JOURNAL OF RESEARCH of the National Bureau of Standards—C. Engineering and Instrumentation
Vol. 66C, No. 2, April–June 1962

Systems of Electrical Units

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(January 19, 1962)

The various systems of measurement, with their respective sets of units, used in the literature on electricity and magnetism are described in detail. Their historical development is summarized. The manner in which each is derived from either of the two alternative points of view of the experimentalist and the theoretician is compared and contrasted. The desirability of recognizing both points of view in international standardization, particularly when discussing rationalization, is pointed out. The present status of the absolute measurements on which all electrical units are based is reported, and tables are included for the conversion of equations and numerical values from one system to another.

Contents

1. Introduction

This paper has been prepared with several objectives in mind. The first is to provide a definite account of the authorities and procedures on which measurements of electrical and magnetic quantities are currently based. A second is to offer a nomenclature in the field of units and standards which is fairly consistent with current usage and which if generally adopted would minimize semantic confusion in the field. A third is to provide a brief historical survey to record the successive steps in the evolution of the various systems of electrical units, together with a systematic tabulation for converting equations and data from one system to another. A fourth objective is to reconcile the current controversy which was triggered by the 1950 decision of the International Electrotechnical Commission to recommend the use in the future of a "rationalized" system of measurement.

The sharpness of this conflict is illustrated concisely by comparing the following statements:

(a) "1 oersted = 1,000 ampere-turns/meter."

(b) "The number of ampere-turns per meter = $1{,}000/4\pi$ times the number of oersteds."

Each of these statements has been made frequently by scientists and engineers of recognized standing. Their apparent contradiction is, in the author's opinion, merely one particularly striking indication of a very deep-seated difference in the points of view and resulting philosophies of two major classes of workers and thinkers in the field of physics. Hence, the fourth objective of this monograph is to reconcile these contrasting philosophies by disentangling them as completely as possible, even at the expense of a possibly excessive amount of circumlocution and repetition in the text.

In the development of this paper, a brief historical summary will meet the third objective; the major division of the two philosophies will then be outlined, the basic principle of the first (experimental) philosophy being also appropriate for introducing digressions to cover the first and second objectives. The contrasting theoretician's philosophy will then be described on the foundation of the quantity calculus. The various systems of equations and units in the electrical field will then be listed and compared. This will be followed by a brief discussion of the subject of dimensions from both points of view. The process of rationalization as seen from the theoretician's point of view will contrast with that described earlier, and the suggestions of various other writers who have recently attempted to correlate or reconcile the philosophies will be discussed. Because of semantic pitfalls, the glossary (sec. 10.3) gives *in extenso* the particular meanings with which certain terms are used in this monograph.

2. Historical Summary

The concepts of quantities, units, standards (see glossary), and their names and symbols constitute in effect an international language by means of which workers in different countries or in different branches of science exchange and compare their ideas and experimental findings. Therefore, it is natural that a history of the development of these subjects should consist mainly of a chronology of proposals by individual workers, or by small groups, of systems of units and names therefor and of actions by larger national and international organizations accepting or rejecting such proposals as parts of an international language. In section 10.2 is given a sketchy chronology listing various milestones in these developments.

The early workers in the electrical field, especially the telegraph engineers, made frequent use of extemporized standards of resistance and of voltage. A table published in 1864 lists the conversion factors between units defined by standards which range from "25 feet of copper wire weighing 345 grains" to "1 German mile (8,238 yards) of iron wire 1/10 inch in diameter", and include Siemen's "column of mercury 1 meter long and 1 sq mm in cross section," as well as units defined in absolute terms as "10^7 feet/second" or as "10^7 meters/second." The Daniell cell was widely used as a standard of voltage until the Clark cell appeared in 1872.

However, in his studies of terrestrial magnetism, Gauss in 1833 had realized the possiblity and desirability of tying his results into the more permanent and widely recognized system of mechanical units. He invented "absolute methods" (see glossary, sec. 10.3) for measuring magnetic moment and magnetic field intensity. His colleague at Göttingen, W. Weber, in 1840 extended the work to the measurement of current by the tangent galvanometer and later by the electrodynamometer and in 1851 to the measurement of resistance [21].[1] Gauss and Weber used the millimeter, milligram, and second as basic units.

A major influence in the development of systems for electrical measurement was exerted for almost half a century by the Committee on Electrical Standards appointed first in 1861 by the British Association for the Advancement of Science. It was active from 1861 to 1870 and was reactivated from 1881 until it turned over its apparatus and responsibilities to the British National Physical Laboratory in 1912 [1]. Under the leadership of Professor William Thomson (later Lord Kelvin), this group contributed both experimental and theoretical points of view to the problem. In its first report, 1862, it recognized as desirable qualities in the units that they:

(1) be of convenient size.
(2) bear a definite relation to the unit of work, "the great connecting link between all physical measurements".
(3) bear a definite relation to other electrical units.
(4) be perfectly definite and not likely to require correction or alteration from time to time.
(5) be reproducible (a) in case the original standard were injured or (b) so that an observer unable to obtain copies might be able to manufacture them.

They also were confronted with the following experimental "facts of life" in the field of electrical measurement:

(a) no artificial reference standard (see glossary) is truly permanent.
(b) errors in reproducing a prototype standard (see glossary) are materially greater than the errors arising in comparing two reference standards of the same nominal value.
(c) errors in calibrating a reference standard by an absolute measurement are usually even greater than those encountered in reproducing a prototype standard.
(d) electrical units germane (see glossary) to either the meter, gram, and second or the foot, grain, and second were very different in magnitude from the electrical quantities of engineering interest.

The committee's response to this situation set the pattern for all future developments. To secure point (3) they immediately stated that "the material relations between these units are, clearly, that a unit electromotive force maintained between two points of a conductor separated by the unit of resistance shall produce unit current, and that this current shall in the unit of time convey the unit quantity of electricity." Also to secure the advantage of point (2) they immediately recognized the immense value of the work of Gauss and Weber and set up basic absolute definitions initially germane to the meter, gram, and second. They also initiated a program of absolute measurements, first of resistance and much later of current. The "BA unit of resistance" which resulted in 1864 corresponded to a mercury column 1 sq mm in cross section and 104.8 cm long, and hence was about 0.986 ohm as we now know it. To meet point (1) they recognized a practical system purely for electrical quantities defined as decimal multiples of the MGS units which they first used. In more modern language, they chose 10^8 CGS electromagnetic units of electromotive force as the practical unit because it was approximately equal to that of the Daniell cell, and suggested the name volt for it. They chose 10^9 CGS electromagnetic units of resistance with the name ohm as the practical unit because it was approximately equal to the Siemens Unit defined by a column of mercury 1 m long and of 1 sq mm cross section. They thus could meet requirement (5) by specifying the proper length of such a column. To meet point (4) in spite of fact (a) they initiated a program of studies on the stability of the resistance of alloys. In recognition of (c) and (b) they constructed a considerable number of standard resistors of the best known construction adjusted as closely as feasible to their "BA unit;" and distributed them internationally and by sale to the public. (Faraday in 1865 was their first paying customer).

Another BA Committee on "The Selection and Nomenclature of Dynamical and Electrical Units"

[1] Figures in brackets indicate the literature references at the end of this paper.

in 1873 decided to base theoretical definitions in both dynamics and electricity on the centimeter-gram-second (CGS) system rather than the meter-gram-second (MGS) system, mainly because in the former the density of water is substantially unity. It also urged the merits of the dynamical over the gravitational units in mechanics, thus making the gram primarily a unit of mass and not of force. They proposed the names dyne and erg and defined the horsepower as approximately "7.46 erg-nines [2] per second." The reporter of this committee, Professor J. D. Everrett published in 1875 a little book "Illustrations of the C.G.S. System of Units." This timely committee action gave such impetus to the CGS system that it has since come to have widespread application in all branches of science and engineering. It has almost met the pious hope of its originators that their selection should "be so made that there will be no subsequent necessity for amending it."

The year 1881 saw the first of a series of international electrical congresses (see sec. 10.2) which for the next quarter of a century served as forums for the discussion of nomenclature, units, etc., and as authorities for the approval and promulgation of those ideas which proved acceptable. The 1881 meeting in Paris approved of the basic status of the CGS units, and of the parallel practical set with the names ohm, volt, ampere, coulomb, and farad. It also set up a Commission which in 1884 recommended a legal ohm defined by a prototype mercury column 106 cm long and 1 sq mm in cross section at 0 °C (i.e., approximately 0.9973 ohm).

By 1893 the 4th International Electrical Congress at Chicago was able to crystallize the situation further by defining the ohm, ampere, and volt in terms of both the decimal multiples of the CGS electromagnetic units and also in terms of prototype standards. It passed a series of resolutions addressed to the various governments represented, urging them to "formally adopt [them] as legal units of electrical measure." The prototype for the ohm was lengthened to 106.3 cm (equivalent to about 1.000 5 ohm). The prototype for the volt was the Clark Zn-Hg cell to which was then assigned the value 1.434 v.

The 6th International Electrical Congress in St. Louis in 1904 recognized the distinction between two aspects of these developments. On the one hand, there was an overriding necessity for prompt official and universal conformity in the sizes of the units used in commercial measurements. This could best be secured by cooperative governmental actions. On the other hand, the improvement and invention of new and more useful nomenclatures and concepts could best be fostered by providing a forum where they could be discussed freely and by which the best usages could be recognized and coordinated. Accordingly two separate resolutions were passed suggesting these two parallel lines of progress. In sequel the first led through several intermediate steps to the inclusion in 1921 of electrical units in the scope of the International Committee on Weights and Measures (ICWM) (see glossary), while the second led to the organization during the next few years on a permanent basis of the International Electrotechnical Commission (I.E.C.).

The next major step in the first line of progress was the International Conference on Electrical Units and Standards at London in 1908, attended by official delegates from 24 countries. It recognized the basic importance of the CGS systems of units and their decimal multiples but also recognized that their experimental realization by absolute measurement could not then be attained with the accuracies desirable for much engineering work. It therefore recommended as representing these and "sufficiently near to them to be adopted for the purposes of electrical measurements and as a basis for legislation" a separate system of "International Electrical Units." The International Ohm defined as the resistance, at 0 °C, of a column of mercury 106.300 cm long and weighing 14.4521 g, and the International Ampere defined as the current which would deposit silver from an aqueous solution of silver nitrate at a rate of 0.00111800 g/sec were basic units of this system. In 1910 delegates from the British, German, and French national laboratories met at the Bureau of Standards in Washington and experimentally intercompared their respective national standard cells and resistors. From the results, values on a unified basis were assigned to the various standards and the units then arrived at were maintained [44, 46] as closely as practicable, disseminated throughout the civilized world, and used in commerce, industry, and science until January 1, 1948.

Pursuant further to the first St. Louis resolution, the 6th International Conference on Weights and Measures in 1921 voted to amend the Convention of the Meter (of 1875)[3] to assume authority over electric and photometric units.

In 1927 an Advisory Committee on Electricity was established to advise the International Committee on electrical problems, and the facilities at the International Bureau were enlarged to enable them to make precise comparisons of electrical standards. Since 1931 (except in time of war) the International Bureau has made intercomparisons of standard cells and standard resistors submitted periodically by the various national laboratories. This enables each laboratory to know how its units as maintained compare with those of the other nations, and to make adjustments on the rare occasions when such may become necessary to restore uniformity.

By 1928 many experimenters felt that the situation had changed since 1908. The availability of calibration services from national laboratories had eliminated the need for convenient reproducibility in protoptyes (desideratum (5)). Also, experience had

[2] See p. 156, footnote 13.

[3] This multi-lateral international treaty established a self-perpetuating International Committee on Weights and Measures consisting of 18 scientists appointed by reason of their individual competence but with the proviso that only one member be appointed from any one nation. This Committee supervises the work of the International Bureau of Weights and Measures which occupies laboratories on a plot of internationalized territory in Sèvres near Paris. The operations of the Committee are reviewed and given formal approval by an International Conference on Weights and Measures which normally meets every six years and on which all nations signatory to the Convention of the Meter are represented.

shown that with modern techniques "fact (c)" was no longer true [4] and that the errors in the absolute measurement of resistance and current probably did not then exceed those of reproducing the units by using the prototype standards. By 1933 the 8th General Conference approved in principle the change back to absolute units and authorized the International Committee to proceed as fast as reliable data became available. World War II, however, intervened and it was not until October 1946 that the International Committee voted to make the change effective January 1, 1948 [43].

Since January 1, 1948, the various national laboratories have continued to maintain their units by groups of standard resistors and standard cells with very satisfactory results, on the newly assigned basis, and with the expectation of occasional revision in the basis as better absolute determinations become available. The comparisons of 1957 at the International Bureau after the lapse of almost a decade showed that the units as maintained in Germany, the United States, France, Canada, Great Britain, Japan, and Russia (see glossary) all still lie within a range of a few microvolts and microhms.

The responsibility for the standardization of definitions and nomenclature covered by the second resolution of the St. Louis Congress has been borne mainly by the IEC. The work has proceeded since 1904 at a necessarily more leisurely tempo and with less precise discussions of detail. The two classic CGS electrostatic and electromagnetic systems sufficed for Maxwell's immortal Treatise of 1873, but long before 1904 a number of improvements had been proposed.

In 1882 Heaviside had complained of the presence of a factor "4π" in many formulas as due to an unwise definition of the unit magnetic pole and in 1891 [111, 112] he initiated a vigorous campaign for the use of what he called a more "rational" system (see sec. 8). His theoretically very elegant remedy would have involved changing the legalized units by factors involving "$\sqrt{4\pi}$" and therefore proved unacceptable to the practical engineer. Alternative partial systems which avoided changes in the units of voltage, current, or resistance, but at the expense of changing the simple choice of unity for the permeability of space were suggested, by Perry, Baily, Flemming, Fessenden, and others [113, 114, 115, 116]. Kennelly has used the adjective "subrationalized" to denote such schemes.

Another improvement on the CGS system which results in a desirable symmetry in the coefficients of electric and magnetic quantities is usually called the "Gaussian" system and was used by Foppl in 1894.[5]

In theoretical developments it is often desirable to express the dimensions (see sec. 7) of electrical quantities in terms of four basic dimensions rather than three.

Consideration of these possibilities led Giorgi to offer, initially in 1901 [51, 53], a "package deal" in the form of the MKS system. This gives rationalization, symmetry, 4 basic units (to which dimensions can be assigned), freedom from memorizing a large number of decimal exponents 10^9, 10^8, 10^{-1} etc., and the possibility of a single system applicable to all branches of science while retaining the firmly entrenched practical electrical units (ohm, volt, . . .). Unfortunately the cost of the package includes using germane units of density and permeability in terms of which water has a density of 1,000 and air a magnetic permeability of $4\pi\cdot10^{-7}$. This major proposal naturally stimulated a great deal of discussion and during the ensuing half century received a gradually increasingly favorable response, primarily in the field of electricity.

At its 1930 Oslo meeting the IEC indulged in a very protracted discussion, apparently resulting from a confusion between the "dimensions" of mathematical variables and the inherent "kinds" of physical quantities. It ended by voting that B and H are different in nature and that Γ_m (see glossary), the "permeability of space," has physical dimensions. In 1935 it voted "that the system with four fundamental units, comprising the three units: metre, kilogramme, second and a fourth fundamental unit to be chosen later be adopted under the name Giorgi system." In 1938 the IEC recommended "as the connecting link between the electrical and mechanical units, the permeability of free space with the value $\mu_0=10^{-7}$ in the unrationalized system, or $\mu_0=4\pi\cdot10^{-7}$ in the rationalized system."

In 1950 the IEC took the final step and recommended the use of the MKS system with the equations in their rationalized form as suggested by Giorgi. It also resolved "that for the purpose of developing the definitions of the units the fourth principal unit should preferably be the ampere."

The 1950 action of the IEC served to trigger off a further protracted discussion on the proper interpretation of rationalization. The Symbols, Units, and Nomenclature (SUN) (see glossary) Committee of the International Union of Pure and Applied Physics (IUPAP), consisting as it does largely of theoreticians, promptly (1951) voted that "in the case that the equations are rationalized, the rationalization should be effected by the introduction of new quantities" [8]. In the IEC, however, the experimentalists who prefer to change units are also represented and long arguments in Philadelphia (1954) [11], Opatija (1956), Stockholm (1958), and Madrid (1959) have failed to bring agreement. It is the hope of the author that this paper may contribute to the reconciliation of the two groups.

3. Fundamental Philosophies

The quantitative development of electromagnetism, like that of any other branch of science, has been marked by the interaction of two distinct, though complementary, kinds of work: experimental operations in the laboratory and theoretical studies applying mathematical reasoning. The interplay

[4] It is interesting to note that "fact (c)" still holds for temperature measurement and that the theoretical Kelvin Thermodynamic Scale still has to be supplemented by the more reproducible International Practical Temperature Scale.

