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Hallén's Method in the Problem of a Cavity-Backed Rectangular Slot Antenna[1]

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The integral equation for the electric field distribution in the slot which is excited by a current source at its center is solved for the longitudinal field variation by Hallén's iteration method. The first order solution of the slot susceptance provides an agreement with computations based on the variational method for cavities as shallow as $\lambda/20$ provided the slot length exceeds $\lambda/2$. There is no agreement for very shallow cavities, where the fields are rapidly attenuated along the slot according to the variational solution. A simple closed-form approximation to the susceptance is applicable if the slot and the cavity are of equal lengths. The first-order slot conductance is accurate only for approximately $\lambda/2$ long slots which are backed by deeper cavities.

1. Introduction

The input admittance of a rectangular slot which is backed by a rectangular cavity has been recently computed using variational techniques [Galejs, 1963]. The somewhat similar problem of rectangular slots cut in waveguide walls has been treated using the Hallén's method of antenna analysis [Hallén, 1938; Watson, 1947; Stevenson, 1948]. Also slots above a dielectric half space have been considered by the same technique [Galejs, 1962].

In this note it will be attempted to compute the admittance of a rectangular cavity backed slot by the Hallén's iteration method [Hallén, 1938; Watson, 1947; Stevenson, 1948; Galejs, 1962]. The electric field in the slot plane is related to the source current by an integral equation. Approximations to the electric field distribution are obtained by successive iterations, and the slot admittance follows as the ratio of source current to the voltage across the center of the slot.

The integral equation is developed in section 2. Its zero order solution is discussed in section 3. The first-order solution is compared with the variational solution in section 4.

2. Integral Equation

Figure 1 depicts the slot which is backed by a rectangular cavity of volume $x_0y_0z_0$. The

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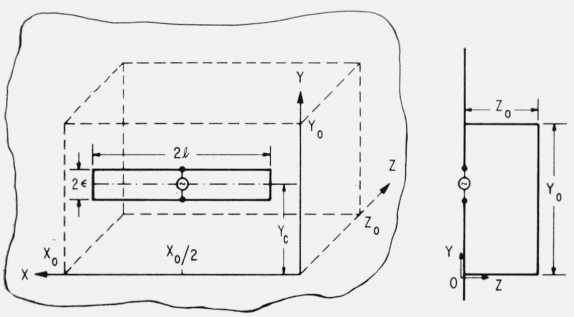

FIGURE 1. *Geometry of the slot and cavity.*

slot is symmetrical with respect to the x-dimension of the cavity. The y-coordinate of the slot center, y_c, is arbitrary.

The H_x^i component of the magnetic field in the $z=0$ plane inside the cavity is readily related to the electric field in the slot plane [Galejs, 1963]. The H_x^0 component of the magnetic field in the $z=0$ plane outside the cavity also depends on the electric field in the slot plane [Levine and Schwinger, 1950; Galejs, 1963]. The source current I_y flows across the slot at $x=x_0/2$. Therefore, H_x^i and H_x^0 are related by

$$H_x^i - H_x^0 = I_y\,\delta\left[x-\frac{x_0}{2}\right]. \tag{1}$$

Substituting the expressions for H_x^i and H_x^0 in (1) results in

$$\left[k^2+\frac{\partial^2}{\partial x^2}\right]\iint_{(\text{slot})} ds'\,E_y(x',y')(G_i+2G_0) = -4\pi i\omega\mu_0 I_y\delta\left[x-\frac{x_0}{2}\right], \tag{2}$$

where Green's functions G_i and G_0 are given by

$$G_i = -\frac{4\pi}{x_0y_0}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\epsilon_m\epsilon_n}{\beta_c}\cot\beta_c z_0\sin\beta_x x\cos\beta_y y\sin\beta_x x'\cos\beta_y y', \tag{3}$$

