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Magnetotelluric Fields in the Frequency Range 0.03 to 7 Cycles Per Kilosecond: Part I. Power Spectra

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Power spectra of the horizontal components of the magnetic field and the telluric field are computed for data recorded on 1 and 2 September 1957 at the Soviet Magnetic Observatory in Tbilisi. Power spectra of the East-West telluric field component are computed for 20 September 1957 for Soviet stations located at Lvov, Tbilisi, and Ashkhabad. All analyses are based on microfilm copies furnished by the IGY World Data Center A. Each power spectrum shows a frequency dependence over a frequency range of 0.03 to 7 cycles per kilosecond of the form P_0f^{-n} where n varies from 1.1 to 2.5. The coherency between orthogonal components of the telluric and magnetic fields is computed for the data from Tbilisi. Magnetotelluric power spectra from USSR, Canada, Texas, and Massachusetts are plotted on a common graph to show the frequency dependence over the range 0.03 to 400 cycles per kilosecond.

1. Introduction

The aim of the present study is the computation of power spectra of the magnetotelluric fields at the earth's surface. One of the authors (CWH) spent a week in August 1960, at the IGY World Data Center A, Washington, D.C., examining the microfilm copies of data available at that time. The available data were examined with a view to finding simultaneous recordings of telluric and magnetic fields at one station so that Cagniard's theory of magnetotelluric prospecting [Cagniard, 1953] could be applied to the earth. A second goal of the study is the comparison of simultaneous recordings at different stations. With these aims in mind the microfilms were searched for days on which the fluctuations were large so that the records could be read easily but which were free of storms and sudden commencements so that the records would approximate stationary time series. This last condition was important since the data would be subjected to spectral analysis.

Of the microfilms available at the time of the visit to the Data Center those for the month of September 1957, for the IGY stations Lvov, Tbilisi, and Ashkhabad,[1] USSR, were most suitable for the present purpose. Figure 1 shows the location of the

[1] There are two transliterations of the Russian word. It is spelled Ashkabad in the publications of the IGY World Data Center A, while Bartholomew's Advanced Atlas gives Ashkhabad.

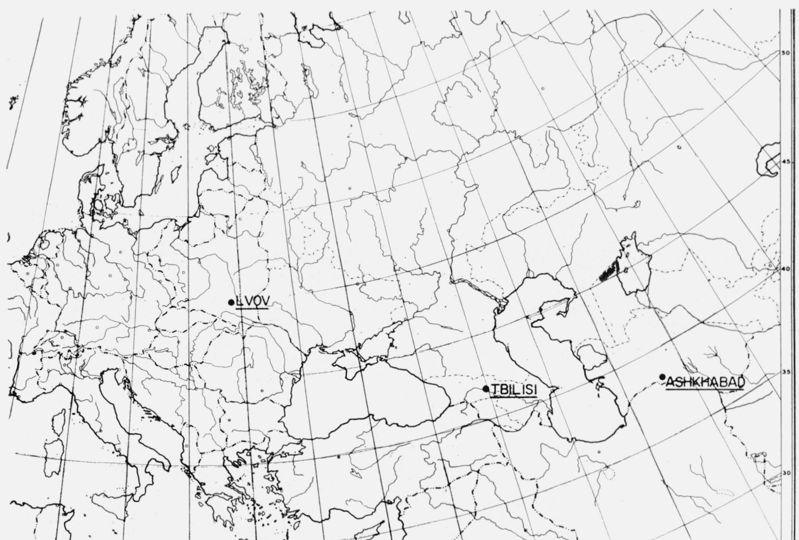

FIGURE 1. *The location of the three Soviet IGY Stations Lvov, Ashkahbad, and Tbilisi.*
Base map copyright by Univ. of Chicago.

stations, and figures 2 and 3 show typical samples of the data. The reader is referred to the publications of the I.A.G.A. [1957] for detailed descriptions of these observatories.

The magnetic and telluric fields recorded at Tbilisi, on 1 and 2 September 1957, were subjected to fairly extensive analysis. The East-West component of the telluric fields at the three stations were analyzed for simultaneous recordings made on 20 September. This gave a cross section in space on one day and a sampling in time at one station for one of the components of the telluric field.

The telluric recordings give the East-West and the North-South components directly. The magnetic recordings, however, give the horizontal intensity H and the declination D. These values were converted to North-South and East-West components for the present analysis.

