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VLF Superdirective Loop Arrays

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Superdirectivity may be achieved with short VLF loop arrays because the beamwidth depends only upon the number of loops and not the length of the array. In addition the usual factors limiting superdirectivity are not so prevalent due to the decoupling between VLF loops.

Expressions are derived for the beamwidth, effective height, reception pattern, amplitude and position of the back lobes and the effects of loop voltage phase and amplitude differences between loops. These equations describe short arrays of any number of loops. The most serious limitation on the directivity of superdirective loop arrays is the voltage phase and amplitude differences between loops. These differences between adjacent loops add up to obscure the nulls and deteriorate the reception pattern.

List of Symbols

E_ϕ=relative voltage received from direction ϕ compared to the voltage from one loop.
ϕ=angle of received signal in the horizontal plane measured from the plane of the loops.
D=distance between loops.
λ=free space wavelength.
δ=delay time between loops.
n=number of loops in the array.
v_0=velocity of light.
ϕ_0=null position.
$2\phi_A$=half power beamwidth.
ϕ_1=side lobe maximum position.
R_0=ratio of front lobe to back lobe amplitude.
R_1=ratio of front lobe to side lobe amplitude.
h_e=effective height of the array compared to one loop in db's.
E_L=amplitude of the voltage from one loop.
E_r=null voltage.
Δe_{12}=voltage amplitude difference between loop 1 and 2.
$\Delta \theta_{12}$=phase difference between loop 1 and 2 in radians.

1. Introduction

The superdirective antenna with the possibility of infinite gain with an infinitesimal small antenna has been discussed by several authors [Schelkunoff, 1943; Schelkunoff and Friis, 1952; Riblet, 1948; Taylor, 1948; Yaru, 1951; Stearns, 1961]. These authors have pointed out that such arrays are impractical due to basic limitation such as narrow bandwidth, high losses, and critical tolerances. Moderate superdirectivity has been achieved [Elliott, 1054; Spitz, 1959] with practical arrays. Spitz shows that superdirectivity can be obtained from radiators that are decoupled from one another. The elements of receiving arrays at VLF can very easily be decoupled from each other since they are so small compared to the wavelength.

At very low frequencies it is quite difficult to obtain high directive antenna patterns. Large tracts of land are needed due to the long wavelengths involved. It appears that arrays can be greatly reduced in size by using the principle of superdirectivity. Considerable directivity can be achieved with a superdirective loop array only a small fraction of a wavelength long. These superdirective receiving antennas are realizable at VLF because the limiting factors such as narrow bandwidth and high losses are minute due to the decoupling between loops [Seeley, 1963]. The critical tolerance of individual loop voltages is the factor that limits the number of loops that can be used and therefore the directivity.

The characteristics and performance of short two- and three-loop arrays have been presented previously [Friis, 1925; Seeley, 1963]. This paper will extend their work to present the radiation pattern characteristics, the effective height, and the effect of phase and amplitude errors on the short n-loop superdirective array.

2. Radiation Pattern Characteristics

The important characteristics of the pattern of the n-loop array will be deduced from corresponding equations of the two- and three-loop arrays. Equations for the position of the side lobes and nulls, the beamwidth, and ratios of side and back lobe to front lobe will be presented.

The horizontal pattern of a horizontal array with the planes of the loops oriented in a vertical plane such as in figure 1 is [Seeley, 1963]

$$|E_\phi|=\cos\phi\left[\frac{2\pi D}{\lambda}(\cos\phi-\cos\phi_0)\right] \tag{1}$$

for two-loop array and

$$|E_\phi|=\cos\phi\left[\frac{2\pi D}{\lambda}(\cos\phi-\cos\phi_0)\right]^2 \tag{2}$$

for three-loop array, where $D<<\lambda$ and there is $(\delta-\pi)$ phase difference between identical adjacent loops. By mathematical induction the pattern of n-loop array is

$$|E_\phi|=\cos\phi\left[\frac{2\pi D}{\lambda}(\cos\phi-\cos\phi_0)\right]^{n-1}. \tag{3}$$