[5] Lorentz [52] refers to this as "associated with Gauss, Helmholtz, and Hertz."

between these processes has been very close and has proved very fruitful. The experimentalist has hit upon new phenomena and recognized the need for new concepts in terms of which to describe them. The theoretician thus stimulated has sharpened the definitions of his concepts, discovered possible relations between them, and suggested further experiments to confirm and extend such predicted relations. During the development of the science each type of worker has evolved an ever more useful and powerful set of tools both in the form of laboratory apparatus and of mathematical methods. In this process even the basic concepts have been modified, not only by the inclusion of new ones, but also by changes in the definitions of certain old ones. A major step suggested long ago by Heaviside but only recently receiving official recognition in this evolution is called "rationalization" and involves the deliberate changing of the coefficients conventionally used in certain equations of electromagnetism. Unfortunately it is often described by the misleading phrase "use of rationalized units." It is this step which has brought into prominence a situation which has existed throughout the development of the science but which has hitherto been safely disregarded. This situation is that the experimenter and the theoretician, in spite of their effective cooperation, have each developed his own specialized nomenclature which is different in some of its connotations from that of the other although he uses the same words. To explain the semantic situation more clearly the following sections will expand in more detail the two distinct points of view and their resulting connotations.

To apply the power of mathematics to any branch of science, the physical relationships involved are best put into the form of equations. There are two ways of doing this.

The first way starts with measurement. The natural phenomena are conceived as describable in terms of a number of definable and measurable physical quantities. These taken together constitute what may be called a physical model of Nature. A particular sample of each kind of quantity (see glossary) involved in the phenomena under study is selected arbitrarily as a physical unit. Operations are developed by which other examples of the same kind of physical quantity can be compared with the physical unit. The result of this operation is a number called the "measure" or the "numerical value" of the physical quantity in terms of the physical unit. The numbers thus obtained by measurement are then written into equations which express the way in which the measures of certain dependent variable physical quantities depend on the measures of other independently controlled physical quantities. By the algebraic manipulation and combination of such measure equations a complete science can be built up.

The second way is to construct a mathematical model which has a certain correspondence at many points with the phenomena studied. The model consists of a number of kinds of mathematical elements which will here be called "symbolic quantities" (see glossary). One element of each kind is assigned a measure 1 and called a "symbolic unit." The equations relating these symbolic quantities in general look like and correspond to the measure equations obtained in the first way, but the letter symbols in the equations represent the symbolic quantities themselves. Such equations are called "quantity equations" and have much to offer in mathematical elegance and convenience.

At first sight there appears to be little difference between these two ways of introducing mathematics. In any one system of units and equations, the relation between each symbolic quantity and unit of the mathematical model and the corresponding physical quantity and unit in the physical system being studied is indeed very close. As a result both the physical quantity and its mathematical model are customarily given the same name (e.g., "electric current") and their units are given the same name (e.g., "ampere"). In a great many circumstances there is no occasion to distinguish between them. However, when, as in this paper, one is concerned with more than one set of equations or of units, the correspondence between the model and the reality is in general different for the different models. Failure to distinguish between the mathematical model and the physical model in such cases has been the basis of a great deal of confusion and misunderstanding.

For this reason in this paper the distinction between the two "levels of abstraction" will be carried to an extreme, and probably unnecessary, extent by the frequent insertion of the adjectives "physical" or "symbolic" (see glossary) to designate respectively the actual physical quantity and its corresponding element in the mathematical model. Also following König [88], who early realized this basic cause of confusion, the words "Realist" and "Synthetiker" (see glossary) will be used to emphasize the distinction in the two philosophies. The words as here used represent the extreme ends of the spectrum. Any living scientist or engineer thinks and speaks sometimes like a Realist and sometimes like a Synthetiker. No harm results even if he applies both types of thinking to the same problem, provided that at each instant he is aware of which type he is using. However, when he slips unconsciously from one type of thinking to the other or when two members of an international committee are simultaneously thinking in different types, then trouble is sure to develop.

The Realist who thinks only in terms of physical quantities and units and considers all his equations to be measure equations, in general exemplifies the operating engineer, tester of materials, writer of specifications, metrologist, laboratory experimenter, or measurer of the constants of nature. The Synthetiker who thinks only in terms of symbolic quantities and units and considers all his equations to be quantity equations, in general exemplifies the college professor, textbook writer, or theoretical physicist.

It is interesting to contrast the backgrounds and motivations of the men who hold these contrasting points of view. The Realists deal experimentally with energized electrical apparatus in the laboratory, the powerhouse, and the industrial plant. Through long familiarity they come to attribute to properties like current, inductance, magnetic field strength as much reality as to their machinery and raw materials. They quite overlook the fact that these electrical quantities are in truth only artificial concepts invented for convenience in describing the natural phenomena concerned. They must deal with a wide range of magnitudes, from microvolts to megavolts, and find it convenient to use a plurality of non-germane physical units in expressing their measured results. Also, in English-speaking countries they must frequently shift results between the British and the Metric systems. Hence they very frequently apply the basic principle that "the measure is always inversely as the unit" and have come to regard it as fundamental in the science of measurement. They therefore cling to it not only when the change is (1) from one non-germane (see glossary) unit to another in the same system or (2) between the germane units of two systems in which the equations are identical but the basic units differ but also (3) even when the change involves a change in the coefficients in their measure equations.

As they are content to write only measure equations, they are quite willing to forego the use of quantity equations and the use of letter symbols to denote their physical quantities. These are small prices to pay for the universality of the principle that the measure is inversely as the unit, and for the comfort of thinking (albeit mistakenly) that they deal with "real" quantities.

In contrast, the Synthetikers realize that both they and the Realists are dealing with conceptual artifacts. With their mathematical background they readily conceive of their symbolic quantities as defined by the equations of the system. They seldom have any use for units, but when they do they recognize the neatness of a set of symbolic coherent units (see glossary), each defined merely by the dimensions (see glossary) of the quantity involved, together with a few basic symbolic units. They rarely use noncoherent units and rarely have occasion to translate a measure from one set of units to another. The sacrifice usually made of the universality of the inverse law relating measure to unit, is a very small price to pay for the elegance of the quantity-calculus with its complete independence of units.

As the Synthetiker group is the more articulate of the two and has already provided most of the literature on systems of electrical units, the arrangement of the present paper has been to give first the whole picture from the side of the Realist to illustrate how complete and effective his approach can be. Then, in the interest of fairness, the Synthetiker's side with its neat elegance is given as a climax.

4. Experimental Approach

In presenting the situation from the point of view of the Realist, it seems advisable first to review in some detail the language of the laboratory. Using the terms there defined, the logical basis of experimental measurement will then be sketched and illustrated, with a detailed digression to give an up-to-date picture of the current basis for electrical measurement. To demonstrate the basic logic of the Realist, his process for establishing physical laws by purely experimental methods is then illustrated. In tables 1, 2 and 3 (sec. 10.1) the overall results of such operations are formally tabulated. Certain warnings as to the mathematical handling of a Realist's results are followed by an outline of the Realist's process for deriving formal definitions for any of his germane systems of measurements.

4.1. Nomenclature of Units and Standards

Before outlining the point of view of the Realist, let us first review the vocabulary he uses to describe his operations by using words such as those italicized in the following paragraphs. He thinks of a *physical quantity* as an example of a measurable (and therefore definable to some desired degree of precision) physical property which possesses the attribute of *magnitude* as well as of *kind*. The *unit* ("physical" in our nomenclature) is a sample of a physical quantity selected arbitrarily, but usually not capriciously, for the purpose of measuring other physical quantities of the same kind. *Measurement* is the act of comparing the magnitude of the *measurand* (the physical quantity the magnitude of which is to be measured) with the magnitude of the unit. The number resulting from this act is the *measure* (or *numerical value*) of the measurand *in terms of* the unit and is always a numeric.

A *physical standard* is a physical system of such a nature that it embodies in definite and usually convenient form one or more examples of one kind of physical quantity, and to which a *value* (or values) has been assigned to indicate the measure of the embodied quantity in terms of some appropriate specified unit.

For any given physical quantity there is usually a large assortment of different units. This situation is the result of many factors including convenience, historical accident, the particulate nature of matter, and especially the numerous different attempts which have been made to secure the advantages which result from the existence of simple systematic relations among the units of different kinds of quantities. In the various proposed logical systems of measurement, the units of a few quantities are selected as *basic* units and defined in terms of artificial or natural standards. The units of the remaining quantities are called *derived* units and are defined by operational procedures by which a value in terms of each new derived unit is assigned to each standard embodying one of the remaining quantities.

Thus as units of electric charge we have among others the statcoulomb (esu), the coulomb, the abcoulomb (emu), the millicoulomb, the electronic charge, the faraday, and the ampere-hour. The relations between the magnitudes of these units are known in some cases by definition and in others as the result of experiment. For each unit there is one *ideal* magnitude fixed by reference to the definition of the unit. However, in actual laboratory operations this ideal is approached only asymptotically as experimental methods are refined. One must therefore recognize the existence at any particular time in any given laboratory of a *unit as maintained* in that laboratory at that time, which in general is not exactly equal to the ideal. Thus, in 1950, the magnitude of the ohm as maintained by the British National Physical Laboratory was smaller by 2.2 μohm than the magnitude of the ohm as maintained at the U.S. National Bureau of Standards. The resistors used in the comparisons between the two laboratories showed no difference as great as 0.1 μohm between their values before and after their two crossings of the Atlantic for the comparison. Hence the observed difference in the two units is probably real, but who is to say which magnitude is closer to the ideal ohm? The units as maintained at some small laboratory in a university or industrial factory may well depart much more widely from the ideal. In addition to the differences resulting from the unknown and unavoidable inaccuracies in measurement, other, and usually larger, differences in units have been produced on certain occasions by the formal actions of international standardizing bodies. An example is the decrease in the ideal magnitude of the ohm by 490 μohm effected January 1, 1948 on the recommendation of the General Conference on Weights and Measures. Such deliberate changes are made only at relatively long intervals and are usually signalled by a change in an adjective in the formal name of the unit. Thus the "Legal Ohm" of 1884 was followed in 1893 by the "Ohm," in 1908 by the "International Ohm," and in 1948 by the "(Absolute) Ohm."

The word *standard* is also used with a variety of meanings both as a noun and as an adjective. Its use as a noun to designate a physical standard (as distinct from printed standards of practice or of safety) should preferably be limited to physical objects or systems which are used or intended for use in the definition or maintenance of a unit and for the calibration of other instruments or measuring devices in terms of that unit. A shop or laboratory instrument, even though of very high accuracy, if used in everyday operations to measure physical quantities should not be designated a *standard.* However a measuring device may compare an unknown measurand with some known quantity in a physical system which temporarily *serves as a standard.* Also a term like *standard resistor* is preferable to *standard of resistance* or *resistance standard,* becuase it stresses the fact that the complete physical structure (alloy wire, terminals, supports, etc.) is meant.

Physical standards are used for a variety of purposes and a correspondingly large variety of adjectives are applied to the noun *standard* to describe these uses. The adjective *prototype* designates members of that very small group of standards which serve to define the basic units of a system of measurement (see glossary). On the assumption that the whole world now uses only measuring systems based on the "International System of Units" [6] fixed in 1958 by the International Committee for Weights and Measures, there currently exist prototype standards for only 5 kinds of quantity. These include one individual artifact, the International Kilogram preserved at Sevres, to the mass of which is assigned the value 1 kg in the International System; the wavelength *in vacuo* of the orange-red line of krypton 86, to which is assigned the value 1/1,650,763.73 m; the tropical year of the earth-sun system, to which for 12h Ephemeris Time of January 0, 1900 is assigned the value 31,556,925.9747 sec; the temperature of the triple point of water, to which is assigned the value 273.16 °K; and the luminous intensity per square centimeter of a blackbody at the melting point of platinum, to which is assigned the value 60 candelas. There is obviously only one prototype standard each of mass and of time, while there are in existence as many prototype standards of length, temperature, and luminous intensity as may happen to be set up and used for standardizing purposes at any given time. Of course if some measurement laboratory is operating in such complete isolation that it is obliged to establish its units quite independently of the present group of cooperating national and international laboratories, the standards which define its basic units will also be properly designated as "prototypes." Huntoon and Fano [45] have suggested the possibility that all prototype standards may ultimately be selected properties of atoms or molecules rather than of macroscopic bodies.

It may be noted here that except for the special case of the prototype kilogram the *value* assigned to a standard need not be 1 unit and may be very different. Even when the standard is constructed with the intention that its *nominal value* shall be one unit and hence that it should embody a quantity the measure of which is exactly 1, errors in manufacture or subsequent changes usually cause its measure to depart slightly from unity. Of course, when the definition of a unit is changed, as in 1948, the *assigned values* of all standards of that kind should be

[6] Care must be taken to distinguish for example between (1) the "ampere" or "absolute ampere" introduced effectively Jan. 1, 1948, defined by an electromechanical experiment, and constituting one of the basic units of this SI (System Internationale) and (2) the older and now obsolete "International Ampere" defined by the London Conference of 1908 by means of the silver coulometer (see p. 161). A similar distinction is needed for the other electrical units.

changed to correspond to the new unit even though there has been no change in the *magnitude* of the quantity embodied by the standard. When the magnitude of the quantity embodied in a standard has been found to have drifted with time, the standard must be assigned a new value.

The process of making appropriate measurements on a standard on which to base a correct assignment of a value is called a *calibration* of the standard. All standards except prototype standards must be calibrated in some way.

This need has caused the development throughout the civilized world of a hierarchy of standardizing laboratories. Each national laboratory (see glossary) maintains a set of units by means of its *national standards*. Periodic intercomparisons at the International Bureau of Weights and Measures help coordinate the activities of the national laboratories and enable them to achieve close agreement among the electrical units as maintained by them. Within each nation other laboratories have their standards calibrated at the national laboratory and in turn use their standards to calibrate other standards and measuring equipment.

Within any one laboratory there is also a hierarchy of standards. The highest in rank are preferably called "*reference standards*" and serve to maintain the corresponding unit in the laboratory. Calibrated by reference to them are the *working standards* which are regularly used to calibrate the *shop instruments* and measuring devices used in the everyday work of the main organization. Another category is that of *interlaboratory standards*, which are those sent periodically to the national laboratory or other source of high accuracy and which then serve to bring the magnitude of each unit to the given laboratory. In some cases some of the reference standards are used as interlaboratory standards, but in other cases it is best to spare the reference standards from the disturbances incident to transportation and to count on the statistical accumulation of data by the repeated round-trip shipments of a rugged interlaboratory standard to build up a high accuracy in the final assignment of a value to the undisturbed reference standard. The adjectives *travelling* (voyageur) and *sedentary* (sedentaire) are used by the International Bureau and sometimes by others to designate these two uses of standards.

In another category of standards are the *transfer* standards, which are of specialized construction so that under widely varying conditions of use they continue to meet the criteria required for defining the quantities which they embody; or, alternatively, experience only a definite and known change in value for which an accurate correction may be made. The most common example is the standard transfer wattmeter, which is so constructed that its deflection for a given active power is the same on alternating current as on direct current. Other examples are resistors which have the same resistance on alternating and on direct current; attenuators which can be calibrated by d-c resistance measurements and used to produce known attenuation in a-c circuits; and resistors capable of carrying very large currents.

Many standards embody only a single example of the quantity concerned and are called *single-valued;* examples are gage-blocks, standard cells, most standard resistors. Others embody a plurality of examples of quantities of the same kind and are called *multi-valued* standards. Examples are graduated scales, decade-type resistance boxes, or capacitors. Still other standards like continuously adjustable air capacitors or inductors may be set to embody any desired value of the quantity within their range with a precision limited only by the readability of their scale and mechanical imperfections in their construction. These are preferably called *continuously adjustable*[7] standards.

The word *standard* is also conveniently applied either as a noun or as an adjective to a class of usually more complex measuring devices often called *standard instruments* which are used in much the same way as simpler physical standards. Typical examples of such *instrumental standards* are thermometers, floating hydrometers, and electrical indicating instruments such as ammeters, voltmeters, wattmeters, etc. Like any other standard (prototypes excepted) they have to be calibrated by some operation higher in the hierarchy. By a slight extension of our concepts each can be said to embody a range of mangitudes of one kind of quantity. Thus when the ammeter is deflected to its 5-amp scale mark a current of 5 amp does then exist in its circuit. Similarly the hydrometer float embodies a definite mass and the measure of this mass if divided by the measure of the immersed volume equals the measure of the density of the liquid in which it floats. The thermometer indicates a particular temperature when its bulb embodies that temperature.

Physical standards of still another type consist of samples of particular materials which embody measurable properties to which definite values have been assigned. One subclass of this type consists of what may be called *standard reference materials*. Each such material embodies some physical quantity, not significantly dependent on its geometrical shape, which has been measured and which can therefore be used for the calibration of measuring devices. A standard reference sample of highly purified benzoic acid offers an almost unique example of a standard embodying three different kinds of quantity (1) its temperature of melting, (2) its heat of combustion, (3) its specific heat.