2. Methods of Processing the Data

After suitable days were selected, photographic enlargements were made of the records and the amplitudes of the traces were read at equally spaced times. The time interval, Δt, between successive samples was chosen small enough to avoid loss of the higher frequency components. The data were processed by the methods described by Blackman and Tukey [1959] whose notation and terminology will be followed closely. Their procedures have been used by other workers in this field [Munk, Snodgrass, and Tucker, 1959; Cantwell, 1960]. Table 1 contains a summary of the parameters associated with each sample of data discussed in this paper.

After the ordinates were read, they were subjected to an arithmetic operator [Horton, 1962] which acted as a high-pass filter. The power spectrum of the filtered values is more nearly constant than that of the unfiltered values so the filter serves as a "prewhitening" filter. The N filtered values y_i were used to compute the mean lagged products

$$C_r=\frac{1}{N-r}\sum_{i=0}^{N-r} y_i y_{i+r}, \quad r=0,1,2, \ldots, 30. \tag{1}$$

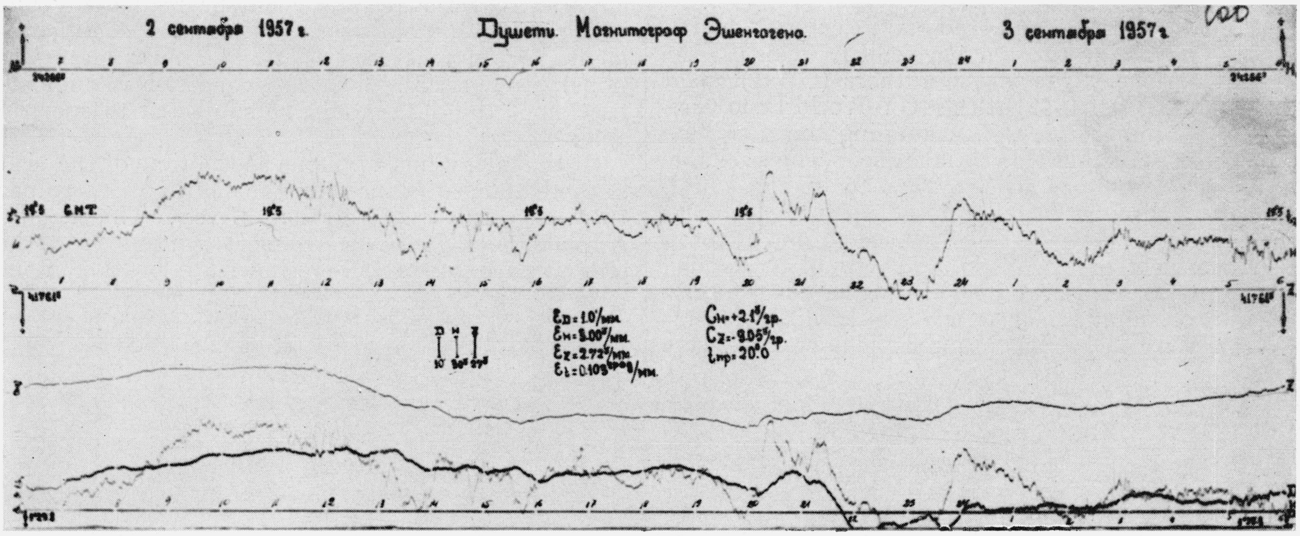

FIGURE 2. *A typical sample of the magnetograms analyzed.*
Record for Tbilisi 2 September 1957.

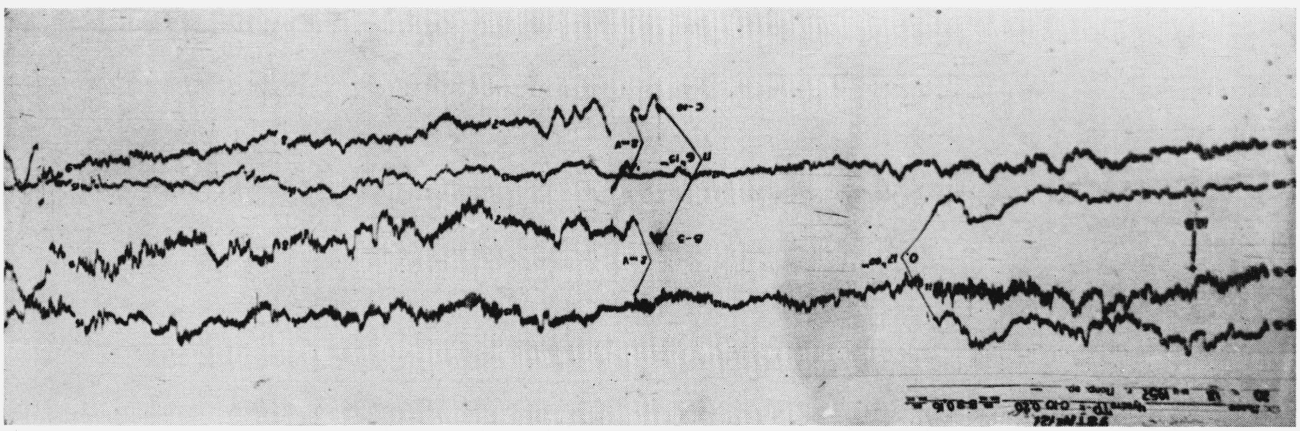

FIGURE 3. *A typical sample of the tellurograms analyzed.*
Record for Lvov 20 September 1957.