$$G_0 = \frac{\exp(ikr)}{r}, \tag{4}$$

and where

$$k=\omega\sqrt{\mu_0\epsilon_0},$$

$$\beta_x=\frac{n\pi}{x_0},$$

$$\beta_y=\frac{m\pi}{y_0},$$

$$\beta_c^2=k^2-\beta_x^2-\beta_y^2,$$

$$\epsilon_m=1,\ \text{if } m=0,$$

$$\epsilon_m=2,\ \text{if } m\neq 0,$$

$$r=\sqrt{(x-x')^2+(y-y')^2}.$$

The field components are assumed to exhibit an $\exp(-i\omega t)$ time variation. Considering (2) as a differential equation for the double integral, its solution is

$$\iint_{(\text{slot})} dx'dy'\,E_y(x',y')(G_i+2G_0) = -\frac{4\pi i\omega\mu_0}{k} I_y u\left[x-\frac{x_0}{2}\right]\sin\left[k\left(x-\frac{x_0}{2}\right)\right] + C_1\sin\left[k\left(x-\frac{x_0}{2}\right)\right]+C_2\cos\left[k\left(x-\frac{x_0}{2}\right)\right], \tag{5}$$

where $u(x)$ is a unit step function. Equation (5) may be rearranged by applying the identity [Watson, 1947; Stevenson, 1948; Galejs, 1962]

$$E_y(x',y')=E_y(x,y')+[E_y(x',y')-E_y(x,y')] \tag{6}$$

into

$$\iint_{(\text{slot})} dx'dy'\,E_y(x,y')(G_i+2G_0)=A(x)\sin\left[k\left(x-\frac{x_0}{2}\right)\right]+B\cos\left[k\left(x-\frac{x_0}{2}\right)\right]-\Lambda(x), \tag{7}$$

where

$$A(x) = -\frac{4\pi i\omega\mu_0}{k} I_y u\left[x - \frac{x_0}{2}\right] + C_1, \tag{8}$$

$$B = C_2, \tag{9}$$

and

$$\Lambda(x) = \iint_{\text{(slot)}} (G_i + 2G_0)[E_y(x', y') - E_y(x, y')]dx'dy'. \tag{10}$$

This rearrangement of the integral equation is the essence of Hallén's method in antenna problems, which makes it possible in our case to obtain $E_y(x)$ on the left-hand side of (7) by substituting less accurate E_y expressions in $\Lambda(x)$. For narrow slots ($\epsilon << l$), it is permissible to use the static approximation of E_y. The electric field across a slot of width 2ϵ in an infinite conducting screen is obtained as [Smythe, 1950]

$$E_y(y) = -\frac{V}{\pi\sqrt{\epsilon^2 - (y - y_c)^2}}, \tag{11}$$

where V is the voltage across the slot. Using this approximation of E_y, it follows that [Watson, 1947; Stevenson, 1948; Galejs, 1962]

$$2\iint_{\text{(slot)}} dx'dy'E_y(x, y')G_0 \approx -4V(x) \log \frac{4l}{\epsilon}. \tag{12}$$

Noting that [Groebner and Hofreiter, 1958]

$$\int_{-b}^{b} \frac{\cos ax}{\sqrt{b^2 - x^2}} dx = \pi J_0(ab) \tag{13}$$

and applying (3) and (11), the integral involving G_i is evaluated as

$$\iint_{\text{(slot)}} dx'dy'E_y(x, y')G_i = \frac{8V(x)}{y_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{\epsilon_m \epsilon_n}{n\beta_c} \cot \beta_c z_0 \sin \beta_x x \cos \beta_y y$$

$$\cdot J_0(\beta_y \epsilon) \cos \beta_y y_c \sin\left(\beta_x \frac{x_0}{2}\right) \sin \beta_x l. \tag{14}$$

The G_0 integral (12) as well as the G_i integral (14) become logarithmically infinite as $\epsilon \to 0$. In case of G_i this is seen most readily after approximating G_i by its small r value [2]

$$G_i \simeq \frac{2}{r}, \tag{15}$$

which makes (14) equal to the approximate value of the G_0 integral in (12). However, this approximation ignores any cavity effects on the G_i integral, which is undesirable in the present investigation. $\Lambda(x)$ of (10) remains finite for $\epsilon \to 0$, and the subsequent analysis simplifies after replacing y and y' of the G_0 expression in (10) by y_c [Watson, 1947; Stevenson, 1948; Galejs, 1962].

