left(\frac{z_0}{d_0}\right)\left[\frac{1}{[k(d_1-d_0)]^{\frac{1}{2}}} - \frac{1}{[kd_1]^{\frac{1}{2}}}\right]. \tag{15}$$

If, in addition, $d_1>>d_0$ corresponding to observations sufficiently far from the coastline, the above formula simplifies, even further, to

$$\delta\theta' \cong \frac{\sin\theta_0(\cos\theta_0)^{\frac{1}{2}}}{4\pi^{\frac{1}{2}}} \frac{z_0}{d_1} \frac{1}{(kd_1)^{\frac{1}{2}}}, \tag{16}$$

which is independent of d_0.

It is interesting to note that $\delta\theta'$ varies approximately as $(d_1)^{-3/2}$, whereas $\delta\theta$ varies approximately as $(d_1)^{-1/2}$. Thus, it is concluded that the bearing error resulting from topographical features at the coastline is probably of minor consequence in comparison with the influence of conductivity contrast.

5. References

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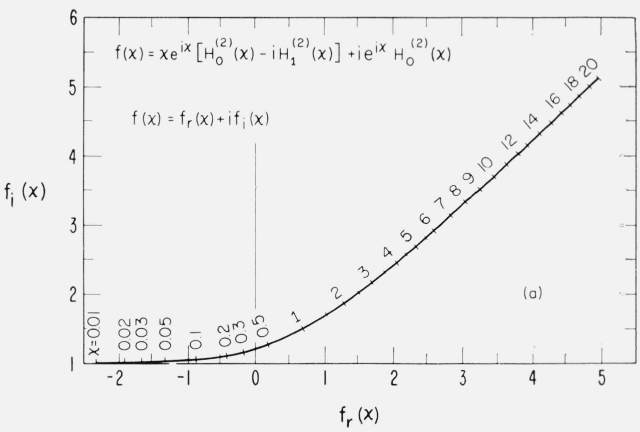

FIGURE 8. *Argand plots of the function* f(χ) *and* g(χ).

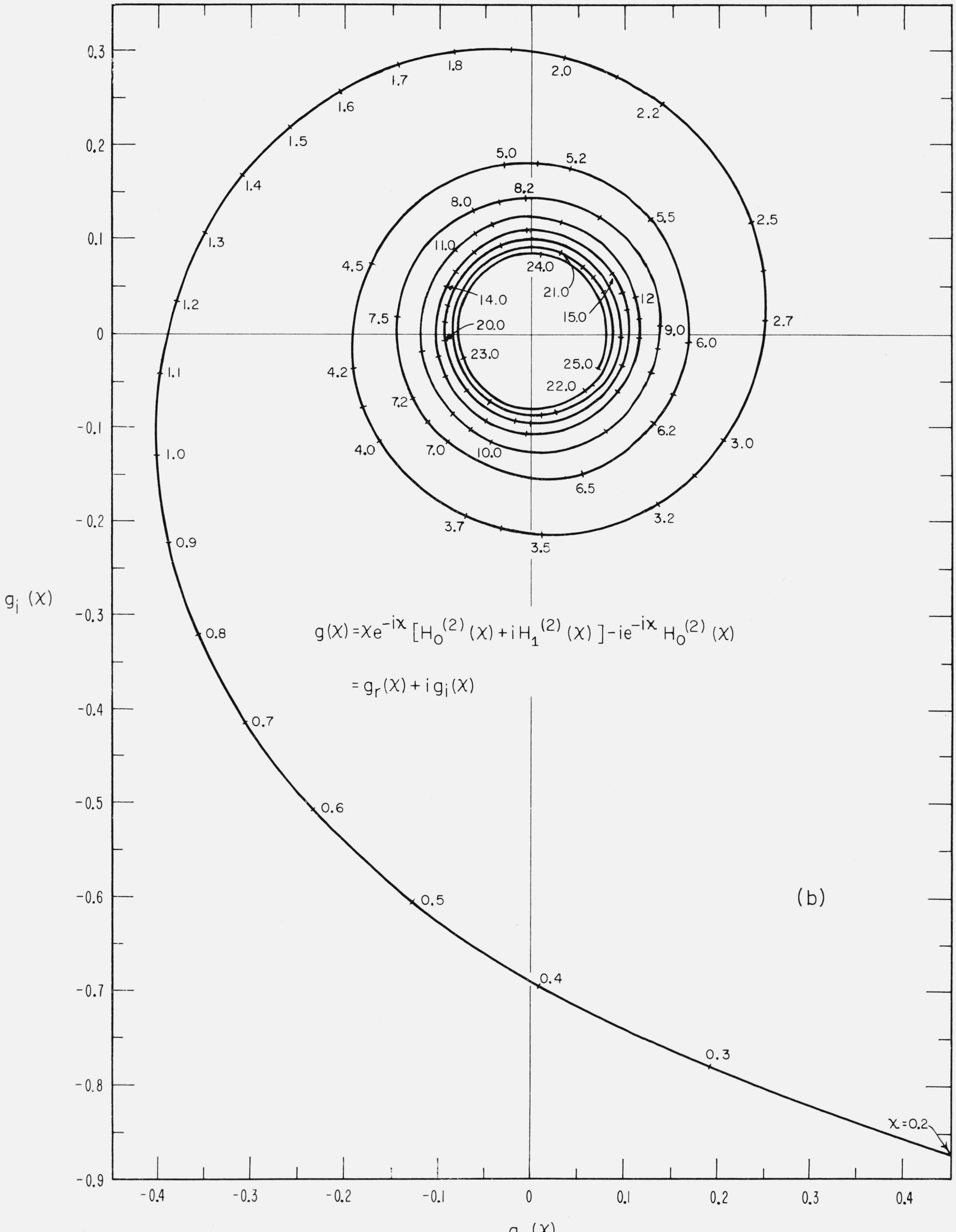

FIGURE 8. *Argand plots of the function* f(χ) *and* g(χ)—Continued.

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