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On the Role of the Process of Reflection in Radio Wave Propagation[1]

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Nature offers numerous examples of irregular stratification of the medium for the propagation of radio waves. A study of the process of reflection in such a medium distinguishes between specular reflection and diffuse reflection. The phenomenon of trans-horizon tropospheric propagation offers an example of the application of such a process, necessary for the interpretation of experimental results. Other examples are those of ionospheric propagation (sporadic-E layer) and propagation over an irregular ground surface (phenomenon of albedo).

1. Introduction

This paper discusses the role of partial reflection in the propagation of radio waves in stratified media. This work differs considerably from other works by different authors about the same problem [Epstein, 1930; Feinstein, 1951 and 1952; Wait, 1952; Carroll et al., 1955; Friis et al., 1957; Smyth et al., 1957].

The authors were led to make such a study in the course of seeking an interpretation of experimental results obtained in trans-horizon tropospheric propagation for which the classical theory of turbulent scattering did not appear to give a satisfactory explanation. The process of reflection in irregular media points to evidence for the existence of two forms, coherent or specular reflection, and random or diffuse reflection. These two phenomena, added to the pheonomenon of scattering, permit one to account for the ensemble of experimental results obtained in trans-horizon tropospheric propagation.

The existence of stratified irregular media, or of layers, is far from being limited to the case of the troposphere. Such layers should, in fact, appear each time that a field of force along a dominant direction is exerted on the medium, creating stratification in a perpendicular direction. This is the case for the troposphere or the ionosphere when subjected to the gravitational field, as well as the magnetosphere in a geomagnetic field. In all these cases, the conclusions reached by a study of the process of reflection should be found to be applicable.

2. Study of the Phenomenon of Reflection in an Irregular Medium

Consider a small stratum of thickness, H, with a horizontal surface, S, and with an average length, L. This volume contains irregularities having a mean surface, s, of average length, l, distributed over the thickness, H.

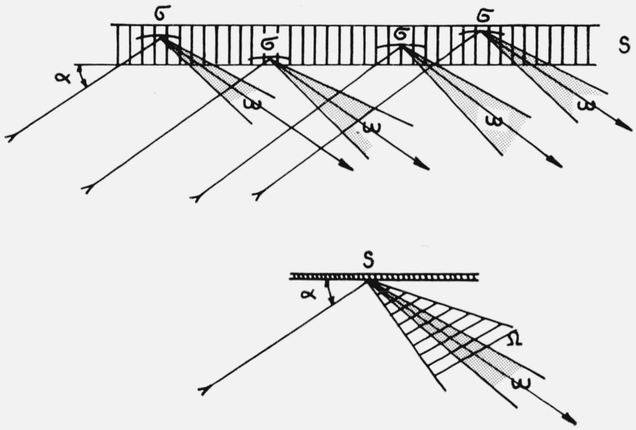

FIGURE 1. *Phenomenon of partial reflection.*

A plane wave incident at an inclination angle α is partially reflected by the surface, S, having a reflection coefficient, ρ, assumed constant at every point of the surface of the irregularities, s (fig. 1).

2.1. Elementary Reflected Power

Each element of a surface s has a certain inclination, so that it reflects an incident ray in a direction $\theta=\alpha+\beta$ relative to it. The element has a certain radius of curvature, R, and an elementary effective reflecting cross section, σ, corresponding to the direction θ with which it can be associated (fig. 2).

Let us take a local system of cylindrical coordinates, a, ϕ, ζ, such that ζ is perpendicular to the bisectrix of angle θ and normal to the surface at a center point, P, of the element. To a point $M(a,\phi)$ of the tangent plane $\zeta=0$, there corresponds an altitude $\zeta=a^2/8R$. At this point M, the radiation is dephased by $2K\zeta$ relative to the center point P, where $K=2/\Lambda$ and $\Lambda=\lambda/(\sin\,\theta/2)$ is the space wavelength.

[1] Translated from the French by P. Guzman-Rivas, Boulder, Colo.

A limiting condition for the element of effective surface, σ, will be given, for example, by a dephasing of less than $\pi/4$. Therefore, for the limiting points, $\zeta=\Lambda/4$, and $a=\sqrt{R\Lambda/2}$, the value of the effective reflecting surface, σ, is

$$\sigma=R\Lambda/2. \quad (1)$$

The validity of this expression evidently assumes that $\sigma<s$, so that

$$R<2l^2/\Lambda. \quad (2)$$

The corresponding reflected power, δp, in the direction θ can then be calculated as a function of the power incident per unit of surface, p_0, by a reasoning analogous to that for antenna theory. The expression for the gain of an antenna with surface σ, in the direction θ, would be

$$\frac{\delta p}{p_0\sigma \sin \theta/2}=\frac{4\pi}{\lambda^2}\sigma(\sin \theta/2)\rho^2,$$

ρ being the amplitude of the reflection coefficient, so that

$$\delta p=p_0\frac{4\pi}{\lambda^2}\sigma^2\rho^2. \quad (3)$$

The surface s can be considered as consisting of a combination of elements of effective surfaces σ, each reflecting an energy δp in a direction θ. It can be assumed that the surface s is small and that its mean radius of curvature R is large, with the radii of curvature of the surfaces σ approximated by the average radius R. In the expression for K, the reflection angles β can be approximated by the angle α, so that $\theta=2\alpha$. One obtains for the radius of curvature R, if H is the maximum height of the irregularity:

$$R=l^2/8H.$$

Hence, we obtain an equation for δp:

$$\delta p=p_0\rho^2\frac{\pi}{16}\frac{l^4}{H} \quad (4)$$

and, for condition (2):

$$H>\Lambda/16.$$

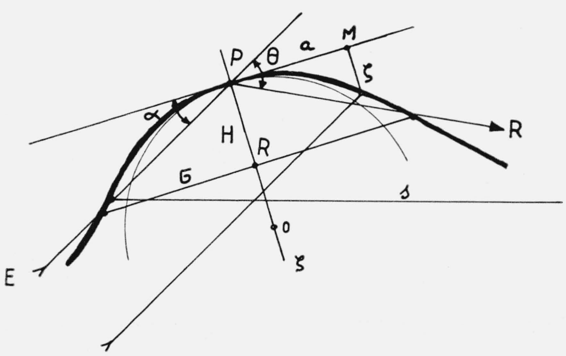

FIGURE 2. *Reflection by an irregularity of a surface.*

These approximations should not obscure the fact that the power is reflected into a solid angle around the direction 2α, which can be characterized by a vertical angle ϕ and a horizontal angle ψ. These angles are related to the curvature of the surface within the latter's limits, and can be written:

$$\phi=\psi=2\frac{l}{2R}=8\frac{H}{l}, \quad (5)$$

if one takes into account small angles ϕ and ψ.

