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Enhancement of the Lunar Tide in the Noon Critical Frequency of the F_2 Layer Over the Magnetic Equator

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The lunar semi-diurnal variations in the midday values of the critical frequency (f_0F_2) and the height of the maximum electron density (h_pF_2) are derived for the two chains of equatorial stations in South America and India for the period 1957–58. The latitudinal variation of the amplitude of the lunar semi-diurnal variation in f_0F_2 shows a sharp maximum over the magnetic equator in both of the longitude zones. There is an indication of systematic variation in the amplitude with longitude along the magnetic equator, the maximum occuring in the South American zone (about 0.63 Mc/s) and the minimum in the Indian zone (about 0.33 Mc/s). Similar longitudinal variations have been indicated in the lunar semi-diurnal variations of the horizontal component of the Earth's magnetic field. The latitudinal variation of the amplitude of the lunar semi-diurnal variation of h_pF_2 is opposite to that of f_0F_2. The enhancement of the lunar variation in the F_2 layer ionization over the magnetic equator appears to be associated with the equatorial electrojet.

1. Introduction

The very large amplitude of the lunar semi-diurnal variation in f_oF_2 found at Huancayo [Martyn, 1947], Christmas Island and Leyte [McNish and Gautier, 1949], and Ibadan [Brown, 1956], has led to the idea that the amplitude increases at lower latitude. Bossolasco and Elena [1960] suggested that the amplitude varies roughly as the cosine function of the magnetic dip. Rastogi [1961] showed that it is a more complex function of latitude. There is a narrow belt within ±3° latitude of the magnetic equator in which the amplitude is abnormally great. There are two wider maxima roughly around ±20° magnetic latitude. The three maxima are separated by zones in which the amplitude is almost constant.

With the establishment of an ionospheric station at Trivandrun in January 1957, the network of the low-latitude ionospheric stations in India was extended southward to the magnetic equator. Further, the establishment of ionospheric stations at Chimbote and Chiclayo, together with the already existing stations at Huancayo and Talara, provided another chain of closely spaced ionospheric stations lying in one longitude zone.

The present article describes the lunar variations in the midday values (mean of 11, 12, 13 hr LST) of f_oF_2 and the height of maximum ionization (h_pF_2) or (h_mF_2) at the equatorial stations operated during the International Geophysical Year.

2. Method of Analysis

From the published hourly values of f_oF_2 daily averages of three hourly values at 1100, 1200, and 1300 LST are computed to obtain the individual midday value of f_oF_2. The monthly means are then subtracted from the individual day values to obtain the deviations for the particular days, Δf_oF_2. These Δf_oF_2 values are then arranged according to the lunar hour approximated to the nearest integral number 00 to 23, derived from the tables given by Bartels and Fanselau [1937]. The data for lunar hour n are combined with those for $n+12$ and the average deviations are found for each lunar hour 00 to 11. The data for days having Ap values greater than 50 are rejected so as to avoid the large perturbations associated with magnetic disturbances. The resultant curves are subjected to Fourier Analysis, the first components of which give the amplitude and phase of the lunar semi-diurnal variation of f_oF_2. The phases are then corrected for the errors due to the movement of the moon between Greenwich and local noon. Similar analyses are done for the derivation of the lunar semi-diurnal variations in the (M3000) F_2 factor, which may be taken as a rough indicator of changes in the height of the maximum ionization in the F_2 layer.

The data of the stations in the American zone used in the present analysis cover the period January to December 1958. The data of some of the Indian stations for 1958 are not satisfactory for such analysis, so to avoid too much interpolation to fill up the missing or doubtful values, these data are not used. Instead, the data for January to December 1957 were used in the case of Indian stations. In order to compare the results of Indian and American stations, similar analysis is done for Ibadan, a station roughly halfway between the longitudes of the two groups of stations; this was done separately for both 1957 and 1958.

