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Effect of Air Drag on the Motion of a Filament Struck Transversely by a High-Speed Projectile

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The effect of air drag on the motion of a filament struck transversely is treated theoretically. The air drag is shown to produce curvature in the transverse wave formed by the impact, to increase the speed of the transverse-wave front, and to increase the strain in the filament. The theory is applied to the case of a nylon yarn impacted transversely at 189 meters/second. The calculated radius of curvature of the yarn was 1.1 meter, which agreed well with 1.2-meter radius obtained by experiment. The calculated effects of air drag on the strain and distance traveled by the transverse-wave front 601 microseconds after impact were small.

When a long filament is subjected to a high-velocity transverse impact, strain waves and transverse waves are generated which travel outwards along the filament away from the point of impact. According to theory [1–6][1] the velocities and strain distributions in these waves can be calculated provided that the stress-strain curve of the filament under the impact conditions is known. Recently [7–9] the inverse problem has been considered for a textile yarn, i.e., the calculation of the stress-strain curve if the velocities or the strain distribution resulting from an impact are known. Unfortunately, the theory is incomplete for this practical application because it neglects the effect of air drag on the yarn. In this paper the theory of transverse impact is extended to include the effect of air drag, and the errors introduced by its neglect are estimated for a typical case.

1. Behavior of the Filament Neglecting Air Drag

Since the effect of air drag on the motion of the filament is treated as a perturbation on the motion without air drag, the theory neglecting air drag is first reviewed. The impact produces longitudinal-strain waves that propagate outward in each direction from the point of impact. These strain waves produce a strain e_0 in the filament. In the region between the strain-wave fronts, material of the filament is set into motion longitudinally toward the point of impact with a velocity of W_0. This material is taken up by a tent-shaped wave of transverse motion, as shown in figure 1. The filament material between the projectile and the transverse-wave front moves vertically with the velocity V of the projectile, and the wave front travels in a longitudinal direction with velocity U_0. The section of filament between the projectile and transverse-wave front is a straight line; the solid and dotted lines in figure 1 show the filament at two different times after impact.

[1] Figures in brackets indicate the literature references at the end of this paper.

The longitudinal velocity of the filament material, W_0, is given by

$$W_0=\frac{1}{\sqrt{m}}\int_0^{e_0}\sqrt{\frac{dT}{de}}\,de \tag{1}$$

where m is the mass per unit length of the unstrained filament, e_0 is the strain, and T is the tension required to produce a strain e.

The transverse-wave front velocity is

$$\lambda_0=\sqrt{\frac{T_0}{m(1+e_0)}} \tag{2}$$

with respect to the unstrained filament, i.e., with respect to a Lagrangian coordinate system. T_0 is the tension in the filament resulting from the impact. These velocities are related to the impact velocity V by the equation

$$V^2=2W_0(1+e_0)\lambda_0-W_0^2. \tag{3}$$

An expression for the transverse-wave front velocity U_0 in laboratory coordinates will now be

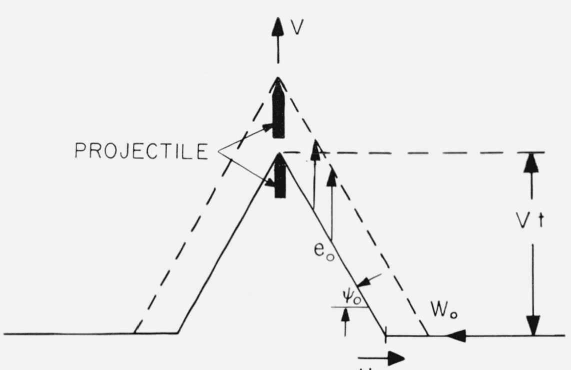

FIGURE 1. *Configuration and motion of a filament impacted transversely at velocity V, air drag neglected.*

