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Schumann Resonances of the Earth-Ionosphere Cavity—Extremely Low Frequency Reception at Kingston, R.I.*

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Since June 1961 magnetic fields of natural origin in the 5 to 20 c/s frequency range have been recorded in Kingston, R.I. The experimental equipment is described briefly, and results are presented. Variations with time of the first resonant frequency of the earth-ionosphere cavity are indicated, and effects of solar activity are discussed. An analysis of the envelope of recorded wave trains shows only fair agreement with existing theory.

1. Introduction

In 1952 it was suggested by Schumann [1952a] that the earth and ionosphere may together act as a cavity resonator for electromagnetic waves and that the first resonance should occur at 10.6 c/s. Refinement of the theory [Schumann, 1952 b and c, 1954, 1957] indicated that losses due to absorption by the ionosphere or radiation through the ionosphere should reduce the resonant frequency by at least 1.5 c/s. Experimental evidence for the existence of the first resonant mode was reported first by Schumann and König [1957] and, in considerably more detail by König [1959].

The theory of ELF resonances was more recently discussed by Wait [1960 a and b], and experimental evidence for the existence of the first and higher resonant modes, based upon measurements of electric fields, was presented by Balser and Wagner [1960 a and b]. Their results were also applied by Raemer [1961] to the calculation of an ionospheric loss parameter which is a function of the height and the conductivity of the sharply bounded ionosphere assumed in the first order theory. Additional experimental results were published recently by König [1961 a and b], by Maple [1961], and by Lokken, Shand, and Wright [1961].

Since May, 1961, we have carried out measurements in this frequency range (8 to 20 cycles), first on the University of Rhode Island campus, and, since June 28, 1961, on an electromagnetically "quiet" field site located a few miles from Kingston, R.I. Our main object was originally to establish whether data obtained at a location in the continental United States bear any resemblance to the records obtained by König in Munich. Like König we are using equipment with a relatively low upper cut-off frequency (20 cycles), and our analysis thus far has been based upon visual examination of individual wave forms recorded on paper tape. It will be shown that such examination of individual noise bursts may be very useful in addition to spectral analysis of noise averaged over several seconds or minutes.

Results, thus far, clearly indicate that relatively strong sinusoidal oscillations occur in the 7 to 10 c/s frequency range, that the "resonant" frequency is not constant with time, that certain characteristic wave shapes occur frequently, and that activity in this frequency range is increased during magnetic storms.

2. Instrumentation

The receiving equipment is located approximately 1,200 ft from the nearest 60-cycle overhead line; it consists of two independent, identical channels, each of which is made up of a receiving coil and suitable filters and amplifiers. The overall frequency response of the entire system, including the pick-up coil, is shown on figure 1.

Each of the receiving coils (fig. 2) consists of 249,000 turns of number 38 isomel insulated wire wound over a length of $2\frac{11}{32}$ in. on a bakelite tube of 1.092 in. o.d. Iron cores are used which consist of a 1 x $\frac{11}{16}$ x 11 in. stack of 0.014 transformer laminations. The self-resonant behavior (due to distributed capacitance) of each coil is indicated by figure 3; the response shown there was obtained by inserting the pickup coil into a long solenoid driven by an ELF oscillator and noting the voltage across the pickup coil on a vacuum tube voltmeter which has an input impedance of 10 megohms. Resonance occurs at 175 c/s without iron core and at 80 c/s with the iron core.

After amplification and filtering the signals are fed to a Sanborn 2-channel recorder which is operated at a paper speed of 5 cm/sec. The voltage gain between the output of the receiving coil and the input of the recorder is 23,000 at 10 c/s. At this frequency an axial magnetic field of 4.25 (10^{-5}) ampere meter^{-1} at the location of the pickup coil produces a 1-v signal at the input to the Sanborn recorder.

