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Optimum Reception Pattern of the Beverage Wave Antenna at Very Low Frequencies

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The theoretical reception pattern characteristics of the wave antenna at VLF, plus an experimental study of the parameters that determine the patterns, are presented. Wave antennas have optimum lengths where the front-to-back ratio is greatest. The first optimum length (highest front-to-back ratio for the shortest length) is considered in this paper. The patterns of the wave antenna at these optimum lengths depend upon only two parameters which can easily be measured at the desired antenna location. These two parameters are the antenna loss and the wave velocity along the antenna. When these two parameters are known, the patterns of all first optimum length wave antennas can be sketched from the curves in this paper.

1. Introduction

In recent years there has been a resurgence of interest in very low radiofrequencies. Very-low-frequency uses that were once regional are now global, indicating a need for long range communication and other long-range electromagnetic systems. Very low frequencies are particularly suited for these systems because of the low propagation losses. The disadvantages in using these frequencies are the high noise environment and the difficulty of radiating power. This makes it imperative that receiving antennas with high directivity and large effective heights be used.

At very low frequencies where wavelengths are measured in kilometers it becomes extremely difficult to obtain high directivity and great effective heights from low structures. The Beverage or wave antenna is one of the simplest structures capable of overcoming this difficulty. In the simplest form it consists of an insulated wire laid on the ground and terminated at both ends by its characteristic impedance.

This paper consists of an analytical study of the horizontal pattern characteristics of the wave antenna plus an experimental study of the parameters that determine the radiation pattern.

2. Pattern Characteristics

The reception pattern of the wave antenna has been derived by Beverage, Rice, and Kellogg [1923], Bailey, Dean, and Wintringham [1929], Wait [1954], and Belousov and Yampolshii [1960], by summing up the incremental currents induced in a long horizontal wire by a tilted wave front of vertically polarized energy. These currents flow toward both ends of the long wire which is terminated in its characteristic impedance, shown in figure 1. The power pattern equation is

$$P_\theta=\frac{l^2E_0^2e^{-\alpha l}\cos^2\theta[\cosh\alpha l-\cos[\beta l(1-n\cos\theta)]]}{2Z_0[(\alpha l)^2+(\beta l)^2(1-n\cos\theta)^2]} \tag{1}$$

where P_θ=wave antenna received power,
E_0=horizontal component of electric field parallel to the wave antenna,
θ=angle between direction of signal and direction of antenna,
$n=\frac{v}{c}$, ratio of wave velocity along antenna to free space wave velocity,

$\beta l = \frac{2\pi l}{n\lambda}$, radians,
λ=free space wavelengths,
l=length of antenna,
αl=attenuation along antenna in nepers,
Z_0=terminating impedance of wave antenna.

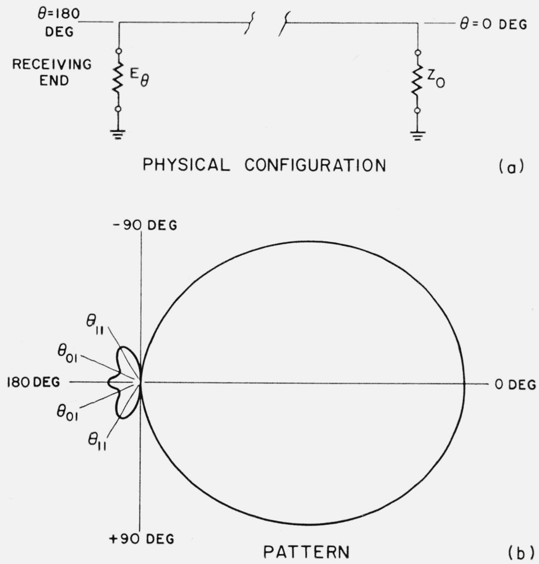


FIGURE 1. *Wave antenna.*
a. Physical configuration.
b. Reception pattern

2.1. Front-to-Back Ratio

One of the most important characteristics of the wave antenna is the front-to-back power ratio, a measure of the directivity of the antenna. This is the ratio of peak power of front to the peak power of the back lobe. High front-to-back ratios are desirable to exclude unwanted signals of atmospheric origin which are especially prevalent at VLF.