2.2. Total Reflected Power

The stratum with a total surface S and having a dimension L consists of a series of surface irregularities with mean dimensions s distributed over a thickness H. With each of these is associated a reflected power δp along a given direction, and a phase term characteristic of its position.

Let us take Cartesian coordinate axes x, y, z from a central point O in the surface S (fig. 3), where x and y are horizontal and z is vertical. Associated with each surface element σ is a phase term kr, where r is the distance to the point O. Separating horizontal and vertical characteristics, one can write, t being the horizontal distance from O of the element σ:

$$r=t+2z(t)\sin\alpha, \quad (6)$$

where:

$$t=x\sin\alpha\sin\phi+y\sin\psi, \quad (7)$$

with ϕ and ψ characterizing the direction of observation β by a vertical angle and a horizontal angle around α, and $z(t)$ describing the position of the elements σ. It amounts to restoring them entirely to a horizontal plane, but conserving for each element its proper phase term $2Kz(t)$. Defining:

$$\gamma(t)=e^{j2Kz(t)}, \quad (8)$$

we obtain a function describing the distribution of irregularities.

a. Specular Reflection

To calculate the total power reflected in a direction θ, two cases come into consideration, depending on whether the elementary terms involved are in phase or not.

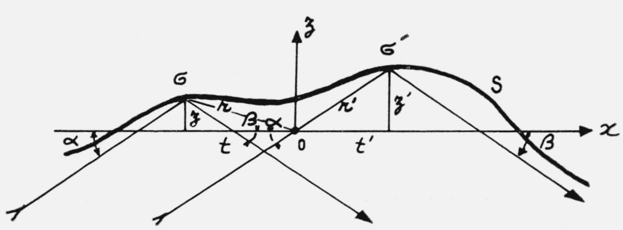

FIGURE 3. *Reflection by an irregular surface.*

For terms in phase, the reflected power, P_s, is obtained by considering elementary energies, i.e., by squaring the phase terms:

$$P_s=\delta p\left[\frac{1}{s}\int_S e^{jkt}\,\gamma(t)d^2t\right]^2. \tag{9}$$

Since the terms are in phase, one must have $kt=2\pi$ or $\phi=\psi=0$. Reflection in phase occurs only along the direction of specular reflection $\beta=\alpha$. Therefore:

$$P_s=\delta p\left[\frac{1}{s}\int_S \gamma(t)d^2t\right]^2. \tag{10}$$

Since $\gamma(t)$ is a cissoidal function, for an approximate value of its integral over S, one can take:

$$\gamma=\frac{\sin u}{u}, \tag{11}$$

where:

$$u=2KH \tag{12}$$

and H is the maximum height of the irregularities $z(t)$. Hence, for *specular reflection*,

$$P_s=\delta p\,\frac{S^2}{s^2}\,\gamma^2. \tag{13}$$

b. Diffuse Reflection

For terms whose phases differ, it is necessary to resort to a reasoning analogous to that which is used in the theory of scattering, and to take into account the random character of the distribution of reflecting elements. In considering two elements corresponding to t and $t+\delta$, the reflected power will be:

$$P_d=\delta p\frac{1}{s}\int_S e^{jkt}\gamma(t)d^2t\,\frac{1}{s}\int_S e^{jk(t+\delta)}\gamma(t+\delta)d^2(t+\delta). \tag{14}$$

This expression can be transformed into:

$$P_d=\delta p\frac{1}{s}\int_S e^{jk\delta}\left[\frac{1}{s}\int_S\gamma(t)\gamma(t+\delta)d^2t\right]d^2\delta. \tag{15}$$

In this form, the correlation function of $\gamma(t)$ is shown as:

$$\nu(\delta)=\frac{1}{s}\int_S\gamma(t)\gamma(t+\delta)d^2t, \tag{16}$$

and its Fourier transform:

$$\Gamma(k)=\frac{1}{s}\int_S e^{jk\delta}\nu(\delta)d^2\delta. \tag{17}$$

However, since the functions $\gamma(t)$ involved are cissoidal functions, it is possible to make an approximation, and to replace the integral in (17) with unity. It should be observed that the reflected power corresponding to the terms in phase appears formally in eq (15). It is necessary to take into account that this power has been selected so that instead of unity one takes $(1-\gamma^2)$.

Diffuse reflection is therefore written as:

$$P_d=\delta p\frac{S}{s}\,(1-\gamma^2). \tag{18}$$

c. Solid Angle of Reflection

Coming back to the physical meaning of this mathematical analysis, a certain number of elements are distributed at random in a stratum, each element reflecting a certain power into a solid angle, ω, around the direction of specular reflection (fig. 1). The distribution of these elements is analyzed by their spectral distribution, $\Gamma(k)$; i.e., with the space wavelength, Λ, in the direction of propagation, and the wavelength, λ, in the transverse direction. Elements whose phases are random give a diffuse reflection in an angle equal to the angle ω. Elements in phase give a more intense reflection in a much smaller angle ω'.

The preponderance of one phenomenon or the other depends on the size of the irregularities. By fixing a limit, $u=\pi/4$, Rayleigh's criterion for an irregular surface is found:

$$H>\Lambda/16,$$

this corresponding to condition (2).

The solid angles involved have limits fixed by the phase variation. In the vertical plane, the phase term, within limits of the surface, is $Kl\sin\phi$, for diffuse reflection, where the surface s is involved, and $KL\sin\phi$ for specular reflection.

The phase varies from π for small angles equal to:

$$\Phi_s=\Lambda/L \qquad \Phi_d=\Lambda/l. \tag{19}$$

In the horizontal plane, the phase is $Kl\sin\psi$. For a variation of π:

$$\psi_s=\lambda/L \qquad \psi_d=\lambda/l. \tag{20}$$

These limits are stricter than the geometrical limits of formula (5) if $H>\Lambda/8$. We note also that reflection creates a lobar structure, as in the case of antennas, and that the results involve only the main lobe.

Several remarks can be made concerning the approximations used. Introduction of the approximation γ instead of $\gamma(t)$ in (11) is tantamount to considering the correlation function $\nu(\delta)$ in (16) equal to 1 when $\delta<l$, and equal to γ as an average. However, since the characteristic function of the irregularities, $z(t)$, appears as an exponent, the choice of the correlation function is not critical. The minima of $\gamma(u)$ will be introduced by the approximation and it is preferable to replace γ^2 by its mean value 1/2, since $u>3\pi/4$, or $H>3\Lambda/16$.

d. Total Reflected Power

Total reflected power can be put in the following form:

$$P_r=p_0\,\frac{4\pi\sin^2\alpha}{\lambda^2}\,\sigma^2\rho^2\left[\frac{S^2}{s^2}\,\gamma^2+\frac{S}{s}\,(1-\gamma^2)\right]. \tag{21}$$

This equation shows the number of reflecting elements which enter into the second power in coherent reflection.

