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# Residual Stresses and Their Relaxation on the Surfaces of Sections Cut From Plastically Deformed Steel Specimens

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Residual stresses were measured on sections cut from steel specimens after plastic extension and after plastic compression, using the inclined incident X-ray beam procedure. Computations based on the conventional assumption of a zero surface-normal stress component were compared with those based on a recently suggested method of allowing for some contribution of the normal component. The conventional calculations indicated an axial residual stress opposite in sign to the preceding deforming stress. The more recent method is said not to do so after compression but was inconclusive in this experiment because of lack of precision arising from microfluctuations of stress about the average. Doubt was cast, moreover, on the validity, except perhaps as to sign, of extending the stress values measured on sections cut from a specimen to the residual stress system existing within the specimen before sectioning.

## 1. Introduction

The calculation of stresses from crystal lattice strains observed in polycrystalline metal specimens by means of X-ray diffraction is a long established procedure [1,2].[1] The change in the Bragg angle of diffraction is a measure of the change in spacing between layers of atoms in the metal crystals, hence a strain, from which certain calculations of stress in the material may be made in the light of various assumptions concerning the stress-strain relationship. This method of stress measurement is of particular interest in those cases where no external force is acting on the body. These instances of internal or residual stresses arise, generally speaking, from some type of inhomogeneity or anisotropy in the metal specimen or in its history of thermal or mechanical treatment.

The change in the peak position of a diffraction line indicates a nonzero average strain in the crystalline material in which the diffraction takes place [3]. If the state of stress is constant over an extended region of the specimen, with the forces involved balanced by opposing system forces in another region, the stresses are called body stresses or macrostresses. If, on the other hand, the stresses are balanced locally, between neighboring crystallite grains or from one part of a grain to another part within the grain, the stresses are known as microstresses or textural stresses. Not only do the macrostresses result in a shift of the X-ray line peak position, but so also do the microstresses under the proper conditions. It is true in the latter case that the average stress in the volume illuminated by the incident X-rays may well be zero, but the diffraction peak position is not the result of the state of the entire volume, nor is it a random statistical sampling of it. It is, on the contrary, a very specialized sampling, dependent upon the crystalline phase, the degree of perfection of the material, and the orientation of the crystallites. If any of these factors bears a relation to the state of stress, the microstress will result in a shift of peak position and not merely in line broadening, which is sometimes mistakenly considered to be the only result of this type of stress.

In this laboratory a few years ago a procedure of X-ray stress measurements on sections cut from a uniformly uniaxially plastically deformed material was employed to demonstrate that the sign of the stress in the interior of the deformed specimen was the same as had been reported on the exterior surface [4]. The residual stress system, therefore, was not a body stress system but a textural stress, or microstress system that had the diffraction angle shift characteristic of a body stress, or macrostress system. This conclusion has been supported by recent work by M. J. Donachie and J. T. Norton [5], with a different X-ray procedure, not involving sectioning. A procedure involving diffraction from the surfaces of sections has been employed by D. M. Vasil'ev [6, 7], along with a new assumption as to X-ray penetration and stress relaxation normal to a surface, with somewhat surprising results that will be discussed later in this paper.

The usual formula for the calculation of a residual stress (or an applied stress), $\sigma$, in a particular direction, $\varphi$, with respect to some arbitrary direction in a surface is [8]

$$\sigma_\varphi = \frac{E}{1+\nu} \cdot \frac{1}{\sin^2 \psi} \cdot \frac{d_\psi - d_\perp}{d_\perp}, \qquad (1)$$

where $E$ is Young's modulus, $\nu$ is Poisson's ratio, $\psi$ is the angle between the normal to the surface and the normal to the crystal planes whose spacing is $d_\psi$,

[1] Figures in brackets indicate the literature references at the end of this paper.

obtained by inclining the incident X-ray beam to the surface, and $d_\perp$ is the spacing of crystal planes diffracting when the X-ray beam is incident normal to the surface. (Strictly speaking, $d_\perp$ should be the spacing between planes parallel to the surface of the specimen.) The usual polycrystalline gross mechanical values of $E$ and $\nu$ are customarily used. There are some assumptions made in the derivation of this formula; the most basic, perhaps, is that the diffracting material is obeying the usual laws of elasticity for an isotropic, homogeneous medium. There are certain checks that can be made on this assumption, which will be discussed later.

Another set of two assumptions has to do with the relaxation of stress components at a surface of the diffracting material. At the very surface, of course, the normal component is zero; and it is usually assumed that the depth of effective penetration of the X-rays is so slight (less than a thousandth of an inch in the case of iron diffracting cobalt radiation, according to G. B. Greenough [1]) that the normal component may be assumed to be zero in the X-ray diffraction stress measurements. The stress components lying in the plane of the surface, on the other hand, are assumed to be totally unaffected by the presence of the surface, in so far as the X-ray diffraction measurements are concerned. This is perhaps an irrelevant point when considering the stress system on the original surface of a specimen; but the assumption is likewise made when the surface is a section cut from the specimen, and it should be recognized as such.