The integral (7) now becomes

$$V(x)c(x) = A(x) \sin\left[k\left(x - \frac{x_0}{2}\right)\right] + B \cos\left[k\left(x - \frac{x_0}{2}\right)\right] - \Lambda(x), \tag{16}$$

[2] The Green's function of free space is equal to r^{-1} for $r \to 0$. The factor 2 of (15) is due to the image beyond the boundary on which $\nabla G = 0$.

where

$$c(x)=-4\log\frac{4l}{\epsilon}+2\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}F_{nm}\frac{\sin\beta_x x}{\beta_x}\sin\left(\beta_x\frac{x_0}{2}\right)\sin\beta_x l \tag{17}$$

$$\Lambda(x)=\int_{x_0/2-l}^{x_0/2+l}\left[-2\frac{e^{ik|x-x'|}}{|x-x'|}+\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}F_{nm}\sin\beta_x x\sin\beta_x x'\right]\cdot[V_0(x')-V_0(x)]dx' \tag{18}$$

$$F_{nm}=4\pi\frac{\epsilon_n\epsilon_m}{x_0y_0\beta_c}\cot\beta_c z_0\cos^2\beta_y y_c J_0(\beta_y\epsilon). \tag{19}$$

3. Zero Order Solution of the Integral Equation

The simplest solution $V_0(x)$ of (16) is obtained by neglecting $\Lambda(x)$. $A(x)$ and B are determined from the condition

$$V\left[\frac{x_0}{2}\pm l\right]=0. \tag{20}$$

This gives

$$V_0(x)=\frac{-2\pi iI_y\sqrt{\mu_0/\epsilon_0}}{c(x)}\left\{\sin\left[k\left|x-\frac{x_0}{2}\right|\right]-\tan kl\cos\left[k\left|x-\frac{x_0}{2}\right|\right]\right\}. \tag{21}$$

The slot admittance may be computed from (21) as

$$Y_0=\frac{I_y}{V(x_0/2)}=\frac{1}{2\pi i}\sqrt{\frac{\epsilon_0}{\mu_0}}\,c\left(\frac{x_0}{2}\right)\cot kl. \tag{22}$$

This slot admittance is purely susceptive and $B_0=Imy_0$ has been plotted in figure 2. A closed form approximation to the double summation of $c(x_0/2)$ has been worked out for $2l=x_0$ in appendix 2. The corresponding susceptance figures B_0 are indicated by dashed curves in figure 2.

The susceptance B_0 is equal to zero for $l=\lambda/4$. If no modes propagate in the cavity or if the cavity reflects an ideal open circuit, it follows that $c(x_o/2)<0$. Short slots of $l<\lambda/4$ are now inductive, and long slots of $l>\lambda/4$ are capacitive. This applies also to the deeper cavities of figure 2. The leading term of the double summation in (17) is positive if the $m=0$, $n=1$ mode propagates. For very shallow cavities, $c(x_0/2)>0$. Hence long slots are inductive, and short slots become capacitive, which may be seen from figure 2. This last conclusion cannot be justified from physical considerations because a shallow cavity should reflect an inductive short circuit regardless of the slot length. The zero order solution of the integral (21), which ignores $\Lambda(x)$ in (18), may be expected to be accurate only for deep cavities. It provides an incorrect susceptance for short slots which are backed by shallow cavities. Furthermore, the zero order solution does not provide the slot conductance.