Examples in the electrical field are liquids of measured volume resistivity or dielectric constant. If such a standard reference material is used in a

[7] The use of "continuously variable" is to be deprecated as it implies the occurrence of variations which are not under the control of the operator.

test cell, a calibration factor is obtained by which the corresponding properties of other liquids used subsequently in the same cell can be computed. Other examples are bars or strips of ferromagnetic materials. The magnetic flux (usually expressed as a flux density using conventionally assumed cross-sectional dimensions) corresponding to a succession of accurately measured applied magnetizing forces is measured in one laboratory. The specimens are then used to verify the calibration of permeameters in other laboratories lower in the hierarchy. Standard reference materials are widely used as standards of viscosity, temperature, refractivity, and chemical composition.

A somewhat different subclass of standard materials, preferably called *standard ingredients* includes samples of substances prepared in large uniform batches for use as ingredients in other materials (e.g., standard fillers for rubber compounds) to eliminate certain manufacturing variables when studying the effects of others.

4.2. Measurement

Having established a vocabulary let us now develop the Realist's approach by considering his major operation which is measurement.

Measurement has been defined as "the assignment of numerals to represent properties in accordance with physical laws." In the present connection we are concerned with a somewhat more specialized operation which can establish what Stevens [3] has classified as a "ratio scale" for each measurable physical property.

To qualify as "measurable" a property must be recognized as having two aspects, both of which must be definite: first its particular physical nature (e.g., electric current, resistance, energy); and second its magnitude. This means that there is an experimental operation for determining quantitatively its relation as smaller than, equal to, or larger than other examples of the same kind of property and by what ratio. Because of this latter feature measurable physical properties are usually called "physical *quantities*."

To be measurable to a given degree of accuracy the physical quantity must first of all be identifiable by particular defining operations, of at least that accuracy which can discriminate between it and other similar but different phenomena. A major feature in the development of any branch of science is the successive recognition of such physical quantities and the continuing improvement in the scope and incisiveness of their definitions.

The process by which a particular concept has been successively refined is exemplified by the concept of electrical resistance. In a general way this was early recognized as that property of a part of an electric circuit by reason of which the current produced by a given voltage is limited in magnitude. In many cases the ratio of the measure of the voltage at the terminals of the circuit element to the measure of the resulting current in it was found to be substantially constant, over a very wide range of currents. This fact justifies the recognition of the ratio as a measurable physical quantity. It was christened *resistance* and circuit elements exhibiting this property prominently are called *resistors*. Further studies showed that the method of measurement should be limited to the use of unvarying current in order to separate out an extraneous effect which is now recognized as a quantity of a different kind called *reactance*. Later extensions of the concept of resistance restored the possibility of measurement using alternating current, provided observations based on phase relations served to discriminate between the a-c resistance and the reactance. The extension to radiation resistance has made the quantity a property of antennas as well as of resistors. To insure that the current resulted only from the applied voltage, procedures such as taking the mean of values before and after reversing the polarity were specified and an additional new concept of internal parasitic emf (electrochemical, thermoelectric, etc.) was invented to complete the description. If the voltage used was so high that corona discharge caused the current to be different in different parts of the resistor, a further specification had to be included to bar observations under such conditions. When the current was so large as to change the temperature materially a similar limitation had to be imposed. This was usually expressed by stating that the measure of the resistance was defined as the limit of the ratio of the measure of the applied voltage to the measure of the resulting current as both approached zero. Even with these limitations, results may be found to be different at different ambient temperatures or with different conditions of mechanical strain. Therefore new additional concepts of temperature coefficient and strain coefficient have to be included in the picture to preserve the desired definiteness of the concept of resistance.

In addition to the basic requirement of definite identifiability just discussed, many physical quantities possess the further useful attribute which we may call *additivity* (see glossary). This permits their use in the direct establishment of a ratio scale. Addivity means that if two examples of the quantity are properly combined the measure of the resultant in any unit must equal the sum of their separate measures in that unit. Many physical quantities have the attribute of additivity. For the simple concept of length the existence of this attribute is almost intuitive, provided that the combination rule is to put the components end to end in the same straight line. For volumes of liquid the rule involves pouring the contents of small containers into a larger one, and must be limited by a clause that no mutual solution of miscible liquids is permitted. Similar additivity is found, for example, in non-inductive resistances connected in series, in direct currents toward (or from) a branch point, and in direct voltages in a series circuit.

For any additive physical quantity, if there is also available some of indicator or detector which can show, with the needed sensitivity, whether or not two examples of the quantity are equal, and if not which is the greater, then it is possible to construct a ratio scale for that kind of physical quantity.

This procedure can best be understood by again considering a particular example, say electrical resistance. For the detector we use the classic Wheatstone bridge circuit with ratio arms A and B across the battery, an adjustable but uncalibrated arm R_1, and a fourth arm X. Here the letters serve merely to identify the four examples of resistance embodied in the four resistors. For simplicity let us assume that resistance B has been adjusted so that the bridge remains balanced when A and B are interchanged. Then if the galvanometer shows a balance it insures (1) that the current in X is the same as the current in R_1 and (2) that the voltage drop in X is equal to that in R_1 (and in A and in B also), and hence that the resistances X and R_1 are of equal magnitude. Initially, of course, no numerical values have been assigned to any of the resistances and the scale on which the adjustable contactor of R_1 moves is unmarked. Now, with the galvanometer on balance, mark the contactor position "x". Replace X with another resistor Y and adjust Y until the bridge is again balanced, thus making the magnitude of Y equal to that of X. Connect X and Y in series in the X arm, and restore the balance by sliding the contactor to a new position R_2. Mark the new position "$2x$". By repetitions of this process a true scale of resistance, in which the resistance of X serves as a temporary unit, can be laid out on R. It should be noted that nothing has been said as to the linearity or otherwise of the resulting spacing of the marks along R. It is necessary merely that for each marked setting the resistance of the arm shall be definite and reproducible enough for use in the applications of the scale in future measurements. With the scale of resistance once obtained it may be applied to the ratio arms of the Kelvin double bridge and thus extended to low values of resistance defined by resistors of four-terminal construction.

When a scale of say 10 equal steps has been established, the total resistance, $10x$, can be used as the basis for building up a second decade the elements of which each have magnitude 10 times those of the preceding decade. The combination of n such decades in series yields a multivalued standard resistor having 10^n discrete values. Assuming the individual elements to have adequate stability this yields a scale precise to 1 part in 10^n (see sec. 6). It then remains only to assign arbitrarily to the resistance x a permanent numerical value to fix the unit of resistance. A consideration of the factors involved in such arbitrary assignments throughout the field of electromagnetics is a major purpose of this paper.

Of course, an additive scale could be established with a minimum of operations but with less convenience by building up a series of components each having only two elements so that n components yield 2^n discrete values. For physical quantities such as voltage, mutual inductance, or mass (using an equal arm balance), which can be either added or subtracted, the scale need only contain powers of 3 (i.e., 1, 3, 9, . . . units). Such schemes require the adjustment of fewer components.

If an experimental situation can be set up in which some quantity for which an adequate ratio scale has been established can be made proportional by a known factor to some other physical quantity, which itself may not be additive, then the latter can be measured directly. A simple example is the measurement of the specific volume (which is not additive) of a liquid by the method of balancing columns. In this method the liquid to be measured and a standard liquid are placed in adjacent open containers. A long inverted U-tube is placed so that one open end is immersed in each liquid. Suction at the bend in the U draws up a column of each liquid. The heights of the columns are measured. Here by definition the specific volume is proportional to the height of liquid column supported by a given difference in pressure. The height for the same pressure difference of the column of standard liquid of known density fixes the factor of proportionality. The heights are directly measurable on the basic scale of length. This principle is the basis for the potentiometer and the voltage divider which measure voltages by use of the scale of resistance. The calibration of a direct reading indicating ammeter or voltmeter establishes a similar proportionality between the reading (not necessarily the deflection) and the current or voltage.

The measurement of a physical quantity by direct reference to its own appropriate ratio scale or some scale arranged to be proportional to it is called a *direct* or *comparative* measurement.

Other definable properties such, for instance, as density and resistivity do not have the attribute of additivity and it is sometimes not easy to set up a simple proportionality between them and some additive property. However, enough properties are additive so that the magnitudes of the other properties can be compared by the indirect process of measuring a plurality of component quantities in terms of which each non-additive quantity is defined and combining their individual measures in accordance with the definition of the new quantity to obtain the measure of the new quantity by what may be called an *indirect, derivational,* or *absolute* measurement. Thus measurement of the mass and volume of a body permits the computation of the measure of its density; measurements of resistance, length, and cross section yield a measure of resistivity, etc. Moreover, often a quantity like electric current, although it is additive, as combined at the branch points of a circuit, is in practice often measured indirectly by using the ratio scale built up by resistances in combination with a standard voltage.

Examples of indirect measurements are the measurement of energy in terms of current, voltage, and time; magnetic induction in terms of flux and area; capacitance in terms of resistance and frequency; inductance in terms of capacitance and resistance, etc. The adjective *absolute* is usually applied only to those operations in which a quantity is measured indirectly and in terms of the ultimate basic units (usually length, mass, and time) of the system of units used.

4.3. Present Experimental Basis for Electrical Units

Each national standardizing laboratory endeavors to maintain a set of electrical physical units which is constant in time and in agreement with the magnitudes recommended in 1946 by the International Committee on Weights and Measures [41]. This Committee had based its recommendation on a careful consideration of all available experimental data obtained by absolute measurements of resistance and of current in terms of the units of length, mass, and time and of the postulated value of $4\pi \ 10^{-7}$ for the magnetic constant Γ_m, in the rationalized MKSA system of measurement.

The basis for the maintenance of the electrical units at NBS involves the construction and preservation of a group of reference standards of the highest quality; the assumption that their secular drifts in magnitude tend on the average to cancel; and their periodic use in the precise measurement of some constant of nature as a check on possible drifts. (For more detail see [44] also.)

For the ohm there is used a group of about 20 standard resistors, made of annealled manganin wire, each mounted in a sealed container of the double-walled type [42]. These are stored in thermostated oil baths and are intercompared annually by a substitution method with a precision of 1 in 10^7. The resulting measures are examined on the assumption that the mean of the magnitudes of a subgroup of 10 of these standards has not changed since the preceding intercomparison. If any individual resistor of the 10 originally chosen for the subgroup is found to show a change considered large compared to those of its fellows, it is rejected and another member of the larger group is used to carry the unit forward. If the measure of each resistor differs from the mean of the subgroup by about the same amount as at the previous intercomparison, a new value is then assigned to each standard in the group. In this new assignment the mean resistance of the ten resistors is assumed to be the same as it was at the preceding intercomparison. The newly assigned value for each individual resistor then differs from this mean by the newly measured amount.

The volt is maintained in much the same way by using a group of 40 cadmium standard cells of the saturated type. Cells of several different forms (i.e., acid and neutral) are included in the group. Comparisons are made to 0.1 μv.

It is seen that the primary reason for expecting the standards and the units based on them to remain constant is merely the simple assumption that examples of these particular physical systems (i.e., pieces of alloy wire, and electrochemical cells) if stored under reasonably constant conditions will not change their physical properties. The basis for confidence in this assumption is found in the reasonably satisfactory, though far from perfect, record of comparisons of groups of such systems during the past half century [44]. This record, as derived from international intercomparisons among the six cooperating national laboratories, between 1910 and 1948 shows that after the lapse of about 20 years the standard resistors of two laboratories had drifted by about 30 μ ohm and had increased by this amount the units they were maintaining. These laboratories then assigned new values to their standards to recover the old unit. Similarly, after 25 years two laboratories found it desirable to increase their volts by about 80 μv to restore their units. Since the reassignment of values for the national standards in 1948 the performance has been better. In 1957, almost a decade after they had been reassigned values on a uniform basis, the units both of resistance and of emf of the six national laboratories compared at the International Bureau of Weights and Measures fell within a range of $\pm 6 \cdot 10^{-6}$ from the mean of all.

To obtain an independent alternative basis for maintaining the electrical units over long intervals, two types of project are currently under way at NBS. The first is to redetermine at desired intervals some "constant of nature" in terms of the units as maintained. If the same measure is obtained at each later periodic redetermination, it gives a strong confirmation that the units have not changed during the interval. Two such constants are the gyromagnetic ratio of the proton and the electrochemical equivalent of silver. The first [47] invloves primarily the measurement of electric current, frequency, and the pitch of a winding on a single-layer solenoid. The second [48] involves primarily the measurement of electric current, time interval, and the mass of the electrochemically corroded silver. Frequency and time can be measured with ample accuracy. The other variables in the gyromagnetic experiment may introduce a random uncertainty of 1 or 2 in 10^6. Although in a determination of the gyromagnetic ratio possible systematic errors may exceed this estimate, a repetition of the experiment under the same conditions after the lapse of 10 years would suffice nevertheless to detect a steady drift in the groups of standards equivalent to only 2 in 10^7 per year. The electrochemical experiment is not quite as reproducible (perhaps by a factor of 4) but offers an entirely independent and therefore very valuable backstop to detect drifts. The precise measurement of current in either of these experiments involves the standards for both the ohm and the volt. It is possible but very improbable that separate drifts in the magnitudes of the two types of standard should be such as to compensate exactly.

Basic projects of the second type include the absolute measurement of resistance and of current. These projects have two objectives. First the determination of any difference which may exist between the unit as maintained by the national reference standards and the ideal absolute unit. The second objective is to detect any change in the unit as maintained since the previous absolute measurement. The accuracy with which the first objective can be attained is currently perhaps not much better than 10 in 10^6, largely because of the possible presence of systematic errors which are not eliminated by using detectors of extreme sensitivity nor by accumulating data through many repetitive observations. However, to the extent that such systematic errors remain truly constant from one use of the apparatus to a subsequent use, they do not limit the accuracy in attaining the second objective. The ability to repeat an absolute measurement after a lapse of 10 years may be as high as 1 in 10^6.

Since the International Committee on Weights and Measures made its decision in 1946 on the recommended values of the units, a number of additional absolute measurements have been made. For the ampere the only recent work published is that by Driscoll and coworkers at NBS [28, 29]. When using a current balance and measuring the force between coaxial single-layer helical coils, they obtained in 1957 0.999992 as the measure of 1 absolute ampere in terms of the volt and the ohm as currently maintained at NBS. When using an electrodynamometer of the Pellat type and measuring the torque between two concentric single-layer helical coils with their axes at right angles, Driscoll obtained in 1957 0.999987 for the NBS measure of 1 absolute ampere. The agreement between the two methods is very gratifying, because it is unlikely that many sources of systematic error would be present to an equal extent in both of two pieces of apparatus which are so different mechanically. However, one source of uncertainty is common to both, namely, the local value of gravity, g. The measures here given are based on Dryden's [22] estimate from his revision of the Potsdam data.

More work has been done on the ohm. In 1949 Thomas, Peterson, Cooter, and Kotter [23] using the Wenner method obtained 1.000006 ± 0.000010 as the measure of an absolute ohm in terms of the unit preserved at NBS with 1-ohm standards since January 1, 1948. In this measurement the biggest single source of error was probably the uncertainty in the distribution of current in the primary winding of the mutual inductor. Current distribution is affected by resistivity-stress relationship in the copper wire. The current-distribution correction used in 1949 was based on resistivity-stress studies made by Kotter in 1940. Later studies made by Wells in 1956 [26] gave additional data which, had they been available in 1949, would have resulted in a value of 1.000003 for the measure of the absolute ohm in terms of the unit maintained at NBS. During the decade 1950–1960 the latter unit agreed with the unit maintained by the International Bureau of Weights and Measures within 1 μohm. In 1953 Rayner [24] of the British National Physical Laboratory, using the Campbell method, reported 0.999996 ± 0.00008 for the measure of an absolute ohm when reduced to the international basis. In 1957 Romanowski and Olson [27] of the National Research Council of Canada reported a result equivalent to 1.000003 ± 0.000020 for the measure of the absolute ohms in terms of the units of the International Bureau.

In 1956 Thompson and Lampard of the Australian National Standards Laboratory discovered a new theorem in electrostatics [25] which can be applied to the computation with very high accuracy of the capacitance of small 3-terminal capacitors. Cutkosky [31] in 1960 completed a measurement using such a capacitor and obtained 0.999997_7 for the measure of the absolute ohm in terms of the unit then maintained at NBS. If there has been no relative drift between the units of NBS and of the International Bureau, this means a measure of 0.999998_7 on the international basis. Cutkosky's method involves stepping up in 4 decimal stages from 1 pf to 0.01 μf; the comparison at that level and at 1,592 c/s ($\omega = 10^4$ radian/sec) of the admittances of a pair of capacitors with the conductances of a pair of 10,000-ohm resistors; and the further stepping down in 4 more decimal stages to 1 ohm. Nevertheless the extreme simplicity of the computable capacitor and the simple self-checking features available in the 10:1 steps limited the uncertainty to ± 3 in 10^6 (50 percent confidence interval). To this estimate an uncertainty of ± 1 in 10^6 in the speed of light makes a significant contribution. This method evidently constitutes a significant "breakthrough" in the field of absolute electrical measurement.