If, for an element of volume in the interior of the deformed specimen as in figure 1, we write the Hooke's Law relationship for one of the principal strains, such as $\epsilon_1$, resulting from the residual principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$, we have

$$\epsilon_1=\frac{1}{E}\left[\sigma_1-\nu(\sigma_2+\sigma_3)\right]. \tag{2}$$

If a surface is cut normal to $\sigma_1$, and hence parallel to $\sigma_2$ and $\sigma_3$ (in this example a cross section), the principal stress normal to the surface will be modified by a factor $k_N$ and the principal stresses lying in the plane of the surface will be modified by a factor $k_S$ in that shallow layer of material in which the effective X-ray diffraction takes place. In a manner similar to eq (1), we may then write for the principal strain normal to the surface

$$\epsilon_{1N}=\frac{1}{E}\left[k_N\sigma_1-\nu(k_S\sigma_2+k_S\sigma_3)\right] \tag{3a}$$

and for the principal strains parallel to the surface

$$\epsilon_{2S}=\frac{1}{E}\left[k_S\sigma_2-\nu(k_N\sigma_1+k_S\sigma_3)\right] \tag{3b}$$

and

$$\epsilon_{3S}=\frac{1}{E}\left[k_S\sigma_3-\nu(k_N\sigma_1+k_S\sigma_2)\right]. \tag{3c}$$

The relaxation factors $k_N$ and $k_S$ must lie between 0 and +1 for obvious physical reasons. They are conventionally taken to be

$$k_N=0 \text{ and } k_S=1,$$

whence eqs 3a, b, and c become

$$\epsilon_{1N}=\frac{-\nu}{E}\left[\sigma_2+\sigma_3\right] \tag{3d}$$

$$\epsilon_{2S}=\frac{1}{E}\left[\sigma_2-\nu\sigma_3\right] \tag{3e}$$

$$\epsilon_{3S}=\frac{1}{E}\left[\sigma_3-\nu\sigma_2\right]. \tag{3f}$$

Another assumption usually made (and implied in fig. 1) but seldom stated is that the directions of the residual principal stresses, $\sigma_1$, $\sigma_2$, and $\sigma_3$ are directly related to the geometry of the specimen and its mechanical history. That is to say, that after a uniform, uniaxial deformation of a specimen of circular cross section, as, for example, the plastic extension for a few percent of a standard tensile specimen, $\sigma_1$, will be axial and $\sigma_2$ will equal $\sigma_3$, lying in a plane normal to $\sigma_1$. If the deformation is truly uniform and if the polycrystalline aggregate is made up of a random distribution of many small crystallites so that the effects of anisotropy may be well averaged out, this assumption is no doubt quite valid.

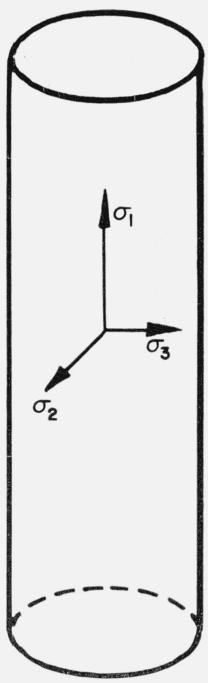


FIGURE 1. *Principal residual stresses in a uniaxially plastically deformed test specimen.*

Hence eq (3d) becomes

$$\epsilon_{1N}=\frac{-2\nu}{E}\,\sigma_2 \tag{4a}$$

and (3e) and (3f) become

$$\epsilon_{2S}=\frac{1-\nu}{E}\,\sigma_2. \tag{4b}$$

The new assumption made by D. M. Vasil'ev, briefly mentioned above, was that the penetrating power of the X-ray beam in the metal specimens was sufficiently great that the stress component normal to the surface was not reduced to zero, and the surface parallel components, as above, were unaffected; that is, eq (3) became

$$0<k_N<1 \text{ and } k_S=1, \tag{5}$$

where $k_N$ may be determined from more measurements of strain made on the original surface and on a surface cut at right angles to the first, as indicated in the following analysis. If we use superscripts $C$ and $L$ to denote cross and longitudinal sections and subscripts 1 and 2 to denote axial and transverse directions as above and in figure 2, we have from Hooke's Law a set of four equations. For the principal strains normal to and parallel to the cross section, we have

$$\epsilon^C_{1N}=\frac{1}{E}\,[k_N\sigma_1-2\nu\sigma_2], \tag{6a}$$

$$\epsilon^C_{2S}=\frac{1}{E}\,[\sigma_2(1-\nu)-\nu k_N\sigma_1]; \tag{6b}$$

and for the principal strains normal to and parallel to the longitudinal section,

$$\epsilon^L_{2N}=\frac{1}{E}\,[\sigma_2(k_N-\nu)-\nu\sigma_1], \tag{6c}$$

$$\epsilon^L_{1S}=\frac{1}{E}\,[\sigma_1-\nu\sigma_2(k_N+1)]. \tag{6d}$$

[Notice that the surface strain component $\epsilon^L_{1S}$ in the longitudinal section is to be measured in the axial plane.]