If (1) with $\theta=0°$ is divided by the same equation with $\theta=180°$ the front-to-back power ratio is

$$R_b = \frac{\cosh \alpha l - \cos [\beta l(1-n)]}{\cosh \alpha l - \cos [\beta l(1+n)]} \cdot \frac{(\alpha l)^2 + [\beta l(1+n)]^2}{(\alpha l)^2 + [\beta l(1-n)]^2}. \tag{2}$$

It should be pointed out that only a lossless antenna ($\alpha l=0$) will have a complete null at $\theta=180°$, unless some form of nulling network is used. In other words (2) could only be infinite, its denominator zero, if $\cosh \alpha l = \cos [\beta l(1+n)]$. This can only be true when α or l is zero. As the antenna loss increases the back lobe will grow until it is as large as the front lobe. This trend is borne out in figure 2.

There are optimum lengths for greatest front-to-back ratio. If (2) is differentiated with respect to antenna length and the differential set $=0$ it can be easily shown that for a lossless antenna the front-to-back ratio is greatest when

$$\frac{l}{\lambda} = \frac{Kn}{n+1} \tag{3}$$

where $K=1, 2, 3, \ldots$.

This is also a very good approximation for the optimum length of lossy wave antennas. This is shown in figure 2 where the front-to-back lobe ratio is plotted as a function of length, with constant values of n, and $\alpha\lambda$. Two different values of loss are used to show that even in the case of the very lossy antenna ($\alpha\lambda=9$) the maximum front-to-back ratio occurs at $l/\lambda=0.32$ very near the optimum predicted by (3), which would be $l/\lambda=1/3$ for the first

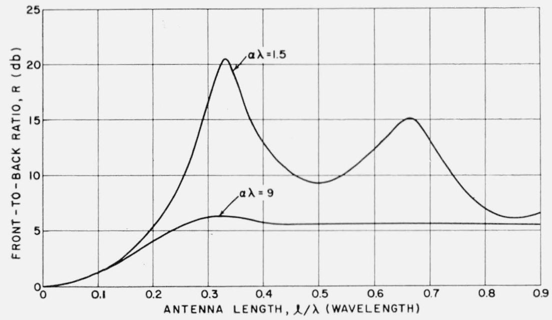


FIGURE 2. *Wave antenna front-to-back lobe ratio* n—0.5.

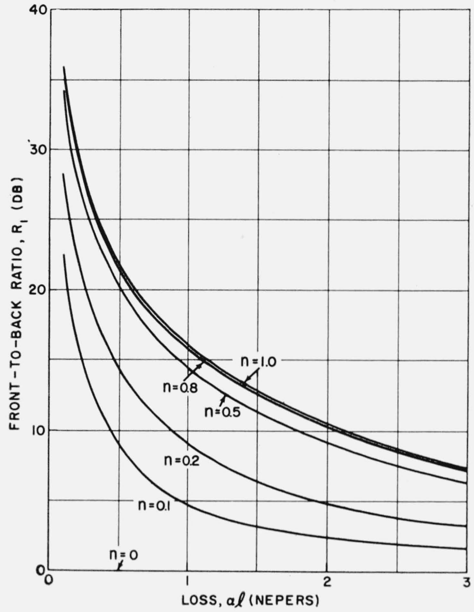


FIGURE 3. *Wave antenna front-to-back lobe ratio, first optimum length.*

optimum length. A wave antenna with that much loss is rendered almost useless by low front-to-back ratio in any case.

It is essential in determining the usefulness of the wave antenna to know the greatest front-to-back ratio that can be achieved with given values of antenna loss and wave propagation velocity. Methods of measuring these two parameters for a land area will be given in a later section. If the first optimum antenna length, $l/\lambda = n/n+1$, is substituted into (2) the greatest front-to-back ratio is obtained,

$$R_1 = \frac{\cosh \alpha l - \cos\left[2\pi\left(\frac{1-n}{1+n}\right)\right]}{\cosh \alpha l - 1} \cdot \frac{(\alpha l)^2 + (2\pi)^2}{(\alpha l)^2 + (2\pi)^2\left(\frac{1-n}{1+n}\right)^2}. \tag{4}$$

Equation (4) gives the best front-to-back ratio for the shortest length antenna. These optimum front-to-back ratios are plotted as a function of antenna loss for several values of wave velocity ratios in figure 3. Very high front-to-back ratios are obtainable for low loss antennas, approaching infinity at zero loss. But as the antenna loss becomes greater the front-to-back ratio decreases rapidly. Typical measured losses for a wave antenna lying on the ground in the southern California desert region at VLF frequencies is 0.4 nepers for the first optimum length. The αl product tends to remain constant over a wide frequency range for the first optimum length. The losses tend to increase with frequency but the optimum length decreases with frequency in a compensatory manner. Combined with a typical measured value of the wave velocity ratio of $n=0.48$ for VLF, the maximum front-to-back ratio of a wave antenna laid on the California desert region is 22db. It appears from figure 3 that the antenna wave velocity should be high and losses should be low for greatest front-to-back ratio.