Limits must be considered in the real surface of reflection (fig. 4). For specular reflection, the hypothesis of a phase phenomenon introduces a Fresnel's ellipse when we no longer have a plane wave. If D is the emission-reception distance, its dimensions are:

$$L_s=\sqrt{\lambda D}/\sin\alpha \qquad L'_s=\sqrt{\lambda D}. \tag{22}$$

Only that zone of S in the Fresnel's ellipse (viz, S_s) will be involved.

In the case of diffuse reflection, the maximum inclination (small) τ of the elements will be involved. This corresponds to a width, $L_d=D\sin\alpha\tau$, where $\tau\simeq H/l$. The possible transverse limit for S_d then is:

$$L_d=D\frac{H}{l}\sin\alpha. \tag{23}$$

Hence, the reflected power (21) should be written in the form:

$$P_r=p_0\rho^2\frac{4\pi\sin^2\alpha}{\lambda^2}\sigma^2\left[\frac{S_s^2}{s^2}\gamma^2+\frac{S_d}{s}(1-\gamma^2)\right]. \tag{24}$$

3. Process of Reflection in Trans-Horizon Tropospheric Propagation

3.1. Characteristics of the Propagation Media

As a consequence of gravity, the atmosphere has a general tendency to become horizontally stratified.

A consideration of movements in the atmosphere shows the possibility of two processes, a laminar flow and a turbulent flow [Misme, 1958]. The limit between these two processes is clearly marked and depends on atmospheric stability.

In a zone of laminar flow, a stratification of the atmosphere appears which can be characterized by layers of a stable nature, with highly variable horizontal as well as vertical dimensions.

In a zone of turbulent flow, elementary unstable blobs appear, which are more or less organized depending upon the thickness of the zone.

A highly plausible representation of the atmosphere would show a spatial superposition of laminar-flow zones (or stable strata) and turbulent-flow zones (or turbulent strata).

Stable strata can be characterized by a discontinuity in the gradient of refractive index, corresponding to a variation δn of the index in a very small thickness, e.

The structure of the stratum is subject to the influence of turbulent flow in neighboring strata and to mechanical vertical movements of the atmosphere. The corresponding irregularities could be expressed by horizontal and vertical mean dimensions l and h, corresponding to a mean surface s, for irregularities of the first order (turbulent-flow) and by average dimensions l_1 and h_1 (corresponding to a mean surface s_1), for irregularities of the second order (mechanical movements), as in figure 5.

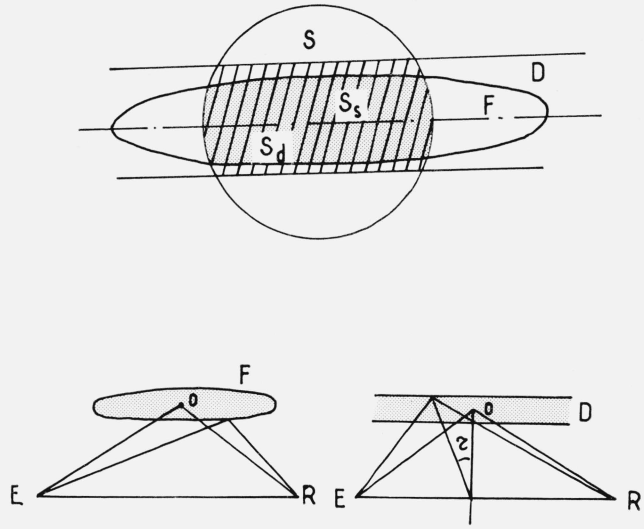

FIGURE 4. *Surface of reflection.*

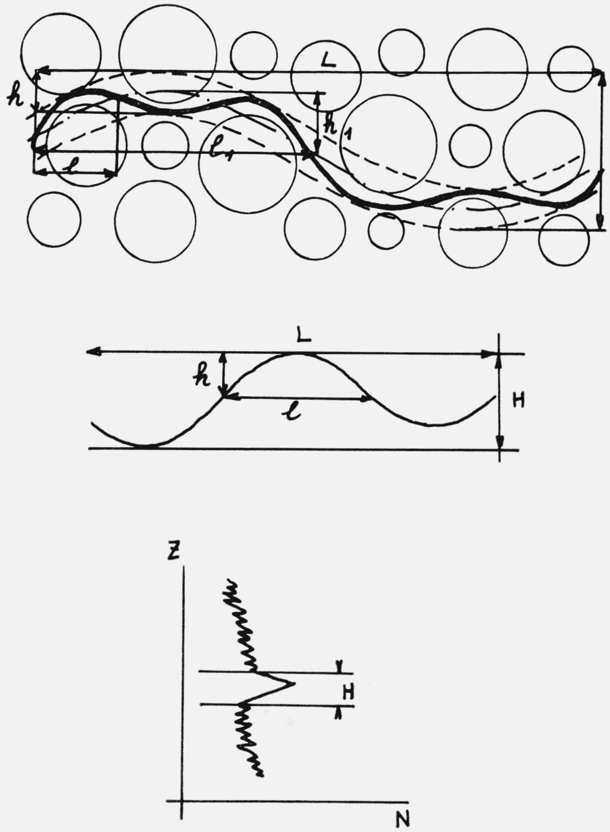

FIGURE 5. *Schematic of the troposphere.*

The stratum has a total thickness, H, comprising all such irregularities. H could be considered the same as h_1 with a total dimension L, corresponding to a surface S.

The orders of magnitude of these different characteristic parameters of the stratification, as far as they can be deduced from experimental observations, are the following:

Discontinuity	$\delta N \sim 10^{-2} N$	$e \sim 10$ cm
Dimensions	$L \sim 10$ km	$H \sim 10$ m
Irregularities	$l \sim 10$ m	$h \sim 1$ m
	$l_1 \sim 1$ km	$h_1 \sim 10$ m

A simplified schematic of the stratification is that wherein irregularities with mean dimensions l and h are distributed through a stratum of dimensions L and H. Such a simple model is considered in the preceding study.

It should be noted that observations of the upper surfaces of cloud layers give evidence frequently of the existence of irregularities such as those recorded here [Misme, 1960].