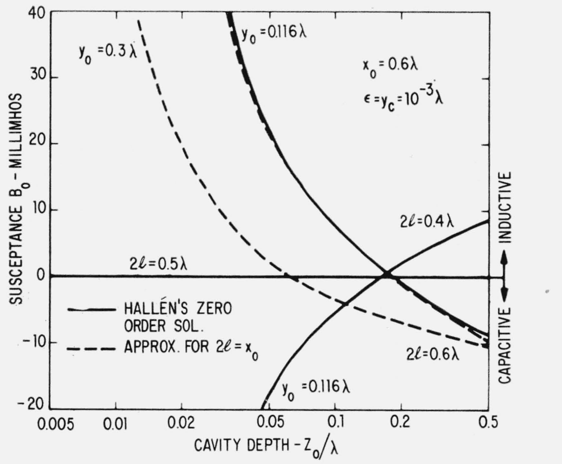

FIGURE 2. *Zero order slot susceptance.*

4. First Order Solution of the Integral Equation

The first order solution $V_1(x)$ may be obtained by using $V_0(x)$ of (21) for computing $\Lambda(x)$ of (18). Because of the difficulties in evaluating the integrals, $V_1(x)$ will be computed with a simpler approximation to $V_0(x)$. For a slot approximately $\lambda/2$ long the voltage distribution along the slot is expected to be nearly cosinusoidal. Assuming a cosinusoidal distribution, the trial function $V_0^t(x)$ becomes

$$V_0^t(x)=B\cos\left[k\left(x-\frac{x_0}{2}\right)\right]\Big/c(x_1), \tag{23}$$

where x_1 may be selected to be equal to $x_o/2$. Details of these computations are discussed in appendix 1.

The resulting slot susceptance $B_1=\mathrm{Im}\,Y_1$ is shown by the solid curves of figure 3. The dashed curves are obtained for $2l=x_1$ by using closed form approximations to Y_1 which are derived in appendix 2. The dotted and dashed curves represent the variational solution [Galejs, 1963]. There is a reasonable agreement between the variational solution and the Hallén's first order solution for $2l>0.5\lambda$ and $z_0>0.05\lambda$. The first order solution for slots of $2l<0.5\lambda$ is accurate only with deep cavities ($z_0>0.25\lambda$), because of the shortcomings of the corresponding zero order solution.

Representative figures of the slot conductance $G_1=\mathrm{Re}\,Y_1$ have been listed in table 1 for $z_0=0.5\lambda$. G_1 is compared with the conductance G_v of the variational solution [Galejs, 1963] and with an admittance estimate G_c based on a complementary dipole configuration. It can be shown that [Kraus, 1950]

$$G_c=2\,\frac{\epsilon_0}{\mu_0}\,R \tag{24}$$

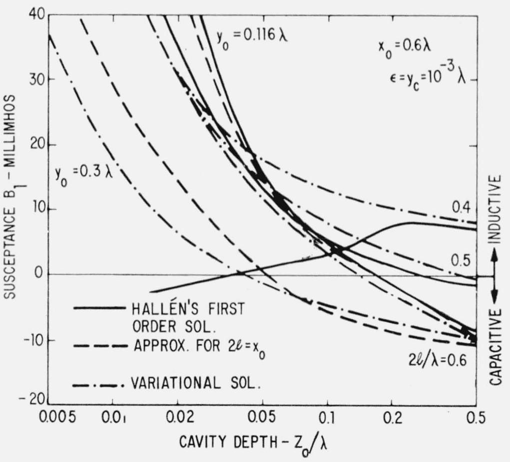

FIGURE 3. *First order slot susceptance.*

TABLE 1. *Slot conductance computed by the Hallén's method* (G_1), *variational method* (G_v) *and from the resistance of a complementary dipole* (G_c). $x_0=0.6\lambda$, $z_0=0.5\lambda$, $\epsilon=y_0=10^{-3}\lambda$

y_0/λ	$2l/\lambda$	G_1	G_v	G_c
		mmho	*mmho*	*mmho*
0.116	0.4	0.83	0.58	0.59
	0.5	1.01	1.11	1.15
	0.6	0.93	2.01	2.34
0.3	0.6	0.93	2.03	2.34

where R is the free space input resistance of a complementary dipole antenna. As pointed out before, $V_0^t(x)$ of (23) may approximate $V_1(x)$ if $2l\approx\lambda/2$. Therefore, G_1 approximates G_v and G_c in table 1 for $2l=\lambda/2$. $V_0^t(x)$ of (23) is too inaccurate for conductance computation at different slot lengths.