2.2. Side Lobes

In addition to the back lobe there is a pair of symmetrical side lobes as shown in the typical pattern in figure 1. These side lobe peaks and associated nulls change their positions over a wide range for different values of wave velocity ratio and loss. The position and amplitude of the side lobes and nulls can be derived from (1). It is necessary to first locate the position of the side lobes and nulls and then the front-to-side lobe ratios and front lobe-to-side null ratio can be determined.

If the differential of (1) with respect to θ is set equal to zero the results are

$$\frac{\left(\frac{\alpha l}{\beta l}\right)^2+(1-n\cos\theta)^2}{\left(\frac{\alpha l}{\beta l}\right)^2+1-n\cos\theta}=\frac{\cosh\alpha l-\cos\left[\beta l\,(1-n\cos\theta)\right]}{\frac{\pi l}{\lambda}\cos\theta\sin\left[\beta l\,(1-n\cos\theta)\right]}. \tag{5}$$

Solutions of (5) for θ gives the positions of the pattern nulls and lobe peaks. In order to obtain the greatest front-to-back ratio with the shortest length antenna only the first optimum length will be considered. If $l/\lambda=n/n+1$ is substituted into (5)

$$\frac{\left[\frac{\alpha l(1+n)}{2\pi}\right]^2+(1-n\cos\theta)^2}{\left[\frac{\alpha l(1+n)}{2\pi}\right]^2+\ 1-n\cos\theta}=\frac{\cosh\alpha l-\cos\left[\frac{2\pi}{n+1}(1-n\cos\theta)\right]}{\frac{\pi n}{n+1}\cos\theta\sin\left[\frac{2\pi}{n+1}(1-n\cos\theta)\right]}. \tag{6}$$

Equation (6) has been solved for various values of wave velocity ratio and loss along the antenna. These solutions result in side lobe peaks and nulls plotted in figure 4. One important observation is that for certain values of n and αl the nulls and peaks occur at the same angle, which indicates that the side lobes disappear. Apparently the amplitude of the side lobe is below the back lobe amplitude at that angle. Another important observation is that the side lobes disappear completely regardless of the value of n when the loss, αl, is greater than 1.376. If the values of αl and n which cause the peaks and nulls to coincide are plotted versus each other, a region of side lobes and a region of no side lobes is established as in figure 5. These curves may be used to determine the existence and position of the side lobes of a wave antenna after a site has been selected and αl and n has been measured at the operating frequencies. A method of measuring αl and n is described in a later section.

Once the side lobe position is established the front-to-side lobe power ratio can be determined by dividing (1) with $\theta=0°$ by the same equation with $\theta=\theta_{11}$. The result at the first optimum length is

$$R_{11}=\frac{\cosh\alpha l-\cos\left(2\pi\frac{1-n}{1+n}\right)}{\cos^2\theta_{11}\left[\cosh\alpha l-\cos\left(2\pi\frac{1-n\cos\theta_{11}}{1+n}\right)\right]}\cdot\frac{(\alpha l)^2+(2\pi)^2\left(\frac{1-n\cos\theta_{11}}{1+n}\right)^2}{(\alpha l)^2+(2\pi)^2\left(\frac{1-n}{1+n}\right)^2} \tag{7}$$

where θ_{11} and R_{11} are the side lobe position and front-to-side lobe ratio respectively at the first optimum length. The depth of the side nulls can be obtained in a similar manner. By replacing θ_{11} with θ_{01} in (7) the front-to-side null ratio is

$$R_{01}=\frac{\cosh\alpha l-\cos\left(2\pi\frac{1-n}{1+n}\right)}{\cos^2\theta_{01}\left[\cosh\alpha l-\cos\left(2\pi\frac{1-n\cos\theta_{01}}{1+n}\right)\right]}\cdot\frac{(\alpha l)^2+(2\pi)^2\left(\frac{1-n\cos\theta_{01}}{1+n}\right)^2}{(\alpha l)^2+(2\pi)^2\left(\frac{1-n}{1+n}\right)^2} \tag{8}$$