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3. Lunar Variations in Critical Frequency of the F_2 Layer

In figures 1 and 2 are shown the variations of Δf_0F_2 with lunar hour at the American and Indian stations separately. The lunar semi-diurnal variation at Ibadan for the respective periods is included in each figure for comparison. The lunar semi-diurnal wave derived from the harmonic analysis is also shown in the figures. Table 1 contains the amplitude (expressed in Mc/s and as percentage of the average f_0F_2), and phase (expressed as the time of maximum positive deviation) of the lunar semi-diurnal variation. Also tabulated are their probable errors derived as described in the appendix.

Referring to figures 1 and 2, the agreement between the lunar semi-diurnal wave and the individual points is very good, except in the case of Madras. From table 1, the amplitude is about ten times its probable error for the American stations and for Ibadan. For Indian stations, the ratio is about 6.5 except for Madras (3.5). According to Chapman

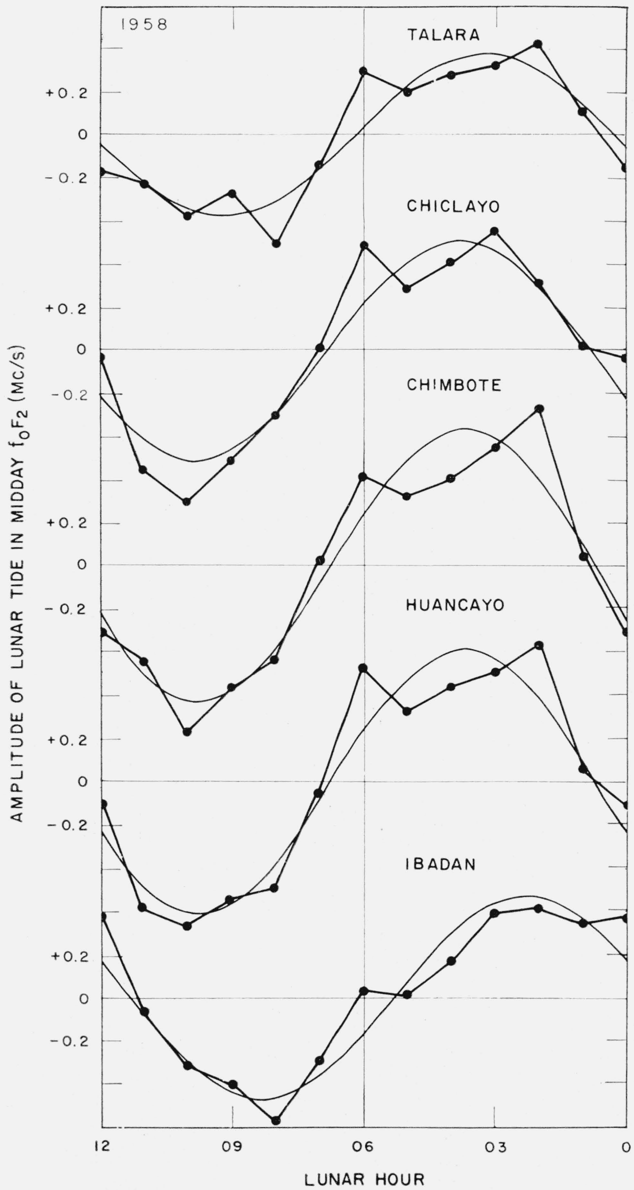

FIGURE 1. *Lunar semi-diurnal variation in midday* f_0F_2 *at American stations.*