derived. Suppose ink marks 1 cm apart are placed on the unstrained filament before the filament is impacted. After impact, the transverse-wave front velocity in cm/sec is λ_0 with respect to the unstrained filament, so the transverse-wave front will pass λ_0 ink marks per second. However, these marks are now $(1+e_0)$ cm apart since the strain in advance of the transverse wave is e_0. The transverse-wave front, therefore, traverses a distance $\lambda_0(1+e_0)$ cm on the filament per second. Since the filament in advance of the transverse wave has a velocity W_0 opposite to the direction of motion of the transverse-wave front, a stationary observer detects a transverse-wave front velocity of

$$U_0=\lambda_0(1+e_0)-W_0. \tag{4}$$

The slope, ψ_0, of the filament behind the transverse-wave front is seen from figure 1 to be given by

$$\tan \psi_0=V/U_0.$$

Equations (1) to (4) describe the motion of the impacted filament in the absence of air drag. These equations, however, must be modified slightly in certain cases. For instance, the above theory is based on the assumption that the stress-strain curve is always concave downwards, whereas many stress-strain curves for textile yarns have concave upward portions. The modification required in this case has been discussed by Smith et al. [10]. At very high impact velocities, part of the region of variable strain in the strain wave occurs in the wake of the transverse-wave front instead of in advance of it as assumed in the discussion above. The modifications required in this rarely occurring case [6] are not considered in this paper.

2. Effect of Air Drag

The force on the filament between the projectile and transverse-wave front due to air drag may be expressed as a component perpendicular and a component parallel to the filament. However, the force component parallel to the filament is small, so only the force component perpendicular to the filament will be considered.[2]

This force produces curvature in the section of filament between the projectile and transverse-wave front, thereby increasing the strain and tension in the filament. This increase in strain produces strain waves of small amplitude that are propagated past the transverse-wave front and along the filament. Since the velocities of these strain waves are usually much greater than the velocity of the transverse wave, the strain, e, in the filament between the projectile and transverse wave is nearly uniform at any time, but varies with time. Thus the Lagrangian velocity of the transverse-wave front is given by

$$\lambda=\sqrt{\frac{T}{m(1+e)}} \tag{5}$$

[2] These force components are compared later in the paper.

where T is the tension in the filament. Since e and T vary with time, λ also varies with time.

In order to compute the length of the filament between the projectile and transverse-wave front, consider ink marks that have been placed 1 cm apart on the filament before impact. The rate at which the transverse-wave front passes these ink marks is λ cm/s. Therefore, the transverse-wave front will have passed $\int_0^t \lambda dt$ marks at a time t after impact. Since the yarn has strain e at time t, the distance between the marks will be $(1+e)$ cm. Therefore, the length of filament between the projectile and transverse-wave front is

$$S=(1+e)\int_0^t \lambda dt. \tag{6}$$

Note that because of the extra strain caused by air drag, this length of filament is no longer a straight line but is curved.

The velocity of the transverse-wave front in laboratory coordinates is given by $\lambda(1+e)-W$ and varies with time. Therefore, the longitudinal distance traveled by the transverse-wave front is

$$X=\int_0^t [\lambda(1+e)-W]dt \tag{7}$$

where the velocity of the material of the filament in advance of the transverse-wave front is given by

$$W=\frac{1}{\sqrt{m}}\int_0^e \sqrt{\frac{dT}{de}}\,de. \tag{8}$$

Equations (5) to (8) will be used to calculate the distances S and X after e and T are given as functions of time.

3. Evaluation of Air Drag

Let the strain e be expanded to give

$$e=e_0+a_0+a_1t+a_2t^2+\ .\ .\ .\ . \tag{9}$$

The strain is assumed to vary only a small amount from the strain e_0 that would occur without air drag, so a few terms of this series are expected to give a good approximation. For strains near e_0, the tension may also be approximated by

$$T=T_0+K(e-e_0) \tag{10}$$

where K is the slope of the tension-strain curve of the filament at e_0. Therefore, the tension in the filament is

$$T=T_0+Ka_0+Ka_1t+Ka_2t^2+\ .\ .\ .\ . \tag{11}$$

With this approximation for the tension, eq (8) becomes

$$W=\frac{1}{\sqrt{m}}\int_0^{e_0}\sqrt{\frac{dT}{de}}\,de+\frac{1}{\sqrt{m}}\int_{e_0}^{e}\sqrt{\frac{dT}{de}}\,de$$