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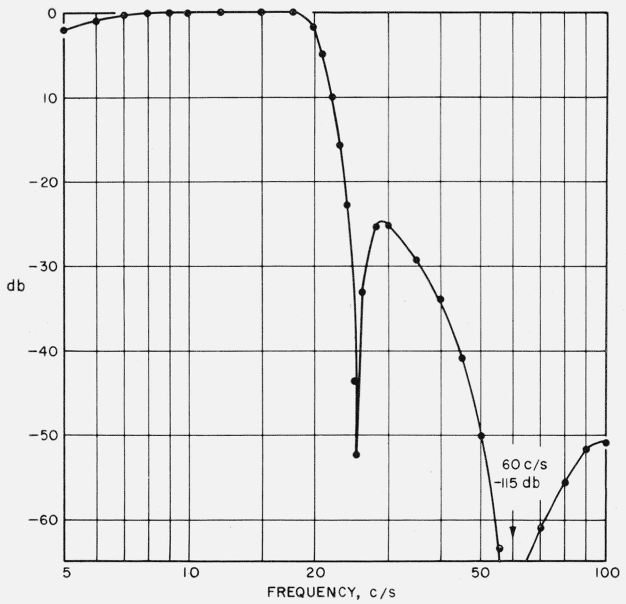

Figure 1. *Response of receiving system (from coil input to recorder input).*

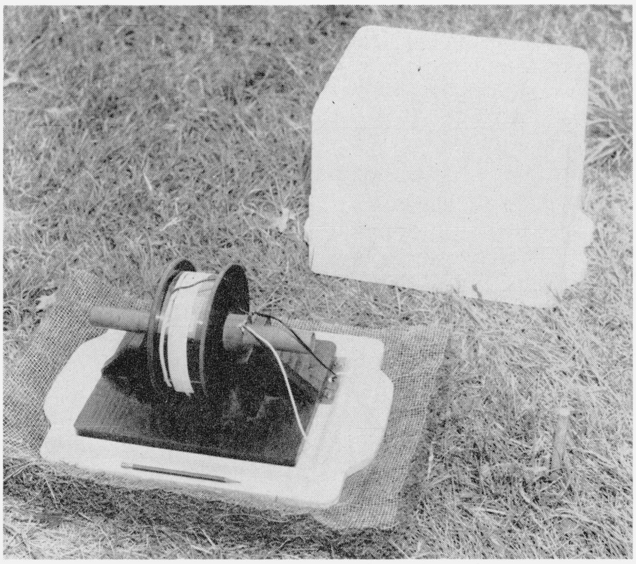

Figure 2. *Close-up of receiving coil and housing.*

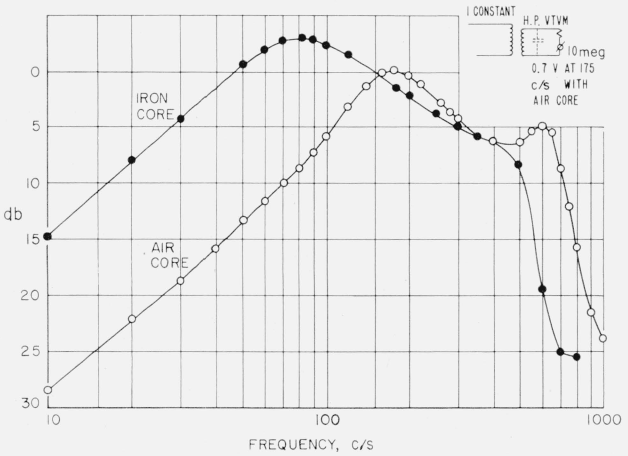

Figure 3. *Response of pick-up coil.*

3. Characteristics of Data

Data have been recorded on the Kingston field site since June 28, 1961. At first records were only obtained in the afternoon (3 PM EDT—1900 UT) and only on one channel. On August 17 a second channel was added permitting simultaneous recording of noise picked up by two coils, one with its axis oriented in the north-south direction and the other oriented east-west. Since the installation of the second channel, recordings have usually been obtained three times per day for at least 2-min periods: in the early morning (1300 UT), in the afternoon (1900 UT), and at night (0200 UT). All records were examined visually to establish the level and the frequency of periodically varying signals and to note other characteristic features. Results were recorded in tabular form and plotted where appropriate.

Sections of rather typical recordings are shown on figure 4a, b. In addition to irregular wave shapes, both records exhibit clearly sinusoidal oscillations which last for several seconds. The records of figure 4b also are very similar in appearance to the "type I" signals identified by König [1959] in Munich. The frequency of oscillation can be established by counting complete cycles between second markers (vertical lines). Time increases from right to left on all records. Records marked N–S and E–W were obtained with the coil axes oriented, respectively, in the north-south and east-west directions. The lower records on 4f, g, i, and j are the output of a 9-cycle filter having a 3 db-bandwidth of 1 cycle.

Figure 4b exhibits the characteristic modulation pattern which could possibly be due to the interference of two very closely spaced frequencies or could simply represent the onset and decay of individual lightning strokes. This pattern is even more clearly apparent on figure 4d and unusually pronounced on figure 4e.

