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Reflection of VLF Radio Waves From an Inhomogeneous Ionosphere.[1] Part II. Perturbed Exponential Model

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The oblique reflection of radio waves from a continuously stratified ionized medium is considered. In this paper the medium is assumed to be isotropic. The height profile of the effective conductivity is a Gaussian curve superimposed on the (undisturbed) exponential form. The reflection coefficient is shown to be influenced by the vertical location of the Gaussian perturbation. In some cases the magnitude of the reflection coefficient is increased while, in other situations, it is decreased. In nearly all cases, insofar as phase is concerned, the presence of the perturbation corresponds to a lowering of the reflection height.

1. Introduction

In a previous communication from the present authors [Wait and Walters, 1963], oblique reflection of (VLF) radio waves from a continuously stratified ionized medium was considered. The profile of the effective conductivity was taken to be exponential in form. Actually, this is a fairly good representation of the actual D layer of the ionosphere under daytime conditions. Henceforth, that paper will be referred to simply as (I).

It is the purpose of the present paper to consider profiles which are no longer exponential in form. Since the objective is to gain insight into the mechanism of reflection from perturbed layers, a number of idealizations are made. First, it is assumed, under quiescent conditions, that the ionospheric conductivity varies exponentially with height. Then the idealized perturbation is assumed to have a Gaussian form. Again, for sake of simplicity, the earth's magnetic field is neglected as in (I). This is well justified when considering effects which result from ionization in the lowest ionosphere.

2. Description of the Profile

The notation follows that used in (I) as closely as possible. Thus the undisturbed profile, as a function of height z, is defined by the conductivity parameter $1/L(z)$ where

$$\frac{1}{L(z)}=\frac{1}{L}\exp(\beta z), \tag{1}$$

and L is a constant, β is a gradient parameter and z is the height above the reference level $z=0$. Under the isotropic assumption, it is known that [Wait, 1962]

$$L=\frac{\omega(\nu+i\omega)}{\omega_0^2}, \tag{2}$$

in terms of the angular frequency ω, collision frequency ν, and plasma frequency ω_0. At VLF, $\nu>>\omega$, and therefore

$$L\simeq\frac{\omega}{\omega_r} \text{ where } \omega_r=\frac{\omega_0^2}{\nu}, \tag{3}$$

to within a very good approximation.

In general, it is seen that $\frac{1}{L(z)}$ is proportional to $\frac{N(z)}{\nu(z)}$ where $N(z)$ and $\nu(z)$ are the electron density and collision frequency regarded as a function of height. The constant β, in the exponent, is a measure of the sharpness of the gradient. For example, $\beta=1$ km^{-1} means that the ratio of $\omega_r(z)$ or $N(z)/\nu(z)$ increases by 2.71 for each km of vertical height. From the recent work of Barrington et al. [1962], Kane [1962], and Belrose [1963], it appears that β for an undisturbed ionosphere may be in the range from 0.2 to 0.8. If the level from about 60 km to 70 km is considered, it appears that $\beta=0.3$ typifies many of these daytime D-layer profiles. A detailed study of the influence of changing β is to be found in (I). For this paper, β is chosen to be 0.3.

Having specified our undisturbed profile, we now wish to introduce the perturbation. It is assumed that the collision frequency profile is unchanged whereas the ionization is to be increased by an amount $\Delta N(z)$ where

$$\Delta N(z)=\Delta N_0 \exp\left[-\left(\frac{z-F}{D}\right)^2\right], \tag{4}$$

and ΔN_0, F and D are constants. Clearly, the maximum value of $\Delta N(z)$ is ΔN_0 which is located at $z=F$. Furthermore, the thickness of this layer is $2D$ which is the vertical distance between the levels where $\Delta N(z)$ drops to $\Delta N_0/\epsilon$.