TABLE 1. *Summary of the data presented in this paper*

Sample No.	1	2	3	4	5	6
Date	20 Sept. 57	20 Sept. 57	20 Sept. 57	2 Sept. 57	2 Sept. 57	31 July 59
Telluric field	E–W	E–W	E–W	E–W	N–S	E–W
Magnetic field	x	x	x	N–S	E–W	N–S
Paper speed	22 mm/hr	90 mm/hr	78 mm/hr	22 mm/hr	22 mm/hr	12 in./min
Length of sample, hr	24	5.5	7.8	10	10	1.25
Δt, sec	150	31.6	47.3	72	72	0.75
No. of samples	579	624	590	501	501	3,030
Decimation	4	4	4	4	4	4
New Δt, sec	600	126.4	189.2	288	288	3
No. of decimated samples	144	156	147	125	125	757
Plotted in figure	5	6	7	8, 9	10, 11	13, 14

The coefficients C_r represent approximations to the autocovariance function whose Fourier (cosine) transform yields the power spectrum. Thirty-one estimates of the power spectral density are obtained from the following cosine transform

$$V_r = \Delta t \left[C_0 + 2 \sum_{q=1}^{29} C_q \cos(qr\pi/30) + C_{30} \cos r\pi \right]. \quad (2)$$

The resulting power spectral estimates are averaged in groups of three to give improved values. This is the process that Blackman and Tukey refer to as hanning. Finally, the effects introduced by the high-pass filter are removed to obtain the values plotted.

In order to increase the resolution at lower frequencies, one must either increase the range of the index r of eq (1) or increase the sampling interval Δt. Before one can safely increase Δt, a low-pass filter must be used to insure that no significant power occurs at frequencies greater than the Nyquist frequency $1/2\ \Delta t$. When the power spectrum is contaminated by power above the Nyquist frequency one sometimes describes this effect as "aliasing." The second alternative mentioned above, which is preferable in that it requires less computation, was followed. The sampled data were filtered with a low-pass numerical operator and decimated by selecting only every fourth value. After the power spectral density estimates were computed, the values were corrected for the effect of the low-pass filter.

The frequency responses (amplitude) of the two filters are shown in figure 4. Although the abscissa of the curve is labeled for a sampling interval of $\Delta t = 72$ sec, it can be corrected to any other sampling interval by a simple change of scale. The low-pass filter is a symmetric 9-term operator given by Munk, Snodgrass, and Tucker [1959]. The high-pass filter is an unsymmetric 18-term operator designed by Horton [1962]. Since the low-pass operator is symmetric, it does not produce a phase distortion. The phase delay of the high-pass operator has no effect on the power spectra. As one may see from figures 2 and 3 the data do not have any d-c drifts and no correction for such drift was necessary.

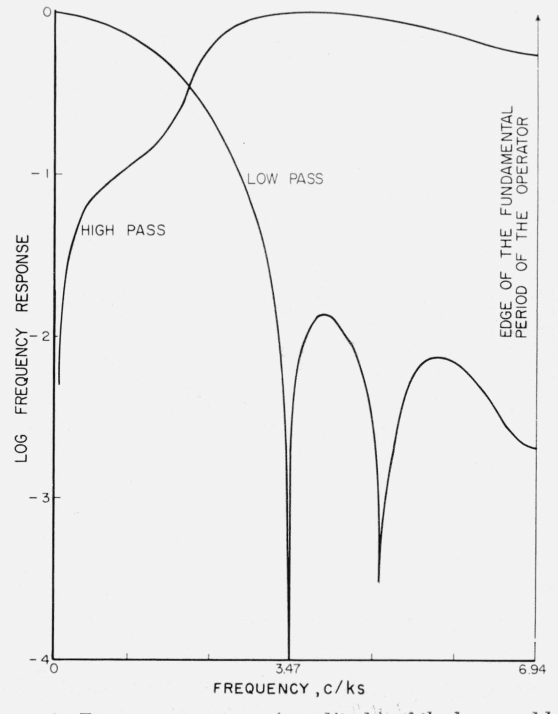

FIGURE 4. *Frequency response (amplitude) of the low- and high-pass filters used in processing the data.*