Occasionally—not more than once in a period of several weeks—a sinusoidal 6 c/s signal appears, as on figure 4c, which has the same type of modulation envelope as the higher frequency signals. This is the wave shape which has been identified as "pearl oscillation" in the literature [Troitskaya, 1961; Tepley, 1961].

Local thunderstorm activity produces either extremely irregular, nonsinusoidal signals, figure 4f, or signals characterized by sudden onset as on the right of figure 4g. Under "normal" conditions, that is in the absence of local thunderstorm activity and in the absence of solar disturbances, the level of signals recorded at night, such as shown on figure

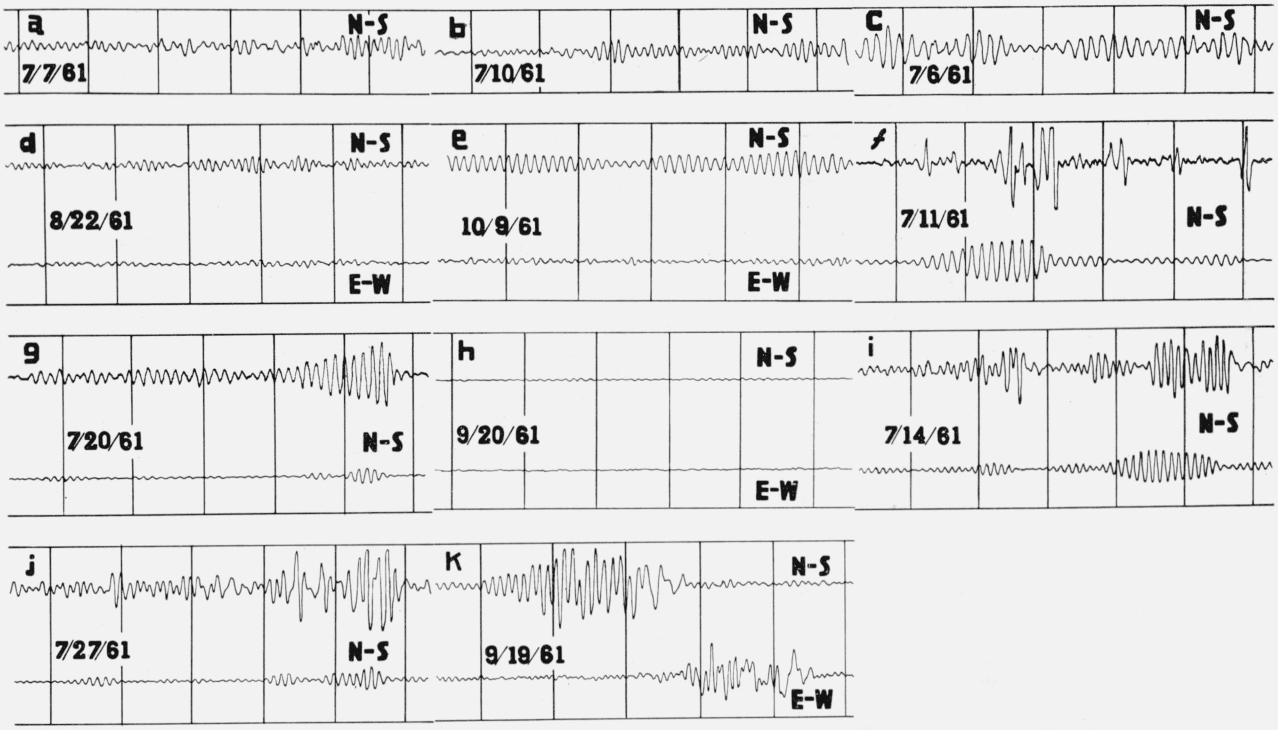

FIGURE 4. *ELF records.*

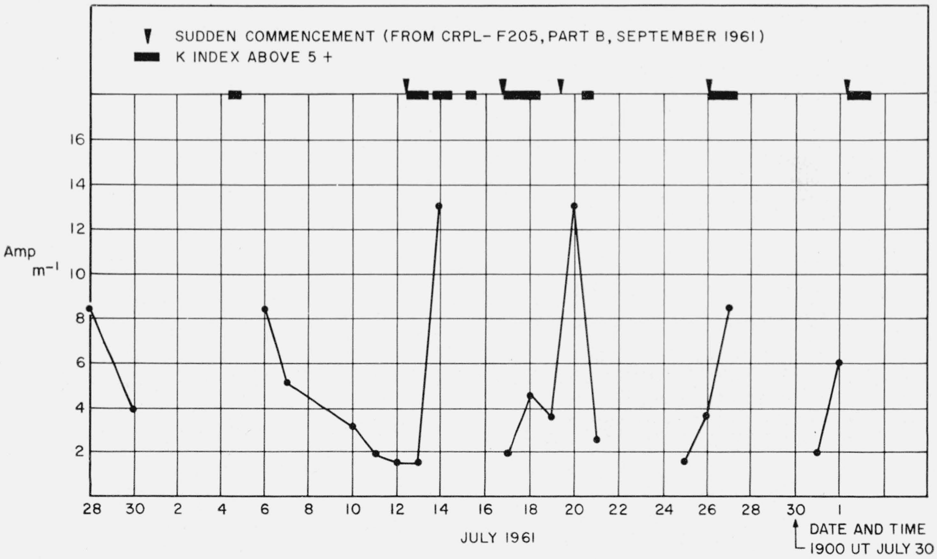

FIGURE 5. *Mean amplitude of received sinusoidal noise (7 to 12 c/s).*
(Ordinate is multiplied by 10^5)

4h, was considerably lower than the level noted during daylight hours. It was often difficult to establish the existence of individual wave trains at night.

During several magnetic storms considerably increased activity, higher average level of received noise in the 8 to 10 cycle frequency range, and rather irregular wave shapes were noted: figure 4i, j, k.