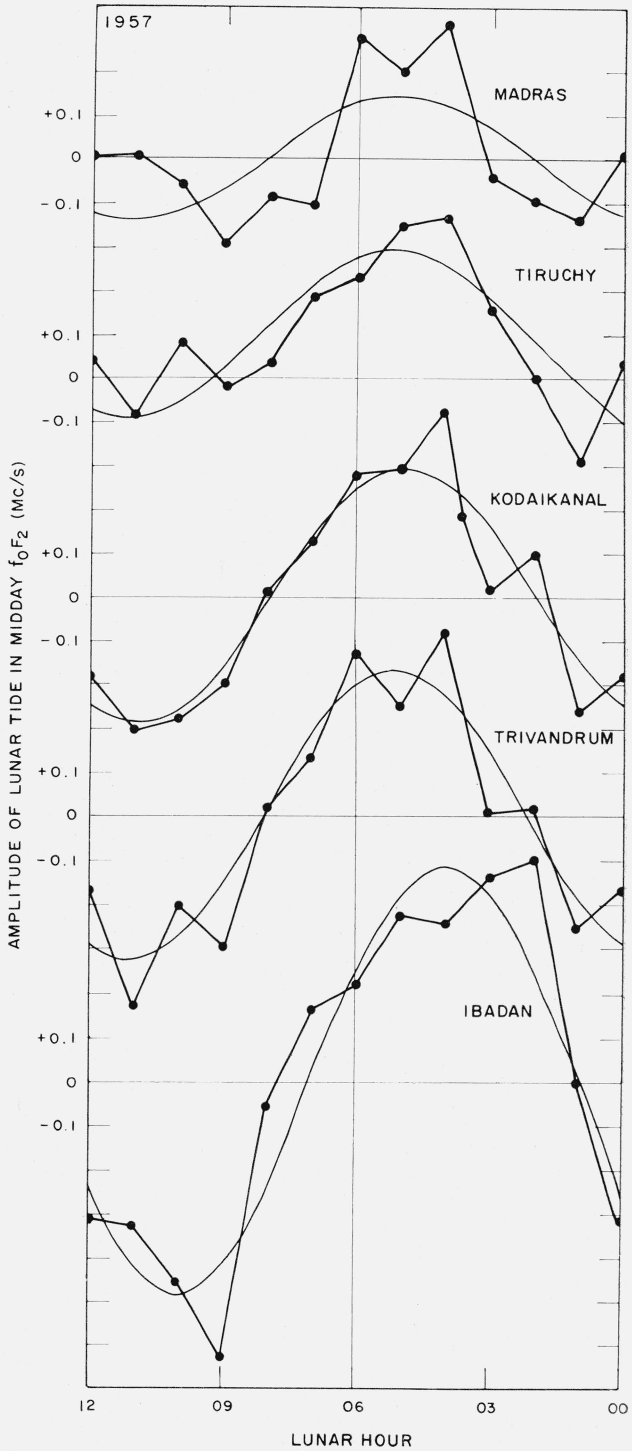

FIGURE 2. *Lunar semi-diurnal variation in midday* f_0F_2 *at Indian stations.*

TABLE 1. *Coefficients of lunar semi-diurnal variations in* f_0F_2 *at the equatorial stations in the American and Indian Zones*

Station	Geog. lat.	Geom. lat.	Mgn. lat.	f_0F_2	Amplitude				Phase		Period studied
					R	R/f_0F_2	Probable error q_R	R/q_R	Time max α	Probable error $q\alpha$	
				Mc/s	*Mc/s*	*Percent*			*Lunar hr*	*Lunar hr*	
Huancayo	75. 3° W	12. 1° S	1° N	12. 65	0. 61	4. 84	0. 055	11. 09	3. 9	0. 17	1958
Chimbote	78. 6° W	9. 1° S	3° N	12. 73	. 63	4. 95	. 053	11. 88	3. 9	. 16	1958
Chiclayo	79. 8° W	6. 8° S	5° N	13. 13	. 51	3. 85	. 058	8. 79	3. 9	. 22	1958
Talara	81. 3° W	4. 6° S	6. 6° N	13. 47	. 38	3. 78	. 043	8. 84	3. 4	. 22	1958
Ibadan	4. 0° E	7. 4° N	2. 5° S	12. 73	. 47	3. 70	. 048	9. 79	2. 3	. 19	1958
Ibadan	4. 0° E	7. 4° N	2. 5° S	12. 63	. 49	3. 90	. 054	9. 07	4. 0	. 23	1957
Trivandrum	77. 0° E	0. 9° N	0. 0	11. 83	. 33	2. 78	. 045	7. 33	5. 0	. 26	1957
Kodaikanal	77. 5° E	10. 2° N	1. 8° N	11. 67	. 29	2. 50	. 044	6. 59	4. 8	. 29	1957
Tiruchy	78. 7° E	10. 8° N	2. 4° N	11. 73	. 19	1. 62	. 034	5. 58	5. 1	. 34	1957
Madras	80. 3° E	13. 1° N	5. 3° N	12. 17	. 14	1. 13	. 040	3. 50	4. 5	. 55	1957