where θ_{01} is the position of the null between back lobe and side lobe and R_{01} is the ratio of front lobe peak to null between the back lobe and side lobe. Equations (7) and (8) are plotted versus αl in figure 6 for various values of wave velocity ratios. The front-to-side lobe ratio is reduced by antenna loss similarly to the front-to-back lobe ratio but not as severely. Unlike the front-to-back lobe ratio high antenna wave velocity tends to reduce the front-to-side lobe ratio. These side lobe levels are at the first optimum length which is the shortest length for the greatest front-to-back ratio. Although the position of the side lobes varies over a considerable angular range (22°), the amplitude varies only moderately (about 3 db).

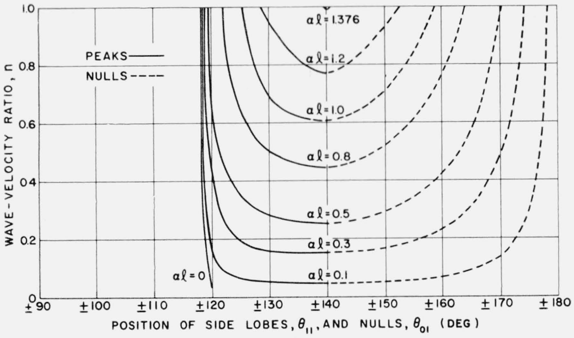


FIGURE 4. *Wave antenna side lobe and null positions, first optimum length.*

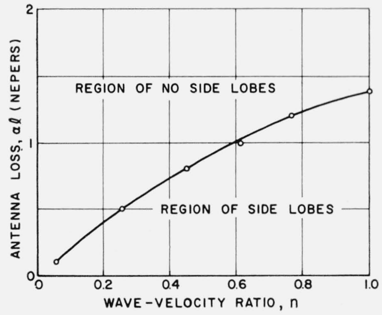


FIGURE 5. *Wave antenna side lobe region, first optimum length.*

2.3. Beamwidth

The beamwidth of the wave antenna is an important indication of its directivity. In a multisource field such as sferics it is particularly important that the main beam be as narrow as possible to exclude unwanted signals. The half-power beamwidth of the wave antenna is derived from (1). If (1) with $\theta = \theta_A$ is set equal to one-half (1) with $\theta = 0°$ the resulting expression in terms of θ_A can be solved for the half-power beamwidth ($2\theta_A$). At the first optimum length this expression reduces to

$$\frac{2\cos^2\theta_A\left[\cosh \alpha l - \cos\left(2\pi\frac{1-n\cos\theta_A}{1+n}\right)\right]}{\cosh\alpha l - \cos\left(2\pi\frac{1-n}{1+n}\right)} = \frac{(\alpha l)^2 + (2\pi)^2\left(\frac{1-n\cos\theta_A}{1+n}\right)^2}{(\alpha l)^2+(2\pi)^2\left(\frac{1-n}{1+n}\right)^2}. \tag{9}$$

Equation (9) has been solved for $2\theta_A$ with various values of antenna loss and wave velocity and the results are plotted in figure 7. There are two important conclusions that can be drawn: The beamwidth is not sensitive to antenna losses, and there is an optimum antenna wave velocity for narrowest beamwidth where the wave velocity ratio is 0.2.

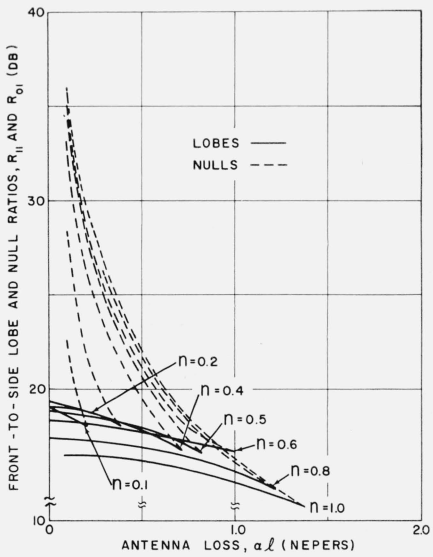


FIGURE 6. *Wave antenna front-to-side lobe and null ratios, first optimum length.*