$$=W_0+\sqrt{K/m}\,(e-e_0). \tag{12}$$

Equations (9) and (11) are now substituted in eqs (5), (6), (7), and (12) to express λ, S, X, and W as series in t. Thus eq (5) becomes

$$\lambda=\frac{1}{\sqrt{m}}\sqrt{\frac{T_0+Ka_0+Ka_1t+\ldots}{1+e_0+a_0+a_1t+\ \ldots}}$$

$$=\sqrt{\frac{T_0}{m(1+e_0)}}\sqrt{\frac{1+ka_0+ka_1t+\ldots}{1+\frac{a_0}{1+e_0}+\frac{a_1t}{1+e_0}+\ldots}}$$

$$=\lambda_0\sqrt{\frac{1+ka_0}{1+a_0/(1+e_0)}}\sqrt{\frac{1+\frac{ka_1}{1+ka_0}t+\ldots}{1+\frac{a_1t}{1+e_0+a_0}+\ldots}} \tag{13}$$

where $k=K/T_0$.

Dividing and taking the square root of the series yields

$$\lambda=\lambda_0\frac{\sqrt{1+ka_0}\sqrt{1+e_0}}{\sqrt{1+e_0+a_0}}+\frac{(k+ke_0-1)\sqrt{1+e_0}\lambda_0a_1t}{2\sqrt{1+ka_0}(1+e_0+a_0)^{3/2}}+0t^2 \tag{14}$$

where $0t^2$ represents terms of the second and higher powers of t. The following equations are similarly derived:

$$\frac{S}{t}=\lambda_0\sqrt{1+e_0}\sqrt{1+e_0+a_0}\sqrt{1+ka_0}$$
$$+\frac{3+(1+e_0+4a_0)k}{4\sqrt{1+e_0+a_0}\sqrt{1+ka_0}}\lambda_0\sqrt{1+e_0}a_1t+0t^2 \tag{15}$$

$$\frac{X}{t}=\lambda_0\sqrt{1+e_0}\sqrt{1+e_0+a_0}\sqrt{1+ka_0}-W_0-\sqrt{\frac{K}{m}}a_0$$
$$+\left[\frac{1+(1+e_0+2a_0)k}{\sqrt{1+e_0+a_0}\sqrt{1+ka_0}}\sqrt{1+e_0}\lambda_0-2\sqrt{\frac{K}{m}}\right]\frac{a_1t}{4}+0t^2. \tag{16}$$

In order to solve for a_0, a_1, etc., a relationship must be found between S and X. This relationship is derived by considering the configuration of the filament between the projectile and transverse-wave front.

The configuration of the filament exposed to air drag is shown in figure 2. Let s be the distance measured along the filament between the projectile and a segment ds of the filament.

The air drag on the segment ds is composed of a frictional component due to the viscosity of the air and a dynamic pressure component due to uneven

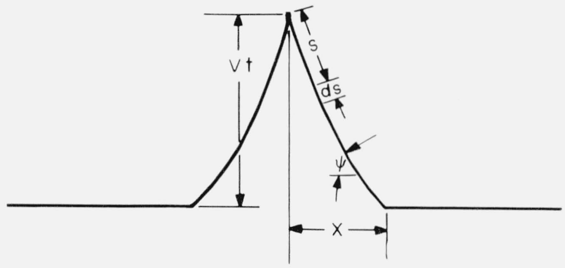

FIGURE 2. *Effect of air drag on the shape of a filament impacted transversely at velocity* V.