3. East-West Telluric Components for 20 September 1957

Figures 5, 6, and 7 are graphs of the power spectra of the East-West component of the telluric field at

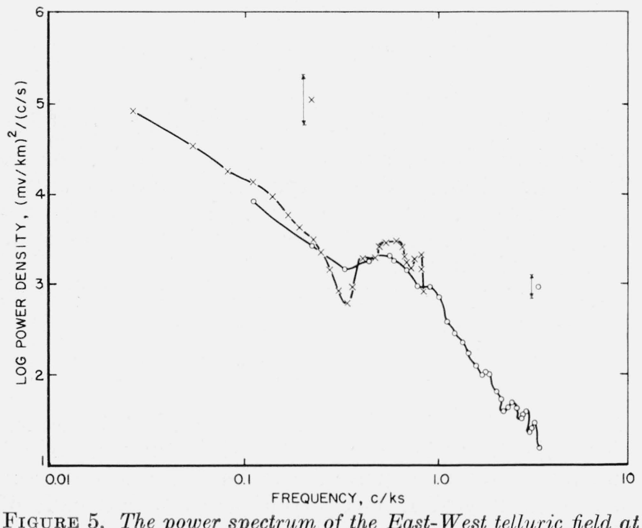

FIGURE 5. *The power spectrum of the East-West telluric field at Ashkahbad 20 September 1957.*
80 percent confidence is indicated.

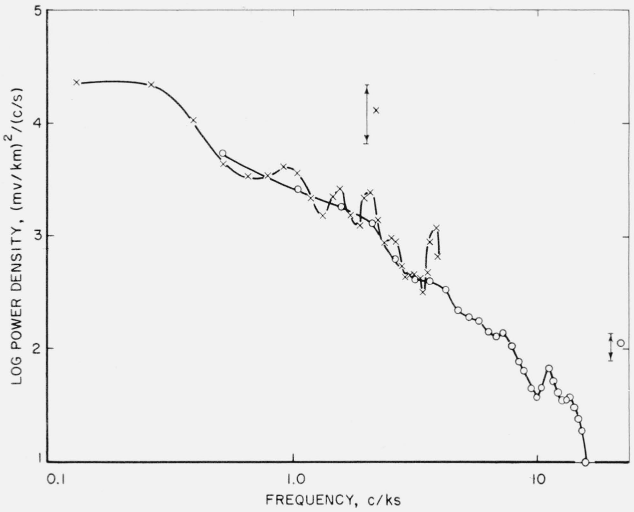

FIGURE 6. *The power spectrum of the East-West telluric field at Lvov 20 September 1957.*

80 percent confidence is indicated.

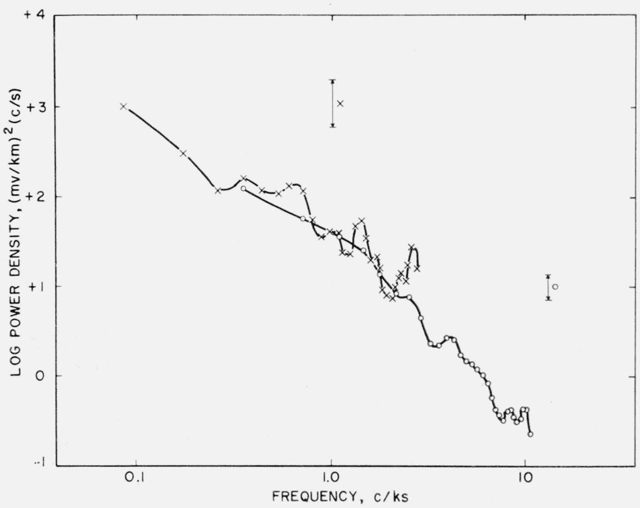

FIGURE 7. *The power spectrum of the East-West telluric field at Tbilisi 20 September 1957*

80 percent confidence is indicated.

Ashkhabad, Lvov, and Tbilisi, respectively, for 20 September 1957. The graphs are self-explanatory and require very little comment. Each of the power spectra obtained from the low-pass filter shows some evidence of aliasing.

The three power spectra can be approximated with a formula

$$P(\mathbf{f})=P_0 f^{-n} \tag{3}$$

with a value of n between 1.5 and 1.6. Actually this formula is not a satisfactory fit since one would select n nearer one for the lower decade of frequency and nearer two at the upper decade of the spectrum. The question of an empirical formula will be considered further in section 6.