3.2. Propagation by Reflection in Tropospheric Strata

In the case of trans-horizon tropospheric propagation, angles of inclination α are always small so that one may take the angle for the sine.

a. Reflection Coefficient

For a discontinuity of the index dN, small compared to an angle of inclination α, the coefficient of reflection of an element, according to Fresnel's formula, can be written:

$$d\rho = dN/2\alpha^2. \tag{25}$$

For a discontinuity thickness e, the coefficient of reflection is:

$$\rho = \int_0^e \frac{dN}{2\alpha^2} e^{j2Kz}$$

or, upon introducing the index gradient $g(z) = dN/dz$:

$$\rho = \int_0^e \frac{g(z)}{2\alpha^2} e^{j2Kz} dz. \tag{26}$$

A limited development permits one to write:

$$\rho = \frac{1}{2\alpha^2} \left\{ \left[j\frac{g}{2K} + \frac{g'}{(2K)^2} - j\frac{g''}{(2K)^3} + \ldots \right] e^{j2Kz} \right\}. \tag{27}$$

For a very small thickness, e, $e << \Lambda$, we will consider a discontinuity gradient $g = \delta N/e$ and write:

$$\rho = \frac{\delta N}{e} \frac{\lambda}{\delta \pi \alpha^3}. \tag{28}$$

b. First Order Irregularities

Irregularities $s(l,h)$, comprised in a surface s, give rise to a specular reflection, p_{1s}, and a diffuse reflection, p_{1d}:

$$p_{1s} = p_0 \rho^2 \frac{4\pi\alpha^2}{\lambda^2} \sigma^2 \frac{j_{1s}^2}{s^2} \gamma^2 \tag{29}$$

$$p_{1d} = p_0 \rho^2 \frac{4\pi\alpha^2}{\lambda^2} \sigma^2 \frac{s_{1d}}{s} (1-\gamma^2) \tag{30}$$

where $\gamma = \sin u/u$ and $u = 4\pi\alpha H/\lambda$. The effective reflecting surface, σ, defined by (1), can be written:

$$\sigma = \lambda l^2/2h\alpha. \tag{31}$$

The mean surface of the irregularities, s, would then be:

$$s = l^2. \tag{32}$$

The total surface of diffuse reflection is equal to s_1, this being small:

$$s_{1d} = l_1^2. \tag{33}$$

The total surface of specular reflection is limited transversely in general by the small diameter of Fresnel's ellipse, $L_s' = \sqrt{\lambda D}$, if D is the total length of the path:

$$s_{1s} = l_1\sqrt{\lambda D}, \sqrt{\lambda D} < l_1 < \frac{\sqrt{\lambda D}}{\alpha}. \tag{34}$$

The expression for the reflected power can be given in relation to the corresponding free-space power, P_0, instead of the incident power per unit of surface, p_0. Then:

$$p_0 = P_0 4/\pi D^2, \tag{35}$$

if D is average distance, close to actual distance, d, such that:

$$1/D = d/4d_1d_2, \tag{36}$$

where d_1 and d_2 are the distances from the reflecting element to the extremes of the path close to $d/2$.

Equations for reflected powers are, therefore:

$$p_{1s} = P_0 a_{1s} \frac{\delta n^2}{e^2} \frac{l_1^2}{h^2} \gamma^2 \frac{\lambda^3}{\alpha^6 D} \tag{37}$$

$$p_{1d} = P_0 a_{1d} \frac{\delta n^2}{e^2} \frac{l^2 l_1^2}{h^2} (1-\gamma^2) \frac{\lambda^2}{\alpha^6 D^2} \tag{38}$$

with numerical values $a_{1s} \sim 6.2\ 10^{-3}$ and $a_{1d} \sim 6.2\ 10^{-3}$.

c. Second Order Irregularities

Over a total surface, S, of a stratum, we assume irregularities $s_1(l_1,H)$ superposed upon irregularities $s(l,h)$. The power of diffuse reflection, p_{1d}, due to an irregularity s_1, can be considered as playing the role of an element of reflected power. Then the sum of these powers will cause a total noncoherent reflection to appear, which will be the total power of diffuse reflection from the surface S, and a total coherent reflection, which can be considered as being the total power of quasi-specular reflection of the surface S.

The first can be written:

$$P_{rd} = p_{1d} S_d / s_{1d}. \qquad (39)$$

The surface of diffuse reflection will be limited transversely by the maximum inclination, τ, of the elements of surface s, i.e., by a transverse dimension $L_d = D\alpha\tau$. With the approximation $\tau = H/l_1$, we have for S_d:

$$S_d = LD\alpha H/l_1. \qquad (40)$$

Then the equation for diffuse reflected power is:

$$P_{rd} = P_0\, a_d \frac{\delta n^2}{e^2} \frac{L l_1 H}{h^2} (1-\gamma^2) \frac{\lambda^2}{\alpha^6 D}, \qquad (41)$$

with a numerical value $a_d \sim 6.2\ 10^{-3}$.

For total power of quasi-specular reflection, one must consider the power p_{1d} as an element of power corresponding to an effective surface σ_1 associated with s_1 and no longer to σ associated with s. The expression for reflected power, therefore, will be:

$$P_{rs} = p_{1d} \frac{\sigma_1^2}{\sigma^2} \frac{S_s^2}{s_{1d}^2} (1-\gamma^2). \qquad (42)$$

For the effective surface σ_1, we have:

$$\sigma_1 = \lambda l_1^2 / 2H\alpha. \qquad (43)$$

As far as the total surface of reflection, S_s, is concerned, the transverse dimension of Fresnel's ellipse, $L_s' = \sqrt{\lambda D}$ will, as before, be involved. Hence:

$$S_s = L\sqrt{\lambda D}. \qquad (44)$$

The expression for the power of quasi-specular reflection will be:

$$P_{rs} = P_0 a_s \frac{\delta n^2}{e^2} \frac{L^2 l_1^2}{l^2 H^2} (1-\gamma^2) \frac{\lambda^3}{\alpha^6 D}, \qquad (45)$$

with a numerically constant value, $a_s \sim 1.2\ 10^{-2}$.

d. General Expressions for Reflected Power

Equations (37), (38), (41), and (45) for the different components of power reflected by a stratum correspond to certain hypotheses dealing, on the one hand, with the nature of the index discontinuity characteristic of the stratum, i.e., the reflection coefficient; and, on the other hand, with the respective dimensions of the different intervening surfaces. It is only in the form of expressions (29), (30), (39), and (42) that the generalization is maintained. Other assumptions would lead to equations a little different from the role played by the different parameters. Moreover, the characteristics and dimensions of atmospheric strata and their irregularities can be markedly variable in time and space. If, then, the assumptions which have been made correspond to average orders of magnitude, the equations which result from them should be considered as having limits in their application for a particular case.

It is essential to keep in mind that reflection phenomena in an irregular medium involve two components: a reflection with coherent character, affecting a limited surface of the stratum and appearing dominant if the irregularities are dimensionally small relative to the space wavelength; and a reflection with diffuse character, involving a large portion of the surface of the stratum, and all the more marked as the irregularities are more important. Rayleigh's criterion gives us the limits within which one or the other component will be preponderant. One other significant characteristic of the propagation by reflection is the fine structure of the phenomenon involving a limited number of reflecting elements.