[1951], the determinations may be considered reasonably good if the length of vector is at least three times its probable error. Under this criterion, the amplitude determination at each of these stations is statistically significant.

It is very clear that the amplitude of the lunar semi-diurnal variation within a particular zone steadily increases with decreasing magnetic latitude. The amplitude at Trivandrum is more than twice that at Madras. Similarly the amplitude decreased from about 0.6 Mc/s at Huancayo to about 0.4 Mc/s at Talara. This confirms earlier suggestions [Rastogi, 1961] that the amplitude of lunar semi-diurnal variation in midday f_0F_2 is greatly enhanced within a narrow belt over the magnetic equator.

However, the amplitude over the magnetic equator itself is about 5 percent in the American zone and only about 3 percent in the Indian zone. The amplitude at Ibadan, lying in the intermediate longitude, is about 4 percent. Thus, there is a suggestion that the amplitude steadily decreases along the magnetic equator between the longitudes of Huancayo and Trivandrum.

It may be pointed out that the ionospheric sounders at Trivandrum, Tiruchy, and Madras are of the manual type, whereas at other stations the equipments are the automatic recording type. However, the amplitude of the lunar variation of f_0F_2 at Kodaikanal, derived from the automatic recordings by CRPL model C–2 recorder, fits so closely on the smooth curve joining the points for other Indian stations that the difference in the amplitudes over the magnetic equator in the two zones seems to be genuine and not due to the different equipments or to the quality of data at different stations.

The longitudinal inequality in the amplitude of the lunar semi-diurnal variation of f_0F_2 at the equatorial stations in the Indian and American zones is further ascertained from the analysis of f_0F_2 data at Kodaikanal and Huancayo for 1954, a year of minimum solar activity. Referring to figure 4 and table 2 one finds the amplitude to be 4.4 percent at Huancayo and only 2.9 percent at Kodaikanal, though the phases at the two places are very nearly the same.

Thus, there is a systematic longitudinal variation in the amplitude of lunar semi-diurnal variation in the critical frequency of the F_2 layer over the magnetic equator, having a maximum in the South American zone.

TABLE 2. *Coefficients of lunar semi-diurnal variations in* f_0F_2 *at Huancayo and Kodaikanal during the year 1954*

Station	f_0F_2	Amplitude				Phase	
		R	R/f_0F_2	probable error q_R	R/q_R	Time of max.	probable error q_α
	Mc/s	*Mc/s*	*Percent*	*Mc/s*		*Lunar hr*	*Lunar hr*
Huancayo	6. 91	0. 304	414	0. 035	8. 7	3. 62	0. 22
Kodaikanal	7. 03	. 203	2. 9	. 030	6. 0	3. 66	. 28