The connection of geomagnetic activity, solar "sudden commencements," and increased level of the 8 to 10 c/s noise is also apparent from figure 5, where signal levels recorded in July 1961 at 3:00 PM local time (1900 UT) are shown together with dates of sudden commencements and dates and durations of geomagnetic disturbances. A similar correlation seems to exist for geomagnetic events which occurred

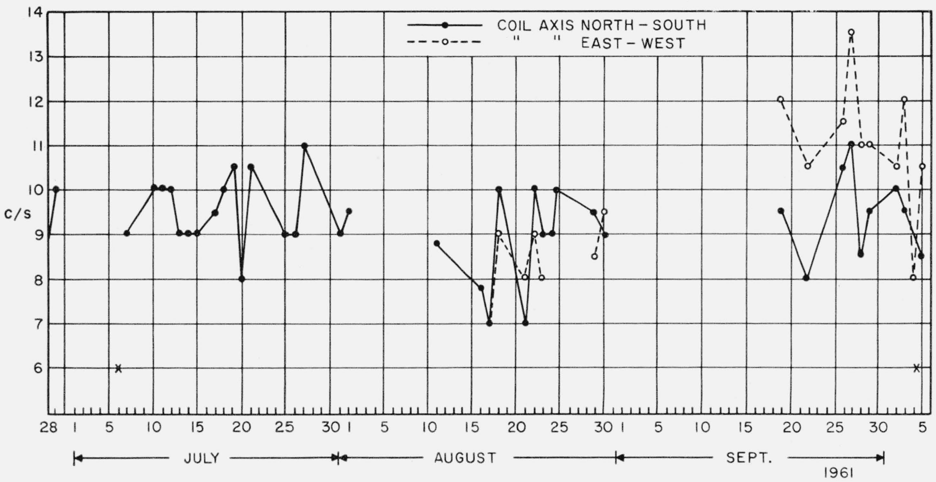

FIGURE 6. *Frequency, afternoon reading (1900 UT), July to Sept. 1961.*

in the latter part of September; detailed analysis of this period will, however, have to be postponed until the pertinent issue of the NBS "Solar-Geophysical Data" [1961] becomes available.

The most prominent frequency of sinusoidal oscillations is not constant and seems to exhibit both short-time variations—apparent from figure 4—and longer period trends which require observation over several months. Figure 6 indicates that for data recorded in the afternoon, the most prominent frequency appeared between 9 and 10 c/s in July, between 7 and 10 c/s during the second part of August, and between 8 and 11 cycles on the north-south channel and between 10 and 13.5 c/s on the east-west channel during the second half of September. It may be pertinent to note that the second part of August was a very quiet period in terms of solar and geomagnetic activity.