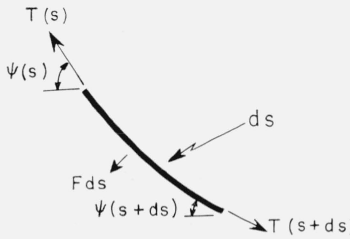

FIGURE 3. *A segment* ds *of the transverse wave portion of the impacted filament showing tensile forces required to balance the air drag.*

air pressure around the diameter of the yarn. The frictional component of the air drag is only a few percent of the dynamic pressure component [11] and so will be neglected.[3] By the "cross-flow principle" [11] the dynamic pressure component of the air drag is directed perpendicularly to every point of the filament. Figure 3 shows the forces on a segment ds of the filament. The equilibrium equations for the horizontal and vertical force components of the segment are

$$\frac{d(T\cos\psi)}{ds}-F\sin\psi=0$$

and

$$\frac{d(T\sin\psi)}{ds}+F\cos\psi=0,$$

where F is the force per unit length on the filament due to air drag. These conditions simplify to

$$\frac{dT}{ds}=0 \tag{17}$$

and

$$\frac{d\psi}{ds}=\frac{-F}{T}. \tag{18}$$

Equation (17) states that the tension is constant along the filament.

The force due to air drag is proportional to the square of the velocity of the segment, to $\cos^2\varphi$ where φ is the slope angle of the segment, and to the length of the segment. Since the case of small air drag is

[3] If the slope of the filament is close to 90°, the pressure component of the drag becomes small.

being considered, the velocity may be approximated by the projectile velocity V and the slope φ by the slope φ_0 that the filament would have if no air drag were present. Therefore, the force on the element ds is

$$Fds = qV^2 \cos^2 \varphi_0 ds. \tag{19}$$

Since F and T are independent of s and therefore constant along the yarn, eq (18) shows that the filament forms an arc of a circle of radius T/F. The angle subtended by the arc S at the center of this circle is given by the length of the arc divided by the radius SF/T, where S is the length of the filament between the projectile and transverse-wave front. The length of the corresponding chord of the circle is then $(2T/F) \sin (SF/2T)$ and is also $\sqrt{X^2+{}^2Vt^2}$ by figure 2. Therefore,

$$(2T/F) \sin (SF/2T) = \sqrt{X^2+V^2t^2}.$$

Since F is very small compared to T, the sine may be approximated by relatively few terms of its infinite series. Squaring both sides of the equation then gives

$$X^2+V^2t^2 = S^2 - \frac{F^2S^4}{12T^2} \tag{20}$$

(neglecting terms higher than F^2), which is the desired relationship between X and S to evaluate the coefficients a_0, a, etc.

Equations (15), (16), and (3) for S, X, and V are substituted in eq (20) to yield the following identity in t.

$$\begin{aligned}&\Bigg[\lambda_0\sqrt{1+e_0}\sqrt{1+e_0+a_0}\sqrt{1+ka_0} - W_0 - \sqrt{K/m}\,a_0 \\ &\quad + \left(\frac{1+(1+e_0+2a_0)k}{\sqrt{1+e_0+a_0}\sqrt{1+ka_0}}\sqrt{1+e_0}\lambda_0 - 2\sqrt{K/m}\right) a_1t/4 \\ &\quad + 0t^2\Bigg]^2 + [2W_0(1+e_0)\lambda_0 - W_0^2] - \Bigg[\lambda_0\sqrt{1+e_0} \\ &\quad \sqrt{1+e_0+a_0}\sqrt{1+ka_0} + \frac{3+(1+e_0+4a_0)k}{4\sqrt{1+e_0+a_0}\sqrt{1+ka_0}}\lambda_0 \\ &\qquad \sqrt{1+e_0}\,a_1t + 0t^2\Bigg]^2 + 0t^2 = 0. \end{aligned} \tag{21}$$

Since this expression is zero for all values of t, the constant part and the coefficient of t must each be zero. Simplification of the constant part gives

$$\begin{aligned}&\left[\frac{Ka_0}{2m} - \sqrt{K/m}\lambda_0\sqrt{1+e_0}\sqrt{1+e_0+a_0}\sqrt{1+ka_0}\right. \\ &\quad \left. + W_0\sqrt{K/m}\right] a_0 + W_0\lambda_0\sqrt{1+e_0}\,[\sqrt{1+e_0} \\ &\qquad \sqrt{1+e_0+a_0}\sqrt{1+ka_0}] = 0. \end{aligned} \tag{22}$$

By inspection $a_0 = 0$. Therefore, by eq (9), the strain just after impact, when t is very small, is seen to be e_0, which is the strain for no air drag. This is reasonable since the length of filament in transverse motion just after impact is too short to be affected by air drag.