In order to estimate correctly the influence of this Gaussian shaped layer, it is necessary to assume

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something about the collision frequency profile. A careful study of the recent literature indicates that an exponential variation of $\nu(z)$ with height z is not unreasonable. The form chosen here is

$$\nu(z)=\nu_0 \exp\left(-\frac{\beta}{2}z\right), \tag{5}$$

where $\beta=0.3$ km^{-1}. Therefore, the resulting conductivity perturbation has the form

$$\frac{\Delta N(z)}{\nu(z)}=\frac{\Delta N_0}{\nu_0}\exp\left(\frac{\beta}{2}z\right)\exp\left[-\left(\frac{z-F}{D}\right)^2\right]. \tag{6}$$

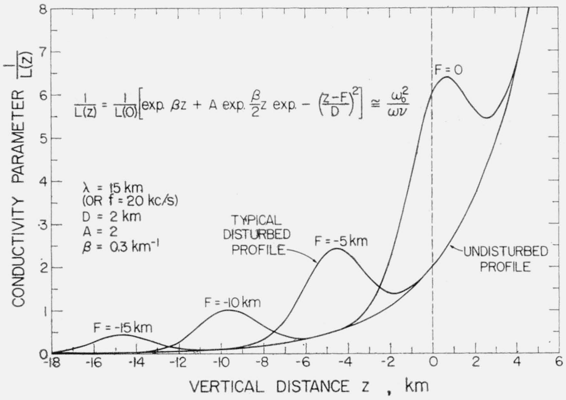


FIGURE 1. *The undisturbed and disturbed conductivity profiles used in this paper.*

The complete profile, under these idealized conditions, is given by

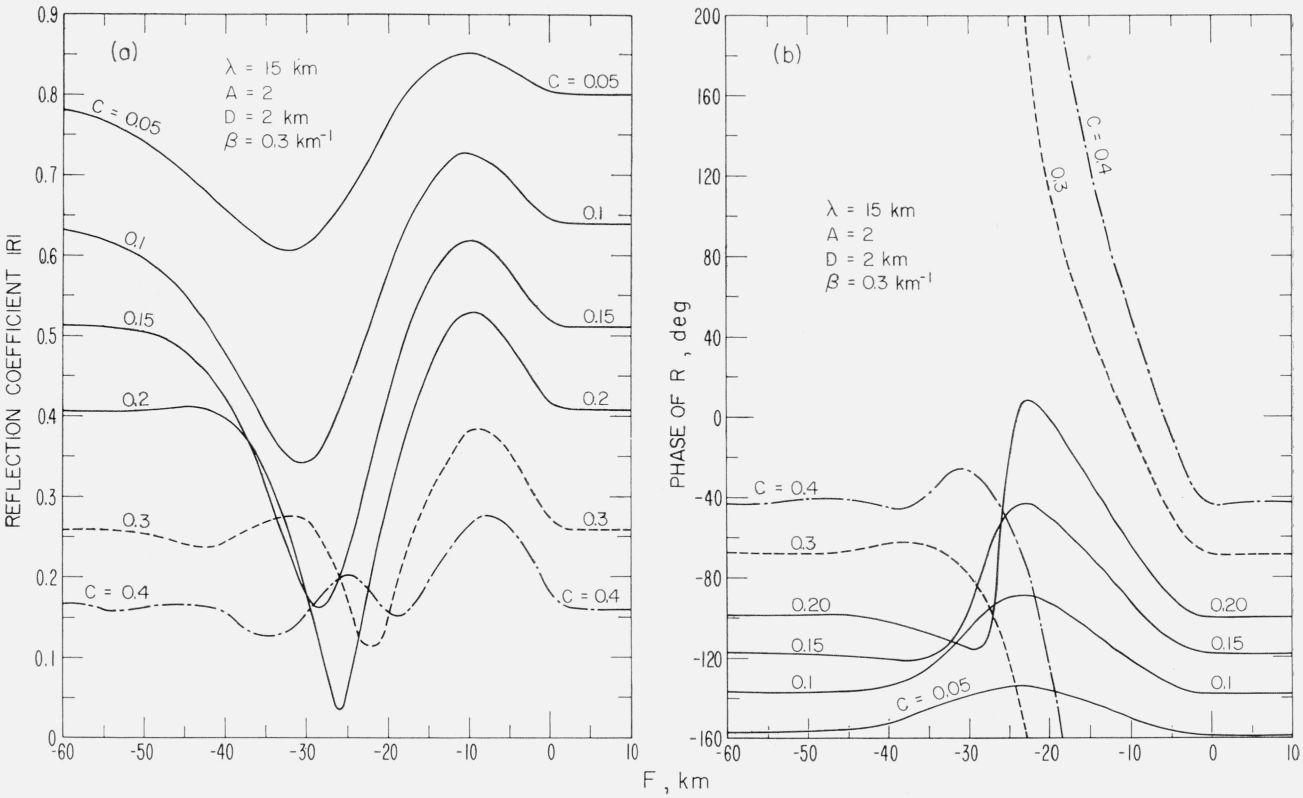
$$\frac{1}{L(z)}=\frac{1}{L(0)}\left[\exp(\beta z) + A\exp\left(\frac{\beta}{2}z\right)\exp\left[-\left(\frac{z-F}{D}\right)^2\right]\right], \tag{7}$$