It is of course the intent of the International Committee on Weights and Measures to keep the electrical units as close as practicable to their ideal values as defined. The adjustment of 1948 must be considered as merely the latest in a series of such adjustments which began with the change from the BA unit to the legal ohm in 1884. The adjustment of 1948 seems to have been chosen very wisely. In fact, the data just quoted suggest that no further change is to be expected for a long time. That is until (a) a materially more accurate value for g becomes available and (b) measuring techniques in science and industry increase materially in their requirements for accuracy and (c) some further revolutionary increases in the accuracy in absolute measurements become attainable which will reduce the present limits of uncertainty materially below the small apparent discrepancies between the ideal and the maintained values. The standardizing laboratory is still confronted with the "fact of life" (b) (p. 138), namely that simple comparison methods will always outstrip absolute methods in accuracy.

4.4. Experimental Establishment of Physical Laws

The modern student, in a world well supplied with calibrated apparatus and recognized systems of measurement, naturally considers experimental research to involve the operations of making measurements on unknown quantities and then expressing their relations to known quantities by appropriate equations. However, a perusal of the writings of the earlier classical workers in any field shows instead that their results were usually stated merely as proportionalities. Thus Newton wrote: "The alteration of motion is ever proportional to the motive force impressed"; Coulomb: "The repulsive force . . . is in the inverse ratio of the square of the distances"; Faraday: "The chemical power of a current of electricity is in direct proportion to the absolute quantity of electricity which passes." The writing of a measure equation or a quantity equation always involves an additional conventional operation.

The nature of these steps by which physical laws are discovered and demonstrated experimentally by the Realist as relations between the measures of physical quantities can perhaps best be understood by considering a couple of examples.

The first extremely simple case illustrates the basic principles and by its contrast with the usual theoretical procedure serves to emphasize the differences between the two philosophies. The second somewhat complex example is offered because it applies to the currently moot question of rationalization.

First let us consider the measurement of area. A Realist supplied with a scale for measuring length, graduated in any arbitrary equal intervals (say for example in inches and 16ths), and a sheet of cross-section paper of any mesh (say for example millimeters) could study the measurement of area as a purely empirical matter without regard to geometry. He would draw various geometric figures of various sizes and measure their dimensions in his scale units (say in inches). He would also count the number of squares of his cross-section paper enclosed by their perimeters. For each rectangle, triangle, circle, and regular hexagon, respectively, he would express his data by the experimental measure equations [8]

$$\{A_r\}_p = 645\{w\}_s\{l\}_s$$

$$\{A_t\}_p = 322\{b\}_s\{h\}_s$$

$$\{A_c\}_p = 2{,}026\{r\}_s^2$$

$$\{A_h\}_p = 2{,}657\{l\}_s^2 \qquad (4.4.1)$$

or in a more general literal form

$$\{A_n\}_p = K_{psn}\{a\}_s\{b\}_s. \qquad (4.4.2)$$

where a and b are appropriate orthogonal dimensions. Here the subscripts p and s denote the use of the arbitrary paper and scale units respectively, and the subscript n, which may take on the values r, t, c, or h, indicates the shape of the area measured.

He notes that the experimental coefficients are very nearly in the ratios:

$$K_{psr}:K_{pst}:K_{psc}:K_{psh}::1:\tfrac{1}{2}:\pi:3\frac{\sqrt{3}}{2}. \qquad (4.4.3)$$

The theorems of plane geometry derived independently by the Synthetiker also show that for these shapes the coefficients K_n would be in these same ratios. The usual textbook also goes on to state dogmatically "$A_r = wl$," thus making the additional tacit and arbitrary assumption that K_r for a rectangle (rather than K_t for a triangle or K_c for a circle) is to be set equal to unity or, in other words, that the unit of area shall be chosen as being equal to the area of a square which has each side of unit length. Our Realist following this suggestion can make his measure equation look like the Synthetiker's quantity equation $A_r = wl$ by arbitrarily choosing 645 of his preliminary square units, as the physical unit of area which is germane (see glossary) both to his physical unit of length, and to the geometric measure equations with their coefficients $K_{psr} = 1$, $K_{psc} = \pi$, etc. (i.e., if his scale unit were 1 in. he would find that his germane unit of area was the square inch).

In strict analogy to the foregoing consider now the more ambitious program of a Realist studying magnetism. He has both a graduated scale to measure lengths in a recognized unit, say the meter, and apparatus for measuring current in a recognized unit, say the ampere. Let a subscript a designate the use of a set of physical units germane to the meter, the ampere, and to the equations defining the ampere.

[8] The reader will appreciate that these particular numerical values will result if the units of the scale and paper happen to be those suggested parenthetically in the text.

The Realist also has several short magnetized needles each suspended by a silk fiber, and a stop watch by which he can measure, in cycles per second (i.e., also system a), the frequency of small oscillations of the needle. He observes both the rest position of the needle and its frequency of oscillation when displaced therefrom, when it is suspended in various definite locations near each of three systems of current-carrying conductors. These locations are (1) at a distance r from a long straight conductor; (2) at the center of each of a set of circles of radii r; and (3) at the center of one of several uniformly wound solenoids of pitch, r_s, and of such length that their open ends subtend an angle 2ϵ at the center. The subscripts l, c, and s respectively denote quantities pertaining to these three kinds of geometric arrangement of conductor.

In any one experiment he finds that the squares of the measures of the frequencies are proportional to the measures of the current. By analogy with a pendulum in the gravitational field of the earth he postulates the existence of a magnetic force field.

For each of the needles he plots the squares of the measures of the frequencies against the quotient of the measure of the current by the measure of the distance, radius, or pitch. He finds these graphs to be straight lines, the slopes of which he designates by S_{nal}, S_{nac}, and S_{nas} for the long wire, the circle, and the solenoid respectively. The subscript n here designates the particular needle and the subscript a indicates that the standard germane units of current and distance were used.

Hence he can write a set of equations of the form

$$\{f_{n,g}\}_a^2 = S_{nag}\{I_g\}_a/\{r_g\}_a \qquad (4.4.4)$$

where the subscript g indicates the possible substitution of l, c, or s to get the measure equation for any of the 3 geometries used. He finds that the slopes S_{nag} can be arranged in an array which has very nearly the form shown below where S denotes the slope found with the first needle and the long straight conductor, SM_2 that with the second needle, etc.

Needle, n / Geometry, g	1	2	. . .	n
Straight, l	$S\cdot 1\cdot 1$	$S\cdot M_2\cdot 1$ . . .	. . .	. . . $S\cdot M_n 1$
Circle, c	$S\cdot 1\cdot \pi$	$S\cdot M_2\cdot \pi$ . . .	. . .	. . . $S\cdot M_n\cdot \pi$
Solenoid, s	$S\cdot 1\cdot 2\pi\cos\epsilon$	$S\cdot M_2\cdot 2\pi\cos\epsilon$ . .	. . .	. . . $S\cdot M_n\cdot 2\pi\cos\epsilon$

(4.4.5)

This shows that for any needle, n, the slopes in a given column are in the ratios

$$S_{nal}:S_{nac}:S_{nas}::1:\pi:2\pi\cos\epsilon \qquad (4.4.6)$$

and for any geometry, g, the slopes in a given row are in the ratios

$$S_{1ag}:S_{2ag}\ \ldots\ S_{nag}::1:M_2:\ \ldots\ M_n. \qquad (4.4.7)$$

From these facts he infers that the measures of the squares of the frequency are proportional to both the measure of a new physical quantity, M, which depends only on the needle, and to the measure of a new physical quantity, H, which depends only on the geometry and size of the circuit and the magnitude of the current. He can factor each slope into a constant S (i.e., the first member), a part K_n which depends only on the needle, and a part K_g which depends only on the geometry, thus getting for any slope

$$S_{nag} = S\cdot K_n\cdot K_g. \qquad (4.4.8)$$

Combining eq (4.4.8) with eq (4.4.4) gives the set of equations

$$\{f_{n,g}\}_a^2/K_n = SK_g\{I_g\}_a/\{r_g\}_a. \qquad (4.4.9)$$

The right member of each of the eq (4.4.9), although it involves the constant S and hence the strength of one needle, is independent of n. Hence the left member must be also independent. Therefore each left member can be considered as an appropriate measure $\{H_g\}_p$ of the new physical quantity H which depends on g, in terms of a preliminary physical unit, ${}_pU_H$. The operational definition for measuring the physical quantity H is that, when the oscillation frequency is 4 times as high, H is to be considered 2 times as large. Also the preliminary physical unit of H is the sample of the physical quantity H existing at 1 m from the straight wire when the measure of the current is $1/S$ amp. The use of this preliminary unit gives the set of 3 measure equations

$$\begin{aligned}\{H_l\}_p &= K_{pal}\{I_l\}_a/\{r_l\}_a\\ \{H_c\}_p &= K_{pac}\{I_c\}_a/\{r_c\}_a\\ \{H_s\}_p &= K_{pas}\{I_s\}_a/\{r_s\}_a\end{aligned} \qquad (4.4.10)$$

where

$$K_{pal} = S$$

$$K_{pac} = \pi S$$

$$K_{pas} = 2\pi S\cos\epsilon.$$

We need not follow the Realist further in his study of the needles or the relation of M with their magnetic moments and moments of inertia. Instead we see that his colleague the Synthetiker from eq (4.4.10) is led to recognize the more general law of Ampere [9]

$$d\{H_A\}_p = K_{paA}\frac{\{I_A\}_a d\{l_A\}_a}{\{r_A\}_a^2}\sin\theta \qquad (4.4.11)$$

where

$$K_{paA} = S/2. \qquad (4.4.12)$$

[9] Here, of course, the geometry subscript "A" designates that the current I_A is in an elementary length dl_A at a distance r_A from the point at which the field strength dl_A is measured, and θ is the angle between the directions of r and dl_A.

From this he can proceed to deduce equations appropriate to still other geometrical arrangements. For example the measure of the magnetic field strength $\{H_h\}_p$ at the center of a hexagon of side h will be

$$\{H_h\}_p = K_{pah}\{I_h\}_a/\{h\}_a \tag{4.4.13}$$

provided

$$K_{pah} = 6S/\sqrt{3}. \tag{4.4.14}$$

The Realist's next step is to eliminate the individualistic factor S due to his preliminary unit for H symbolized by the subscripts p. He does this by (1) choosing some readily described geometry which we may designate by d and for which he writes

$$\{H_d\}_a = K_{ad}\{I_d\}_a/\{r_d\}_a \tag{4.4.15}$$

and (2) assigning some simple coefficient K_{ad} to this particular geometry. He then (3) derives a new physical unit $_aU_H$ germane to the basic units of system a and to the chosen coefficient K_{ad}. This means that

$$_aU_H = {_pU_H} \cdot \frac{K_{pad}}{K_{ad}}. \tag{4.4.16}$$

This step reduces the data obtained with the uncalibrated needles and the temporary preliminary unit $_pU_H$ to measures based on a single physical unit $_aU_H$, which is germane to the more basic units $_aU_I$ and $_aU_l$ and to the simple arbitrary coefficient K_{ad}[10].

It should be noted here, however, that the change in units indicated by eq (4.4.16) is not the only possible procedure for obtaining the desired value of K_{ad}. An alternative would have been to consider that the revised measure equation (4.4.15) and the desired coefficient K_{ad} gave the measure in the old provisional unit $_pU_H$ of a newly conceived physical quantity H' of the same kind as H but of a magnitude related to the physical quantity H as formerly conceived by the relation

$$\{H'\} = \frac{K_{ad}}{K_{pad}}\{H\}. \tag{4.4.17}$$

This alternative usually seems repugnant to the Realist.

It is by processes of the general nature here illustrated that the Realists have built up the whole discipline of electromagnetics into a collection of measure equations. Table 1 (see sec. 10.1 for all tables) lists a number of these equatons in which by common agreement the coefficient K_{ad} is unity. Table 2 lists other equations in which the coefficients are different in different systems of measurement. In both tables to economize space the { }'s are omitted but each quantity symbol should be regarded as merely a measure as long as we are viewing the equations merely as established experimentally by the Realist. Later we shall see that the identical equations without the { }'s are used by the Synthetiker to show relations between his symbolic quantities. In table 2 the equations are written in column 2 with a number of arbitrary parameters in their coefficients. By this device [57, 58, 60] it is possible to assign various sets of parameters in such a way that each set yields the set of equations to which one of the many alternative proposed systems of measurement is germane. The correlation between the sets of parameters and the systems of measurements is indicated in table 3. It will be noted that Γ_s (column 4) serves to distinguish symmetrical from unsymmetrical systems (see sec. 6.2). Γ_r (column 5) serves to distinguish unrationalized from rationalized systems.The reasons for particular choices of coefficients are primarily of concern to only the Synthetiker and will be mentioned specifically in section 6. The Realist working in any one system with one set of parameters and a small set of basic units proceeds to define and realize experimetally his derived germane physical units for each of the other quantities involved as shown in section 4.5 below.

After the Realist has expressed his experimental laws as measure equations, he is free to combine them by any desired mathematical operations, because the symbols in the equations represent numbers for which such operations are permissible. This is the rigorous basis for his algebra and he should preferably stick to it. However as a short cut he often finds it desirable to use mathematical phraseology to obtain conciseness in describing his experimental operations. When he combines the lengths of two gage blocks by wringing them together he says he has "added" them. When by measurement he has ascertained that the length of his desk is three feet, instead of the rigorous measure equation "$\{L_{\mathrm{desk}}\}_{\mathrm{ft}}=3$" he writes "$L_{\mathrm{desk}}=3$ ft." His replacement of the verb "is" by the symbol "=" is more than a mere substitution, and introduces mathematical connotations. It leads him to call his abbreviated statement an "equation" and to say that he has "multiplied" the physical unit "foot" by the number "3." Such a "multiplication" of a symbolic unit by a number lies at the very heart of the Synthetiker's quantity-calculus, but to a pure Realist it means primarily that the noun "feet" is modified by the adjective "three." He may also write "L (in inches)$=12L$ (in feet)" and may generalize the combined information by writing "1 ft= 12 in.," and "the ratio of 1 ft to 1 in. is 12." This leads some writers to state as useful principles: (1) two of the Realist's physical quantities, if of the same kind, may be "added" or "subtracted"; (2) one physical quantity may be divided by another of *the same kind*; (3) a physical quantity may be multiplied by a number. To this extent physical quantities can be said to be amenable to some of the principles used by the Synthetiker in the quantity-calculus of symbolic quantities, and presented in section 5. However the Realist must stop at this point. He *cannot* multiply together two physical

[10] The process here outlined is not so very unlike the actual historical sequence which started with Biot's experimental proportionality like the first eq (4.4.10). Laplace proceeded to eq (4.4.11) choosing K_{aA} equal to 1. In 1893 Heaviside suggested that in effect it was more "rational" to set $K_{aA}=1/4\pi$. This change in the choice of this and of certain other proportionality factors constitutes the act of "rationalization."

quantities (even if of the same kind) nor divide one physical quantity by another of a different kind. He can, and should perform these latter operations *only* on the *measures* of his physical quantities. On the other hand the Synthetiker (as will be seen) can perform these operations on his *symbolic* quantities.

The temptation to describe the Realist's operations in such mathematical sounding language is very great and yielding to it often saves words and space. However it is just the possibility of doing this to a limited extent which has led many writers (with their readers) to slip unconsciously beyond the pale from being Realists to being Synthetikers. This usage hides the fundamental distinction between physical quantities and symbolic quantities. Either the measures of the Realist or the symbolic quantities of the Synthetiker are amenable to *all* the familiar operations of algebra. For the Realist to consider that he is applying some of them to his physical quantities, although it may be justifiable, is dangerous, because, like an alcoholic, he may not realize when to stop. The wise Realist considers each letter symbol used in describing his physical operation to be either a measure (i.e., a number) in a measure equation or an abbreviation in a sentence. If he wants to play with mathematical operations he should go the whole way, become a Synthetiker, and realize that he is using only symbolic quantities and not physical quantities.

4.5. Derivation of Germane Systems of Measurement by the Realist

To describe and predict phenomena on the basis of the proportionalities discovered experimentally between various measurable physical quantities by the method exemplified in the preceding section 4.4, the Realist must set up measure equations. For this purpose he must have chosen and defined operationally (a) a set of N different kinds of physical quantities and also (b) a set of N physical units, one for each kind of physical quantity.

The choice of the units in the set might conceivably be entirely capricious. In this case each of the resulting measure equations would contain an experimentally determined numerical coefficient. Many equations in engineering handbooks are the result of this process, particularly where the sizes of the units have been selected to be of the same order of magnitude as the quantities concerned.