Substituting $a_0 = 0$ in eq (21), calculating the coefficient of t and simplifying gives

$$\begin{aligned}&[(\lambda_0(1+e_0) - W_0)(\lambda_0 + \lambda_0(1+e_0)k - 2\sqrt{K/m}) \\ &\qquad - \lambda_0^2(1+e_0)(3+k+ke_0)]a_1 = 0. \end{aligned} \tag{23}$$

This equation gives $a_1 = 0$ since the coefficient does not vanish.

Since a_0 and a_1 are zero, the terms in higher powers of t must be evaluated in order to calculate the effect of air drag. Thus eqs (9) and (11) become

$$e = e_0 + a_2t^2 + a_3t^3 + \ldots$$

$$T = T_0 + Ka_2t^2 + K_3a_3t^3 + \ldots. \tag{24}$$

The quantities X and S in eqs (15) and (16) are recalculated to higher powers in t and substituted in eq (20) to give

$$\begin{aligned}&\frac{3}{1}[(4\lambda_0 + 2\sqrt{K/m} + kW_0)(1+e_0) \\ &\quad + W_0(1 - 2\sqrt{K/m}/\lambda_0)]a_2t^2 - \frac{1}{12T_0^2} F^2\lambda_0^3(1+e_0)^4t^2 \\ &\quad + \frac{1}{4}[(6\lambda_0 + 2\sqrt{K/m} + kW_0)(1+e_0) \\ &\qquad + W_0(1 - 2\sqrt{K/m}/\lambda_0)]a_3t^3 + 0t^4 = 0. \end{aligned}$$

Putting the coefficient of t^2 equal to zero yields

$$a_2 = \frac{F^2(1+e_0)^4\lambda_0^3/(4T_0^2)}{(4\lambda_0 + 2\sqrt{K/m} + kW_0)(1+e_0) + W_0(1 - 2\sqrt{K/m}/\lambda_0)} \tag{25}$$

and putting the coefficient of t^3 equal to zero yields

$$a_3 = 0. \tag{26}$$

Thus the strain in the filament is given by

$$e = e_0 + a_2t^2 + 0t^4. \tag{27}$$

Substituting this expression into eq (7) and integrating gives for the distance traveled by the transverse wave

$$X = U_0t + [\lambda_0k(1+e_0) + \lambda_0 - 2\sqrt{K/m}]a_2t^3/6 + 0t^5 \tag{28}$$

where a_2 is given by eq (25).

4. Comparison With Experiment

This theory is compared with an impact study of a nylon yarn by Smith et al. [12]. Their multiflash