From these equations Vasil'ev derived the following expressions: $k_N$ is found to be

$$k_N=\frac{(\epsilon^L_{2N}-\epsilon^C_{1N})-\nu(\epsilon^C_{2S}-\epsilon^L_{1S})}{-\nu(\epsilon^L_{2N}-\epsilon^C_{1N})+(\epsilon^C_{2S}-\epsilon^L_{1S})}, \tag{7}$$

and the equations for the principal stresses are

$$\sigma_1=\frac{-E}{(1+\nu)(1-k^2)}\,[\epsilon^L_{2N}-\epsilon^L_{1S}+k(\epsilon^C_{1N}-\epsilon^C_{2S})] \tag{8a}$$

$$\sigma_2=\frac{-E}{(1+\nu)(1-k^2)}[k(\epsilon^L_{2N}-\epsilon^L_{1S})+\epsilon^C_{1N}-\epsilon^C_{2S}]. \tag{8b}$$

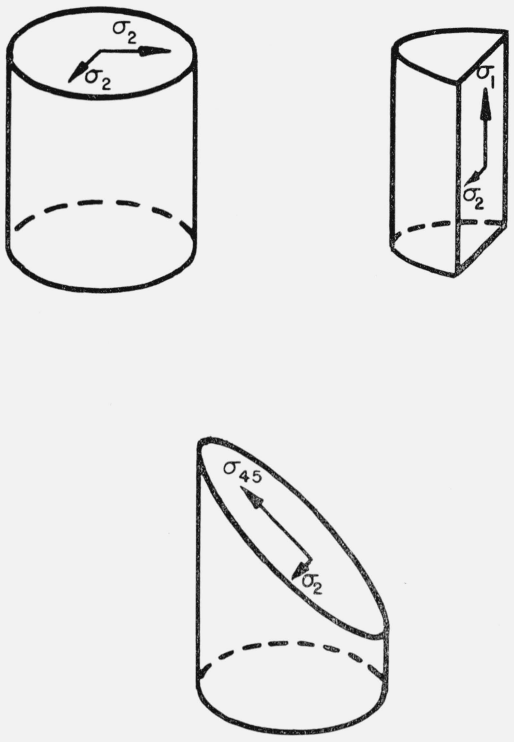


FIGURE 2. *Sections, with surface stresses, cut from deformed test specimen.*

Vasil'ev found, using various combinations of X-radiation and diffracting material, that $k_N$ varied from 0.3 to 0.7. He also found, after preliminary extension of the specimen, the residual stresses were

$$\sigma_1<\sigma_2<0 \tag{9a}$$

and after preliminary compression

$$\sigma_2<\sigma_1<0. \tag{9b}$$

In this latter case, to have the axial residual stress negative after preliminary compression is contrary to the usual observation [9].

In the technique for determining residual stress by the two-exposure method, the strain normal to the surface and the strain at some inclined angle $\psi$, usually 45 deg, rather than at 90 deg, lying in the surface, are usually measured. It is possible to modify eqs (7) and (8) for this procedure. Using these equations and a typical residual stress system such as one previously found in iron with cobalt radiation [4], one may calculate the precision with which the lattice spacing must be determined to distinguish between a value of $k_N$ of zero and a value, for example, of 0.4. It appears that changes in the fourth decimal place of the $d$-value are critical; and it can be shown that changes in the $2\theta$ diffraction angle of about 0.03°, therefore, must be detectable. Since the change in the $2\theta$ angle can, under favorable circumstances, be measured with a precision of $\pm 0.02°$, it appeared that a check of a typical $k_N$ value might be possible. This was to be the principal objective of the present study.

In addition to the attempt to find a value of the surface normal stress relaxation constant, there was a desire in this study to investigate carefully after a plastic compression the sign of the axial component of residual stress, which we had previously found to be positive [4, 9], in contradistinction to Vasil'ev's result. It was hoped, moreover, with the means at hand for a greater precision of measurement of diffraction angles, that a more critical study would be made of the validity of extension of the stress values measured on sections cut from the specimen to the stress system existing within the specimen before sectioning.

## 2. Experimental Material and Procedure

The precision of the X-ray method of stress measurement is improved if the diffracting material has grains sufficiently small (about 0.05 mm or somewhat less in diameter) to give reasonably smooth powder diffraction lines, and if the combination of lattice constant and characteristic X-ray wavelength is such as to give a large diffraction angle, preferably a 20 over 150 deg. A material satisfying these conditions and having some commercial importance was found to be 4130 steel. In addition to iron, the material contained the following constituents, in percent:

| | | |
|---|---|---|
| C—0.28 | Si—0.26 | Cu—0.10 |
| Cr— .92 | Mo— .20 | S— .013 |
| Mn— .50 | Ni— .10 | P— .006 |

A sample of the "as received" bar stock probably in the hot-worked condition, was given a 1 hr annealing treatment at 1,300° F. Diffraction patterns from this material showed continuous Debye-Scherrer rings with partial resolution of the $k\alpha_1-\alpha_2$ doublet both with Co and Cr X-radiation. Another prerequisite of the material is that it exhibits a measurable residual stress system giving a detectible line-shift after a degree of plastic deformation less than that which might lead to excessive line broadening that results from severe fragmentation or random microstresses. This requirement was found also to be met reasonably well by the 4130 steel.

Tensile testing specimens, with a reduced diameter of 0.505 in. and a gage length of 2 in., and compressive testing specimens, right circular cylinders, of 0.505 in. diam and 1 in. length, were prepared from the bar stock material and annealed as specified above. The plastic deformation first employed was 10 percent, but this was found to give excessive broadening of the diffraction lines. A plastic deformation of 3 percent was then tried and found to be satisfactory. The yield points of the specimens were found to be in the neighborhood of $70\times10^3$ psi, and the stresses producing 3 percent permanent deformation were about $82\times10^3$ psi.