It is interesting to compare the power density at each of the stations at a frequency of 1 c/ks. If one uses Lvov as the reference, the power densities at Ashkhabad and Tbilisi are -6.3 db and -18.8 db, respectively. These relative values undoubtedly reflect the underlying continental structure. For example, the topography at Tbilisi is higher and more rugged than the other two stations. The moderately saline Black Sea (salinity 17.6 to 22.2 $^0/_{00}$) is between Lvov and Tbilisi but nearer the latter.

4. Magnetotelluric Spectra at Tbilisi

As mentioned above the records made at Tbilisi on 1 and 2 September 1957 were subjected to a detailed analysis. The reader is referred to the Ph. D. thesis of Hoffman [1962] for more graphs and tabular values of the power spectra Most samples were 10 hr long. A few computations were made for 20-hr samples but these did not show enough difference from the 10-hr samples to warrant their presentation. For some reason that cannot be discovered, when the East-West component of the magnetic field and the North-South component of the telluric field were analyzed with the aid of the low-pass filter, the power spectra were erratic and unreliable. This showed up both in numerous negative values for the power spectral densities but also in ridiculously large values in the coherency. These low-passed data were rejected and the responses of the high-pass filters only are used in this paper.

The individual magnetic and telluric power spectra for 2 September are illustrated in figures 8 through 11. The spectra of the North-South magnetic and the East-West telluric components do not show any striking departures in behavior from the curves for 20 September. The empirical formula in eq (3) can again be made to fit the magnetic spectra with $n=2$ and the telluric spectra with n between 1.1 and 1.2.

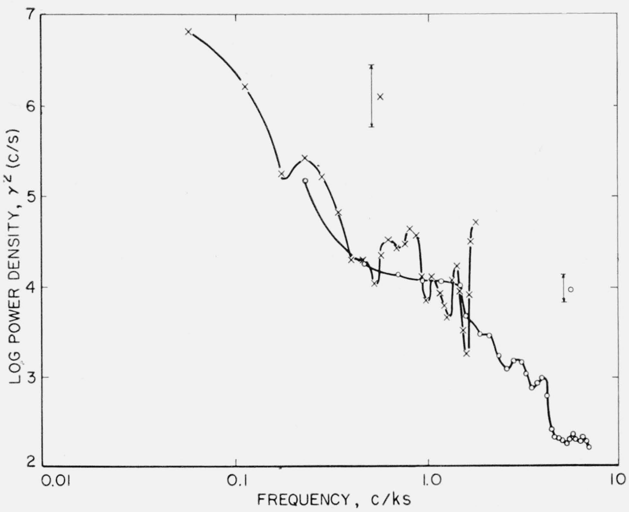

FIGURE 8. *The power spectrum of the North-South magnetic field at Tbilisi 2 September 1957.*

80 percent confidence is indicated.

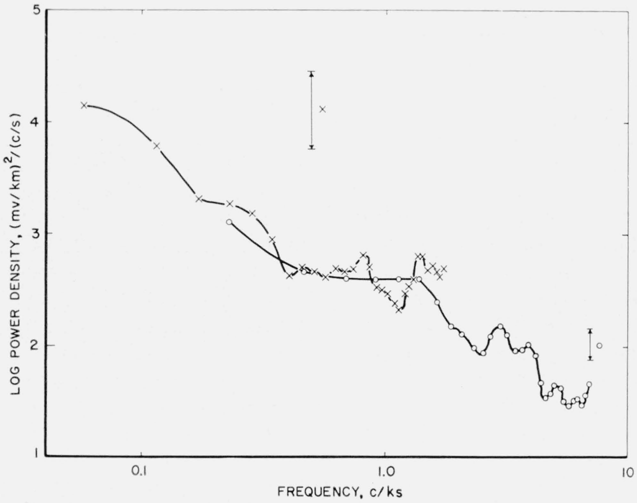

FIGURE 9. *The power spectrum of the East-West telluric field at Tbilisi 2 September 1957.*
80 percent confidence is indicated.