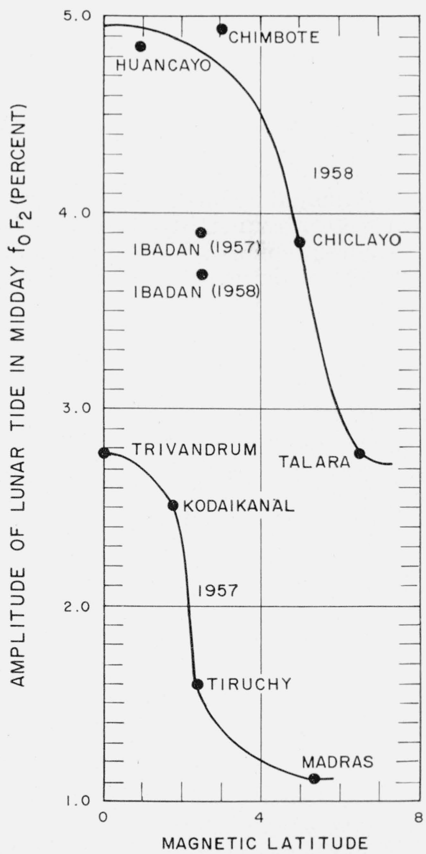

FIGURE 3. *Latitudinal variation of the amplitude of lunar semi-diurnal variation in* f_0F_2 *at the equatorial stations in the American and Indian zones.*

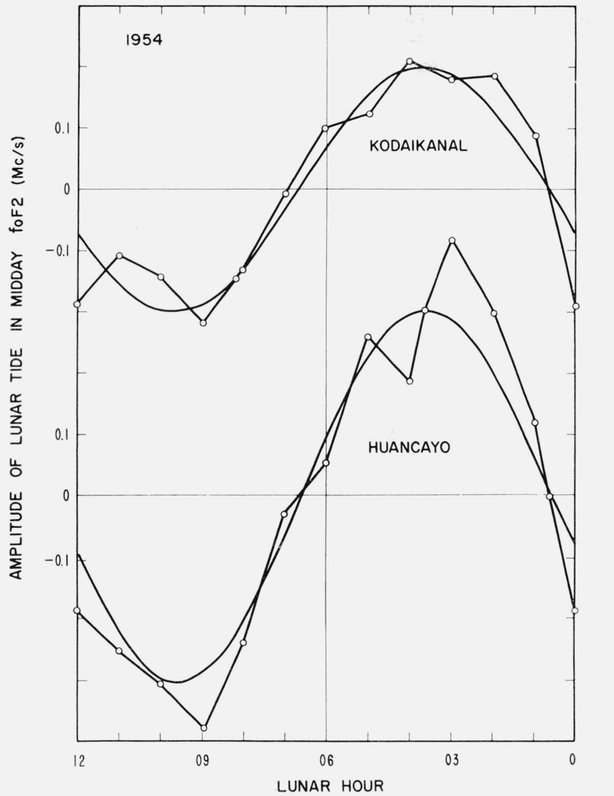

FIGURE 4. *Lunar semi-diurnal variation in midday* f_0F_2 *at Huancayo and Kodaikanal during the minimum sunspot year 1954.*

4. Lunar Variation in Height of the F_2 Layer

The published ionospheric data for Indian stations excepting Kodaikanal include h_pF_2, the virtual height of the F_2 layer at a frequency 0.834 times f_0F_2. This quantity may be taken as a measure of the variation in the height of maximum electron density in the F_2 layer [Wright and McDuffie, 1960].

The distributions of the percentage deviation from the mean value of h_pF_2 with the lunar hours for Madras and Trivandrum are shown in figure 5. Again solar hours 11, 12, and 13 were used. The amplitudes and phases of the lunar semi-diurnal components are collected in table 3. The amplitude of the h_pF_2 variation is about 8 km at Trivandrum and about 14 km at Madras, both reaching the maximum value at about 07 lunar hours.