After the test specimens had been deformed, three different diffraction specimens were cut from each, as in figure 2, so that the residual interior stress system could be investigated on three different sections: (1) a cross section, (2) a longitudinal section, and (3) a 45° inclined section. These specimens were mounted in Bakelite with the appropriate section surface exposed, mechanically polished, and finally electropolished in several stages with intervening inspections by X-ray diffraction photographs until no surface distortion that might have resulted from specimen preparation remained. Attempts were initially made to study the residual stress from photographically recorded patterns of the diffracted X-rays, with the incident beam normal and inclined to the surface of the specimen. Although changes in the Bragg angle could be observed and residual stresses could be calculated, it was found that the breadth and diffuseness of the lines, or rings, on the films were such that the precision was very low, and nowhere near that mentioned above as being necessary in this study.

The precision can be significantly improved in many cases by means of a point by point X-ray intensity counting procedure with a diffractometer, in which case the diffraction pattern is detected not with a photographic film but with an electronic device, in this case a proportional counter.

Before the standard commercial instrument can be used for the inclined incidence method of X-ray stress measurement, two modifications are necessary: (1) the specimen holder must be free to change the angle of incidence of the X-ray beam upon its surface, independently of the position of the counter, and (2) the receiving slit for the diffracted beam entering the counter must be moved forward because of the change in focusing conditions. The first modification is not too difficult to make with the aid of a new specimen mount. The second modification would present much greater difficulty if the ideal arrangement of continuously variable and accurately radial positioning of the slit were attempted. Fortunately it was possible to use a permanent compromise forward position of the slit, as described by B. D. Cullity [10]. The compromise may introduce a small additional increment in the Bragg angle change, but it may be corrected by means of observations from a standard, well-annealed specimen that is very nearly stress-free.

The X-ray intensities, measured at small intervals of $2\theta$ across the peak of an X-ray diffraction line, were used to locate the peak position in the fashion described in the SAE Information Report, TR–182, "Measurement of Stress by X-ray," edited by A. L. Christenson [11]. This involved first finding the approximate peak position, then measuring with high precision the X-ray intensity at three points straddling the peak and at least 80 percent up from background. This choice of points minimizes the effects of poor resolution of the $k\alpha_1-\alpha_2$ doublet. The intensity values obtained at the three points, which were set usually 0.10° apart with a precision of about $\pm0.002°$, are then corrected for the Lorentz and polarization factors and for the absorption effect and finally put into a formula to yield the axis position of a parabola that passes through the three given points. The intricacies of this procedure

make it difficult to calculate the expected precision with which the resulting Bragg angle is finally known; but in cases where a measurement was repeated, a reproducibility of 0.02° or better was usually observed.

The coefficient of thermal expansion of iron is approximately $11\times10^{-6}$/°C. It is apparent that large changes of temperature of the specimen must be avoided when small changes of Bragg angle are to be detected in the study of residual stress. In this case, the temperature of the specimen holder was constantly monitored. After the diffraction equipment had approximately reached thermal equilibrium, it was possible by controlling the temperature of the room to keep the specimen holder to within about half a degree of 25 °C. Hence, any thermal expansion error was negligible.

The instrumental variables of the X-ray equipment were carefully kept constant during the use of each of two different characteristic radiations, from targets of cobalt and chromium. Both the voltage and current of the X-ray tube were closely regulated. The counting response of the proportional counter was visually monitored on a strip chart recorder in order to reject any preset count interval that included spurious "noise" counts. This source of error in intensity measurements was further reduced by using the average value of three or more intervals. With cobalt radiation the individual counting intervals were for 40,000 counts, yielding an intensity precision of the order of 0.5 percent; with chromium radiation, since a more intense beam was available, the intervals were for 100,000 counts, with precision of about 0.3 percent.

## 3. Results

Equation (1) can be written in terms of the change in strain observed normal to a surface, $\epsilon_N$, and the strain parallel to the surface, $\epsilon_S$, as

$$\sigma_\varphi=\frac{E}{1+\nu}(\epsilon_S-\epsilon_N). \tag{10}$$

The strain, moreover, can be written in terms of the change in Bragg angle:

$$\epsilon=\frac{d_i-d_A}{d_A}=\frac{-\cot\theta}{2}[2\theta_i-2\theta_A], \tag{11}$$

where the subscript $A$ refers to a well-annealed specimen and $i$ indicates the quantity pertaining to a strained state. Hence, the stress in the surface becomes

$$\sigma_\varphi=\frac{E}{1+\nu}\cdot\frac{\cot\theta}{2}[(2\theta_{i,N}-2\theta_{A,N})-(2\theta_{i,S}-2\theta_{A,S})]$$

$$=\frac{E}{1+\nu}\cdot\frac{\cot\theta}{2}[(2\theta_N-2\theta_S)_i-(2\theta_N-2\theta_S)_A]. \tag{12}$$

Now ordinarily the last term,

$$(2\theta_N-2\theta_S)_A=0;$$

but using the compromise position of the receiving slit on the diffractometer may cause the appearance of a small nonnegligible spurious difference here, which is the correction needed for the change of focusing conditions [10]. With cobalt radiation, this difference was vanishingly small; with chromium radiation it was 0.06°. Since the angular measurements are made in degrees, there is a further factor in the expressions to convert from degrees to radians, which is 0.01745 radians per degree. When all of the constants are put into the equations, we have in the case of cobalt radiation

$$\sigma_\varphi=(3.37\times10^4\ \text{psi/deg})[2\theta_N-2\theta_S], \tag{13a}$$

for the (310) diffraction, which occurs with a $2\theta$ equal approximately 161°; and in the case of chromium radiation, we have

$$\sigma_\varphi=(4.334\times10^4\ \text{psi/deg})[(2\theta_N-2\theta_S)-0.06], \tag{13b}$$

for the (211) diffraction, which occurs with a $2\theta$ near 156°. In a similar fashion, it is possible to replace the differences of strains in Vasil'ev's eqs (8a) and (8b) with differences in diffraction angles.

