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Magnetotelluric Fields in the Frequency Range 0.03 to 7 Cycles Per Kilosecond: Part II. Geophysical Interpretation

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The power spectra computed in part I for Tbilisi, USSR, are used to compute apparent resistivity in accordance with a formula developed by Cagniard. It is found that both components of the telluric field yield a value of 6 ohm meters for the apparent electrical resistivity of the earth. The data do not extend over a sufficiently wide range of frequency to permit conclusions about the stratification. There is no evidence of horizontal anisotropy of the earth's resistivity.

The magnetic power spectrum for the earth's ambient field is corrected for the transmission losses in the ionosphere to yield the power spectrum of the magnetic field incident on the earth. The major part of the variation with frequency is eliminated by this correction.

1. Method of Analysis

Cagniard's paper [1953] on the magnetotelluric field produced a renewal of interest in the use of magnetotelluric field components for the interpretation of the electrical structure of the earth. His proposal involves the plotting of an apparent resistivity versus the square root of the period of the wave but he offers no suggestion of how one might resolve the experimental observations which resemble a stochastic time series into frequency components. Scholte and Veldkamp [1955] applied Cagniard's theory to data recorded at magnetic observatories by searching the records for intervals during which the oscillations were nearly sinusoidal. In this manner they were able to obtain a few points on a frequency curve without making a spectral analysis. These authors also examine the effect of the curvature of the earth and showed that it was negligible.

Cantwell [1960] considered the magnetotelluric field components as samples of stationary stochastic processes and applied the numerical methods of Blackman and Tukey [1959] to the data. Cantwell showed that Cagniard's basic formula for the apparent resistivity may be expressed in terms of power spectra as

$$\rho_a = 0.2\, T P_E(\mathrm{f})/P_H(\mathrm{f}), \tag{1}$$

where

ρ_a = apparent resistivity, ohm meters,
T = period in seconds,
$P_E(\mathrm{f})$ = power spectral density of the telluric field component in (millivolts/kilometer)^2 per cycle per second,
$P_H(\mathrm{f})$ = power spectral density of the orthogonal component of the horizontal magnetic field in (gammas)^2 per cycle per second,
0.2 = dimensional constant chosen to make the equation correct.

Cagniard provides a template for interpreting the apparent resistivity in terms of electrically conducting layers when the data are plotted as a graph of ρ_a versus $T^{1/2}$ on log-log paper. However, although the present data extend over more than two decades of frequency, they do not show evidence of an electrical interface. Consequently the data are plotted versus frequency, and the vertical scale is compressed to reduce the scatter of the points and to facilitate the drawing of a curve through the points. Figure 1 contains plots of apparent resistivity based on the East-West telluric field and the North-South magnetic field for 1 and 2 September. Figure 2 contains a similar plot based on the other pair of components of the magnetotelluric field. The latter data are restricted to the outputs of the high-pass filter because of the uncertainties in the low-passed data.

The four curves plotted in figures 1 and 2 are mutually consistent and they indicate an apparent resistivity of 6 ohm m over the frequency range 0.1 to 7 c/ks. The apparent resistivity is essentially constant over this frequency range and there is no evidence of stratification of the earth's conductivity. The difference between the apparent resistivities obtained from the two components of the telluric field are not sufficiently different to suggest any horizontal anisotropy in the earth's resistivity at the relevant depths.

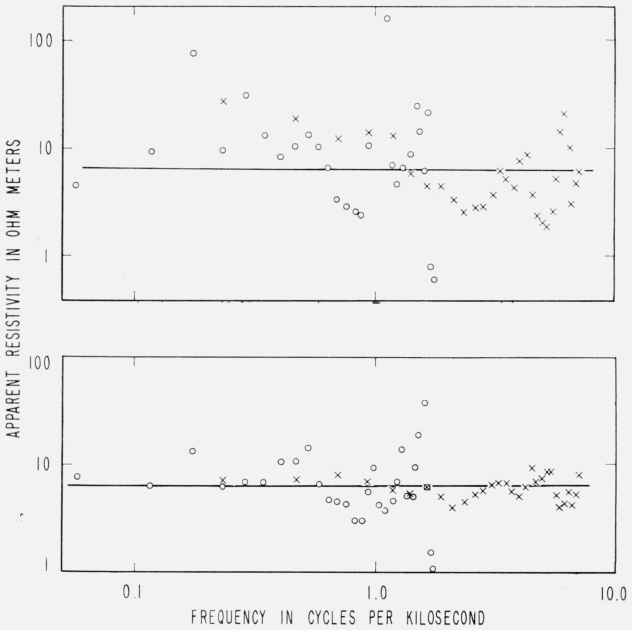

FIGURE 1. *Graphs of apparent resistivity versus frequency determined from the East-West component of the telluric field at Tbilisi 1 and 2 September 1957.*