5. Conclusions

The first order slot admittance computed by the Hallén's method appears to corroborate the slot admittance of the variational method [Galejs, 1963] in the cases where the voltage distributions of (21) and (23) are valid. Obvious disagreements occur for very shallow cavities, where the voltage is rapidly attenuated along the slot [Galejs, 1963].

More accurate solutions of the Hallén's method may be obtained by trial functions $V_0^t(x)$ which resemble $V_0(x)$ of (21) more closely than $V_0^t(x)$ of (23), or by going to higher order solutions $V_2(x)$, $V_3(x)$, ... This involves a considerable analytical effort and will not be attempted in this note.

The author thanks P. Kimball and T. W. Thompson for computational assistance.

6. Appendix 1. First Order Solution of the Integral Equation

$\Lambda(x)$ of (18) is evaluated using the trial function $V_0^t(x)$ of (23). Substituting (23) in (18) and considering $x=x_0/2+\xi$ and $x=x_0/2\pm l$, (where $\xi\neq l$), it follows that

$$\Lambda\left[\frac{x_0}{2}+\xi\right]=-\frac{B}{c(x_1)}\left[e^{ik\xi}\left\{\log[k(l+\xi)]+Ei[2ik(l-\xi)]+\frac{i\pi}{2}-\log 2-\gamma\right\}\right.$$
$$+e^{-ik\xi}\left\{Ei[2ik(l+\xi)]+i\frac{\pi}{2}-\log 2-\gamma+\log[k(l-\xi)]\right\}$$
$$\left.-2\cos k\xi\{Ei[ik(l+\xi)]+Ei[ik(l-\xi)]-2\gamma+i\pi\}\right]$$
$$+\frac{B}{c(x_1)}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}F_{nm}\sin\left[\beta_x\left(\frac{x_0}{2}+\xi\right)\right]\sin\left(\beta_x\frac{x_0}{2}\right)$$
$$\cdot\left\{\frac{\sin l(\beta_x-k)}{\beta_x-k}+\frac{\sin l(\beta_x+k)}{\beta_x+k}-2\frac{\sin\beta_x l\cos k\xi}{\beta_x}\right\}. \tag{25}$$

$$\Lambda\left[\frac{x_0}{2}\pm l\right]=-\frac{2B}{c(x_1)}\left\{\frac{e^{ikl}}{2}\log(2kl)+\frac{e^{-ikl}}{2}\left[Ei(i4kl)-\log 2-\gamma+\frac{i\pi}{2}\right]\right.$$
$$\left.-\cos kl\left[Ei(i2kl)-\gamma+\frac{i\pi}{2}\right]\right\}$$
$$+\frac{B}{c(x_1)}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}F_{nm}\sin\left[\beta_x\left(\frac{x_0}{2}\pm l\right)\right]\sin\left(\beta_x\frac{x_0}{2}\right)$$
$$\cdot\left\{\frac{\sin[l(\beta_x-k)]}{\beta_x-k}+\frac{\sin[l(\beta_x+k)]}{\beta_x+k}-2\frac{\sin\beta_x l\cos kl}{\beta_x}\right\} \tag{26}$$

where $\gamma=0.5772\ldots$, $Ei(x)=Ci(x)+iSi(x)-i\frac{\pi}{2}$, $Si(x)$ and $Ci(x)$ are the sine and cosine integrals respectively, and where B, $c(x_1)$ and F_{nm} are defined by (9), (17), and (19) respectively.

Substituting $\Lambda[(x_0/2)\pm l]$ in (16) $A(x)$ and B may be determined from the condition

$$V\left[\frac{x_0}{2}\pm l\right]=0. \tag{27}$$

This results in

$$B=\frac{4\pi i\omega\mu_0}{k}I_y\sin kl\left\{2\cos kl-\left[\Lambda\left(\frac{x_0}{2}+l\right)+\Lambda\left(\frac{x_0}{2}-l\right)\right]B^{-1}\right\}^{-1} \tag{28}$$

and

$$A(\pm|x|) = -\frac{B}{\sin(\pm kl)}\left[\cos kl - \frac{\Lambda\left(\frac{x_0}{2}\pm l\right)}{B}\right]. \tag{29}$$