where the right-hand side is proportional to the effective conductivity of the medium as a function of height above (or below) the reference level at $z=0$. The coefficient A defines the strength of the perturbation. In fact,

$$A=\frac{\Delta N_0}{N_0},$$

where N_0 is the electron density of the undisturbed profile at the reference level $z=0$. In this paper, as in (I), $L(0)=7.5/\lambda$ where λ is the wavelength in kilometers.

It is admitted that other ways to define a perturbation in the profile may be preferable. Here the electron density anomaly, for a given value of A, does not change with its vertical location F. Consequently, we may anticipate that the influence of this type of perturbation will be diminished at sufficiently low heights because of the increasing



FIGURES 2a and b. *The reflection coefficient as a function of the vertical location,* F, *of the Gaussian perturbation, for various angles of incidence.*

collision frequency. However, we shall see the problem is not quite this simple as other factors come into play.

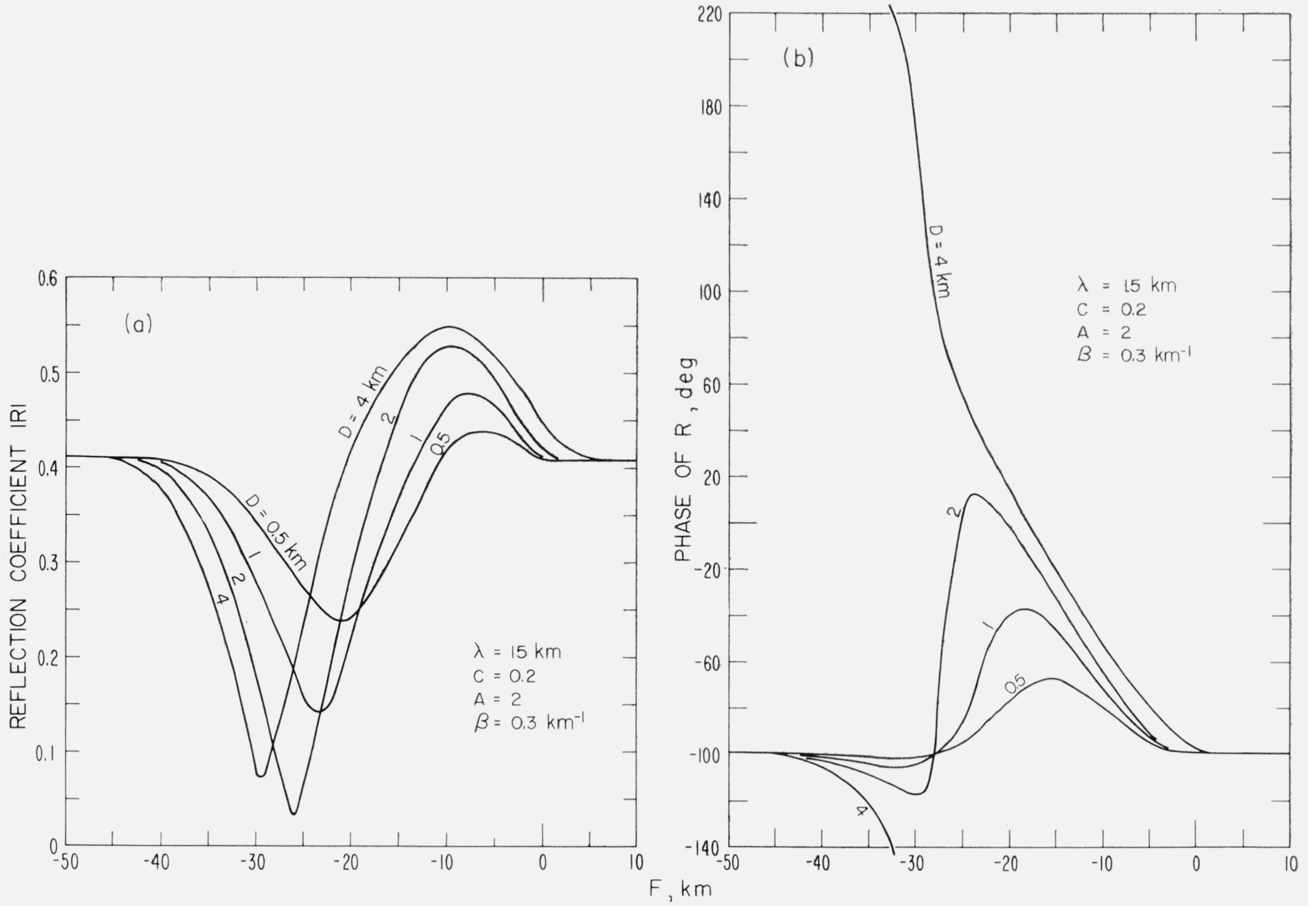
A sketch of the profiles used is given in figure 1. The undisturbed profile is the exponential form, while the disturbed profiles have the superimposed Gaussian "bump." The location of the "bump" for five typical profiles is specified by the appropriate value of F.