TABLE 3. *Coefficients of lunar semi-diurnal variations in the height of the maximum ionization density in the* F_2 *layer*

Station	$(M3000)F_2$	Ampl. $R \times 10^{-2}$	$\frac{R}{(M3000)F_2}$	Time max.	hmF_2 or h_pF_2	Ampl. R	Time of max.
			Percent	*Lunar hr*	*Kilometer*	*Kilometer*	*Lunar hr*
Huancayo	2.16	1.7	0.8	2.4	514	5.4	8.4
Chimbote	2.13	2.9	1.4	3.1	524	9.5	9.1
Chiclayo	2.13	3.6	1.7	4.4	524	11.8	10.4
Talara	2.15	5.6	2.7	4.4	517	14.8	10.4
Trivandrum	------	------	------	------	535	7.9	6.9
Madras	------	------	------	------	562	14.0	7.0

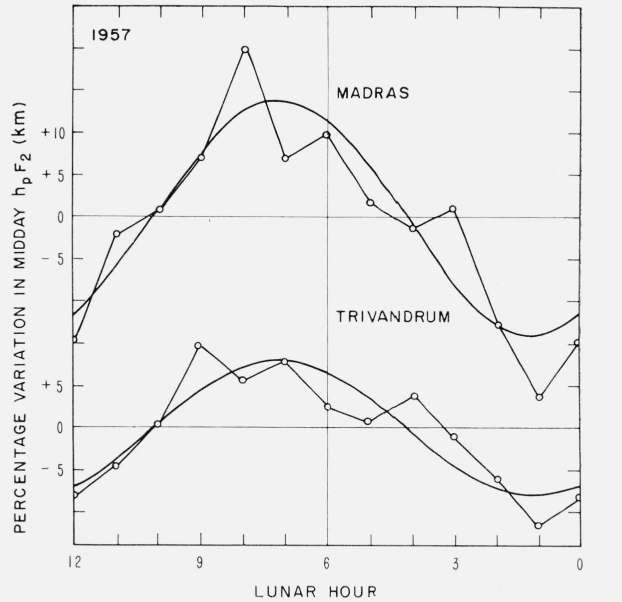

FIGURE 5. *Lunar semi-diurnal variation in midday* h_pF_2 *at Madras and Trivandrum during 1957.*

Lunar tidal analyses have been performed at the National Bureau of Standards on electron density profile data from Huancayo and Talara. These results show that the height of maximum electron density in the F_2 layer, $h$max, has a lunar semi-diurnal variation of amplitude 14.5 km, phase 8.4 lunar hour for Huancayo, and 21.0 km at 8.6 lunar hour for Talara during the period January to May 1960 [Matsushita et al.].

The published hourly ionospheric data of the American stations do not include h_pF_2. However, the height of the F-region peak can be estimated from the $(M3000)F_2$ factor which is regularly scaled at these stations. Wright and McDuffie [1960] have shown that for daytime values at low and medium latitudes the relationship between $(M3000)F_2$ factor and h_mF_2 is approximately

$$h_mF_2 = \frac{1411}{(M3000)F_2} - 169 \ldots\ldots$$

The lunar semi-diurnal variations of the midday value of $(M3000)F_2$ factor at the American stations are shown in figure 6. The average value and the coefficients of the lunar semi-diurnal variation of the height of the F_2 layer peak are derived from the corresponding values and coefficients of the $(M3000)F_2$ factor. These are collected in table 3.

One notices that the amplitude af the lunar semi-diurnal variation in the $(M3000)F_2$ factor gradually decreases with the decrease of magnetic latitude between Talara and Huancayo. Correspondingly the variation of h_mF_2 has an amplitude of about 15 km at Talara gradually decreasing to about 5 km at Huancayo. The amplitudes of the lunar semi-diurnal variation in $h$max as determined from the electron density profile data for Huancayo and Talara are much larger than the ones derived from the $(M3000)F_2$ factor. This may be due to the