Some numerical examples permit one to account for the orders of magnitude involved, and the limits of approximation used.

For a distance $D = 200$ km, the minimum value for the angle of inclination is about $\alpha = 12.5 \cdot 10^{-3}$ rad; for $D = 400$ km, $\alpha = 25 \cdot 10^{-3}$ rad. The space wavelengths for radiation of wavelength $\lambda = 1$ m or $\lambda = 10$ cm are, respectively, for 200 km, $\Lambda = 800$ m or $\Lambda = 80$ m; for 400 km, $\Lambda = 400$ m or $\Lambda = 40$ m.

For a discontinuity thickness, $e = 2$ or 3 m, we have always $e << \Lambda/8$, and formula (28) for the reflection coefficient is generally valid.

Rayleigh's criterion for an irregular surface shows, on the contrary, that it is sufficient that the thickness of the irregularities of a stratum, H, be greater than 10 m for the surface to appear irregular to radiation of $\lambda = 10$ cm. However, the thickness has to be greater than 50 or 100 m, according to the distance, for the surface to appear irregular to radiation of $\lambda = 1$ m.

The dimensions of Fresnel's ellipse vary from $L_s = 38$ km and $L_s' = 450$ m for $D = 200$ km and $\lambda = 1$ m, to $L_s = 8$ km and $L_s' = 200$ m for $D = 400$ km and $\lambda = 10$ cm. For stratum dimensions on the order of $L = 1$ km, the assumption that $L_s' < L < L_s$ is, therefore, no longer valid.

An inclination of an irregularity in the stratum of 10° relative to the horizontal, i.e., $\tau = 0.17$ rad, results in a transverse dimension of the diffuse reflection surface of $L_d = 400$ m at 200 km, and $L_d = 1{,}700$ m at 400 km. It should be remarked that generally $L_d > L_s$. The assumption that $L_d < L$ is less general than $L_s' < L$, and the actual surface of diffuse reflection is much larger than the real surface of specular reflection. Note that an inclination of 10°, corresponding to a ratio of stratum thickness, H, to irregularity dimension, l, close to $H/l = 0.2$, is plausible. It results in $H = 20$ m for $l = 100$ m, and $H = 200$ m for $l = 1$ km.

3.3. Consequences of the Reflection Theory

a. Effect Due to Frequency

Equations (37) and (38), or (41) and (45), for specular and diffuse reflection powers point to evidence that there exists a decrease of specular reflection with frequency (λ^3) more rapid than the case involving diffuse reflection (λ^2).

It must, however, be remarked that, while the frequency increases, irregularities of the medium become more apparent under radiation, to the detriment of specular reflection. On the other hand, eq (28) for the coefficient of reflection ceases to be valid for frequencies that are too high, and the ensuing terms of the development introduce much higher exponents of λ.

b. Effect Due to Distance

Upon admitting a proportionality of α to distance, the expressions for reflected power point to an evident and rapid decrease of specular reflection with distance as compared to diffuse reflection. If one, however, takes into account the fact that the dimensions of the strata appear to increase with altitude, diffuse reflection will tend to be dominant at very great distances.

c. Statistical Properties of the Signal

Fluctuations of signals are due essentially to instantaneous interference between signal components. The differences in path lengths can be more significant, therefore, the greater the dimensions of the atmospheric zone which are involved. It can be seen that the limits of the useful surfaces differ for specular reflection (S_s transversely limited by Fresnel's ellipse, $L_s=\sqrt{\lambda D}$) and for diffuse reflection (S_d limited transversely by the inclination of reflecting elements, $L_d=D\alpha\tau$). In thickness, the atmospheric zone is limited by the thickness, H, of the stratum.

The result is that the fading range will be less than with Rayleigh's law. Upon considering the number, n, of reflecting surfaces, equation (21) for reflected power could be put in the form:

$$P_r=P[n+n(n-1)\gamma^2]. \tag{46}$$

This form corresponds to the expression for the distribution of a vector sum, whose phase is distributed in a random manner between two limits, w and $-w$ [Beckmann, 1957], with:

$$\gamma=\sin w/w.$$

The fading rate is a function of the velocity of relative displacement of reflecting elements, each relative to the others, resulting in average movements, of velocity u, and vertical movements, of velocity v, of the atmosphere. This fading rate can be expressed by the number N of crossings of the median level. An approximate expression as a function of the velocity of mean radial displacement, v_r, is:

$$N=3\tau 2\alpha v_r/\lambda. \tag{47}$$

The factor, τ, which is the maximum inclination of the strata, is introduced by the transverse limits of the reflection surface.

d. Space Selectivity of Reflected Power

The diversity distance is related to the measurements of the useful zone of the atmosphere, more exactly, to the angle at which the zone is seen from the point of emission.

In a transverse direction, diversity distance δ_H would be equal to:

$$\delta_H=\frac{\lambda}{2\alpha}\frac{1}{3\tau}. \tag{48}$$

In the vertical direction, however, the small thickness, H, of the stratum is involved, hence:

$$\delta_v=\frac{1}{8}\frac{\lambda D}{H}, \tag{49}$$

increasing with the frequency.

e. Frequency Selectivity of Reflected Power

The dimensions of the useful zone of the atmosphere involved, in limiting the bandwidth transmissible by reflection, will be the maximum dimensions in a horizontal, rather than vertical, plane, by reason of the small thickness of the stratum.

In this manner, the equation for the bandwidth Δf is in the form:

$$\Delta f=\frac{b}{D^3}\frac{1}{\tau^2}. \tag{50}$$

For a correlation sufficiently close to 1, when Δf is descriptive of the transmissible bandwidth, the value of the numerical coefficient is close to $b=10$, if Δf is in Mc/s and D is in 100-km units.

f. Fine Structure of the Field

The instantaneous characteristics of the reflected fields differ sensibly from the character of the scattered fields.

In a rapid spatial analysis, diffusion will appear only as a large bright zone, depending upon the dimensions of the scattering volume. Reflection, on the contrary, can cause very narrow zones to appear, corresponding to surfaces of reflecting strata. These will appear as *"brilliant points,"* by virtue of the reduced openings of the antenna lobes. Experiences in rapid swinging of a narrow beam of an antenna are, in this connection, a confirmation of the possible existence of these two phenomena of propagation [Waterman, 1958].

In a rapid frequency analysis, the statistical properties of the frequency structure will similarly be different for scattering and reflection. Besides an increase in the maxima due to differences in the transmissible bandwidth, and besides a reduced fading rate in the case of reflection, it is possible to relate the spectral characteristics of fluctuations to the atmospheric zones involved. The existence of different types of spectral distributions, within the range of experiments in rapid frequency analysis, also afford a verification of the existence of several propagation phenomena [Landauer, 1960; Biggi et al., 1960].