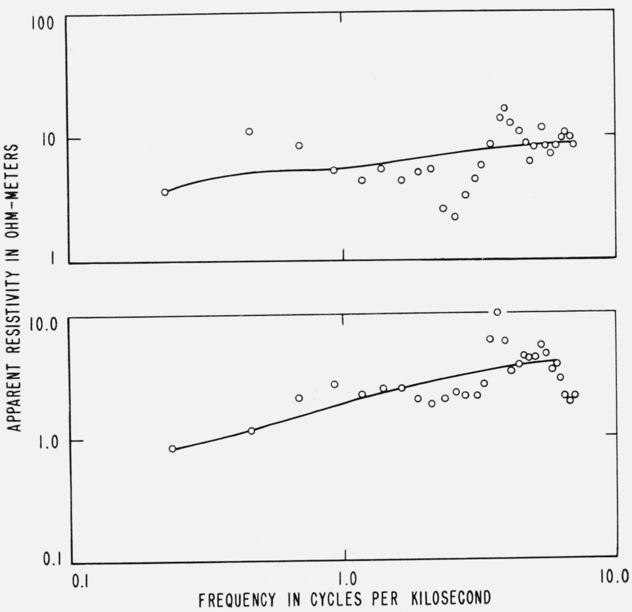

FIGURE 2. *Graphs of apparent resistivity versus frequency determined from the North-South component of the telluric field at Tbilisi 1 and 2 September 1957.*

2. Interpretation of the Earth's Conductivity

The concept of apparent resistivity as introduced by Cagniard involves the assumption of plane electromagnetic waves impinging on a horizontally stratified earth. There is some discussion in the literature [Wait, 1954; Cagniard, 1954] regarding the validity of these hypotheses for magnetotelluric fluctuations. If they be true, the earth at Tbilisi responds to these waves in the same manner as a uniformly conducting earth of resistivity 6 ohm meters. This value is typical of the earth at depths near 500 km [Tozer, 1959] which is the penetration that one would expect for waves of this frequency.

The frequency range of the data plotted in figures 1 and 2 is not great enough to permit one to deduce the variation of resistivity with depth at Tbilisi. For this purpose one would need to extend the data to much higher frequencies. Cantwell [1960] interprets the resistivity profile in Massachusetts as a layer 70 km thick with a resistivity of 8,000 ohm meters followed by a zone of much higher conductivity. If this model is applied to Tbilisi, it is found that the apparent resistivity has a much greater dependence on frequency than shown by the data plotted in figures 1 and 2. In order to apply a two layer earth model to the present data, one must postulate a layer only 7 km thick with resistivity 8,000 ohm meters or some intermediate combination of thickness and resistivity. This reduction of the thickness of the resistive layer could be attributed to thermal insulation due to the Caucasus mountains.

3. Magnetic Fields Above the Ionosphere

The data presented in figure 13 of part I showed that the North-South component of the magnetic field for a wide range of locations and times were mutually consistent and could be fitted rather well by an empirical curve. If one assumes that the origin of these fields lies outside of the ionosphere, one can make a correction for the transmission of the ionosphere and arrive at a spectrum for the magnetic fluctuations incident on the ionosphere. It must be remembered that the earth is a sufficiently good conductor at these frequencies so that the magnitude of the magnetic field at the earth is almost exactly twice the magnitude of the incident radiation. Hence, one concludes from eq (3), part I, that the power spectrum of the radiation incident on the earth's surface is

$$P_H(\mathrm{f})=2.5\times10^4[1+25f^2+f^4+1.3\times10^{-4}f^6]^{-1} \quad (\text{gammas})^2 \text{ per c/s } 0.1<f<400 \text{ c/ks}, \qquad (2)$$

where f is expressed in cycles per kilosecond.

Akasofu [1960] has computed transmission coefficients of the ionosphere and concludes that it is nearly transparent for frequencies less than 10 c/ks. Column 2 of table 1 contains the power transfer constants obtained by Akasofu while column 3 shows the power density incident at the earth's surface. The last column of table 1 lists the power spectral densities of the magnetic field incident on the ionosphere.

The only conclusion that one may reach is that the tremendous decrease in P_H(f) with increasing frequency found at the earth's surface may be due to the attenuation in the ionosphere and that the magnetic field incident on the ionosphere has a much more nearly flat spectra than the fields observed at the earth's surface.

TABLE 1. *Magnetic power densities incident on the ionosphere*

f	Power transmission coefficient of ionosphere	P_H(f) incident on earth	P_H(f) incident on ionosphere
c/ks		*Gammas*^{2} *c/s*	*Gammas*^{2} *c/s*
1	1	9.25×10^2	925
10	0.104	1.98	19.0
100	8.01×10^{-4}	1.09×10^{-4}	0.136
1000	6.76×10^{-12}	[a] 1.91×10^{-10}	28.3

[a] Extrapolated from eq (2).

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4. References

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