All of these equations call for the strain or the diffraction angle measured parallel to the surface, which, of course, it is physically impossible to obtain directly; however, a value can be obtained by extrapolation. The diffraction angle is measured at several values of $\psi$, the angle between the normal to the diffracting planes and the normal to the surface of the X-ray specimen, as shown in figure 3. The

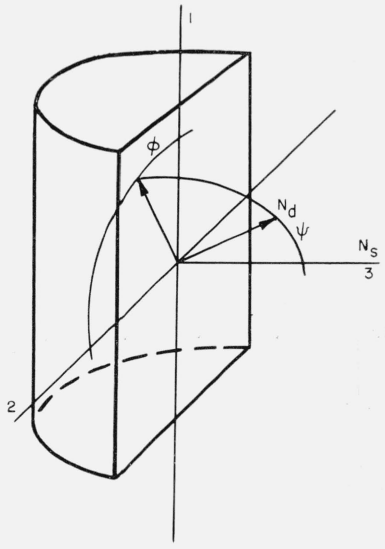


FIGURE 3. *Typical longitudinal section.*

$N_S$: Normal to surface
$N_d$: Normal to diffracting planes
$\phi$: Azimuth angle
$\psi$: Co-altitude or inclination angle

diffraction angle is found to be a linear function of the square of the sine of $\psi$, which is in itself an indication that the X-ray diffraction effects are related to a stress system obeying the usual laws of elasticity for a homogeneous isotropic medium. Then, using a least root mean square error procedure, one obtains best values of $2\theta$ for $\psi$ equal zero and $\psi$ equal 90 deg.

It does not seem necessary to present all of the detailed data in this report, but four samples are presented in the accompanying table 1 and figures 4 to 7, consisting of the $2\theta$ values versus $\sin^2 \psi$ with cobalt and chromium $k\alpha_1$ radiation for the annealed specimen and the cross section of the tensile specimen. The precision of the individual values of $2\theta$ is about $\pm 0.02°$. The scatter, which will be discussed later, is considerably greater than this, about $\pm 0.08°$ with Co radiation. Therefore, the precision of the best fit extrapolated values of $2\theta$ is somewhat questionable. One might estimate an intermediate value for it, of the order of $\pm 0.05°$.

TABLE 1. *Examples of diffraction angle, 2θ, as a function of plane normal inclination Ψ*

| Cobalt radiation | | | | Chromium radiation | | | |
|---|---|---|---|---|---|---|---|
| $\psi°$ | $\sin^2 \psi$ | $2\theta°$ (Ann) | $2\theta°$ (C-S) | $\psi°$ | $\sin^2 \psi$ | $2\theta°$ (Ann) | $2\theta°$ (C-S) |
| 0 | 0.0000 | 161.25 | 161.71 | 0 | 0.0000 | 156.08 | 156.36 |
| 25 | .1786 | 161.24 | 161.48 | 10 | .0302 | ------- | 156.30 |
| 45 | .5000 | 161.24 | 161.46 | 20 | .1170 | 156.04 | 156.22 |
| 60 | .7500 | 161.25 | 161.29 | 30 | .2500 | 156.04 | 156.16 |
| | | | | 40 | .4132 | 156.04 | 156.08 |
| | | | | 50 | .5868 | 156.03 | 156.01 |
| Best fit extrapolated values | | | | | | | |
| 0 | 0.0000 | Average | 161.66 | 0 | 0.0000 | 156.06 | 156.32 |
| 90 | 1.0000 | 161.24 | 161.18 | 90 | 1.0000 | 156.00 | 155.76 |

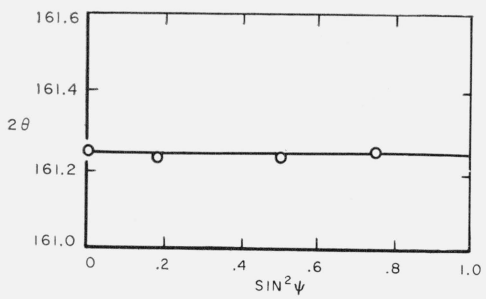


FIGURE 4. *Diffraction angle, 2θ, versus sin² ψ for annealed specimen,* Co $k\alpha_1$ *radiation.*

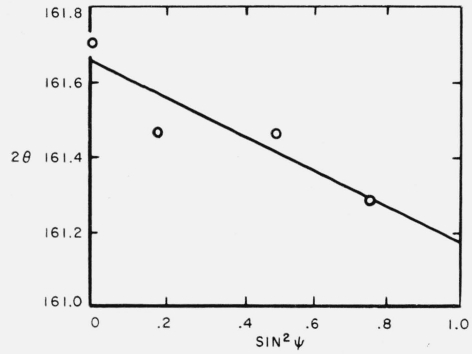


FIGURE 5. *Diffraction angle, 2θ, versus sin² ψ for cross section of tensile specimen,* Co $k\alpha_1$ *radiation.*