3. Results of the Calculations

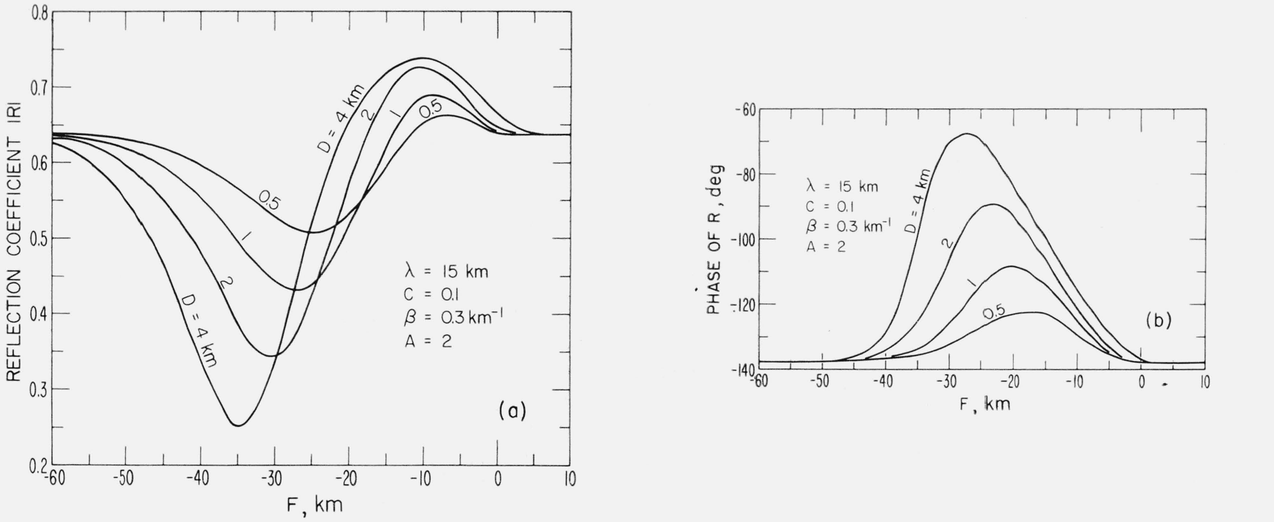
The method used to calculate the reflection coefficient R has been described in detail in (I). The quantities considered are the amplitude $|R|$ and the phase of R for a vertically polarized plane wave incident at an angle whose cosine is C. The reflection coefficient is evaluated in the free space region corresponding to $z \rightarrow -\infty$. However, it is important to remember that the *phase is referred to the level* $z=0$.