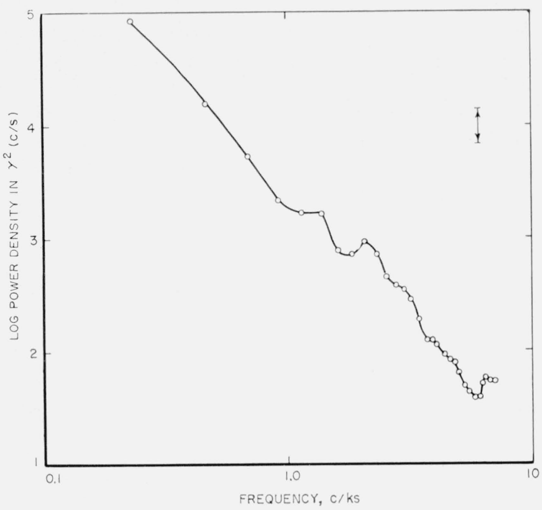

FIGURE 10. *The power spectrum of the East-West magnetic field at Tbilisi 2 September 1957.*
80 percent confidence is indicated.

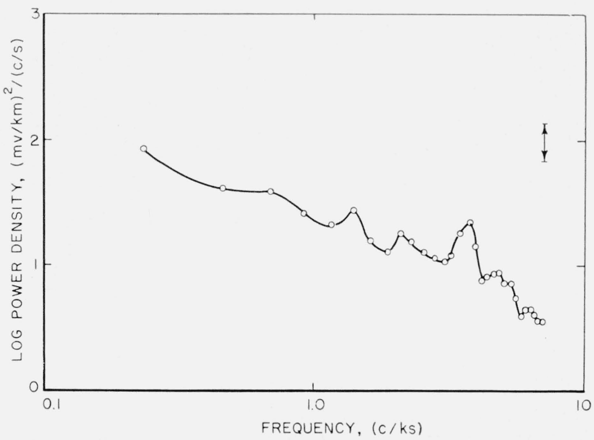

FIGURE 11. *The power spectrum of the North-South telluric field at Tbilisi 2 September 1957.*
80 percent confidence is indicated.

If we compare power densities at 1 c/ks and use 1 September as the reference, the power levels of the East-West telluric field are 0 db, +0.6 db, and −0.4 db on 1, 2, and 20 September, respectively. The power levels of the North-South magnetic fields are 0 db and +0.6 db on 1 and 2 September, respectively.

5. Coherency of the Magnetotelluric Field at Tbilisi

It has been known for some time [Chapman and Whitehead, 1923] that any horizontal component of the telluric field is related causally to the orthogonal horizontal component of the magnetic field. The degree of correlation between two such components can be evaluated from the coherency defined as

$$\mathrm{Coh}_{EH}(\mathrm{f})=\frac{|\Phi_{EH}(\mathrm{f})|}{\sqrt{\Phi_{EE}(\mathrm{f})\Phi_{HH}(\mathrm{f})}} \tag{4}$$

where $\Phi_{EH}(\mathrm{f})$ is the cross spectrum between the two field components and $\Phi_{EE}(\mathrm{f})$ and $\Phi_{HH}(\mathrm{f})$ are the corresponding power spectra of the individual series. The cross spectrum is approximated from the sampled data with the aid of the lagged cross products

$$C_r=\frac{1}{N-r}\sum_{i=0}^{N-r} x_i y_{i+r}, \qquad r=0, \pm 1, \ldots, \pm m. \tag{5}$$

When x and y are different time series, $C_{-r}\neq C_{+r}$, in general, and the Fourier transform is complex. Therefore eq (2) must be supplemented by a similar expression with sines. Thus, the corresponding power spectral density estimates Φ_{EH} are complex and one must use absolute values in eq (4). Figure 12 contains a few plots of the coherency between components of the magnetotelluric fields at Tbilisi on 1 and 2 September.

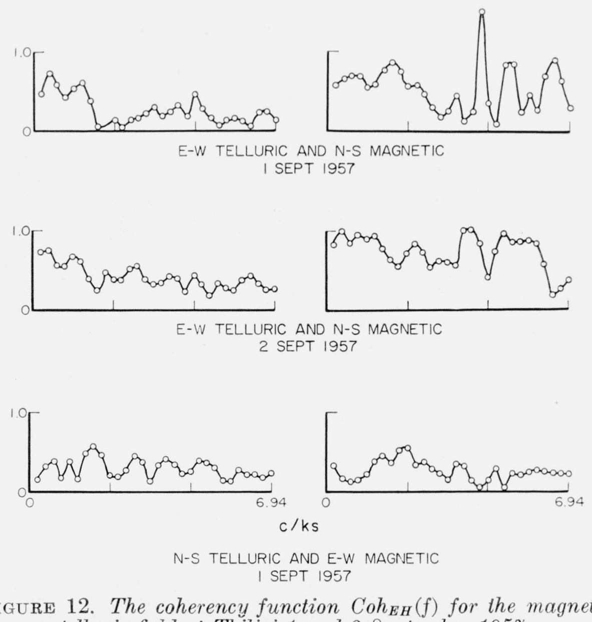

FIGURE 12. *The coherency function $Coh_{EH}(f)$ for the magnetotelluric field at Tbilisi 1 and 2 September 1957.*

The Nyquist frequency of each spectrum is 6.94 c/ks except for the upper two curves on the right. The Nyquist frequency for these two curves is 1.74 c/ks.