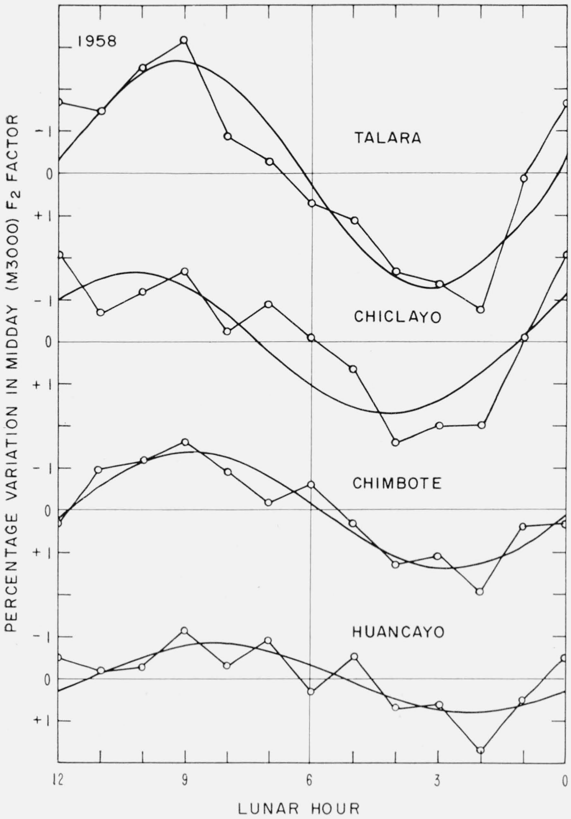

FIGURE 6. *Lunar semi-diurnal variation in midday (M3000)* F_2 *factor at American stations during 1958.*

inherent removal of some accidental errors in the process of the computation of the electron density profile data or partly due to the different period studied. However, there is a definite indication that the amplitude of the lunar semi-diurnal variation of f_0F_2 and h_pF_2 at the equatorial station are inversely related to each other.

5. Other Phenomena Associated With the Magnetic Equator

The solar diurnal variation of the horizontal component of the Earth's magnetic field, H, has been found to be abnormally large at stations close to the magnetic equator, such as Huancayo [McNish, 1937], Uganda [Chapman, 1948], Kodaikanal [Thiruvegadathan, 1954], and Ibadan [Onwumechilli and Alexander, 1959]. The great enhancement of the dirunal range in H is confined to a narrow belt roughly about 10° wide over the magnetic equator [Egedal, 1947, 1952; Pramanik and Hariharan, 1953; Forbush and Casaverde, 1961]. It was ascribed by McNish [1937] to a locally concentrated electric current flowing from west to east above the magnetic equator, to which the name electrojet was given by Chapman [1951].

In figure 7 are plotted the dirunal variations of H averaged for five quiet days of December 1957, March 1958, and June 1958 at Huancayo and Trivandrum. The diurnal range of H is seen to be larger at Huancayo than at Trivandrum, being about

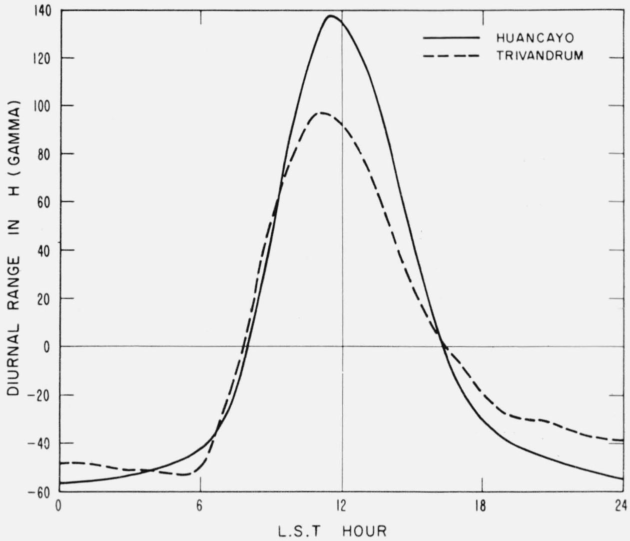

FIGURE 7. *Diurnal variation of the horizontal component of the Earth's magnetic field at Huancayo and Trivandrum during 1957–58.*

196 γ and 139 γ, respectively. Rastogi [1962] has shown the range of diurnal variation of H over the magnetic equator varies systematically with longitude, being maximum over South America and minimum over India.