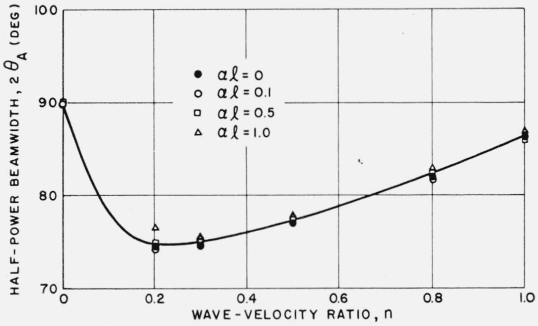


FIGURE 7. *Wave antenna beamwidth, first optimum length.*

3. Experimental Measurements

The shape of the reception or radiation pattern of the optimum length wave antenna is dependent upon only two parameters, the total loss along the antenna, αl, and the wave velocity ratio along the antenna, n. The latter is normalized to free space velocity.

These two parameters vary with the conductivity and dielectric constant of the soil under the antenna. Both parameters can easily be determined by measuring the input impedance of a portion of the antenna with the far end open circuited. Impedance maxima and minima will occur periodically if the antenna input impedance is measured over a range of frequencies. From transmission line theory, n can be calculated from the position of the impedance peaks by

$$n=\frac{2l}{K\lambda_p} \tag{10}$$

where λ_p is the wavelength at the frequency of the peak input impedance, and $K=1, 2, 3, \ldots$.

The expression for the antenna loss deduced from lossy transmission line equations is [Beverage et al., 1923]

$$\tanh \alpha l=\sqrt{\frac{Z_{\min}}{Z_{\max}}}. \tag{11}$$

The correct antenna terminating impedance, Z_0, can also be computed from the input impedance curves. It is

$$Z_0=\sqrt{Z_{\max}\, Z_{\min}}. \tag{12}$$

Impedance measurements were made on wave antennas at two different locations with a wide variety of soil characteristics. The first was located at EMDAR Oasis in the Johnson Valley region of the California desert and consisted of 20,500 ft of R_g 58 A/μ cable (only the shield used) lying on the ground in a straight line with termination ground planes at both ends. The input impedance was measured and plotted versus frequency in figure 8 for open termination on the far end. Using (10), (11), and (12) the parameters of the wave antenna are calculated and tabulated in table 1.

TABLE 1

f	$Z_{\max}$	$Z_{\min}$	Z_0	αl	n	αl (first optimum length)
kc/s				*Neper*		*Neper*
11.3	960	80	278	0.29	0.472	0.405
17.5	740	120	300	.43	.485	.386
23.0	620	185	338	.60	.479	.406
29.0	535	230	351	.77	.483	.415
34.5	470	260	350	.96	.480	.433

It can be concluded from these data that the wave velocity and loss do not vary greatly over the VLF spectrum for the optimum length wave antenna.

The second wave antenna was located at the U.S. Naval Ordnance Laboratory, Corona, Calif., and was similar to the first, except that it was shorter, 4,760 ft long. Initial input impedance curves are shown in figure 9. These data were taken before the seasonal rains began. More impedance measurements were taken during and after the rains and figures 10, 11, and 12 give some indication of the wide variation in wave antenna parameters affected by different amounts of moisture in the soil. These are qualitative measurements inasmuch as the soil conductivity was not measured to correlate it with the change in wave antenna parameters. It began raining January 28 and continued off and on throughout the measurements. Generally the more moisture in the soil the greater the antenna losses and the slower the antenna wave velocity becomes. One definite conclusion is that changes in soil moisture content will markedly change the back lobe structure of the wave antenna.

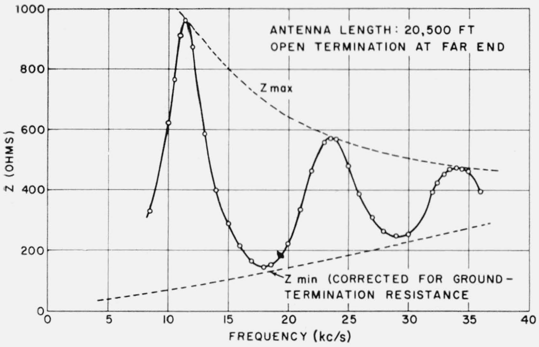


FIGURE 8. *Wave antenna impedance, antenna located at Johnson Valley field site.*