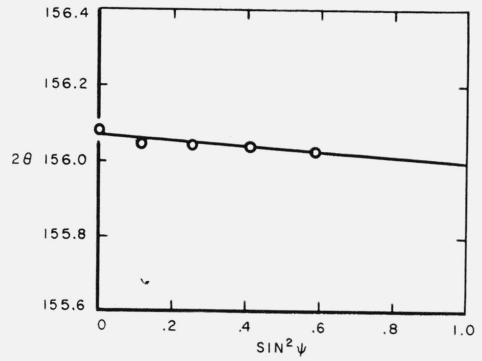


FIGURE 6. *Diffraction angle, 2θ, versus sin² ψ for annealed specimen,* Cr $k\alpha_1$ *radiation.*

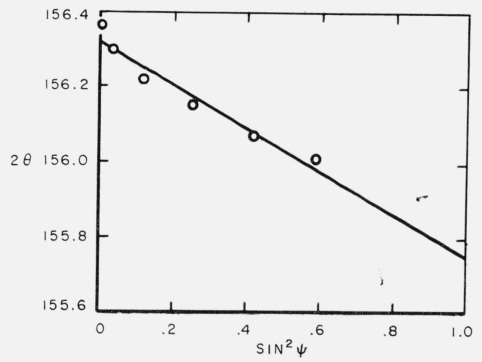


FIGURE 7. *Diffraction angle, 2θ, versus sin² ψ for cross section of tensile specimen,* Cr $k\alpha_1$ *radiation.*

A summary of the "best fit" extrapolated values of the diffraction angles and the lattice strains measured on the eight specimens with the two radiations is presented in table 2. The precision of the diffraction angles is thought to be about $\pm 0.02$ of a degree, although that of the extrapolated values given in table 2 is perhaps not this good. The precision of the strain values in this table is estimated to be about $\pm 1 \times 10^{-4}$.

TABLE 2. *Extrapolated diffraction angles and lattice strains*

| Specimen | $\psi°$ | Cobalt radiation | | Chromium radiation | | |
|---|---|---|---|---|---|---|
| | | $2\theta°$ | $\epsilon(\times 10^{-4})$ | $2\theta°$ | $(\epsilon\times 10^{-4})$ | |
| Annealed | 0 | 161.24 | ---------- | 156.06 | ---------- | |
| | 90 | 161.24 | ---------- | 156.00 | ---------- | |
| Tensile C–S | 0 | 161.66 | −6.0 | 156.32* | −4.8 | $\epsilon^{C}_{1N}$ |
| | 90 | 161.18 | 1.0 | 155.77* | 4.2 | $\epsilon^{C}_{2S}$ |
| Tensile L–S | 0 | 161.08 | 2.4 | 156.01 | 1.0 | $\epsilon^{L}_{2N}$ |
| | 90 | 161.74 | −7.2 | 156.18 | −3.4 | $\epsilon^{L}_{1S}$ |
| Tensile 45°–S | 0 | 161.47 | −3.2 | 156.12 | −1.0 | $\epsilon^{45}_{45N}$ |
| | 90 | 161.81 | −8.2 | 156.46 | −10.0 | $\epsilon^{45}_{45S}$ |
| Compr. C–S | 0 | 160.83 | 5.9 | 155.91 | 2.9 | $\epsilon^{C}_{1N}$ |
| | 90 | 161.27 | −0.4 | 156.11 | −2.1 | $\epsilon^{C}_{2S}$ |
| Compr. L–S | 0 | 161.49 | −3.5 | 156.19 | −2.3 | $\epsilon^{L}_{2N}$ |
| | 90 | 161.52 | 10.4 | 155.82 | 3.3 | $\epsilon^{L}_{1S}$ |
| Compr. 45°–S | 0 | 161.22 | 0.3 | 156.15 | −1.6 | $\epsilon^{45}_{45N}$ |
| | 90 | 162.07 | −11.9 | 156.17 | −3.3 | $\epsilon^{45}_{45S}$ |

C–S: Cross section.
L–S: Longitudinal section.
45°–S: 45° section.
*These differ from values in table 1 because more data included in an average

In table 3 are tabulated the extrapolated values of the diffraction angles and residual stresses in the plane of the surfaces of the three sections of the tensile specimen, measured at three different azimuths, $\varphi$, in each plane (see fig. 3). The radiation employed was the $k\alpha_1$ of chromium. A stress on each surface calculated from elastic theory is also given in this table.

TABLE 3. *Extrapolated diffraction angles and stresses for different azimuths on tensile sections with chromium radiation*

| Specimen | Azimuth $\varphi°$ | 2θ° | | Stress ($\times 10^3$ psi) | |
|---|---|---|---|---|---|
| | | $\psi=0°$ | $\psi=90°$ | Observed $\sigma_\varphi$ | Calc. $\sigma_{45}$ |
| Tensile C–S | 0 | 156.32 | 155.76 | 21 | (Av: 20.3) |
| | 45 | 156.32 | 155.77 | 20 | |
| | 90 | 156.33 | 155.79 | 20 | |
| Tensile L–S | 0 | 156.02 | 156.18 | −10 | −4.5 |
| | 45 | 156.02 | 156.06 | −5 | |
| | 90 | 155.99 | 155.91 | 1 | |
| Tensile 45°–S | 0 | 156.14 | 156.46 | −17 | −7.5 |
| | 45 | 156.13 | 156.25 | −8 | |
| | 90 | 156.09 | 155.94 | 2 | |

Table 4 contains the conventionally calculated principal stresses and the 45° inclined stress component from the tensile and compressive specimens, observed with cobalt and chromium radiation. It is not possible to give a rigorous value of the precision of the values of stress presented in these tables, since the true precision of the angular differences upon which they are based is not known, but it can be estimated at about 1 or $2\times 10^3$ psi.