Bartels and Johnson [1940] showed that the lunar variation in H is abnormally large at Huancayo. Very large amplitudes of lunar semi-diurnal variation in H has been found at other equatorial stations. Kodaikanal [Raja Rao and Sivaraman, 1958] and Ibadan [Onwumechilli and Alexander, 1959b]. Forbush and Casaverde [1961] have shown that the abnormally larger amplitude of lunar-tide in H at Huancayo arises from the electrojet.

The amplitude of the lunar semi-diurnal variation in H at Huancayo as determined by Bartels is still the largest so far determined for any other equatorial station. The amplitude averaged for the three seasons is 5.43 γ at Huancayo, 3.96 γ at Ibadan, and 2.34 γ at Kodaikanal. This indicates a systematic longitudinal variation in the amplitude of the lunar semi-diurnal variation in H similar to that in f_0F_2 over the magnetic equator.

The similarities between the latitudinal variations of the lunar semi-dirunal variation of f_0F_2 and the solar diurnal variation of H, and further longitudinal inequalities in the amplitudes of the lunar semi-diurnal variation in f_0F_2 and in H, strongly suggest that the abnormally large amplitude of the lunar semi-diurnal variation in the F_2 layer ionization over the magnetic equator is closely associated with the equatorial electrojet. The mechanism of such an association is still to be understood.

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6. Appendix. Formulas for Probable Errors of Amplitude and Phase

We make use of standard procedures such as those given by Whittaker and Robinson [1944]. The lunar semi-diurnal wave (period of 12 lunar hours) may be expressed by

$$R \sin\left(\frac{\pi t}{6}+\alpha\right)=a \cos(\pi t/6)+b \sin(\pi t/6)$$

where R is the amplitude, t is the time in lunar hours, α is the phase, and a and b are the cosine and sine coefficients for the 12-hr wave obtained by a Fourier analysis of the means of the Δf_0F_2 values for each integral lunar hour. As usual in such analyses, $t=0$ corresponds to an upper or lower culmination of the moon at the longitude of the station.

The variances of a and b are

$$\sigma_a^2=\left(\frac{1}{6}\right)^2 \sum_{p=0}^{11} \sigma_p^2 \cos^2(\pi p/6)$$

and

$$\sigma_b^2=\left(\frac{1}{6}\right)^2 \sum_{p=0}^{11} \sigma_p^2 \sin^2(\pi p/6),$$

where p is the lunar hour and σ_p^2 is the variance of the mean value of Δf_0F_2 for that hour.

For simplicity, the (weighted) mean variance σ^2 for all lunar hours was substituted for σ_p^2 in the above formulas. Thus if

$$\sigma^2=\left(\sum_{p=0}^{11} n_p\sigma_p^2\right)\Big/\sum_{p=0}^{11} n_p$$

where n_p is the number of values of Δf_0F_2 for the lunar hour p, we take

$$\sigma_a^2=\left(\frac{1}{6}\right)^2 \sigma^2 \sum_{p=0}^{11} \cos^2(\pi p/6)=\sigma^2/6$$

and

$$\sigma_b^2=\left(\frac{1}{6}\right)^2 \sigma^2 \sum_{p=0}^{11} \sin^2(\pi p/6)=\sigma^2/6.$$

Then, provided that $\sigma_R<<R$ the variances of R and α are given approximately by

$$\sigma_R^2=\sigma_a^2=\sigma_b^2=\sigma^2/6$$

and

$$\sigma_\alpha^2=(\sigma_R/R)^2.$$

Assuming that R and α are normally distributed (approximately valid if $\sigma_R<<R$), their probable errors (error likely to be exceeded in 50 percent of the cases) are approximately

$$q_R=0.675\sigma_R=0.275\sigma$$

and

$$q_\alpha=q_R/R \text{ radians}.$$

In lunar hours, q_α is

$$q_\alpha=(6/\pi)(q_R/R)=1.91q_R/R.$$

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