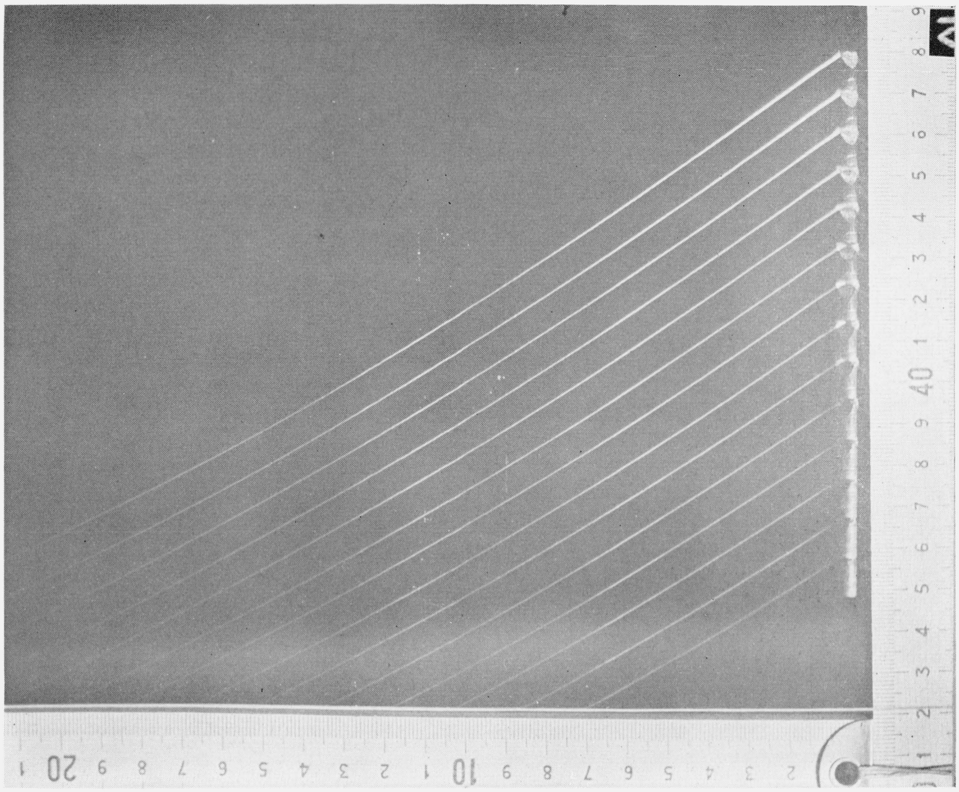

FIGURE 4. *Multiflash photograph of a high tenacity nylon yarn struck by a projectile.*

photograph of a high tenacity nylon yarn, impacted transversely by a projectile with a velocity of 189 m/sec is shown in figure 4. The yarn is photographed before impact, 201 μsec after impact, and every 50 μsec thereafter. Only half of the yarn was photographed in order to show greater detail. The projectile is seen along the right-hand edge of the photograph.

The curvature of the yarn in the transverse wave due to air drag can be seen in figure 4. The radius of curvature of the yarn for the first 10 exposures was measured and found to be approximately constant at 1.2 m.[4]

The linear density of this nylon yarn is 92.7 tex or 9.27×10^{-5} kg/m. Its tension-strain curve, measured at a rate of strain of 4800 percent/sec by Smith et al. [9], is shown in figure 5. Using this curve, the strain e_0 without air drag was computed from eqs (1), (2), and (3). For the impact velocity $V=189$ m/sec, $e_0=2.031$ percent.

In order to calculate the effect of air drag, the drag coefficient, F, is calculated using eq (19). The value of q in this equation is defined by Hoerner [11] as

$$q=\frac{1}{2}\rho d C_D \tag{29}$$

where ρ is the density of air, d is the diameter of the

[4] After the first 10 exposures, the reflected strain wave interacts with the transverse wave and produces curvature in the yarn in addition to the curvature produced by air drag [4].

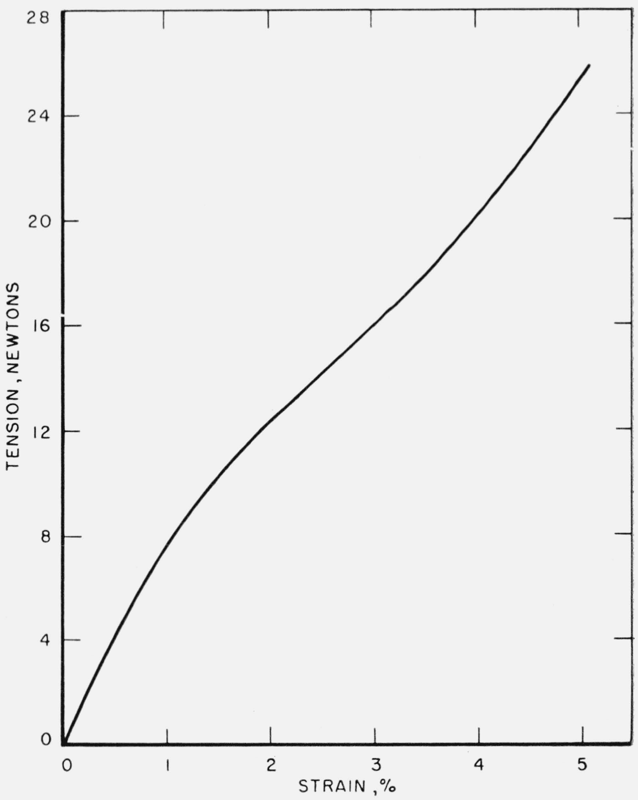

FIGURE 5. *Tension-strain curve of high tenacity nylon yarn for a rate of straining of 4800 percent/second.*