$V(x)$ follows from (16), with $A(x)$, B and $\Lambda(x)$ determined from (29), (28), and (25) respectively. The slot admittance Y_1 may be computed from (16) as

$$Y_1 = \frac{I_y}{V\left(\frac{x_0}{2}\right)} = \frac{c\left(\frac{x_0}{2}\right)}{\frac{B}{I_y}}\left[1 - \frac{\Lambda\left(\frac{x_0}{2}\right)}{B}\right]^{-1}. \tag{30}$$

Appendix 2. Approximate Evaluation of the Double Summations

The numerical calculation of the admittances Y_0 and Y_1 of (22) and (30) is rather laborious because of the infinite double summations of (17), (25), and (26). These double summations may be simplified if the slot is extended over the length of the cavity ($l=x_0/2$). The simplified summations may be approximated by integrals or summed directly.

Thus $c(x_0/2)$ of (17) becomes

$$c(x_0/2) = -4\log\frac{2x_0}{\epsilon} + \frac{2x_0}{\pi}\sum_{m=0}^{\infty}\sum_{p=0}^{\infty}\frac{(-1)^p}{2p+1}F_{pm} \tag{31}$$

where $n=2p+1$. The double summation of (31) will be simplified by means of several approximations. For the cutoff cavity modes $\cot\beta_c z_0 \approx -i$. For $m=0$ and $p\neq 0$, $\beta_c \approx i\beta_x$. For $m\neq 0$, $\beta_c\approx i\beta_y$. The last approximation becomes inaccurate for larger values of p. However, the individual terms of the p summation become small for p large, which tends to minimize this error. These approximations result in

$$\begin{aligned} c(x_0/2) \approx{} & -4\log\frac{2x_0}{\epsilon} + \frac{16}{y_0}\,\frac{\cot\left(\sqrt{k^2-(\pi/x_0)^2}\,z_0\right)}{\sqrt{k^2-(\pi/x_0)^2}} + \frac{16x_0}{\pi y_0}\sum_{p=1}^{\infty}\frac{(-1)^{p+1}}{(2p+1)^2} \\ & + \frac{32}{\pi}\sum_{p=0}^{\infty}\frac{(-1)^{p+1}}{2p+1}\sum_{m=1}^{\infty}\frac{\cos^2\left(\frac{m\pi}{y_0}y_c\right)J_0\left(\frac{m\pi}{y_0}\epsilon\right)}{m}. \end{aligned} \tag{32}$$

The summation over m is approximated by an integral as

$$\sum_{m=1}^{\infty}\frac{\cos^2\left(\frac{m\pi}{y_0}y_c\right)J_0\left(\frac{m\pi}{y_0}\epsilon\right)}{m} \approx \int_{1/2}^{\infty}\frac{J_0\left(\frac{\pi\epsilon}{y_0}m\right)\cos^2\left(\frac{\pi y_c}{y_0}m\right)dm}{m}, \tag{33}$$

on the assumption that $\cos(m\pi y_c/y_0)\neq 0$. It follows that

$$I = \lim_{\delta\to 0}\left\{\frac{1}{2}\int_0^{\infty}\frac{m}{m^2+\delta^2}\left[1+\cos\left(\frac{2\pi y_c}{y_0}m\right)\right]J_0\left(\frac{\pi\epsilon}{y_0}m\right)dm - \int_0^{1/2}\frac{m\,dm}{m^2+\delta^2}\right\} = \left[\log\left(\frac{4y_0}{\pi\epsilon}\right)-\gamma\right] \tag{34}$$

if $\cos(\pi y_c/y_0)\approx 1$ and $J_0\left(\frac{1}{2}\pi\epsilon/y_0\right)\approx 1$. If $y_c=y_0/2$, all the $m=$odd terms of the summation are equal to zero and the summation is approximated by $I/2$. The summation over m may be generally represented by

$$\sum_{m=1}^{\infty}\frac{\cos^2\left(\frac{m\pi}{y_0}y_c\right)J_0\left(\frac{m\pi}{y_0}\epsilon\right)}{m} = qI \tag{35}$$