TABLE 4. *Conventionally calculated stresses*

| Specimen | Stress component | Stress ($\times 10^3$ psi) | |
|---|---|---|---|
| | | Co Rad. | Cr Rad. |
| Tensile | Axial | −22 | −10 |
| | Transverse | 16 | 21 |
| | 45° | −12 | −17 |
| Compressive | Axial | 32 | 13 |
| | Transverse | −15 | −12 |
| | 45° | −29 | −4 |

In table 5 are shown the very doubtful results of the calculations of the surface normal relaxation constant, $k$, and the principal stresses using the formulas of Vasil'ev. No values are given for the stresses after extension from the measurements with cobalt radiation because of the impossible value of $k$, larger than 1, obtained in that case.

TABLE 5. *Surface relaxation constants and principal stresses after the method of Vasil'ev*

| Radiation | Tensile spec. | | | Compressive spec. | | |
|---|---|---|---|---|---|---|
| | $k$ | Stress ($\times 10^3$ psi) | | $k$ | Stress ($\times 10^3$ psi) | |
| | | Axial | Transverse | | Axial | Transverse |
| Co | 1.04 | ---------- | ---------- | 0.79 | 53 | 29 |
| Cr | 0.87 | 16 | 32 | .93 | 15 | 2 |

## 4. Discussion

It is evident that one of the principal objectives of this study, an investigation of Vasil'ev's method of X-ray stress measurements, was not fulfilled in a very positive manner. For ferrous materials, with Co and Cr radiation, Vasil'ev calculated values of $k$ that ranged from 0.3 to 0.5. The $k$ values listed in table 5 are all larger than would seem reasonable, and they show no valid correlation with the penetration power of the X-radiation. The stresses in this table also are in disagreement with Vasil'ev's results. If we denote that axial residual stress by $\sigma_1$ and the transverse residual stress by $\sigma_2$, we should, according to Vasil'ev, have the following inequalities:

$$\text{after an extension: } \sigma_1 < \sigma_2 < 0$$
$$\text{and after a compression: } \sigma_2 < \sigma_1 < 0.$$

In this table, on the contrary, we find

$$\text{after an extension: } \sigma_2 > \sigma_1 > 0$$
$$\text{and after a compression: } \sigma_1 > \sigma_2 > 0.$$

In view of the unlikely values of $k$ that entered into the calculations of these stresses, however, one should not try to infer too much from any seeming relationship among them, except to say that the result apparently is not in agreement with Vasil'ev's results.

The explanation of the failure in this study to obtain meaningful values of $k$ probably lies in the lack of precision with which the extrapolated values

of the diffraction angle were known. Although the precision of individual values of $2\theta$ was high enough, of the order of $\pm 0.02°$, there was a scatter about the straight line function of $2\theta$ versus $\mathrm{Sin}^2\psi$, amounting in the case of the cobalt radiation to a root mean square error of about $\pm 0.08°$. This was reflected in the precision of the extrapolated values upon which the calculation of $k$ was based and hence precluded a significant result for this factor. This scatter does not arise from a lack of precision in the angular measurements themselves, but is inherent in the granular nature of the diffracting polycrystalline material, the statistical fluctuations about a mean stress from one grain to another, and the sampling nature of the X-ray diffraction process. It would seem that Vasil'ev's method could be checked only in situations where this statistical microfluctuation of the stress was at a considerably lower level than was the case in this study.

The results obtained by means of the conventional equations for residual stress calculations, on the other hand, appeared to be quite reasonable and self-consistent. The basic assumption, that the changes in Bragg angle reflected lattice strains or stresses that satisfied the basic laws of elasticity, that is, satisfied the equation of the stress or strain ellipsoid, [12] was borne out in two separate stages of the experiments. The first corroboration was the linearity of the $2\theta$ versus $\sin^2\psi$ measurements, which has long been taken as the proof of a nonrandom stress distribution in the region under analysis. The second was the conformity to the stress ellipsoid on the part of stress components measured in different directions lying in a given surface; three such instances, each involving three surface azimuths are shown in table 3. The stress ellipsoid equation yielding the stress in any direction is

$$\sigma = a_i^2\sigma_i + a_j^2\sigma_j + a_k^2\sigma_k, \qquad (15)$$

where the $a$'s are the direction cosines and the $\sigma$'s are the principal stresses. For the surface case, $a_k$ is zero, and the equation becomes

$$\sigma_\varphi = \sigma_i \cos^2\varphi + \sigma_j \sin^2 \varphi,$$

where the angle $\varphi$ is measured from the direction of $\sigma_i$. If $\varphi$ is taken to be 45°, this becomes

$$\sigma_{45} = 1/2\ (\sigma_i + \sigma_j). \qquad (16)$$

This relationship was checked on each of the three sections cut from the tensile specimen. The three directions chosen on the cross section would be arbitrary; that is, because of the circular symmetry of the specimen and the uniaxial loading, all stress components on this surface should be equal. On the other hand, the directions on the longitudinal and 45° inclined sections are not arbitrary. In these two cases the $\varphi = 0°$ direction was chosen to lie in the direction or the projection of the direction of the original axis of deformation loading of the specimen, and the $\varphi = 90°$ was taken to be at right angles to this. It is seen in table 3 that the agreement between the observations and the stress ellipsoid calculations was excellent.