6. Comparison With Other Magnetotelluric Data

Although each of the spectra plotted in figures 5 through 11 cover 2.4 decades of frequency, it is evident that this is a small section of the total magnetotelluric spectrum. In order to obtain a more comprehensive picture of the magnetotelluric spectrum selections from the present data and other spectra available to the authors are combined on a composite plot in figures 13 and 14.

Since the Canadian data presented in these figures are new data that have not been published before, a brief description of the experimental arrangement is given. In the summer of 1959 Sir Charles Wright, Dr. J. H. Duffus, and other staff members of the Pacific Naval Laboratory, Esquimault, Canada, established a field station at the Suffield Experimental Station near Medicine Hat, Alberta, Canada, to measure magnetic field fluctuations. Dr. K. Vozoff of the University of Alberta set up earth current electrodes at this station for the measurements of telluric fields. Mr. Jerry Joyner, an employee of Texas Instruments, Inc., Dallas, was a member of the field party. Mr. Joyner very kindly supplied the authors with simultaneous recording of the East-West telluric field and of the North-South magnetic field. The component of the magnetic field was measured with a metal-cored solenoid so that the recording showed the time derivative of the magnetic field. The computed power spectrum was corrected to eliminate the effect of this derivative. The telluric recording showed the potential difference between two copper electrodes separated 1,000 ft. The paper speed of the recorder was 12 in. per minute and the records were sampled at an interval $\Delta t = 0.75$ sec. A sample of the data was analyzed which started 0424 Z, 31 July 1959 Greenwich time or 9:24 p.m. 30 July local time and lasted 75 min. The results of the low-pass filter only are included in figures 13 and 14. No absolute sensitivities are available to the authors so the vertical positions of these two curves were adjusted to fit the other data; the frequency dependence is still meaningful, however.

The magnetotelluric data for Littleton, Mass., were recorded by Dr. Thomas Cantwell on 5 December 1959. These spectra are given in his thesis [Cantwell, 1960] as Case 27–3.

The magnetotelluric data for Austin, Tex., were recorded 27 March 1961 by Dr. H. W. Smith and Mr. F. X. Bostick of the Electrical Engineering Research Laboratories, The University of Texas.

The locations of these various stations are shown in figure 15. Despite the fact that the data were recorded over a period of 2½ years and extend over a longitude range of 170°, the data appear to fit a uniform trend. It is suggested that the trends established in figures 13 and 14 are a first approximation to the ambient magnetotelluric field at the earth's surface. Although fluctuations of the power density at one station may vary 10 db (Dr. H. W. Smith, personal communication) and the level between two stations may be as great as 20 db (Lvov and Tbilisi), these variations are small compared to the variation of 80 db in the telluric field plotted

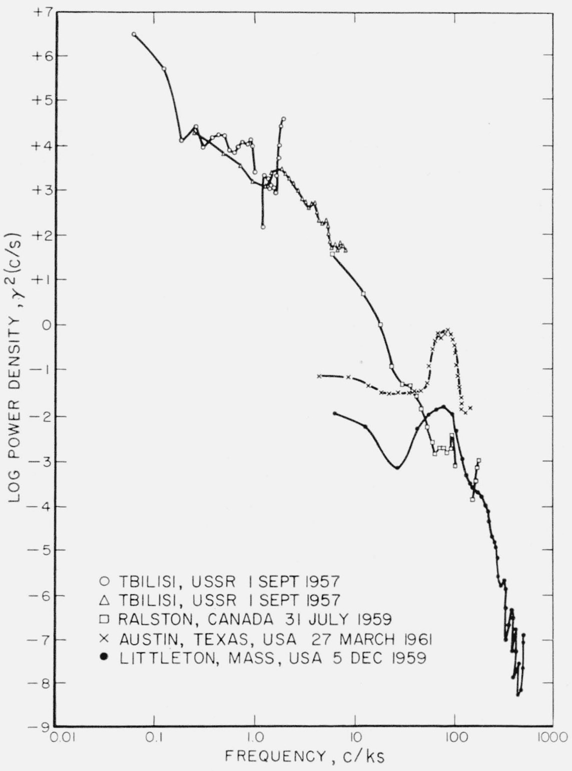

FIGURE 13. *A composite graph of the power spectra of the North-South component of the magnetic field for USSR, Canada, Massachusetts and Texas.*

An absolute calibration of the Canadian data is not available so these points can be displaced vertically an arbitrary amount.