The plan of the calculations is to vary the value of one parameter while keeping the others constant. To obtain a complete understanding of the various phenomena, an enormous number of calculations is needed. In order to keep the problem within reasonable bounds and to reduce the expense of the computation, only a limited number of cases was considered. These results are shown in graphical form in figures 2 to 6. In all cases $\beta=0.3$ km^{-1}.

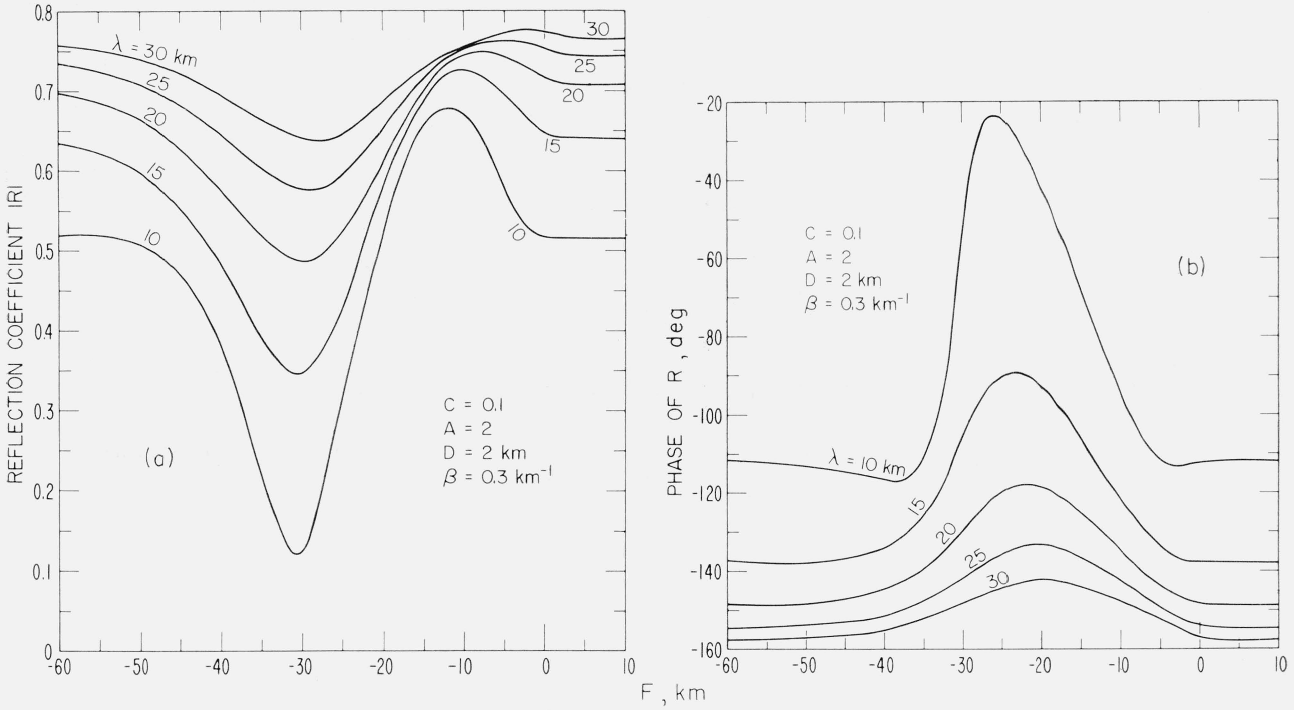
In figure 2a the amplitude of the reflection coefficient is plotted as a function of F for $\lambda=15$ km ($f=20$ kc/s), $A=2$, $D=2$ km, and C values varying from 0.05 to 0.4. Small values of C here correspond to angles near grazing. For long distance propagation of VLF radio waves, values of C near 0.1 are most important. For this case, it is interesting to note, when F is near or above zero, that $|R|$ takes the same value as for the undisturbed profile. As the "bump" or perturbed layer is lowered, the reflection coefficient first increases then decreases. Eventually, as the "bump" is brought down to very low heights, $|R|$ returns to its undisturbed value. The other curves for highly oblique incidence have a similar behavior. Thus, the "bump" may either improve or degrade the reflection. Presumably, at the lower heights the Gaussian layer is acting as an absorber whereas, at greater heights, it enhances the reflection. At the steeper angles of incidence, the situation becomes more complicated. It is probable that this results from interference between multiple reflected rays between the upper side of the "bump"