The third phase of the application of the theory of the stress ellipsoid, however, was not successful. This was the attempt to relate the principal stresses, as determined on a cross section and a longitudinal section, to the stress measured at zero azimuth on a 45°-inclined section. Examples of these measured stresses are given in table 4; and it can be seen that the 45°-section component, which again should be given by eq (16), is nowhere near the theoretical value.

Another somewhat disturbing result that appears when one compares the computed stresses on the different sections is the lack of agreement evident in table 3 between the values of the transverse stress found on each of the three sections. This stress is any one of the three equal values on the cross section and the value of stress at $\varphi = 90°$ on the longitudinal and 45° sections. The latter two values are very small, an order of magnitude smaller than the cross-section value, and are indeed near the limit of precision of the method of measurement. However, all three values, being positive, do agree in sign.

The points of disagreement between these experimental results and the results expected if the sectioning process did not disturb the planar stress components, as originally postulated, have placed this assumption in serious doubt. Sectioning at 45° to the deformation axis in particular seems also to be questionable from the theoretical standpoint, for in this case the actual directions of two of the principal stresses or strains must be rotated through 45°, so that one is normal to the new surface and one is lying along the projection of the axis in the new surface, while the direction of the third is presumably unchanged in the plane of the surface. Following this modification of the internal stress ellipsoid, it is not improbable that the magnitudes of the stresses have undergone change. Although the sectioning along principal planes, as was done to obtain the cross and longitudinal sections in this work and that of Vasil'ev commented upon in this paper, does not change the directions of the principal stresses, the experimental results described above do cast doubt on the constancy of even the planar stress components. It does appear that, although the residual stresses measured by a conventional X-ray method on any one particular surface conform very satisfactorily to the laws of elasticity for a homogeneous, isotropic medium, the stress components determined by this same method on different sections of a single, uniformly deformed specimen cannot in this sense be combined.

In spite of this lack of conformity to an interior stress ellipsoid, presumed to have existed before sectioning, the conventionally calculated principal stresses do, however, present a self-consistent pattern. The stresses revealed by the two radiations bear the same sign in all cases, with those revealed by chromium, the less penetrating radiation, being in most cases smaller in magnitude than

those revealed by cobalt. The axial residual stress after plastic extension is compressive, and after plastic compression, in contradiction with Vasil'ev's results calculated with his equations using nonzero $k$ factors, it is tensile. These results agree with those obtained previously in this laboratory [4, 9].

The opposition of the sign of a residual principal stress, measured by X-rays, to the sign of the preceding plastic deformation is in accordance with the usual two-material explanation for the origin of residual stresses under these circumstances. Briefly, this theory holds that, when a specimen is plastically deformed, for example in tension, some regions, $A$, deform sooner, that is, support a lower elastic stress before slip or other deformation begins, than do other regions, $B$. When the external load is released, the harder regions, $B$, containing higher elastic stresses, try to contract more than do the more plastically deformed, softer regions, $A$. Hence, there are balanced residual stresses in the specimen, compressive in regions $A$ and tensile in regions $B$. If the preceding deformation has been compressive, the signs of the residual stresses in $A$ and $B$ are reversed.

The distinction between region $A$ and $B$ is not one simply of crystallite orientation, since qualitatively the same result is obtained from different groups of grains using different radiations; nor is it a matter of body stresses balancing on the surface against the interior of a specimen, since the same sign is obtained on the surface and interior [5]. Apparently the hardness difference, in terms of flow stress, of regions $A$ and $B$, is ascribable to structure. The harder regions $B$ are very likely regions of high disorder or imperfection, regions of high density of impeded dislocations. Among such regions might be the boundaries of grains or subgrains. The softer regions, $A$, are regions of lower density of imperfection, of lower resistance to the passage of dislocations; such regions might be the interior of grains. This picture is supported by the fact that the X-ray measurements indicate the stress condition, not of the average, but of the softer regions $A$. It is quite logical that the more disordered regions $B$ should contribute less to the coherent diffraction of the X-rays.

In conclusion, therefore, the following may be summarized concerning the X-ray measurement of residual stresses on sections cut from uniformly plastically deformed polycrystalline specimens, using the conventional formulas in computations. Although the stresses on any given section do conform very closely to a stress ellipsoid for an ideal elastic system, the stresses computed from measurements on the various sections cannot be combined to yield a wholly consistent picture of a unique internal stress ellipsoid. Nevertheless, the theory and conventional formulas applied to the two principal sections do yield values of the principal stress that are reasonable in magnitude and consistent with the geometry and deformation history of the specimen. That is to say, the axial and transverse residual stresses have opposite signs; and, when the prestrain is changed from extension to compression, the signs of the stresses change, although there is little change in their magnitude.

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