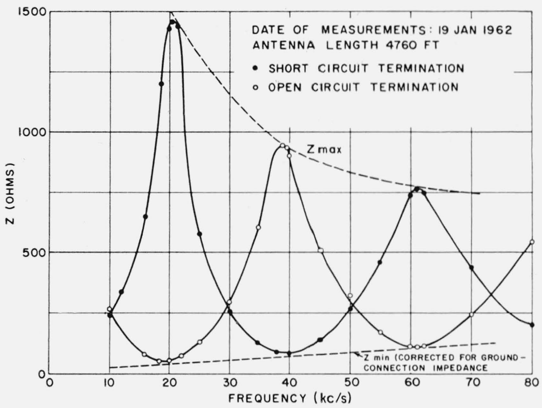


FIGURE 9. *Wave antenna impedance, antenna located at Naval Ordnance Laboratory, Corona, Calif.*

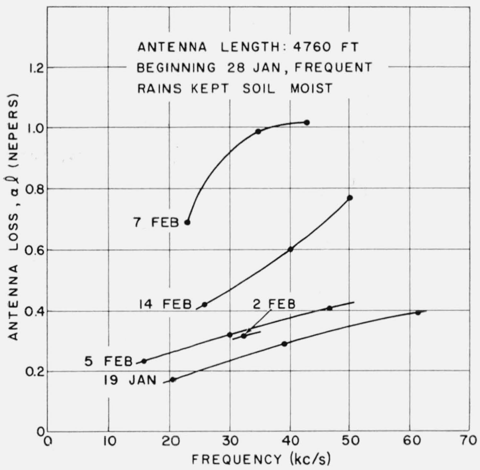


FIGURE 10. *Wave antenna losses.*

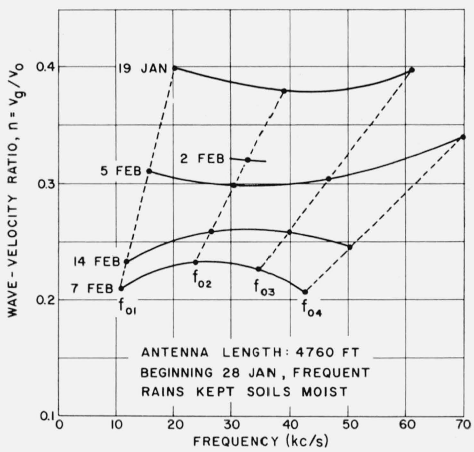


FIGURE 11. *Wave antenna wave velocity.*

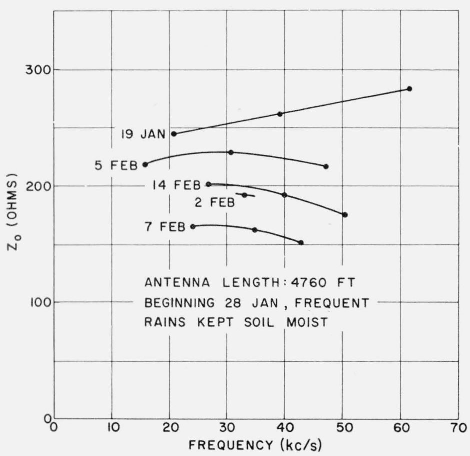
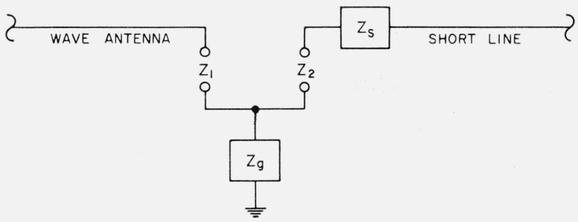


FIGURE 12. *Wave antenna characteristic impedance.*

The input impedance measurement must be corrected for the ground connection impedance since the connection to the ground side of the antenna must be made by burying wires in the earth rather than connecting to a discrete terminal which is the real ground. This ground connection impedance, Z_g, must be subtracted from the measured input impedance. A method of measuring Z_g has been devised by Bailey et al. [1929]. A short line must be laid down in the opposite direction from the wave antenna as in the figure below.

Circuit for measuring $\mathbf{Z_g}$.



The input impedance, Z_2, is adjusted, by means of Z_s, to the same value as the input impedance of the wave antenna, Z_1. Z_1, and Z_2 are then connected in parallel and the resultant impedance measured which shall be designated Z_p. It can then easily be shown that

$$Z_g=2Z_p-Z_1. \tag{13}$$

The ground connection impedances have been measured and are tabulated below in table 2.