FIGURES 3a and b. *The reflection coefficient, as a function of* F, *for various widths of the Gaussian pertubation when* C=*0.2 (i.e., angle of incidence is 78°).*



FIGURES 4a and b. *The reflection coefficient, as a function of* F, *for various widths of the Gaussian perturbation when* C=*0.1 (i.e., angle of incidence is 84°).*

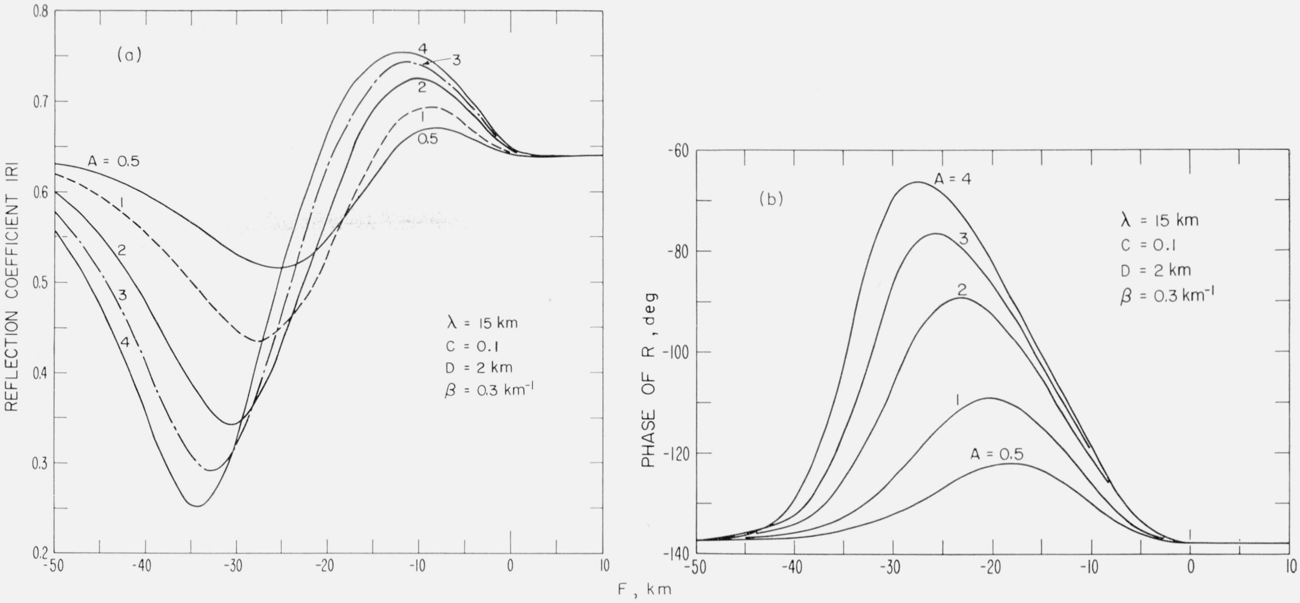


FIGURES 5a and b. *The reflection coefficient, as a function of* F, *for various wavelengths (from 10 to 30 km).*

and the exponentially varying layer. Such an interference phenomenon becomes more pronounced at steeper incidence because the vertical component of the wavelength is becoming comparable with typical values of F.