4. Individual Wave Shapes and Resonance Behavior

The frequency spectrum corresponding to impulse excitation of the earth-ionosphere cavity has been given by Schumann [1957] and Wait [1960a]

$$E(i\omega)=K\{i\omega+\alpha(i\omega)^{1/2}\}(i\omega)\sum_{n=0}^{\infty}P_n(\cos\theta)\,\frac{2n+1}{\omega_n^2-\omega^2+\alpha(i\omega)^{3/2}} \qquad (1)$$

where K is a constant depending upon the strength of the exciting source, the distance, h, between the earth's surface and the assumed sharp lower boundary of the ionosphere, the dielectric permittivity of the air space, and the earth's radius. The quantities ω_n correspond to the resonant frequencies of the loss-free cavity as given by Schumann [1957] or Wait [1960a]; for example, $\omega_0=0$, $\omega_1=2\pi(10.6)$, and $\omega_2=2\pi(18.3)$. Losses in the ionosphere which depend upon the conductivity, σ, of the ionospheric reflecting layer enter into (1) together with the height, h, as the parameter α:

$$\alpha=\frac{1}{h(\sigma\mu)^{1/2}}. \qquad (2)$$

In the neighborhood of the first resonance ($\omega_1=2\pi$ 10.6), and for sufficiently small α the series of (1) may be approximated by its first four terms. If one employs reduced variables

$$u=\frac{\omega}{\omega_1} \qquad (3)$$

$$\delta=\frac{\alpha}{\omega_1^{1/2}} \qquad (4)$$

$$E(u)=\frac{1}{ABCD}\{BCD+3ACD\cos\theta+2.5(3\cos^2\theta-1)\ ABD+3.5\cos\theta(5\cos^2\theta-3)ABC\} \qquad (5)$$

where

$$A=-u^2-\delta u\sqrt{\frac{u}{2}}+i\delta u\sqrt{\frac{u}{2}}$$

$$B=1+A$$

$$C=2.98+A$$

$$D=5.96+A$$

θ=angle between source and point of measurement.

The resulting frequency spectra $|E(u)|$ exhibit clear maxima at "resonant" frequencies which are lower than the frequency $f_1=10.6$ c/s ($u=1$) which would be obtained with a loss-free ionosphere. On figure 7 these spectra are plotted for several values of the loss-parameter α, and for $\theta=45°$ which corresponds to the assumption of excitation in the equatorial thunderstorm belt when recordings are made at temperate latitudes.

If the spectra are to be compared with experimental results, they must be corrected for the response of the receiving equipment, figure 1; and they must also be slightly modified to take into account the spectrum of individual lightning strokes. Following Raemer [1961] we use for this purpose the curve of figure 8. The resulting curves, which differ only slightly from those shown on figure 7, may be approximated in the neighborhood of the frequency u_1' (where maximum amplitude occurs) by

$$E(u)=e^{-\frac{(u-u_1')^2}{4k^2}}\,e^{-iA(u-u_2)}. \tag{6}$$

In this expression k is determined by fitting the curves of figure 7 as corrected above. A single pair of values u_1' and k correspond to each value of the loss parameter α. A and u_2 depend upon the phase of $E(u)$ as given by eq (5); these parameters can be evaluated by computing the phase variation of $E(u)$ in the neighborhood of u_1'. The frequency u_2 where the spectrum $E(u)$ assumes zero phase lies usually below u_1'. Since k is a measure of the width of the resonant curve, it is simply related to the cavity Q for the first resonant mode. It is easily shown from eq (6) that

$$Q=0.425\,\frac{u_1'}{k}. \tag{7}$$

The values of resonant frequency u_1' and of the parameter k which were obtained from (5) and its approximation eq (6), can now be compared with experimental results. The time function corresponding to (6) is

$$f(t\omega_1)=\frac{k}{\sqrt{\pi}}\,e^{-k^2(t\omega_1-A)^2}\cos\{u_1'(t\omega_1-A)+\Phi\}. \tag{8}$$

Thus, the envelope of individual recorded wave trains should be determined by k and the frequency should be $u_1'\,\frac{\omega_1}{2\pi}=u_1'(10.6)$ if the receiving equipment admits only frequencies in the neighborhood of the first resonance. It is, of course, assumed that the spectrum of an individual lightning stroke does not deviate too much from figure 8, and that the model of a single-layer, sharply bounded ionosphere is valid at frequencies of the order of 10 c/s.