yarn, 0.7 mm, and C_D is the dimensionless drag coefficient. C_D is a slowly varying function of the Reynolds number of the flow. For the case considered, the Reynolds number is 8.8×10^3 and the corresponding drag coefficient, C_D, for a cylinder is 1.10. However, the structure of a yarn affects the drag coefficient. Hoerner [11] results yield an average drag coefficient of 1.09 at a Reynolds number of 10^4 for two metal cables that have a structure similar to a textile yarn. Since the drag coefficient of a cylinder is 1.19 for a Reynolds number of 10^4, the drag coefficient of the yarn is given by

$$C_D = \frac{1.09}{1.19} \times 1.10 = 0.99.$$

By eqs (29) and (19) the force acting on the yarn is $F = 11.46$ newtons/m. The radius of curvature of the yarn, T/F, is calculated to be 1.1 m in good agreement with the experimental value of 1.2 m.

By eqs (25), (27), and (28) the strain, e, and distance, X, transversed by the transverse wave may be calculated. For 601 μsec after impact, the strain is 2.12 percent and the distance, X, traveled by the transverse-wave front is 19.07 cm, compared with a strain of 2.03 percent and an X of 19.06 cm, calculated by neglecting air drag. Thus the effect of air drag on the strain in the yarn and the distance traveled by the transverse wave is small for these conditions, although it produces noticeable curvature in the yarn.

The effect of air drag increases slowly with the velocity of impact. For example, an impact of 300 m/sec on the same yarn produces a strain of 4.44 percent and a distance X of 24.24 cm without air drag, and for air drag a strain of 4.54 percent and distance X of 24.38 cm at 601 μsec after the impact. Thus the effect of air drag is small for the examples considered.

5. References

[1] G. I. Taylor, The plastic wave in a wire extended by an impact load, British Official Report R.C. 329 (1942).

[2] J. D. Cole, C. B. Dougherty, and J. H. Huth, Constant strain waves in strings, J. Appl. Mech. **20,** 519 (1953).

[3] J. W. Craggs, Wave motion in plastic-elastic strings, J. Mech. Phys. Solids **2,** 286 (1954).

[4] F. L. McCrackin, Impact tests of textile filaments, Doctor's dissertation, Lehigh University, Bethlehem, Pa. (1957); Dissertation Abstracts 17, No. 10 (1957).

[5] F. D. Ringleb, Motion and stress of an elastic cable due to impact, J. Appl. Mech. **24,** 417 (1957).

[6] J. C. Smith, F. L. McCrackin, and H. F. Schiefer, Stress-strain relationships in yarns subjected to rapid impact loading: 5. Wave propagation in long textile yarns impacted transversely, J. Research NBS **60,** 517 (1958); Textile Research J. **25,** 520 (1955).

[7] J. C. Smith, F. L. McCrackin, and H. F. Schiefer, Characterization of the high-speed impact behavior of textile yarns, J. Textile Inst. **50,** T55 (1959).

[8] D. R. Petterson, G. M. Steward, F. A. Odell, and R. C. Maheux, Dynamic distribution of strain in textile materials under high-speed impact, Part I: Experimental methods and preliminary results on single yarns. Part II: Stress-strain curves from strain position distributions, Textile Research J. **30,** 411–431 (June 1960).

[9] J. C. Smith, P. J. Shouse, J. M. Blandford, and K. M. Towne, Stress-strain relationships in yarns subjected to rapid impact loading: 7. Stress-strain curves and breaking-energy data for textile yarns, Textile Research J. **31,** 721–734 (Aug. 1961).

[10] J. C. Smith, J. M. Blandford, and K. M. Towne, Stress-strain relationships in yarns subjected to rapid impact loading: 8. Critical velocities, Textile Research J. **32,** 67–76 (Jan. 1962).

[11] S. F. Hoerner, Fluid dynamic drag (1958) published by S. F. Hoerner.

[12] J. C. Smith, private communication.

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