In most scientific work, however, it has been found much more desirable to make the choice of the N physical units for quantities of different kinds in a systematic fashion. To do this the Realist selects a small number, p, (usually 3 or 4 in electromagnetism) of basic physical units. Each is defined by reference to a prototype standard. He then writes a set of $n(n=N-p)$ independent measure equations, each of which is based on a proportionality established experimentally as illustrated in section 4.4, and each of which contains a proportionality constant K_g. The n values of K_g can be chosen arbitrarily and are usually taken as unity for some simple geometric arrangement. Historically this choice of the K_g's has been the work of the Synthetiker and is accepted without challenge by the Realist. The complete process has involved a total of $2N$ arbitrary choices, namely N operationally defined physical quantities, p basic physical units, and $n=N-p$ coefficients, K_g. Then by the process exemplified in eq (4.4.16) above the Realist defines the set of n germane derived physical units for the $N-p$ remaining physical quantities. He also, as needed, defines other nongermane units of his system as specified multiples or fractions of each germane unit. The entire ensemble of 4 sets of components namely (1) N physical quantities, (2) p basic physical units, (3) n independent measure equations, and (4) n derived germane physical units is called a *measurement system.* To this may be added any convenient nongermane physical units.

As we shall see in section 5, the Synthetiker in his mathematical model also constructs a complete measurement system with 4 sets of components plus a set of N dimensions. However, the sequence and conception of his quantity equations and symbolic units and quantities is essentially different from those of the Realist.

A convenient way by which the Realist can be assured of the independence of his n measure equations and define the n derived germane physical units of a system is to use a sequential procedure. He starts with one of the n equations which involves measures in terms of 2 or more of the p basic physical units of the system, together with the measure of only one new physical quantity (i.e., one of the n quantities the units of which are to be derived). For example:

$$\{x\}_a=K_g\{y\}_a\cdot\{z\}_a. \qquad (4.5.1)$$

Then for the geometry appropriate to K_g the germane unit of x (i.e., ${}_aU_x$) is $1/K_g$ times the example of x present when $\{y\}=1$ and $\{z\}=1$. By selecting a sequence of measure equations at each of which a single new physical quantity is introduced, a complete system of measurement with its germane set of physical units can be built up. This is further exemplified in section 6.3.

If in this sequence of operations an equation is introduced which involves 2 new physical quantities, N is thereby raised by 2 while n is raised by only 1. Therefore p must be increased by 1 also, and the Realist must select an additional basic physical unit for one of the two new physical quantities with an appropriate prototype standard to define it.

The initial choice of the number, p, of basic physical units is somewhat arbitrary. Even in mechanics there is no particular "magic" in the use of the usual 3, length, mass, and time. This can be seen by considering the Newtonian equation for gravitation

$$\{F\}=G\{m_1\}\{m_2\}/\{r\}^2. \qquad (4.5.1)$$

On a 3-basic system ($p=3$) the coefficient, G, appears as an experimental "constant of Nature" which has been found to have the magnitude $6.670 \cdot 10^{-11}$ newton $(\mathrm{m})^2$ per $(\mathrm{kg})^2$.

As a second alternative, however, it would be entirely possible to set up a consistent set of mechanical units with only the meter and the second as basic units together with the choice of $6.670 \cdot 10^{-11}$ as the number assigned to G in eq (4.5.1). Then the derived germane physical unit of mass (kilogram) would be defined as the mass which when placed 1 m from an equal mass experiences a gravitational acceleration of $6.670 \cdot 10^{-11}$ m/(sec)^2. Such a system with only two basic units is occasionally used in astronomy. Its rejection in physics stems, of course, from the low accuracy obtainable in experimentally assigning values to mass standards in terms of the units thus defined.

A third alternative should not be overlooked. Empty space might be considered to have a gravitational property and to constitute a prototype standard to which there might be assigned the value $6.670 \cdot 10^{-11}$ in terms of a third basic unit of gravitation. This unit, together with the meter and the second, would be the three basic units of this system. The kilogram would again be a derived unit defined by eq (4.5.1), but the system would have three instead of two basic units. The last two alternative interpretations of eq (4.5.1) appear whimsical in the field of mechanics but have been mentioned here because they are strictly analogous to parallel relations which have been seriously discussed in the electrical field (see pp. 157–58).

5. Theoretical Approach

In section 4 procedures have been described by which the Realist can develop a number of complete germane systems of measurement using physical units and the resulting measures related by measure equations. Each system is characterized by its set of coefficients and its set of basic units. Any one such development satisfies the needs of the experimenter, the engineer, and the businessman. In the present section the alternative development, which is preferred by the mathematician and by writers concerned with theoretical relations in electromagnetics, is set forth.

5.1. The Mathematical Model

The Synthetiker, being aware of the concepts invented by his colleague, the Realist, to describe the properties of the latter's physical systems, and knowing the proportionalities found experimentally between the measures of these properties, sets up for each of the Realist's measurement systems a mathematical model which he so designs that the model for each particular system bears a one-to-one correspondence with the Realist's system and thus with the actual physical universe. The correspondence may be different for different systems of measurements.

For each of the N kinds of physical quantity conceived by the Realist, the Synthetiker sets up, for each particular measurement system, a class of physico-mathematical quantities or mathematical elements which for contrast will herein be called "symbolic quantities." The members of any one of these N classes are characterized by having a common dimensionality (i.e., the quotient of any two elements of the same class is a numeric), and a magnitude relative to the other members of the same class. This means that any one of the symbolic mathematical elements may be written

$$Q = \{Q\}_a \langle Q \rangle_a \tag{5.1.1}$$

where $\langle Q \rangle_a$ is that member of the class Q to which is assigned unit magnitude (i.e., it is the symbolic unit of Q in the unit system identified by the subscript a), while $\{Q\}_a$ is the number which is the measure of Q in terms of the symbolic unit $\langle Q \rangle_a$.

The Synthetiker then proceeds to write equations which express the desired relations between his mathematical elements (i.e., his symbolic quantities). He can write n such quantity equations each corresponding to, and being identical in appearance to, one of the n measure equations which the Realist has developed experimentally as described in section 4.4. The Synthetiker, however, regards the letter symbols in his equations as denoting not the numerical measures but the complete mathematical elements. Such equations are called "quantity equations," and their use "quantity calculus." Justification for their use may be traced back to D. Gregory, Boole,[11] and Maxwell [81, 82, 83, 84]. Their use has been revived by Wallot [85], Landolt [86], Page [89, 90], and others in recent decades but is still not often explicitly stated or widely appreciated in engineering circles.

The inherent elegance and simplicity of this approach can be illustrated by writing Ohm's Law first as a measure equation

$$\{V\}_a = \{I\}_a \cdot \{R\}_a \tag{5.1.2}$$

which is equivalent to the statement that the measure of voltage, $\{V\}_a$, in a particular set of units a is numerically equal to the product of the measures of the current and of the resistance in the *same germane set* of units. By contrast the quantity equation

$$V = IR, \tag{5.1.3}$$

which might also be written in greater detail, as

$$\{V\}_a \langle V \rangle_a = \{I\}_b \langle I \rangle_b \cdot \{R\}_c \langle R \rangle_c \tag{5.1.4}$$

is true regardless of the units employed. Thus (5.1.3) makes the general statement that "the mathematical element which corresponds to the potential difference at the terminals of a resistor is equal to

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[11] Thus in "The Mathematical Analysis of Logic" (Oxford, 1847) Boole writes: "They who are acquainted with the present state of the theory of Symbolical Algebra, are aware that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same process may, under our scheme of interpretation represent the solution of a question on the properties of numbers, under another that of a geometrical problem, and under a third, that of a problem in dynamics or optics. . . ."

the product of the elements which correspond respectively to the current in the resistor and to the resistance." The sets of units indicated by the subscripts a, b, and c, can be quite unrelated. To give a typical, more specific interpretation of (5.1.4), it may be considered for example as equivalent to the following statement, "The measure of the voltage in kilovolts times one kilovolt equals the measure of the current in milliamperes times one milliampere multiplied by the measure of the resistance in ohms multiplied by one ohm." As the names "current" and "resistance" here denote symbolic quantities, the Synthetiker is quite agreeable to postulating that their product results in a voltage. In contrast the Realist dealing with the physical quantities finds it meaningless to talk about multiplying a procession of electrons by a property of some alloy which resists such a procession.

The Synthetiker normally selects the same $N-n$ quantities for which the Realist has chosen basic physical units and regards the corresponding $N-n$ classes of mathematical elements as being basic symbolic quantities.[12] From this base he defines in succession the other derived symbolic quantities (elements in his mathematical model) which correspond to the physical quantities of the Realist. As an example, suppose y and z are two of the Synthetiker's basic quantities and $\langle y\rangle$ and $\langle z\rangle$ are particular samples of each quantity to which he arbitrarily assigns the measure 1 and which therefore are two of the symbolic basic units of his system. For some particular experimental or geometrical situation, here dinoted by a subscript g, he notes that experiment has shown the proportionalities indicated by the measure equation

$$\{x\}=K\{y\}\cdot\{z\}. \tag{5.1.5}$$

The Synthetiker then writes the quantity equation

$$x=K_g yz, \tag{5.1.6}$$

choosing a convenient coefficient K_g which he considers appropriate to the geometry, g. He also writes another quantity equation

$$\langle x\rangle=\langle y\rangle\cdot\langle z\rangle \tag{5.1.7}$$

in which $\langle x\rangle$ symbolizes the coherent symbolic unit of x. This is commonly called a "unit equation."

Eqs (5.1.6) and (5.1.7) together serve to define the meaning of the operation of multiplication of an element of y by an element of z. The result of this operation is the creation of an element of x. Also inserting $\langle y\rangle$ and $\langle z\rangle$ in place of y and z in eq (5.1.6) shows that the product of a unit of y by a unit of z in geometry g produces an amount of x to which is to be assigned the measure K_g. This joint action of the two equations defines both x and $\langle x\rangle$. Of course, either equation establishes the dimensionality of x and the dimensional equation (see sec. 7)

$$[x]=[y]\cdot[z]=[y\cdot z]. \tag{5.1.8}$$

By successive applications of processes similar to this, the Synthetiker builds up the complete set of N symbolic quantities, one corresponding to each of the physical quantities of the Realist. The key to the correspondence is the parallelism in form between the Realist's measure eq (5.1.5) and the Synthetiker's quantity eq (5.1.6). The value of K in (5.1.5) is immaterial as it can be readily adjusted by the Realist to be equal to K_g by his choice of his physical unit of x.

It should be noted that the usual correspondence of basic symbolic quantities with basic physical units is purely a matter of convenience and not a logical necessity. Also one or more of the symbolic quantities which the Synthetiker prefers to consider "basic" may correspond to a physical quantity which the Realist measures by using a derived (i.e., nonbasic) physical unit (e.g., the ampere in the MKS systems).

The Synthetiker, using quantity equations, in the establishment of which the concept of "units" entered only briefly, can combine, extend, and manipulate his initial n defining equations to deduce new and valuable relationships between elements in his mathematical model. Most theoretical textbooks present such developments first and introduce a chapter entitled "Units and Dimensions" only somewhere in the last quarter of the volume, if at all.

Although the Synthetiker has infrequent need of units as such, he is much concerned by the differences between the various measurements systems listed in section 6 because these systems differ in the coefficients in their n defining equations (as tabulated in table 3), as well as in the size of their basic units. Thus, as usually treated, electric current in the CGS electrostatic system (symbolized by I_s) is related to electric current in the CGS electromagnetic system (symbolized by I_m), in the Heaviside-Lorentz system (symbolized by I_h), and in the unrationalized MKSA system (symbolized by ${}_nI_f$) by

$$I_s=cI_m=I_h/\sqrt{4\pi}=c\,{}_nI_f\sqrt{{}_n\Gamma_m}. \tag{5.1.9}$$

He must be careful to distinguish, by using subscripts or a similar device, between these 4 different symbolic quantities all labelled "electric current," and all corresponding in different systems to a single physical quantity, also called "electric current" for which the Realist uses the abbreviation "I." Similar relations involving positive and negative powers of c, Γ_m, and $\sqrt{4\pi}$ relate the other symbolic quantities used in the various systems. Thus

$$V_s=V_m/c=\sqrt{4\pi}V_h={}_nV_f/c\sqrt{{}_n\Gamma_m} \tag{5.1.10}$$

and

$$R_s=R_m/c^2=4\pi R_h={}_nR_f/c^2\,{}_n\Gamma_m \tag{5.1.11}$$

and so on.

[12] See however the exceptional departure from this simplicity in the MKSA System, third interpretation (Sec. 6.3).

Page [90] has suggested an alternative procedure by which the Synthetiker may avoid changing his symbolic quantities when rationalizing or shifting from an electrostatic to an electromagnetic system. This procedure is to introduce dimensions in the "geometric factor," K_g, in the quantity equation and thus adjust the symbolic unit in terms of which the measure of the symbolic quantity is computed.

It is unfortunate that the names of the various measurement systems are primarily based on the sizes of the basic units (e.g., CGS, MKSA, etc.) when the features which are really more important, at least from the point of view of the Synthetiker, are the coefficients in the n equations. The Realist dealing with physical quantities and units can happily lump all changes between systems as changes in "unit" but the Synthetiker must discriminate between effects of the coefficients in changing his "quantities" and the effects of choices of basic units which change his symbolic coherent units.

5.2. Coherent Abstract Units

The sequence of "unit equations," of which (5.1.7) is an example, when each is combined with the quantity equation like (5.1.6) appropriate to some geometry g serves to define a sequence of symbolic units, $\langle x\rangle$, which are independent not only of the particular geometry, g, chosen in the defining process but also of the particular coefficients, K_g, used in the quantity equations of the system. As an example of such a step in mechanics the quantity equation for the constant linear acceleration which causes a point to move a distance s in time t is

$$\{a\}\langle a\rangle=2\{s\}\langle s\rangle/\{t\}^2\langle t\rangle^2. \tag{5.2.1}$$

The measure equation is

$$\{a\}_n=2\{s\}_n/\{t\}_n^2. \tag{5.2.2}$$

As noted above, (5.2.1) is true for any assortment of units, but (5.2.2) must have consistent units here indicated by the subscript n, in all terms. Dividing (5.2.1) by (5.2.2) yields the unit equation

$$\langle a\rangle_n=\langle s\rangle_n/\langle t\rangle_n^2. \tag{5.2.3}$$

Equation (5.2.3) is a quantity equation as is eq (5.2.1) but unlike (5.2.1) it must *not* be interpreted as meaning that a point having constant unit acceleration will move a distance s in time t. It does state that if, for instance, the unit length in system n is the meter and that of time is the second, then the coherent unit of acceleration is the meter/(sec)^2. (Williams [68] has referred to this type of treatment as "an algebra of names.")

An alternative statement is that (5.2.1) defines the mathematical operation of dividing a symbolic quantity, distance, s, by the square of the symbolic quantity, time, t, as creating such an amount, $a/2$, of the symbolic quantity, acceleration, that if it were doubled it would correspond to the physical acceleration which does, if maintained constant, move a point a physical distance whose measure is $\{s\}$ in a physical time whose measure is $\{t\}$. In contrast with this (5.2.3) states that the operation of division of a symbolic unit of distance, $\langle s\rangle_n$, in system n by the square of a symbolic unit of time, $\langle t\rangle_n$, in the same system produces that amount of the symbolic quantity, acceleration, which is to be taken as the coherent symbolic unit in system n, namely $\langle a\rangle_n$.

The application of this process to the n defining equations yields n independent unit equations. The Synthetiker also selects, as a symbolic unit for each of the $N-n$ basic symbolic quantities, an example which corresponds in the model to the basic physical unit of the Realist. The n unit equations then suffice to deduce formally the n derived symbolic coherent units for the other mathematical elements.

Inspection of eq (5.2.3) shows that this, and any of the n unit equations, might have been written by inspection by assigning to each basic symbolic unit factor on the right side an exponent equal to the dimensional exponent (see sec. 7) appropriate to the quantity whose symbolic unit appears on the left side. The coefficients of all such unit equations are necessarily 1.

While this procedure suffices for the formal writing of the symbols for the symbolic coherent units by the Synthetiker, it appears to the Realist as an unsatisfactory guide for any analogous steps in the laboratory. The Synthetiker therefore offers an alternative procedure for defining the same symbolic coherent units. This alternative is to go back to an equation such as (5.2.2), set $\{t\}_n=1$ and $\{s\}_n=1/2$ for the particular case of constant linear acceleration of a point for which (5.2.1) is appropriate. The example of the symbolic quantity a (i.e., the mathematical element) which corresponds to the physical acceleration then existing constitutes by definition $\langle a\rangle_n$, the symbolic coherent unit of a. Although this type of definition, like the corresponding definition (sec. 5) of a germane physical unit, refers to some particular case, it also involves the coefficient (in this case 2) for the same case; and hence the resulting symbolic unit will be independent of what particular case is chosen. This independence of the particular case is true of germane physical units also.