3.4. The Complex Phenomenon of Trans-Horizon Tropospheric Propagation

Propagation phenomena by scattering and reflection each play their part in the complex processes of propagation in a troposphere consisting of turbulent layers and stable strata.

The phenomenon of specular or quasi-specular reflection, which presupposes specific conditions of regularity in the strata, will appear generally only a small percentage of the time, and will be weaker as distance increases and frequency rises. This is the origin of the high and little fluctuating levels observed in trans-horizon paths.

The phenomenon of diffuse reflection and scattering will be essential elements in trans-horizon propagation as long as distance is sufficiently large or frequency high enough to make the effect of the earth and its relief neglible compared to the effect of the troposphere.

Although the fundamental characteristics of these two phenomena are quite close, as long as the phenomena are incoherent, specific properties are attached to each, and it will be necessary to resort to one or the other in order to interpret all the experimental results [du Castel, 1960].

A comparative study of these principal properties can be made. Results based on very general hypotheses for each of the phenomena are shown in figure 6.

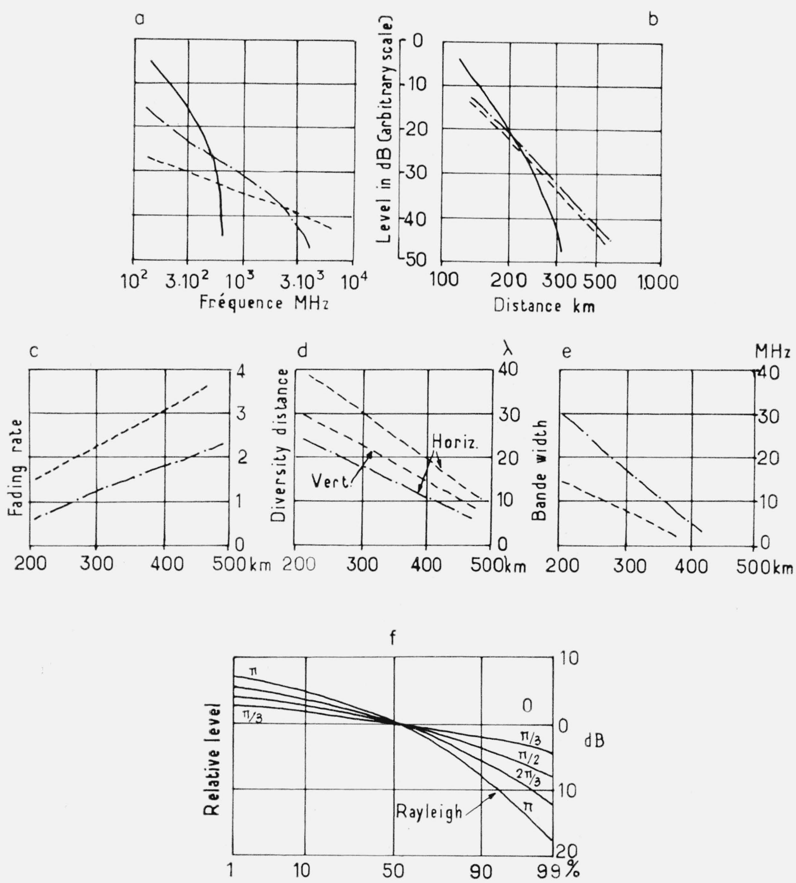

FIGURE 6. *Phenomenon of trans-horizon tropospheric propagation.*

Diffusion (broken line), specular reflection (solid line).
Diffuse reflection (dot-and-dash).
a, According to frequency; b, According to distance; c, Fading speed; d, Diversity distance; e, Transmissible bandwidth; f, Distribution of fading range.

4. Process of Reflection in Other Propagation Media

General considerations concerning reflection processes in irregular media can be applied to numerous instances where the existence of a field of force in a dominant direction leads to some stratification of the propagation medium. The troposphere is an example, but there are others. Following the considerations that have been made in section 3, we shall briefly consider the case involving the ionosphere.

Moreover, if an irregular stratification leads to a phenomenon of reflection, the same result will evidently exist for reflection on an irregular ground surface. We shall similarly deal with this latter case succinctly, as another example of application of the more general analysis.

4.1. Process of Reflection in Ionospheric Propagation (Sporadic-*E* Layer)

The possible existence in the lower ionosphere of strata of ionization is, at present, an experimental fact [Bowles, 1959; Seddon, 1960]. However, interpretations of the phenomenon are not all in agreement. For some, this stratification would result from propagation through the atmosphere of gravity waves, creating vibrations opposite in phase for points separated by a half-wavelength. Such vibrations would create strong gradients of horizontal velocity. In transitional zones between two oppositely directed horizontal currents, the electrified particles which are transported are subject to vertical magnetic forces of opposite sense because of the horizontal component of the earth's magnetic field. Thus, a reinforced ionization will result, corresponding to sporadic-*E* layer [Hines, 1960; Whitehead, 1960].

Such stratification would not be uniform. Under the influence mainly of turbulent movements in lower adjacent layers, structural irregularities would appear [Voge, 1961].

Let us consider a model sporadic-*E* layer, characterized by a gradient of electronic density, corresponding to a variation of density, δN (on the order of 10^4 electrons per cm^3), over a small thickness, e (on the order of 10 m), the average density at the level under consideration being N_0 (on the order of 3×10^3 electrons per cm^3).

Horizontally, the dimension of the stratum is L (on the order of 10,000 m) and vertically H (on the order of 1,000 m). The surface shows irregularities of average dimension, horizontally l (on the order of 1,000 m) and vertically h (on the order of 100 m). The stratum is at a mean altitude D (on the order of 100 km).

What is the significance in this case of the reflection formula of section 2 for vertical transmitting power, P_e, corresponding to an ionospheric sounding?

Ionospheric plasma has a cutoff frequency, f_0, such that:

$$f_0^2=N_0(e_0^2/n^2m_0\epsilon_0), \tag{51}$$

e_0 being the electron charge; m_0, electron mass; ϵ_0, the dielectric constant in vacuum. In practical units, we have approximately:

$$f_0(\text{Mc/s})=10^{-2}\sqrt{N(\text{cm}^{-3})}. \tag{52}$$

For $N_0=3\ 10^3\ \text{cm}^{-3}$, $f_0=0.5$ Mc/s. For any frequency less than f_0, the reflection is total, and the reflected power is:

$$P_0=P_e(1/4\pi D^2). \tag{53}$$

At a frequency f higher than f_0, the ionization gradient δN introduces a partial reflection, corresponding to an index variation δn.