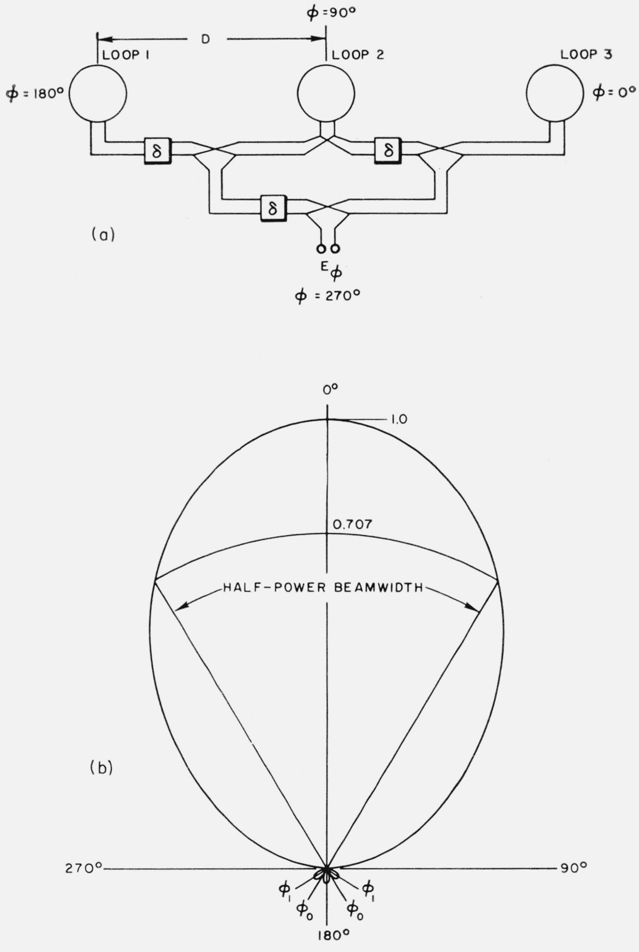

FIGURE 1. *Superdirective loop array.*

Equation (3) is expressed in terms of distance between loops in wavelengths, D/λ, and the null position, ϕ_0, located between the back lobe and the side lobes (see fig. 1). The null position depends upon the delay between adjacent loops and the free space propagation time between loops

$$\phi_0=\text{arc}\cos\frac{\delta}{D/v_0} \tag{4}$$

for the n-loop array. It may be moved about the back half of the pattern to reject unwanted signals by varying the delay (δ) between adjacent loops.

The beamwidth is a measure of the directivity of an antenna. The half-power beamwidth ($2\phi_A$) of the n-loop array in terms of the null position is

$$\cos\phi_A=0.707\left[\frac{1-\cos\phi_0}{\cos\phi_A-\cos\phi_0}\right]^{n-1} \tag{5}$$

when $D<<\lambda$. The narrowest beamwidth occurs when the null position approaches 90°. Then

$$2\phi_A=2\text{ arc}\cos\sqrt[n]{0.707}. \tag{6}$$

When the null position is at 180° the beamwidth is broadest. The beamwidths at these two extremes are plotted in figure 2 as a function of the number of loops in the array. There is a spread of only about 10° between the extremes. The beamwidth is not a function of the distance between the loop and herein lies the possibility of obtaining superdirectivity.

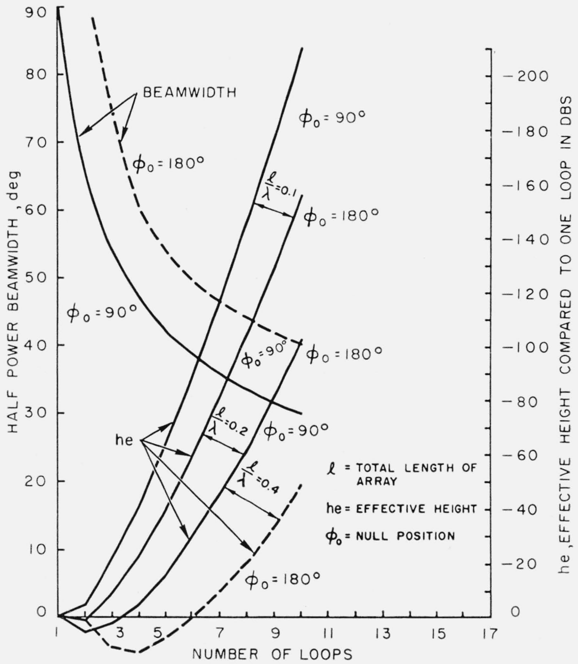

FIGURE 2. *Superdirective loop arrays.*

The amplitudes of the back lobes are an indication of the undirectional properties of the array pattern. There are three back lobes, one at $\phi=180°$ and two symmetrically located about this one (see fig. 1). The lobe at $\phi=180°$ will be called the back lobe and the two lobes on either side of the back lobe are called the side lobes. The ratio of the front to back lobe derived from (3) is