As examples in the electrical field let us consider the symbolic units of charge in the classic CGS electrostatic and electromagnetic systems. In the former we write the quantity equation for two equal point charges at a separation r *in vacuo*

$$F=\frac{Q_s^2}{r^2} \tag{5.2.4}$$

and the measure equation

$$\{F\}_s=\frac{\{Q_s\}_s^2}{\{r\}_s^2}. \tag{5.2.5}$$

Dividing eq (5.2.4) by (5.2.5) and rearranging gives

$$\langle Q_s\rangle_s = \langle r\rangle_s \sqrt{\langle F\rangle_s} = \mathrm{cm}^{3/2}\mathrm{g}^{1/2}\ \mathrm{sec}^{-1}. \qquad (5.2.6)$$

In the electromagnetic system we write

$$F = \frac{c^2 Q_m^2}{r^2} \qquad (5.2.7)$$

and

$$\{F\}_m = \frac{\{c\}_m^2 \{Q_m\}_m^2}{\{r\}_m^2}. \qquad (5.2.8)$$

Dividing eq (5.2.7) by (5.2.8) and rearranging now gives

$$\langle Q_m\rangle_m = \langle t\rangle_m \sqrt{\langle F\rangle_m} = \mathrm{cm}^{1/2}\mathrm{g}^{1/2}. \qquad (5.2.9)$$

In terms of the alternative form of definition of symbolic units, $\langle Q_m\rangle_m$ is one of the two equal examples of Q_m present when $r=1$ cm and $F=9.10^{20}$ dynes. However, from (5.2.4) and (5.2.7) we find that in general for the same physical situation

$$Q_s = cQ_m; \qquad (5.2.10)$$

also from (5.2.5) and (5.2.8) for the same situation

$$\{Q_s\}_s = \{c\}_{s,m}\{Q_m\}_m. \qquad (5.2.11)$$

Dividing (5.2.10) by (5.2.11) yields, since $\langle v\rangle$ is the same in systems s and m,

$$\langle Q_s\rangle_s = \langle v\rangle_{s,m}\langle Q_m\rangle_m. \qquad (5.2.12)$$

In other words, the two symbolic units, each coherent with the dimensions of its symbolic quantity and both coherent with the same basic CGS units, differ by $\langle v\rangle_{\mathrm{CGS}}$ that is by having dimensions which differ by the dimension of velocity. This is to be expected because each of the units is an example of the corresponding kind of symbolic quantity. The two symbolic quantities also differ by the numerical factor 3.10^{10} (approx) while the two symbolic units differ only by the factor 1 cm/sec.

These relations are in marked contrast to those which exist between the physical quantities and units of the Realist. Both philosophies agree on the measure eq (5.2.11), but the Realist considers only the single physical quantity Q and measures it by either of two physical units ${}_sU_Q$ or ${}_mU_Q$ which differ by a numerical factor of 3.10^{10}. Thus, in the electrostatic system, the Realist regards $\{Q\}_s$ as the measure of a physical quantity Q in terms of ${}_sU_Q$, while the Synthetiker gets $\{Q_s\}_s$ as the measure of a symbolic quantity Q_s in terms of $\langle Q_s\rangle_s$. In the electromagnetic system, the Realist regards the smaller $\{Q\}_m$ as the measure of the same Q in terms of the larger unit ${}_mU_Q$. To the Synthetiker the smaller $\{Q_m\}_m$ is the measure of a symbolic quantity which differs from Q_s by a factor 3.10^{10} cm/sec, in terms of a symbolic unit which differs from $\langle Q_s\rangle_s$ by a factor of only 1 cm/sec. A similar shift in the correspondence between the mathematical models and the physical quantities is found in the shift from an unrationalized to a rationalized system (see sec. 8).

6. Systems of Measurement and Representation

As outlined in sec. 2, the progressive improvements in experimental procedures by the Realist and the invention of more useful concepts by the Synthetiker have led to the use during the past 100 years in the technical literature of an unfortunately large variety of different systems of measurement. The more significant of these are described or listed in this section. Although systems based on the centimeter, the gram, and the second as basic mechanical units were historically the first to come into use and are still widely used in many branches of science, systems based on the meter, the kilogram, and the second are currently favored in electrical engineering and are gaining favor in physics. In this section the latter group will be described first to exemplify the alternative modes of developing a system of measurement.

6.1. Sizes of Units

For commercial and engineering purposes it is very desirable that the measures dealt with in daily operations be numbers not too far removed from unity. Hence units should be available of roughly the same order of magnitude as the quantities to be measured. It usually matters very little in commercial transactions whether there is a simple relation between the units for physical quantities of different kinds. In practice no one carries in his head or cares to know how the inch, the mile, the acre, the gallon, or the kilowatthour are related. On the other hand, in scientific work a very great convenience and reduction in burden on the memory is obtained if units are related in systematic fashion. Hence a unique set of units germane to the equations to be used and to a few arbitrary basic units is the primary desideratum and the insertion where needed of integral powers of ten as factors is no hardship.

In the past a great deal of effort has been wasted in attempts to satisfy both sets of requirements by the same set of units. A much wiser procedure is to start with a germane set as a basis. The needs of the engineer and the marketplace can then be met by applying decimal factors as needed to create an assortment of non-germane units. An internationally recognized set of prefixes for such decimally related units is given in table 5.[13] The prefixing of these

[13] The prefixes from "micro" to "mega" seem to have been proposed at the initial invention of the metric system. In 1870 the BA Committee on the Nomenclature of Dynamical and Electrical Units approved a system suggested by Dr. G. Johnstone Stoney for higher decimal multiples. In this system the cardinal number of the exponent of 10 is added after the name of the germane unit for positive exponents and the ordinal number is prefixed to the name of the unit if the exponent is negative. Thus "10^9 grams" is written as "1 gram-nine" and 10^{-11} gram is written as "1 eleventh-gram." This logical system was used very little and has been replaced by the additional prefixes nano, pico, giga, and tera.

syllables to the names of a germane unit is widely recognized as producing the name of a decimally related larger or smaller unit of the same kind.

Exceptions to these arguments are found in certain specialized fields such as atomic physics. Here measurement systems have been proposed in which certain atomic constants, e.g., electron charge, proton mass, Bohr magneton, etc., have been chosen as basic units of a system [45, 61]. Similarly, in astronomy the mean radius of the earth's orbit, the mass and luminosity of the sun, and the speed of light have been used. The motive in these cases has been not so much to avoid large decimal factors as to correlate directly similar measurements on different atoms or celestial objects without any reference to standards of human dimensions.

6.2. Nomenclature of Systems of Measurement

Systems of measurement may be classified in various ways. If the sequence of derivation is started by assuming a conventional value for a magnetic quantity such as Γ_m the system is called *electromagnetic*. In this case Γ_e becomes a constant of nature to be determined experimentally. If the sequence is started by assuming a conventional value for an electrical quantity, the system is called *electrostatic*. If the coefficients in the defining equations for geometrical configurations having spherical or cylindrical symmetry involve explicit factors of 4π and of 2π respectively, while such explicit factors are absent in those equations pertinent to rectilinear geometries, the system is said to be *rationalized*. A system is *symmetrical* if the coefficients in the equations are such as to exhibit a symmetry between electric quantities on the one hand and magnetic quantities on the other. For example energy density in the symmetrical Heaviside-Lorentz system is

$$\{u\}=\frac{\epsilon}{2}\{E\}^2+\frac{\mu}{2}\{H\}^2$$

while in the unsymmetrical, unrationalized CGS electromagnetic system

$$\{u\}=\frac{\epsilon}{8\pi\{c\}^2}\{E\}^2+\frac{\mu}{8\pi}\{H\}^2.$$

The adjective *absolute* is often applied to the term "system of measurement" or "set of units" to indicate that the units are chosen systematically and based on the units of length, mass, and time, to distinguish it from a system in which the units are based on more arbitrary prototype standards such as the properties of particular materials, e.g., the resistivity of mercury and the electrochemical equivalent of silver.

A system is *complete* or *comprehensive* if it is designed to be extended to cover the whole range of physical quantities by a single logical system. It is *partial* or *incomplete* if its systematic use is limited to only a portion of the entire field.

6.3. Development of MKSA Systems

The process by which a Realist builds up a systematic set of physical units in the currently popular rationalized MKSA system is as follows. The process starts by selecting the meter, the kilogram, and the second as the 3 basic mechanical units, each being defined by means of the prototype standards listed in section 4.1 above. A set of germane mechanical units is derived from them by choosing values (usually unity) for the K's in the experimental measure equations of mechanics.

from $\{A_{\text{rect}}\}=K_1\{w\}\{l\}$ — The unit of area (square meter) is the area of a rectangle of which the product of the measures of the sides is 1 ($K_1=1$).

from $\{v\}=K_2\{l\}/\{t\}$ — The unit of velocity (meter/second) is the velocity of a uniformly moving point which traverses a distance whose measure is 1 m in a time whose measure is 1 sec ($K_2=1$).

from $\{a\}=K_3\{l\}/\{t\}^2$ — The unit of acceleration (meter/(second)^2) is the uniform acceleration which moves a point initially at rest a distance whose measure is ½ m in a time whose measure is 1 sec ($K_3=2$).

from $\{F\}=K_4\{m\}\{a\}$ — The unit of force (newton) is that force which imparts to a mass whose measure is 1 kg an acceleration whose measure is 1 m/(sec)^2 ($K_4=1$).

It may be noted that the Realist is free to use any one of the many possible measure equations to define a unit. He might have used $\{A_{\text{circle}}\}=\pi\{r\}^2$, or $\{a\}=\{\Delta v\}/\{\Delta t\}$ equally well.

The next step in building up the MKSA electromagnetic system is to select an equation involving both mechanical and electrical effects. The usual choice is eq (12) of table 2 and to write

$$\{F\}=\frac{\{{}_r\Gamma_m\}\{I\}^2\{l\}}{2\pi\{r\}} \tag{6.3.1}$$

for the measure of the force *in vacuo* between elements of length l of two infinitely long parallel conductors spaced r meters apart, and carrying a current I.

This step is an example of the case mentioned above, in that the measures of two new physical quantities I and ${}_r\Gamma_m$ have been introduced simultaneously. As in the case of the gravitational constant, G (see sec. 4.5), the magnetic constant, ${}_r\Gamma_m$, introduced in eq (6.3.1) has at least three possible interpretations.

On the first interpretation (which is analogous with the 2d alternative in the gravitational case) the Realist replaces $\{{}_r\Gamma_m\}$ by the numeric $4\pi\cdot10^{-7}$. Thus he defines the germane physical unit of current (ampere) as that constant current which, "if maintained in two straight parallel conductors of infinite length, of negligible circular section, and placed 1 m apart in a vacuum will produce between these conductors a force equal to 2×10^{-7} MKS units of force per meter of length." [14] On this first interpretation the ampere is a derived physical unit and the use of $\{{}_r\Gamma_m\}$ involves no increase in the number (3) of basic units. The choice of a conventional (as opposed to an experimental number) for the coefficient in an electromagnetic (opposed to an electrostatic) equation entitles the system to be called electromagnetic. The factor $\{{}_r\Gamma_m\}$ serves as the primary link between electromagnetic and mechanical units in the system. This was emphasized in the IEC resolution of 1938. This interpretation is tabulated in row 1c of table 3.

A second interpretation (which is analogous to the third alternative in the gravitational case) is to consider that empty space is a prototype standard embodying the property of magnetic permeability and having a measure $4\pi\cdot10^{-7}$ in terms of a fourth basic unit (viz, the henry/ampere) of the system. This second interpretation is the one often given by Synthetikers and is tabulated in row 1a of table 3. However, the Realist has no reason to prefer one or the other. The desire of many writers to introduce a fourth unit as basic really stems from a confusion between the basic units of a set of physical units and the basic generators of a system of physical dimensions (see sec. 7).

What is in effect a third interpretation of eq (6.3.1) and the significance of Γ_m, is embodied in Giorgi's early proposals of the MKSA system [51, 53]. Approaching the problem from the Synthetiker's point of view (in which "dimensions" and "units" are closely linked), Giorgi pointed out the desirability of considering his system as based on 4 basic units and suggested that either the ampere or the ohm might be chosen as the "fourth unit." Presumably he thought of the silver coulometer or the column of mercury as possible prototype standards on which to base the experimental realization of his set of units. Later Campbell [59], realizing that the stability of high-grade alloy resistors exceeded the reproducibility of the mercury column, urged the adoption of what he called the "Definitive System of Units" in which the basic units were the meter, kilogram, second, and an ohm defined and maintained by a prototype standard resistor which would be kept in the custody of the International Bureau of Weights and Measures. This proposal is listed in row 11 of table 3. The "definitive coulomb" would be derived from this and the mechanical unit of energy (joule) by setting $K_1=1$ in the equation

$$\{w\}=K_1\{Q\}^2\{R\}/\{t\}. \qquad (6.3.2)$$

[14] This is a translation of the wording of the International Committee on Weights and Measures.

Campbell's suggestion was welcomed by Giorgi as a desirable modification of his earlier suggestions. However, it was opposed by the national standardizing laboratories because they realized the impracticability of constructing a prototype standard of resistance which would be adequately stable, and of measuring power or energy by mechanical means with adequate accuracy. The relegation of the magnetic constant Γ_m to the status of an experimentally determined quantity (in analogy to the first interpretation of G) in what was otherwise an electromagnetic system was a further objection. In 1938 the IEC definitely rejected the "Definitive System" in favor of using a conventional value of Γ_m as the "link" between mechanical and electrical units. Giorgi's espousal of this third interpretation has created an ambiguity in the name "Giorgi System" and thus has led many writers to prefer the less ambiguous name "MKSA System." Even the name "MKSA" is a bit misleading because while the meter, kilogram, and second are basic physical units of the system, the ampere is a derived physical unit on any of the three interpretations set forth above. The IEC in urging this name in 1950 was perhaps swayed by the feeling that current was a convenient fourth dimension and that therefore its unit, the ampere, should appear in the name of the system. In this third interpretation (listed in row 1b of table 3) the Synthetiker chooses the ampere as a basic coherent symbolic unit of current and defines it, not by a prototype standard, but by specifying that its magnitude is such as to make the measure of ${}_r\Gamma_m$ by the rationalized eq (6.3.1) exactly equal to $4\pi\cdot10^{-7}$. Thus in his $2(N-n)$ arbitrary choices of basic quantities and units he chooses 3 mechanical quantities each with its own unit, but he selects electric current as one basic quantity but the unit of permeability as the basic unit. It seems to the writer that the choice of both the quantity and the unit of permeability as in the second interpretation (as on line 1a, table 3) is the more elegant. An alleged objection to this is that the use of permeability (or of resistance) as a basic dimension leads to fractional dimensional exponents, which are avoided by using current or charge as basic. However in the practical application of dimensional analysis the user is free to use any set of dimensions he may choose regardless of those used as basic in defining symbolic coherent units.

Having defined the ampere by eq (6.3.1), the next steps are to define the other germane physical units, viz, volt, ohm, coulomb, farad, henry, weber, and tesla, by using in sequence eqs (6), (7), (8), (9), and (10) of table 1, eq (14), col 3 of table 2, and eq (12) of table 1 in that order. This process leads to the sequence of definitions reading:

The Volt —The volt is the difference of electric potential between two points of a conducting wire carrying a constant current of 1 amp, when the power dissipated between these points is equal to 1 w.

The Ohm —The ohm is the electric resistance between two points of a conductor when a constant difference of potential of 1 v, applied between these two points produces in this conductor a current of 1 amp, this conductor not being the seat of any electromotive force.

The Coulomb—The coulomb is the quantity of electricity transported in 1 sec by a current of 1 amp.

The Farad —The farad is the capacitance of a capacitor between the plates of which there appears a difference of potential of 1 v when it is charged by a quantity of electricity equal to 1 coulomb.

The Henry —The henry is the inductance of a closed circuit in which an electromotive force of 1 v is produced when the electric current in the circuit varies uniformly at a rate of 1 amp/sec.

The Weber —The weber is the magnetic flux which, linking a circuit of 1 turn produces in it an electromotive force of 1 v as the flux is reduced to zero at a uniform rate in 1 sec.

The Tesla —The tesla is a flux density of one weber/m^2.

It must be kept in mind that neither the choice of equation (those listed from table 1 have K=1, but this is immaterial) on which to base the new germane unit for each new quantity, nor even the sequence in which the units are developed is of importance. If the equations are mutually consistent the same germane physical unit will result from any sequence. The names of these germane physical units are listed in column 3 of table 4. The choice of a sequence for any particular purpose depends largely on that purpose. That listed here was chosen by the International Committee on Weights and Measures because of its convenience for concise legal wording. A quite different sequence might be prepared by a teacher in his first explanation to a student. The Realist is guided largely by the attainable accuracy, convenience, or availability of particular apparatus in his experimental realization of a derived unit in terms of others.