TABLE 2. *Ground connection impedance*

Frequency	Z_g
Wave antenna at Johnson Valley	
kc/s	
10	$15+j0$
30	$19+j2$
Wave antenna at Naval Ordnance Laboratory, Corona	
10	$10+j0$
19	$8+j0$
40	$9-j2$
60	$12-j4$

4. Conclusion

The wave antenna has certain optimum lengths where the front-to-back lobe ratio is maximum. In this paper only the shortest or first optimum length is considered. The characteristics of the optimum length wave antenna depends only on antenna loss and antenna wave velocity. High loss and slow wave velocity along the antenna causes large back lobes. Antenna loss and wave velocity have moderate effects on side lobe position (22° change) and small effects on amplitude (3 db change). The side lobes become small and disappear within the back lobe for certain combinations of antenna loss and wave velocity values establishing a region of side lobes and a region of no side lobes. The wave antenna main beamwidth is affected very little by antenna loss and moderately by antenna wave velocity. The narrowest beamwidth occurs at a wave velocity ratio of 0.2.

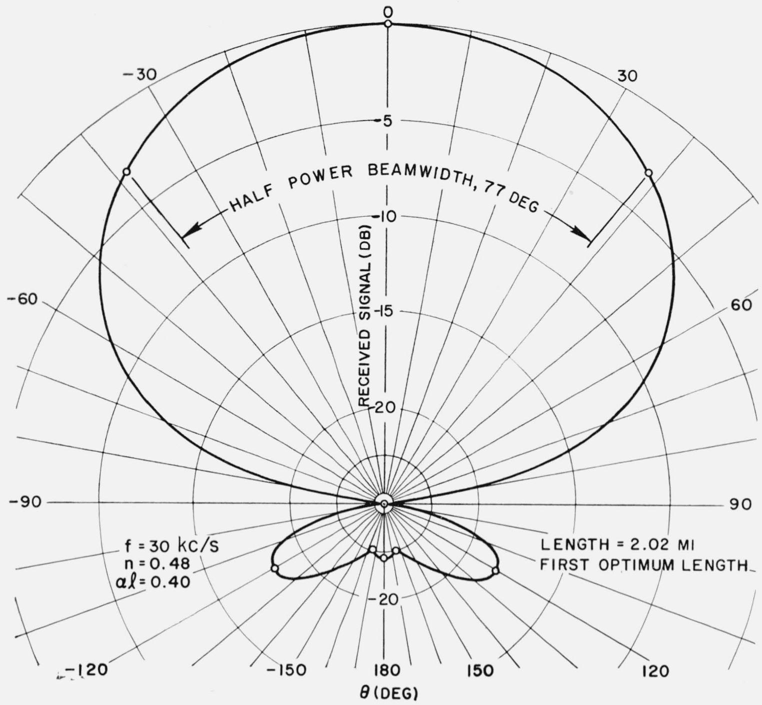


FIGURE 13. *Wave antenna reception pattern, antenna located at Johnson Valley field site.*

The patterns of all wave antennas whose lengths are the first optimum length can be sketched from the curves above. Ten points on the pattern can be taken from the curves in this paper. For example the pattern of a wave antenna in the California desert where the antenna loss $\alpha l=0.40$ and the wave velocity ratio $n=0.48$ is sketched in figure 13. Starting at $\theta=0°$ where the received signal is normalized to 0 db the next pair of points are at the half power beamwidth (−3 db), $\theta=\pm 38.5°$ taken from figure 7. There will be complete nulls at $\theta=\pm 90°$ as seen from (1). The third set of points (−18.2 db), $\theta=\pm 120.5°$ at the peaks of the side lobes and the fourth set (−22.5 db), $\theta=\pm 167°$ at the null between the side lobes and the back lobe are taken from figures 4 and 6. The last point (−22.2 db), $\theta=180°$ at the peak of the back lobe is taken from figure 3. Thus all the peaks and nulls in the pattern are located.

The parameters that determine the radiation pattern of the wave antenna can easily be measured. These parameters remain reasonably constant for the first optimum length wave antenna over the VLF spectrum. This does not mean the wave antenna pattern is broadband, for the first optimum length varies with frequency. The wave antenna pattern will change a great deal with the moisture content of the soil. For best pattern stability, the wave antenna should be used in either a very dry climate region or in a body of water where the soil conductivity does not change.

The author thanks Vernon Kamp and Ivan Rainwater for their efforts in calculating the curves in the figures.

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