We have:

$$n^2=1-f_0^2/f^2 \tag{54}$$

or, in practical units:

$$n^2=1-10^{-4}N(\text{cm}^{-3})/f^2(\text{Mc/s}) \tag{55}$$

so that:

$$\delta n=5.10^{-5}\delta N/f^2. \tag{56}$$

The change in the index takes place over a thickness, e, to which corresponds a gradient of index change:

$$g=\delta n/e.$$

For example, for $\delta N=10^4\ \text{cm}^{-3}$, and $e=10$ m, $g^2=2.10^{-3}$ at $f=1$ Mc/s, and $g^2=2.10^{-7}$ at $f=10$ Mc/s.

Consider the term for specular reflection in eq (24). We will have, for vertical sounding:

$$P_s=P_e\frac{1}{4\pi D^2}g^2\frac{\lambda^2}{64\pi^2}\frac{4\pi}{\lambda^2}\frac{\lambda^2 l^4}{256h^2}\frac{S_s^2}{l^4}. \tag{57}$$

The surface of specular reflection involved is limited by a condition of phase coherence, i.e., a Fresnel's ellipse, and has the value:

$$S_s=\lambda D/4. \tag{58}$$

Therefore:

$$P_s=P_e 6.10^{-6}g^2\frac{\lambda^4}{4h^2}. \tag{59}$$

For the term of diffuse reflection in eq (24), we have here:

$$P_d=P_e\frac{1}{4\pi D^2}g^2\frac{\lambda^2}{64\pi^2}\frac{4\pi}{\lambda^2}\frac{\lambda^2 l^4}{256h^2}\frac{S_d}{l^4}. \tag{60}$$

The surface of diffuse reflection involved is limited by the inclination of the elementary surfaces of reflection, and has a value:

$$S_d=L(DH/L). \tag{61}$$

Therefore:

$$P_d=P_e 6.10^{-6}g^2\lambda^2\frac{l^2}{h^2}\frac{H^2}{L^2}. \tag{62}$$

With orders of magnitude as follows: $h=100$ m, $l=1{,}000$ m, $H=1{,}000$ m, $L=10{,}000$ m, the numerical values indicated in figure 7 are obtained for reflected power as a function of frequency. Transition between specular reflection and diffuse reflection corresponds to a frequency f, such that:

$$\lambda'=2Hl/L. \tag{63}$$

Here, $f'=1.5$ Mc/s. It should be noted that we have not considered here the term γ of section 2, characterizing the respective importance of the two reflection terms by:

$$\gamma=\sin u/u$$

where $u=4\pi H/\lambda$. The term γ would introduce a frequency of transition f', such that:

$$\lambda''=4\pi h. \tag{64}$$

Here, $f''=0.15$ Mc/s. In fact, frequency f'' involved is less than frequency f' because of the dominant role played by the different actual reflection surfaces, S_s and S_d.

In an ionogram, there will appear an occlusion frequency $f_0=0.5$ Mc/s corresponding to total reflection, and a limiting frequency f_1, corresponding to the reception threshold, hence dependent upon the sensitivity of the sounding equipment. If it permits reception with an attenuation of 60 db, then one would, for example, have $f_1=3$ Mc/s, according to figure 7.

4.2. Diffuse Reflection by an Irregular Ground Surface

We shall study the case of an irregular ground surface illuminated by a source at infinity in a direction making an angle α with the ground. We shall calculate the reflected energy received at a point close to the ground, at altitude Z. This is, for example, the calculation for the earth's albedo under the influence of solar radiation [Spizzichino and Beckmann].

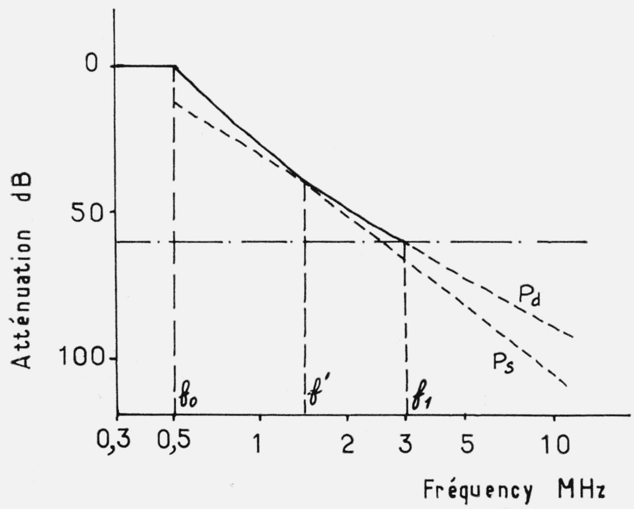

FIGURE 7. *Energy reflected by an irregular sporadic-E layer, as a function of the frequency.*

P_s = specular reflection; P_d = diffuse reflection.

The general schematic of the ground surface will be analogous to that of models previously adopted, with irregularities of dimension l and height h, distributed over the surface with a thickness H (fig. 8).

Each surface element dS, such that its normal is a bisector of directions of the emitter (at infinity) and receiver, will reflect energy as given by eq (24):

$$dP_r=\frac{p}{4\pi r^2}\,\rho^2\,\frac{4\pi\sin^2\alpha}{\lambda^2}\,\sigma^2\,\frac{dS}{s}, \tag{65}$$

where P_0 is the incident energy per unit of surface; r is the distance of the surface element to point of reception; ρ is the coefficient of reflection equal to unity.

The effective surface of reflection has a value according to equation (1):

$$\sigma=\lambda l^2/16h\sin\alpha. \tag{66}$$

Eq (65) will then be written:

$$dP_r=\frac{1}{256}\,\frac{l^2}{h^2}\,\frac{dS}{r^2}. \tag{67}$$

It can be remarked that the ratio l^2/h^2 is, in effect, proportional to inclination τ of the surface element dS on the ground surface, so that (fig. 8):

$$\tau=4h/l. \tag{68}$$

The equation for the reflected energy can then be written, for the case of a unit incident energy:

$$P_r=\frac{1}{16\tau^2}\int_S\frac{dS}{r^2}. \tag{69}$$

The integration should be extended over all the real surface of reflection, S, i.e., over all the portion of the surface of the ground for which the angle between normal and vertical is less than the angle of inclination of the reflecting elements, τ.