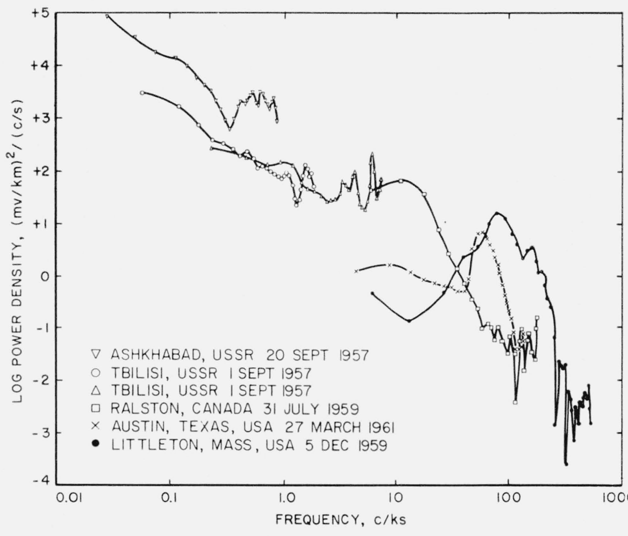

FIGURE 14. *A composite graph of the power spectra of the East-West component of the telluric field for USSR, Canada, Massachusetts, and Texas.*

An absolute calibration of the Canadian data is not available so these points can be displaced vertically an arbitrary amount.

FIGURE 15. *A world map showing the locations of stations referred to in figures 13 and 14.*
Base map copyright by Univ. of Chicago.

in figure 14. The corresponding variation in the power density of the magnetic field plotted in figure 13 is 150 db.

The following empirical formulas represent useful approximations to the data presented in figures 13 and 14. When f is expressed in cycles per kilosecond, the magnetic field can be expressed by

$$P_H(f)=10^5[1+25f^2+f^4+1.3\times10^{-4}f^6]^{-1}\ (\text{gammas})^2/\text{c/s},\ 0.1\ \text{c/ks}<f<400\ \text{c/ks},\ldots \quad (6)$$

Superimposed on this background spectrum is an intense band of power at 80 c/ks (the frequency of pc's) and a smaller one at 180 c/ks. There is slight evidence of the power band at 80 c/ks in the Canadian data. The electric field can be expressed by (again f in c/ks)

$$P_E(f)=10^5[1+100f+20f^2+f^3]^{-1}\ (\text{mv/km})^2/\text{c/s},\ 0.03\ \text{c/ks}<f<400\ \text{c/ks},\ldots \quad (7)$$

The experimental data for P_E(f) scatter more strongly than the data for P_H(f). This is undoubtedly due to the influence of the earth's structure on the telluric field.

It is a pleasure to acknowledge the help of many people and organizations who have contributed to this work. All of the numerical calculations were carried out at The University of Texas Computation Center and the time of the computor was paid for by the University Research Institute. Mr. Hoffman received support from the Department of the Navy through Contract No. 16498, Task II with the Defense Research Laboratory, The University of Texas. The writers thank the Defense Board of Canada, Pacific Naval Laboratory for permission to use the Canadian data from which the power spectra were computed, Mr. Jerry Joyner of Texas Instruments, Inc., and Dr. K. Vozoff of the University of Alberta, Canada, for their work in recording this data, and Mr. David Andrews, a summer employee of Texas Instruments, Inc., who read 6,000 ordinates from the Canadian records. Sir Charles Wright and Dr. H. J. Duffus were very helpful in connection with the visit to the Field Party and the obtaining of the Canadian data. One of the authors (CWH) was able to spend a week at the Suffield Experimental Station and a week at the IGY World Data Center A in Washington through the generosity of Texas Instruments, Inc. The authors express their thanks to Mr. J. W. Gettemy and Mr. W. Paulishak of the U.S. Coast and Geodetic Survey for their help and courtesies during a visit to the IGY World Data Center A in Washington, D.C. Finally, the authors express their thanks to Dr. A. W. Straiton, Director, and Dr. W. H. Smith of the Electrical Engineering Research Laboratory, The University of Texas, and to Dr. Thomas Cantwell, Department of Geology and Geophysics, Massachusetts Institute of Technology for their permission to reproduce the data from Texas and Massachusetts, respectively, shown in figures 13 and 14. The base maps used in figures 1 and 15 are from the Goode Base Map Series, Department of Geography, University of Chicago.

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