where I is given by (34) and where $q \approx 1$ for $y_c = \epsilon$ (slot near the edge of the cavity) and $q \approx 0.5$ for $y_c = y_0/2$ (slot near the center of the cavity). After evaluating the p summations

$$c(x_0/2) \approx -4 \log \frac{2x_0}{\epsilon} + \frac{16}{y_0} \frac{\cot(\sqrt{k^2-(\pi/x_0)^2}z_0)}{\sqrt{k^2-(\pi/x_0)^2}} + \frac{16x_0}{\pi y_0}(1-\lambda) - 8q\left[\log\left(\frac{4y_0}{\pi\epsilon}\right) - \gamma\right] \tag{36}$$

where $\lambda = 0.91596 \ldots$ is the Catalan's constant.

With $l = x_0/2$, $\Lambda(x_0/2)$ of (25) becomes

$$\Lambda\left(\frac{x_0}{2}\right) = -\frac{B}{c(x_0/2)}\left\{2 \log(kx_0/2) + 2\left[Ei(ikx_0) + \frac{i\pi}{2} - \log 2 - \gamma\right] - 4\left[Ei(ikx_0/2) + \frac{i\pi}{2} - \gamma\right]\right\} + S_1 \tag{37}$$

where

$$S_1 = 2\sum_{m=0}^{\infty}\sum_{p=0}^{\infty} F_{pm}(-1)^p\left[\cos(kx_0/2)\frac{\beta_x}{\beta_x^2-k^2} - \frac{1}{\beta_x}\right]. \tag{38}$$

S_1 is evaluated using similar approximations as in the evaluation of the double summation of (31). This results in

$$S_1 \approx 16\left[\cos(kx_0/2)\frac{(\pi/x_0)^2}{(\pi/x_0)^2-k^2} - 1\right]\left\{\frac{\cot(\sqrt{k^2-(\pi/x_0)^2}z_0)}{y_0\sqrt{k^2-(\pi/x_0)^2}} - q\frac{2}{\pi}\left[\log\left(\frac{4y_0}{\pi\epsilon}\right) - \gamma\right]\right\} + \frac{16}{\pi}[\cos(kx_0/2) - 1] \cdot \left\{\frac{x_0}{y_0}(1-\lambda) + 2\left[\log\left(\frac{4y_0}{\pi\epsilon}\right) - \gamma\right] q\left(1 - \frac{\pi}{4}\right)\right\}. \tag{39}$$

For $l = x_0/2$ the double summation of (26) is equal to zero, which greatly simplifies the computation of B in (28).

8. References

Galejs, J., Admittance of a rectangular slot which is backed by a rectangular cavity, IRE Trans. on Ant. and Prop. **AP-11,** No. 2 (1963).

Galejs, J., Excitation of slots in a conducting screen above a lossy dielectric half space, IRE Trans. on Ant. and Prop., **AP-10,** No. 4, 436–443 (1962).

Groebner, W., and N. Hofreiter, Integraltafel, Part II, eq. 333.76c (Springer Verlag Wien, 1958).

Hallén, Erik, Theoretical investigations into the transmitting and receiving qualities of antennae, Nova Acta Regiae Soc. Sci. Upsaliensis, Ser. IV **11,** No. 4, 1–44 (1938).

Kraus, J. O., Antennas, 369–371 (McGraw-Hill Book Co., New York, N.Y., 1950).

Levine, H., and J. Schwinger, On the theory of electromagnetic wave diffraction by an aperture in an infinite plane conducting screen, Comm. Pure and Applied Math. **3,** No. 4, 355–391 (1950).

Smythe, W. R., Static and dynamic electricity, see Problem 4.21 (McGraw-Hill Book Co., New York, N.Y., 1950).

Stevenson, A. F., Theory of slots in rectangular waveguides, J. Appl. Phys. **19,** 24–38 (1948).

Watson, W. H., The physical principles of waveguide transmission and antenna systems, sec. 10 (Oxford at the Clarendon Press, England, 1947).

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