The phase of R is shown in figure 2b for the same conditions as in figure 2a. Again, it is apparent that, when the Gaussian "bump" is such that F is near 0 or above, the phase of R attains its undisturbed value. When the angle of incidence is highly oblique the phase undergoes an increase (i.e., decrease of lag) as the "bump" comes down to lower heights. Sufficiently far below the reference level, the phase of R returns to its undisturbed value. It is well to note that as C becomes small (i.e., approaching grazing incidence), the phase of R is approaching $-180°$.

For highly oblique incidence the influence of the "bump" is to lower the effective height of reflection for the whole range of F. However, a very interesting phenomenon occurs at steeper incidence. As can be seen in figure 2b, when $C=0.2$ the phase undergoes a rather rapid change as F varies from about -22



FIGURES 6a and b. *The reflection coefficient, as a function of* F, *for various amplitudes* A *of the perturbation.*

km to −27 km. As C is increased further there is an apparent discontinuity when the phase changes by 360°. Such a change of 2π radians is quite permissible since the ordinate is arbitrary to within any integral number of 2π radians. Thus, the phase curves for $C=0.3$ and 0.4 could have been drawn in the range below −160°.

The curves in figure 2b, even if they show nothing else, demonstrate that phase shifts in reflection phenomena may have some unusual cycle ambiguities.

The influence of the width of the Gaussian perturbation or "bump" is shown in figure 3a for the amplitude $|R|$ and in figure 3b for the phase of R. Here $A=2$, $\lambda=15$ km, and $C=0.2$. The amplitude curves show that, when D is increased, the overall influence of the layer becomes somewhat greater. There is some tendency for the thinner layers (i.e., smaller D) to be more effective at greater heights. The corresponding phase curves show that the thicker layers always produce a larger phase change. Furthermore, as D exceeds 2 km, a point is reached where the "360° jump" takes place.

The curves in figures 4a and b are for the same conditions as figures 3a and b except that now $C=0.1$, corresponding to nearer grazing incidence. The amplitude curves have a very similar shape. The phase curves are also similar except that the "360° jump" is no longer present.

The wavelength dependence of the reflection coefficient is shown in figures 5a and b. For these $C=0.1$, $A=2$, and $D=2$. The wavelengths chosen (10, 15, 20, 25, 30km) correspond to frequencies of 30, 20, 15, 12, and 10 kc/s. Qualitatively, the curves have a very similar shape. There is some tendency for the shorter wavelengths to be accompanied by more pronounced changes. In all cases the "bump" acts as an absorber at low heights while it enhances the reflection at greater heights.

Finally, in figures 6a and b, the influence of A, the relative magnitude of the anomalous electron density, is shown. As expected, the individual curves are similar in shape with the larger values of A corresponding to an increased change over the undisturbed values. It is important to note that the phase anomaly is almost directly proportional to A.

4. Final Remarks

The results given here constitute a small portion of extensive computations dealing with reflection of waves from inhomogeneous media. In subsequent parts to this series other types of profiles will be considered. Also, the applications to the mode theory of VLF propagation are to be described in some detail.

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5. References

Barrington, R., B. Landmark, O. Holt, and E. Thrane (June 1962), Experimental studies of the ionospheric D-region, Report No. 44, Norwegian Defence Research Establishment, Kjeller, Norway.

Belrose, J .S. (1963, Low-frequency propagation (to be submitted to Proc. IEE, London).

Kane J. A. (1962), Re-evaluation of ionospheric electron densities and collision frequencies derived from rocket measurements, Chap. 29 in Radio Wave Absorption in the Ionosphere (Pergamon Press, Oxford).

Wait, J. R. (1962), Electromagnetic waves in stratified media (Pergamon Press, Oxford).

Wait, J. R., and L. C. Walters (May–June 1963), Reflection of VLF radio waves from an inhomogeneous ionosphere. Part I. Exponentially varying isotropic model, J. Research NBS **67D** (Radio Prop.) No. 3, 361–367.

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