$$R_0=\left[\frac{1-\cos\phi_0}{1+\cos\phi_0}\right]^{n-1}. \tag{7}$$

It is obvious from (7) that a large number of loops in the array cause the back lobe to be much smaller than the front lobe. The front-to-side lobe ratio can be derived also from (3). But first the position of the side lobe maximum must be determined by differentiating (3) and equating the results to zero. Having done this the position of the side lobes is

$$\phi_1 = \arc\cos\left(\frac{1}{n}\cos\phi_0\right). \tag{8}$$

Equation (8) can now be used in conjunction with (3) to derive the front-to-side lobe ratio, which is

$$R_1 = \left(\frac{n}{\cos\phi_0}\right)^n \left[\frac{1-\cos\phi_0}{1-n}\right]^{n-1}. \tag{9}$$

From (7) and (9) it is obvious that the loop array patterns become very unidirectional as the number of loops is increased. This is important in applications where the loop array is in a field of multiple sources such as sferics at VLF.

3. Effective Height

Quite narrow beams can be achieved with a moderate number of loops (see fig. 2), but in return the effective height is reduced. The effective height (in db's) of an n-loop array derived from (3) is

$$h_e = 20(n-1)\log_{10}\left[\frac{2\pi D}{\lambda}(1-\cos\phi_0)\right]. \tag{10}$$

This is the effective height compared to one loop. The effective height is plotted versus the number of loops for three array lengths in figure 2. The area between equal-array-length curves indicates the range of effective heights resulting from positioning the null from 90° to 180°.

The very small effective heights of the shorter arrays could make them impractical unless very high effective loops are used to make up the array. Large ground return inverted loops or perhaps short Beverage antennas would be a practical element to use because of their large effective heights.

4. Effect of Phase and Amplitude Errors

The analysis above has assumed signals of equal amplitude and the proper phase from each loop to cancel at the null angles. As the number of loops is increased any departure from this assumption will reduce the null depths. An analysis of the null voltage with small loop voltage phase and amplitude inequalities has been made [Seeley, 1963]. The resultant null voltage is

$$E_r = \left|\sum_2^n \Delta e_{n-1,n} - jE_L \sum_2^n \Delta\theta_{n-1,n}\right| \tag{11}$$

which is the sum of the amplitude differences between adjacent loops in quadrature with the sum of the phase differences between adjacent loops. These voltage differences between adjacent loops will tend to deteriorate the reception pattern as the number of loops is increased. The feasibility of a practical array will depend on how accurately the individual loops and delay lines can be matched. This can only be determined experimentally.

5. Conclusion

Superdirectivity may be achieved with short VLF loop arrays because the beam width is not a function of the length of the array but the number of loops in the array. Also, the usual superdirective limiting factors such as narrow bandwidth and high losses are minute due to the decoupling between loops at these long wavelengths. The directivity is limited by the critical tolerance of adjacent loop voltages.

The effective height and all the pattern characteristics of the short array such as beamwidth and back lobe amplitude and position can be expressed in terms of the selected null position and the distance between loops for a given number of loops. The assumption is made that the distance between loops is much smaller than a wavelength, which is valid at VLF. The directivity is increased by the number of loops used in the array. Equations (6), (7), and (9) bear this out in that the beamwidth and back lobes become smaller for increasing number of loops.

The two limiting factors on the directivity are effective height and unequal voltages between loops. The very small effective heights of the shorter arrays could make them impractical unless very large loops are used. The most serious limitation on directivity of arrays with large number of loops is the voltage amplitude and phase differences between loops. These differences between adjacent loops will obscure the nulls and deteriorate the reception pattern. Equation (11) shows that the differences add up as the number of loops is increased. The feasibility of designing highly directive loop arrays will depend on the accuracy with which the individual loops and delay lines can be matched. This can only be determined experimentally.

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