In order to calculate this surface S, the system u,v of angular coordinates could be employed, as defined in figure 9. The equation for the limiting curve of the surface is then:

$$2\cos u=\cos v/\cos\alpha+\cos\alpha/\cos u-tg^2\tau\,\frac{\sin v+\sin\alpha}{\cos v\cos\alpha}. \tag{70}$$

This curve should be limited by the conditions:

$$\alpha-2\tau<v<\alpha+2\tau$$

or

$$0<v<\pi/2$$

and

$$-\pi<u<\pi. \tag{71}$$

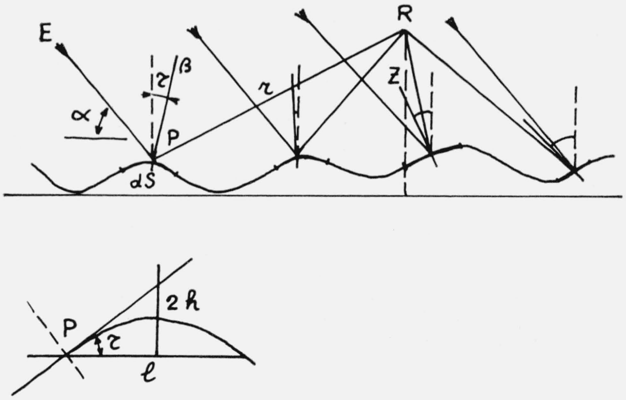

FIGURE 8. *Reflection by irregular ground surface.*

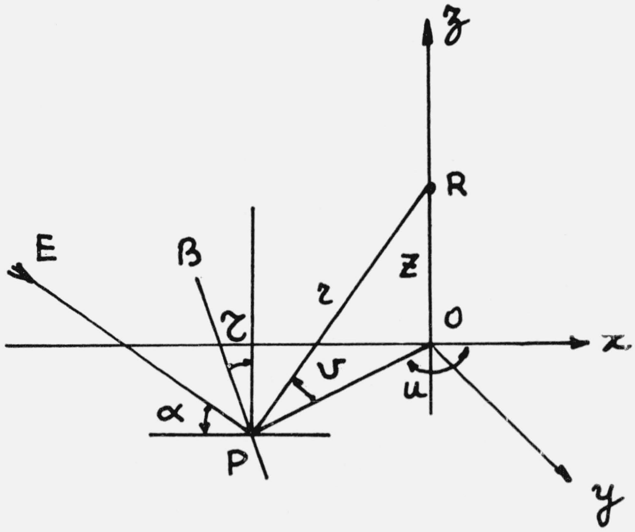

FIGURE 9. *Actual surface of reflection by an irregular surface.* System of angular coordinates.

The equation for the limiting curve is therefore of the form:

$$u = u(v, \alpha) \tag{72}$$

and the reflected energy will be:

$$P_r = (1/16\tau^2) \int_{v_1}^{v_2} u(v, \alpha) \cot v \, dv. \tag{73}$$

It must be noted that this expression is independent of the altitude Z of the point of reception.

According to the limits v_1 and v_2 introduced by conditions (71), the limits of the surface S can be very different. Figure 10 gives some idea of cases which can be encountered, while figure 11 gives the behavior of variations of the corresponding reflected energy.

We note that the calculation assumes a plane ground surface and neglects the earth's curvature. The approximation is valid if the altitude of the reflection point remains small. However, the actual surface of reflection should be, nevertheless, limited to the horizon of the point. A restrictive hypothesis is equally implied, and the equations for reflected power cease to be valid when $\alpha \ll \tau$, obliging one to limit the curves in figure 11 to small values.

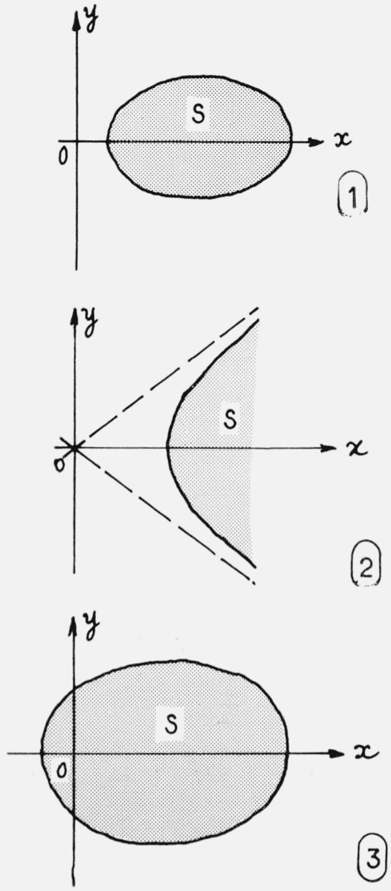

FIGURE 10. *Different examples of actual surface of reflection by an irregular ground surface.*

5. Conclusions

The general study of reflection in irregular media (section 2) and the discussion of applications, complete for the case of trans-horizon tropospheric propagation (section 3), but only summarily made for other cases (section 4), show the importance of reflection phenomena in numerous problems of propagation.

The problems brought up here have been discussed in different French publications. A series of articles on the subject, published 1958–60, have been edited [du Castel, Misme, Voge, Spizzichino, 1960]. A book studying trans-horizon tropospheric propagation was recently published [du Castel, 1961].

The authors, within the framework of their work at the C.N.E.T., envision other applications of this study, namely, to problems of ionosphere and exosphere propagation of decameter waves.

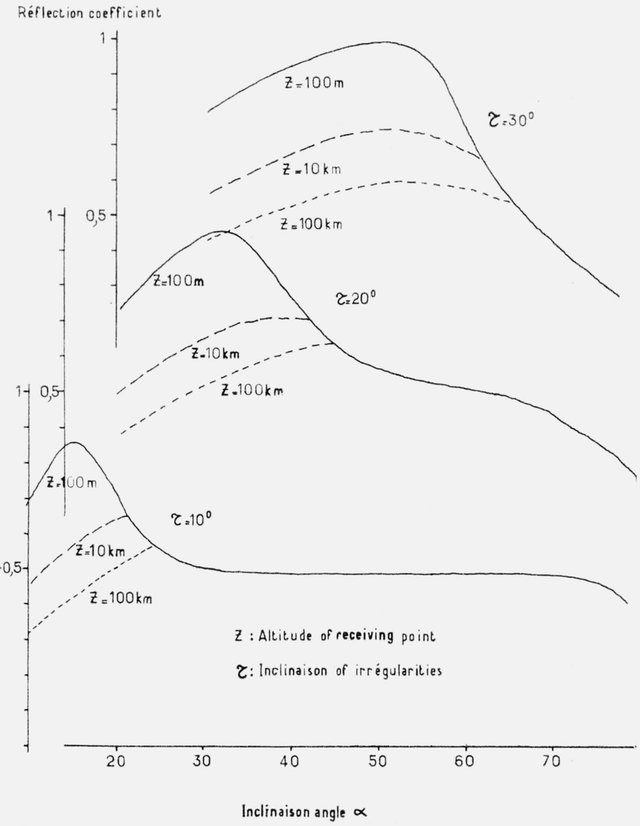

FIGURE 11. *Variation of energy reflected by an irregular ground surface.*

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