The Synthetiker, starting with the symbols m, k, s, and a for his basic symbolic units, can use the dimensional exponents present in the same equations listed in table 1 and table 2, column 2 and write down by inspection the symbols for the derived coherent symbolic units as listed in column 9 of table 4. Each abstract unit listed in column 9 corresponds in the rationalized system to the physical unit listed in the same row in column 3 and also in the unrationalized system to the physical unit listed in column 4.

Although Giorgi initially urged the use of rationalized equations, the delay in the IEC between their acceptance of his basic units in 1935 and their advocacy of rationalization in 1950 has permitted the accumulation of a considerable literature expressed in a nonrationalized MKSA System. The measure equations of this system are obtained by inserting the values of the parameters in row 2 of table 3 in the appropriate places in column 2 of table 2. A development in sequence similar to that in the rationalized case will yield the appropriate set of germane physical units. These will be the same as in the rational system except that as shown in column 4, table 4 the physical units of $D, \psi, H, \mathscr{F}, \mathcal{R}$ are smaller in the unrationalized system by the factor 4π while that of magnetic polarization, J, is greater by this factor. Also the constants ${}_n\Gamma_m=10^{-7}$ and ${}_n\Gamma_e=10^7/\{c\}^2=1.11\cdot10^{-10}$ approximately in the unrationalized system. The measures of the electric and magnetic susceptibilities of any given substance are smaller in the nonrationalized system by the factor 4π, but because these physical properties are defined by simple numerics, even the Realist is content to consider that he is describing this property by different physical quantities in the two systems.

6.4. CGS Systems

The impetus given to the CGS systems of measurement by the British Association Committee in 1873 was so great that they have received a justified worldwide recognition and use. The equations for the two classic systems based on the centimeter, gram, and second as basic units are listed in table 1 and in columns 4 and 6 of table 2. The values of the parameters as given in rows 3 and 4 of table 3, when substituted in column 2 of table 2, will also give the equations for the CGS electrostatic and for the CGS electromagnetic systems respectively. It should be pointed out here that, in his Treatise, Maxwell used a definition for electric displacement density, D, which was smaller by a factor 4π than the value fixed by the more symmetrical definition used by most of the other early writers. The coefficients of D in his equations are therefore always greater by 4π than in the classic equations.

To derive the physical units germane to the equations of the electrostatic system the Realist begins with eq (10) of table 2, which with the appropriate parameters is

$$\{F\}=\frac{\{Q_1\}\{Q_2\}}{\epsilon\{r\}^2}. \tag{6.4.1}$$

For empty space he sets $\epsilon=1$ and also for an unrationalized electrostatic system sets $\{\Gamma_e\}=1$ and derives the unit of charge as that charge which when placed 1 cm from another equal charge in vacuum experiences a repulsive force of 1 dyne. The derivation of the units of current, electric field strength, voltage, capacitance, etc., then can follow from eqs (8), (17), (20), (9), etc., of table 1. The names of resulting germane physical units are listed in column 5 of table 4. Kennelly suggested prefixing the syllable "stat-" to the names of units of the practical (i.e., now MKSA) system to obtain "statcoulomb, statvolt . . ." as names for the units thus defined. This practice is widely used in the USA but not in Europe.

Similarly for the CGS electromagnetic system the Realist starts historically with eq 11, table 2 for the force between two equal magnetic poles and sets μ_r and Γ_m each equal to 1 *in vacuo* thus defining a physical unit of magnetic pole strength. Equation (18) of table 1 then defines a unit of H. The third step is then to use the equation

$$\{H\}=K_m\frac{\{I\}}{\{r\}} \tag{6.4.2}$$

for the magnetic field strength at the center of the circular conducting loop in which there is a current whose measure is $\{I\}$. Setting $K_m=2\pi$ for this system gives the electromagnetic unit of current ${}_mU_I$ as germane to eq (6.4.2) and the centimeter, gram, and second.

Using eqs (6), (7), (8), (9), and (10) of table 1, eq (14) of table 2, and eq (12) of table 1 in sequence then yields the germane physical CGS electromagnetic units for voltage, resistance, charge, capacitance, inductance, flux, and flux density, respectively.

The names of the resulting germane physical units are listed in column 6 of table 4. The prefix "ab-" has been suggested for application to the names of the units of the MKSA system to give names appropriate to the electrical units of the CGS electromagnetic system. Thus one obtains abampere, abvolt, abohm, abcoulomb, abfarad, abhenry, etc. These names are commonly used in the United States but not in Europe.

Unfortunately this notation has not been extended to magnetic units. In 1900 the AIEE had suggested for consideration by the Paris Congress as names of the CGS units the "gilbert" for magnetomotive force, "oersted" for reluctance, "maxwell" for flux, and "gauss" for flux density. However, the Paris Congress of 1900 instead reported only two names, viz, "maxwell" for flux and "gauss" for field intensity. In 1930 the IEC confirmed the name "maxwell" for flux but shifted the name "gauss" to flux density, the "oersted" to magnetic field strength, and approved the "gilbert" for magnetomotive force. These assignments of names to the CGS magnetic units broke down the earlier system by which the units named after scientists had all been in the "practical" system. One suggested way to remedy this was to use the names "pra-maxwell" and "pra-gilbert" for the practical units of flux and magnetomotive force. However, in 1935 the IEC adopted the name "weber" for the MKSA unit of flux (10^8 maxwells) and in 1954 it adopted "tesla" for 10^4 gausses.

The actions in 1900 also had the effect of favoring the use of magnetic units which were not germane to the practical (or International) system. Hence CGS electromagnetic units are still widely used in specifying the properties of ferromagnetic materials and the introduction of MKSA units for this purpose has been retarded.

The 1930 IEC action had assigned different dimensions (sec. 7) to flux density and magnetic field strength and this was the motive for giving different names to the units for these two quantities.

In terrestrial magnetism the name "gamma" (symbol γ) is applied to a unit equal to 10^{-5} oersted and is widely used.

The discussions since 1930 on the theoretical advantages of basing a system of measurement on 4 rather than 3 dimensions have led some writers to advocate modifying the two classic 3-dimensional CGS systems by introducing what they usually call a "fourth unit" as basic. Guggenheim [63, 109] and Fleury [64] have suggested the name "franklin" for a basic CGS electrostatic unit of charge, and deBoer [118] has suggested "biot" as the name for a basic CGS electromagnetic unit of current. To the Realist these are merely synonyms for "statcoulomb" and "abampere" (or "dekaampere") respectively but to the Synthetiker they are very convenient as building blocks for forming the names of two complete modern sets of coherent symbolic electrical units on the basis of the universal and time-honored CGS mechanical foundation. The equations for these systems when written with the constants Γ_m and Γ_e appearing explicitly are symmetrical in form like the MKSA equations. The Realist however must distinguish the CGS-F, as an electrostatic and the CGS-B as an electromagnetic system. Fortunately the high accuracy to which c is currently known makes this distinction rather academic (see sec. 6.2).

Suggestions have also been made to rationalize the 3-dimensional CGS systems but this step is usually combined with the introduction of symmetry as in the Heaviside-Lorentz system.

The original pair of CGS electrostatic and electromagnetic systems each had the very great convenience that either the permittivity, or alternatively the permeability, of space (and also of many real materials) was assumed to be unity. This makes each system very useful for certain problems but very unhandy for others. Many textbooks use both systems shifting from one to the other as needed. Helmholtz and Lorentz attribute to Gauss the credit for realizing the logic of assigning the same physical dimensions to electric charge and to magnetic pole strength because both Q^2/r and m^2/r represented work. Maxwell showed by combining eqs 13 and 14 (table 2) that electromagnetic phenomena may be propagated in space by waves having the speed given by

$$\{c\}=\frac{\{\Gamma_s\}}{\sqrt{\{\Gamma_m\}\{\Gamma_e\}}}. \tag{6.4.3}$$

If we set $\{\Gamma_m\}=1$ and $\{\Gamma_e\}=1$ and $\{\Gamma_s\}=\{c\}=3.10^{10}$ approximately, we get the list of parameters in row 5 of table 3. If these are inserted in the equations in column 2 of table 2 the resulting equations (given in column 5) will be found to have various analogous electric and magnetic quantities appear in symmetrical fashion. These equations are usually called "Gaussian." In this system the units for electrical quantities are the same as those of the CGS electrostatic system while those for magnetic quantities are the same as those of the CGS electromagnetic system. The values of these physical units are identified in columns 5 and 6 of table 4 by the notation "(gaussian)."

About 1900 H. A. Lorentz took the further step of applying the rationalization as suggested by Heaviside to the symmetrical Gaussian equations and used the resulting "Heaviside-Lorentz" equations and units germane to them in his writings. His example has been followed in the textbooks of many Synthetikers. Most of the physical units in this system differ from those in the other CGS systems by factors involving powers of $\sqrt{4\pi}$. No individual names have been assigned to these units, but the magnitudes of the germane physical units are listed in column 7 of table 4.

6.5. Practical and International Systems

In contrast to the Gaussian and Lorentz systems, which are of great convenience and elegance for theoretical work but which are never used directly in experimental operations, there are the "practical" and the "International" systems invented for and used by the electrical engineers. The 6 electrical units (volt, ohm, ampere, coulomb, farad, henry) and the 2 mechanical units, joule and watt, of the practical system were defined as exact decimal multiples of the germane units first (in 1862) of an MGS electromagnetic system and later (since 1873) of the CGS electromagnetic system. In the resolutions of the 1893 Chicago Congress and in the British and American legislation which immediately followed, the practical units were assumed to be indistinguishable from units defined by the mercury column and the silver coulometer. However, the London Conference of 1908 definitely restored the distinction. The practical system was always recognized as being limited in applicability to electrical quantities. If extended in logical fashion retaining the magnetic constant $\Gamma_m=1$, the mechanical units germane to it are found to be 10^7 m, 10^{-11} g, and 1 sec and are seen to be very "impractical." It was therefore occasionally referred to as the "Quadrant-Eleventh-gram-Second (QES) System." With the coming into use of the MKSA systems the use of the name "practical" has faded out, but the same physical units, to which have been added the "weber" for magnetic flux and the "tesla" for magnetic induction, continue in constant use.

From the Realist's point of view the germane physical units of the MKSA system are identical in kind, magnitude, and name with those of the old practical set. The Synthetiker dealing with coherent symbolic units is careful to note that the practical units being defined in terms of the CGS electromagnetic system must be considered as 3-dimensional while the MKSA symbolic units are considered 4-dimensional.

The units of the "International" set recognized explicitly by the London Conference of 1908 differed in magnitude from the corresponding practical units only by the small discrepancies present in the results of the absolute measurements available at the turn of the century. With the benefit of later determinations the International Committee on Weights and Measures in 1946 [41] decided that the mean magnitudes of the International ohm and volt as then maintained at the 6 cooperating national laboratories were related to the absolute (i.e., practical) units as follows:

"1 mean International ohm = 1.00049 absolute ohms
1 mean International volt = 1.00034 absolute volts."

In the United States the units as previously maintained and certified by the National Bureau of Standards had differed slightly from the mean of the units of all the national laboratories. Hence the changes made January 1, 1948 [43] to pass from the International to the absolute (practical or MKSA) units in the United States were:

1 International ohm, or henry = 1.000495 absolute ohms, or henrys
1 International volt or weber = 1.000330 absolute volts, or webers
1 International ampere, or coulomb = 0.999835 absolute ampere, or coulomb
1 International farad = 0.999505 absolute farad
1 International watt, or joule = 1.000165 absolute watts, or joules

Although the "International" units were usually considered as limited in application to electric and magnetic measurements it is quite possible to consider them as part of a complete system in which the basic units are the centimeter, the second, the "International ampere," and the "International ohm." The unit of mass germane to these units and the usual electromagnetic equations is approximately 10^7 grams and the unit of force is approximately 10^7 dynes. Because of the convenience and accuracy in measuring power and energy by electrical means, this international system did in effect constitute the basis for practically all precise scientific and industrial measurements for half a century.

6.6. Miscellaneous Systems

In addition to the systems discussed in sections 6.3, 6.4, and 6.5, many others have been suggested and in some cases used to a limited extent. In his widely used textbooks Karapetoff [54, 55] used what he called the "Ampere-Ohm System of Units." The parameters of the equations of this system are listed in row 12 of table 3. It used rationalized equations.

In 1916 Dellinger [56] pointed out explicitly that the engineering fraternity were in effect using the complete system of "International Electrical Units" as listed in row 10 of table 3. He also pointed out the desirability of rationalization, and being a Realist suggested that the desirable rationalized measure equations relating magnetic field strength and current could easily be obtained by using the ampere-turn as a non-germane unit of magnetomotive force in place of the gilbert. However he was obliged in consequence to write $\{B\}_{\mathrm{gauss}}=\frac{4\pi}{10}\mu\{H\}_{\mathrm{a\text{-}t/cm}}$. He also wrote $\{\Phi\}_{\text{maxwell}}=10^8\int\{E\}_{\text{volt}}d\{t\}_{\text{second}}$ because the germane "International" unit is the volt-second and not the maxwell.

A more recent proposal which makes rather fundamental changes in the electromagnetic equations has been made by Löbl [69]. He suggests removing Γ_m and Γ_e from their usual positions in the relations between **B** and **H** and **D** and **E** respectively. Instead he introduces a corresponding pair of dimensional constants c_m and c_e into Maxwell's circuital equations. He thus gets curl $\mathbf{H}=\mathbf{J}+c_e\dfrac{d\mathbf{D}}{dt}$ and curl $\mathbf{E}=-c_m\dfrac{d\mathbf{B}}{dt}$. If equations with these coefficients, which Löbl has called "Paritätische," are used with the meter, kilogram, and second as basic units the concrete units of this system are identical with those of the rationalized MKSA system. The dimensions of several symbolic quantities and of the corresponding units are however different from those of the symbolic quantities in the usual MKSA system. This proposal offers certain advantages but it remains to be seen whether it will be adopted by the Synthetikers of the future.

6.7. The "Fourth Unit" Problem

In his original "Treatise" Maxwell had found it desirable to introduce as separate concepts and therefore as distinct kinds of physical quantities the members of the pairs (1) magnetic induction, **B**, and magnetic field strength, **H**, and (2) displacement density, **D**, and electric field strength, **E**. He wrote $\mathbf{B}=\mu\mathbf{H}$ and $\mathbf{D}=\epsilon\mathbf{E}/4\pi$, but later writers to preserve the analogy between electric and magnetic equations wrote $\mathbf{D}=\epsilon\mathbf{E}$. In the classic CGS electromagnetic and electrostatic systems, the coefficients μ and ϵ respectively were assumed to be numerics and to have *in vacuo* the measure 1. This meant that the symbolic quantities **B** and **H** were of the same dimensions. Hence their symbolic units were identical and were both called "gauss." A similar situation existed for **D** and **E**. It became customary to write

$$\mathbf{B}=\mathbf{H}+4\pi\mathbf{I} \tag{6.7.1}$$

and to state that **H** was that part of **B** produced by the known macroscopic currents in the system, while the intensity of magnetization, **I**, was the effect of "concealed" Amperian currents.

Rücker [102] in 1889 suggested that the classic systems of measurement previously considered as 3-dimensional could be extended to become 4-dimensional by attributing dimensions other than numeric to permittivity or to permeability. Although he was apparently motivated by a mistaken belief that there was some "mystic" inherent connection between dimensions and kinds of physical quantities his suggestion aroused considerable interest. Other early writers, notably Heaviside, also were careful to discriminate between "absolute" and "relative" permeability and permittivity, and to regard only the latter as a pure numeric. It was included by Giorgi in his early advocacy of the MKSA system.

In 1930 the IEC discussed these ideas at great length. The discussion was unusually acrimonious because of the (at that time unrecognized) differences in the habits of thought of the Realists and the Synthetikers who participated. The latter finally prevailed and voted officially "that the formula $\mathbf{B}=\mu_0\mathbf{H}$ represents the modern concept of the physical relations for magnetic conditions *in vacuo*, it being understood that, in this expression, μ_0 possesses physical dimensions."

This action really involved more than a mere choice of a convenient dimensional label, but was meant to recommend the practice of regarding the physical quantities "magnetic induction" and "magnetic-field strength" as differing in kind. Hence their physical units were entitled to distinguishing names. It also required, as implied by the resolution, that new coefficients Γ_m and Γ_e (in their notation "μ_0" and "ϵ_0") should be written explicitly in all appropriate equations. Because of lack of appreciation of the distinction between units and dimensions, this action also initiated a demand for the official adoption of a "fourth basic unit." In 1938 the IEC recommended that the assumption of 10^{-7} in the unrationalized and $4\pi\cdot10^{-7}$ in the rationalized MKSA system as the, not necessarily dimensionless, value for Γ_m gave a sufficient link between electrical and mechanical units. In spite of this, the mistaken demand for an official selection of a particular "4th unit" continued until 1950 when the IEC recommended "that, for the purpose of developing the definitions of the units, the fourth principal unit should preferably be the ampere as defined by the General Conference on Weights and Measures." This is a minor convenience if the ampere is considered merely as a fourth basic symbolic unit from which to derive mathematically the other symbolic units of the MKSA system. It is, however, from the point of view of the Realist definitely erroneous to consider the ampere as a basic physical unit because it must be experimentally derived from the basic mechanical units using an arbitrarily assigned value for Γ_m.