On figure 9 values of k are plotted against u_1' for several values of α, for $\theta=0°$, $\theta=45°$, and $\theta=180°$ (solid lines). The crosses on this figure show the results obtained by taking ten samples of frequency and wave shape from the records of figure 4. It is apparent that for values of u_1' larger than about 0.8 ("resonant" frequencies greater than 8.5 c/s) fair agreement exists between theory and recorded wave shapes, particularly if one considers that the actual spectrum of individual lightning strokes—as opposed to the average spectrum of figure 8—is unknown. Six of the experimental points on figure 9 would correspond to values of the loss-parameter α between 1.2 and 4. Four points, corresponding to resonant frequencies between 8.00 and 9 c/s, $0.76\leq u_1'<0.85$, seem to indicate a lower value of k (or higher Q) than predicted by eq (1).

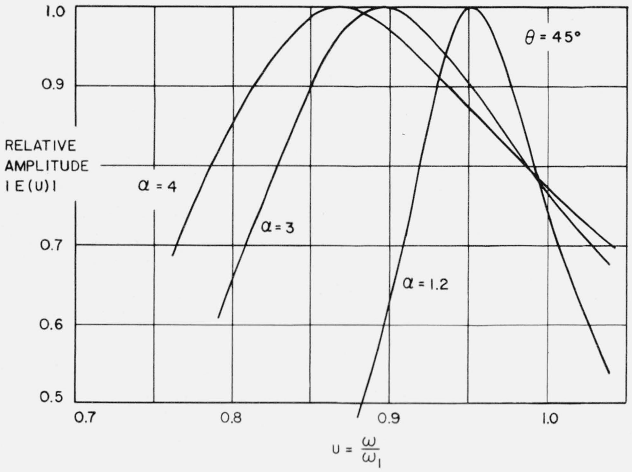

FIGURE 7. *Spectra of impulse excited earth-ionosphere cavity.*

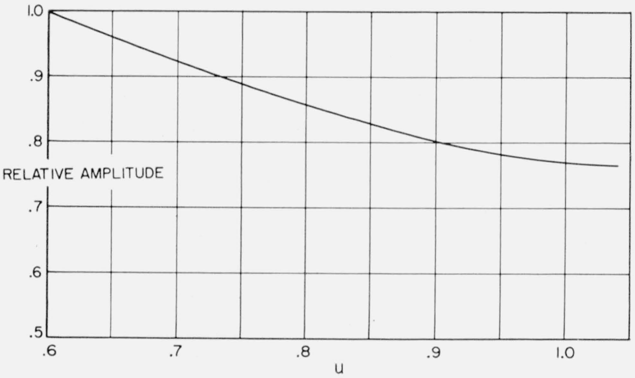

FIGURE 8. *Spectrum of sequence of return strokes in a lightning flash (Raemer and Williams).*

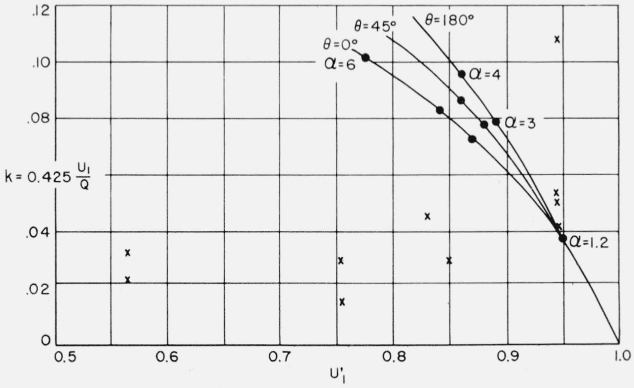

FIGURE 9. *Calculated and experimental values of wave enelope parameter* k.

It is also apparent from figure 9 that the two points at $u_1'=0.565$ ("resonant" frequency 6 c/s) are far removed from the theoretical curve. Either a different phenomenon than the excitation by lightning of the earth-ionosphere cavity is involved at 6 c/s, or eq (1) is invalid for the large values of the loss-parameter α which would be required to reduce the resonant frequency by a factor of 0.565.

5. Conclusions

The existence of comparatively strong natural oscillations in the 7 to 10 c/s frequency range which was first noticed by König [1954] has been confirmed. The "resonant" frequency is not constant; it exhibits short-time variations and possibly also longer period trends. Signals at night are considerably weaker than during daylight hours and stronger than usual during periods of geomagnetic activity. A preliminary, approximate analysis of wave shapes and frequencies within individual noise bursts gives close enough agreement with the existing theory to warrant further, more detailed work along these lines, including careful statistical analysis of a large number of individual wave trains. The examination of envelopes of such wave trains allows evaluation of a parameter which would be lost in simple spectrum analysis of the received noise.

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