However, the shift in the chosen number of basic dimensions from 3 to 4 involved no change in the coefficients in the equations, nor in the magnitudes of the physical units, germane to any given set of basic units. Hence the International Conference on Weights and Measures was careful later to avoid any reference to dimensions in its announcement of the shift in 1948 from the "International" to the "absolute" set of units. The only immediate effect on the Realist in 1930 was the change in the name of the physical unit of magnetic field strength from "gauss" to "oersted," and the discontinuance of the previous unofficial use of "oersted" as the name of the CGS unit of magnetic reluctance.

The effects of the change in dimensions on the Synthetiker are much greater and more complex than might be thought at first sight. In the first place the distinction between the concepts of "relative" and "absolute" permittivity and permeability had to be explicitly recognized in all pertinent equations. New symbols ϵ_r and μ_r were introduced to denote the relative quantities and ϵ_0 and μ_0 to denote the particular values of ϵ and μ applicable *in vacuo*. This system of symbols has not proved entirely satisfactory because many subscripts other than r are needed to denote particular states or components of physical systems to which the values of permeability or permittivity apply. Even the subscript $_0$ is used to denote initial permeability of ferromagnetic materials. Also the concepts denoted by ϵ_0 and μ_0 owe their primary significance to the fact that in any system of measurement they are conventionally chosen constants characteristic of the system. An international movement has therefore started to give ϵ_0 and μ_0 the names "electric constant" and "magnetic constant" respectively. In furtherance of this change the new distinctive symbols Γ_e and Γ_m have been used in this paper and elsewhere.

Further effects of the explicit recognition of these distinctions can be seen by considering the eq (6.7.1) relating the magnetic flux density, **B**, in a material to the magnetic field strength, **H**, and the intensity of magnetization, **I**, at any point. On the new basis in a rationalized system we must write either

$$\mathbf{B}=\mu\Gamma_m\mathbf{H}=\Gamma_m\mathbf{H}+\mathbf{J} \qquad (6.7.2)$$

or

$$\mathbf{B}=\mu\Gamma_m\mathbf{H}=\Gamma_m(\mathbf{H}+\mathbf{M}). \qquad (6.7.3)$$

The IEC in 1954 instead of choosing between (6.7.2) and (6.7.3) preferred to recognize both **J** and **M** as useful concepts. They have been named "magnetic polarization" and "magnetization" respectively. In a rationalized system **J** is identical with the "intrinsic induction" usually denoted by $\mathbf{B}_i$, while in an unrationalized system

$$\mathbf{B}_i=4\pi\mathbf{J}. \qquad (6.7.4)$$

In all cases

$$\mathbf{J}=\Gamma_m\mathbf{M}. \qquad (6.7.5)$$

In the older 3-dimensional system the volume integral of the intensity of magnetization taken over a magnetized body was defined as the "magnetic moment." In the 4-dimensional system this concept also becomes bivalent. Thus the volume integral of **M** has been called the "area moment" of a magnet or of a current loop, and for a plane loop is equal to the product of the current by the area. The volume integral of **J** has been called the dipole moment, and in the case of a long, slender permanent magnet is equal to the product of its pole strength by its length. It would, of course, be possible to push this duality a bit further and define two kinds of magnetic poles. However, this step has not received any formal support. A more recent proposal is to call the volume integral of **M** the "electromagnetic moment" and to ignore dipole moment. The torque on a magnetized body would be the product of this "electromagnetic moment" by the induction, **B**.

A similar duality of course exists in the electrostatic case. Usually one writes in the rationalized system

$$\mathbf{D}=\Gamma_e\epsilon\mathbf{E}=\Gamma_e\mathbf{E}+\mathbf{P}, \qquad (6.7.6)$$

where **P** is called "electric polarization." The name "electrization" has been suggested [65, 66] for the quantity P/Γ_e but no formal action has been taken as yet.

Still another effect of the use of 4 basic dimensions is, of course, to introduce different dimensional labels for many other quantities. This is a matter of slight importance to the Realist. For the Synthetiker, however, it means he must discriminate between the various mathematical elements (symbolic quantities) which in the various systems correspond with a single given physical quantity. Also, the symbolic coherent units for these symbolic quantities will change in dimensionality and symbolism though not in magnitude.

In 1930 the IEC had in fact formally assigned a 4-dimensional nature to the CGS electromagnetic system, prior to its adoption of the MKS system (1935) and its adoption of subrationalization (1950). However, it had not explicitly amended or rescinded any actions of earlier organizations which had clearly recognized the classic systems as 3-dimensional. To minimize ambiguity, the introduction of a pair of 4-dimensional CGS systems, one electrostatic and the other electromagnetic, has been urged to replace the classic CGS systems. The names "franklin" for the 4-dimensional basic symbolic unit of charge in the electrostatic system and "biot" for the 4-dimensional basic symbolic unit of current in the electromagnetic system have been proposed (see also p. 160). On this basis in 1951 the SUN Committee of the International Union of Pure and Applied Physics [8] recommended the introduction of such a pair of systems, though not of the particular new unit names. The sizes of the franklin and the biot are chosen so that $\Gamma_e=1$ in the CGS-Franklin system, while $\Gamma_m=1$ in the CGS-Biot system. Hence 1 biot$=\{c\}$ franklins/sec. In other words, these systems are identical to the systems proposed by Rücker except for the choice of which units are called "basic." They are symmetrical but not rationalized, although they might be.

The choice of equations in the CGS Biot and Franklin systems makes all the symbolic quantities in them identical with the corresponding quantities in the unrationalized MKSA system. The differences in the measures in the CGS-Biot and the unrationalized MKSA systems arise only from the decimal differences in the sizes of their basic units. This produces corresponding decimal differences in both the coherent symbolic and the germane physical units. When compared with the rationalized MKSA (Giorgi) system, the symbolic quantities in some cases differ by a factor of 4π as well as by decimal factors. This is also true of the germane physical units of the CGS-Biot system. The coherent symbolic units however differ only by decimal factors from those of the rationalized MKSA system.

7. Dimensions

The concept of dimensions initiated by Fourier in 1822 [101] is so closely related to and so often confused with that of units that a brief discussion of this concept from our two alternative points of view seems desirable. A dimension may be described as a label of convenience attached to a symbolic quantity to give some, but not complete, information about its relations to other quantities. The name "dimension" originated from the elementary application of the concept in geometry in which surfaces and volumes, being quantities measured by multiplying together two or three lengths measured in mutually orthogonal directions, were said to have "dimensions" "2" or "3" with respect to length. This meant that if the symbolic unit of length were imagined to decrease by, say, 1 percent, the measures in terms of the resulting decreased coherent symbolic units of area and of volume would be increased by 2 percent and 3 percent respectively, to a first approximation.

These notions are readily extended to all symbolic quantities. It can be shown (Buckingham [103], Bridgman [104]) that any of the n quantity equations by which a derived symbolic quantity, Q, is defined in terms of some or all of the $N-n$ basic symbolic quantities A, B . . ., present in some particular system can be put in the form

$$Q=KA^{\alpha}B^{\beta}\ldots. \tag{7.1}$$

Here K is a constant not affected by any change in the basic symbolic units of the system provided coherence is maintained. (K may, of course, depend upon the relative magnitudes of all quantities of each kind in the system, e.g., the shape of parts, the ratios of resistances in various arms of a network, etc. When rewritten as a measure equation, (7.1) becomes

$$\{Q\}_a=K\{A\}_a^{\alpha}\cdot\{B\}_a^{\beta}\ldots \tag{7.2}$$

where the subscripts a denote that these measures are in terms of a particular coherent set of units. Let us now assume a shift to a new set of coherent units, denoted by b, in which the new basic symbolic unit of A is decreased by a factor X while the other basic symbolic units are unchanged. Then

$$\{A\}_b=X\{A\}_a, \tag{7.3}$$

but since eq (7.1) is still true regardless of arbitrary changes in sizes of basic units we must also have

$$\{Q\}_b=K\{A\}_b^{\alpha}\{B\}_a^{\beta}\ldots. \tag{7.4}$$

By (7.3)

$$\{A\}_b^{\alpha}=X^{\alpha}\{A\}_a^{\alpha}, \tag{7.5}$$

hence for (7.4) to remain true

$$\{Q\}_b=X^{\alpha}\{Q\}_a \tag{7.6}$$

or

$$\langle Q\rangle_b=X^{-\alpha}\langle Q\rangle_a. \tag{7.7}$$

Hence the exponent α indicates the relative rates at which the measures and inversely the coherent symbolic units of Q and A must vary.

It is customary to summarize the relations of one quantity, Q, to the group of basic quantities A, B, etc., by writing

$$[Q]=[A^{\alpha}B^{\beta}\ldots]. \tag{7.8}$$

This is often called a "dimensional equation" and is in effect a concise form for encoding the dimensional exponents α, β, . . . in relation to the quantities each connects. Either member of eq (7.8) is called "the dimension of Q." In the particular case where $\alpha=\beta=\ldots=0$, Q is said to "have the dimension of a numeric" or in common parlance to be "dimensionless." It will be noted that the information contained in a dimensional equation such as (7.8) is illustrated in section 4.4 by its prediction of the form of the experimental eq (4.4.1) but that it fails to give the information in the proportionality 4.4.3 (p.149). It places no limitation on the value of K_{psn}.

Although (7.1) was assumed to relate Q only to basic quantities A, B, etc., of the system, this limitation is not necessary and A, B, etc., can equally well be members of any other convenient alternative group of independent quantities not normally considered basic. The resulting dimensional exponents are then equally useful in checking for blunders in algebraic manipulations and in dimensional analysis. The symbolic units listed in columns 9, 10, 11, 13 and 15 of table 4 show the exponents for a number of quantities in reference to several measurement systems and to alternative sets of basic dimensions.

Buckingham's [103] Π-theorem shows that with certain restrictions any complete physical equation relating symbolic quantities can be put in the form

$$\psi(\Pi_1, \Pi_2, \ldots \Pi_i)=0 \tag{7.9}$$

where each of the Π's is a product of powers of some of the symbolic quantities involved, raised to such exponents that the entire product has the dimension of a numeric. Here ψ indicates any function of the independent arguments Π_1, Π_2, . . . and i is the maximum number of independent dimensionless products which can be found by combining in various ways the N quantities involved in the particular problem. This number of dimensionless products (or independent arguments of the function ψ) is equal to the excess of the number of quantities involved in the particular problem over the number $N-n$ of basic dimensions of the system. The smaller the number of Π's the more definite is the information that can be obtained by dimensional analysis. It is partly for this reason that systems of measurement considered to involve 4 rather than 3 basic dimensions are much preferred by Synthetikers.

The other practical application of dimensions (i.e., to the detection of blunders) is of interest to the Realist as well as the Synthetiker. A measure equation must remain true if expressed in a set either of germane physical or of coherent symbolic units, even though the sizes of the basic units are changed. Hence in any equation as a check one substitutes for each quantity or measure the dimension of its corresponding symbolic unit as given in table 4. In the resulting dimensional equation the dimensional exponents for each of the basic dimensions will be found to be the same in all the terms. A failure to meet this test indicates an error in the original equation. Unfortunately the converse is not true and this dimensional check does not guarantee the correctness of the numerical coefficients of the terms.

In past literature much confusion will be found which has originated in the unwarranted assumption that in some mystic fashion the dimensions assigned to a quantity were related to the physical nature of the quantity. This is not necessarily true. In terms of the dimensional system commonly used it appears that, for example, both rationalized and unrationalized magnetic field strength have the same dimensions $[Il^{-1}]$, although the true Synthetiker regards them as different quantities. Resistance and reactance as well as magnetomotive force and current illustrate other pairs of quantities usually considered as isodimensional but which are considered by the Realist to be physically quite different in nature. There is therefore no direct general connection between dimensions and physical units and quantities. A recent suggestion by Page [90] which assigns to plane angles a dimension different from that of numerics offers an escape from these apparent inconsistencies. The physical quantity, electric charge, though conceived as unique by the Realist was assigned different dimensions (i.e., $[L^{3/2}M^{1/2}t^{-1}]$ and $[L^{1/2}M^{1/2}]$) in the two classic CGS systems.

On the other hand the dimensional exponents for a given symbolic quantity, relative to a set of more basic quantities, are identical with the exponents in the unit equation which relates the coherent symbolic unit of that quantity to the symbolic units of the more basic quantities. Hence the dimension of a given quantity can be thought of as a sort of generalized symbolic unit which retains some of the information specified by the latter but which is not limited to any particular choice of the sizes of the basic units.

To summarize these relations, we see that the dimensional exponents (α, β, etc.) appearing in either an experimental measure equation or the corresponding pairs of symbolic quantities, form the label ($[Q]=[A^{\alpha}\ B^{\beta}\ \ldots]$), or dimension appropriate to all examples of that kind of quantity. For any given system of basic symbolic units the insertion of the same dimensional exponents gives the corresponding unit equation $\langle Q\rangle_a=\langle A\rangle_a^{\alpha}\cdot\langle B\rangle_a^{\beta}\ \ldots$, (e.g., $\langle W\rangle_{\mathrm{CGS}}=\mathrm{cm}^2\ \mathrm{g}^1\ \mathrm{sec}^{-2}$). Corresponding to each symbolic unit we have a germane physical unit defined as that example of the physical quantity existing when it has the measure "1" in terms of the more basic physical units.

As an example of the application of dimensional analysis consider the braking action of a drag magnet on the rotating disk of a watthour meter. We may assume that for a series of geometrically similar combinations of magnet and disk, the retarding torque, τ, depends only on the angular speed, ω, of the disk, the average flux density, B, under the magnet poles, the resistivity, ρ, of the disk, and some linear dimension, D, which fixes the mechanical size of the structure. Attacking the problem first with 3 basic dimensions which we choose as force, length, and time we write in column 2 of table 6 the dimension of each of the 5 symbolic quantities. Since the number of variables, 5, exceeds the number of basic dimensions, 3, by 2, we find by the methods of Buckingham or Bridgman [103, 104] that the situation is describable by an equation of the form

$$\psi(\Pi_1,\Pi_2)=0 \tag{7.10}$$

with two dimensionless products. These are

$$\Pi_1=D^{-3}B^{-2}\tau \tag{7.11}$$

and

$$\Pi_2=\rho D^{-2}\omega^{-1}. \tag{7.12}$$

Hence we can write without loss of generality $\Pi_1=\psi_2(\Pi_2)$ or

$$\tau=B^2D^3\psi_2(\rho D^{-2}\omega^{-1}). \tag{7.13}$$

Only by using additional information, such as experimental data showing that τ varies as ω^{+1}, can we infer that $\Phi_2(x)=x^{-1}$ and find how τ varies with ω and D.

TABLE 6

Quantity	Dimension in	
	F, L, T	F, L, I, T
Torque ---- τ	$[F\ L]$	$[F\ L]$
Angular speed ---- ω	$[T^{-1}]$	$[T^{-1}]$
Flux density ---- B	$[F^{1/2}L^{-1}]$	$[F\ L^{-1}I^{-1}]$
Resistivity ---- ρ	$[L^2T^{-1}]$	$[F\ L^2I^{-2}T^{-1}]$
Size ---- D	$[L]$	$[L]$

In contrast to this let us use an analysis employing 4 dimensions, choosing F, L, I, and T as basic. These yield the dimensions in the last column of table 6. Since the number of variables is greater by only 1 than the number of basic dimensions, there exists only the single dimensionless product

$$\Pi=\tau D^{-5}B^{-2}\rho\omega^{-1}. \tag{7.14}$$

Hence we get directly

$$\tau=KD^5\,B^2\,\omega/\rho. \tag{7.15}$$

As a means of obtaining a more satisfying symmetry and also perhaps in order to get more effectiveness in dimensional analysis some writers have proposed the use of 5 basic dimensions in defining sets of symbolic quantities and units. The present status of these suggestions is summarized by Stille [10].

8. Rationalization

A major cause of the proliferation of the unduly large number of alternative systems of measurement in the field of electricity has been the quest for what Heaviside called "rationalization." The underlying ideas can best be illustrated by considering the concepts used in electrostatics. The results of experiments using arbitrary preliminary physical units can be expressed by the equation

$$\{F_1\}_p = K_1\{Q_1\}_p\{Q_2\}_p/\{\epsilon\}\{r\}_p^2 \tag{8.1}$$

which introduces the measures of a new quantity, electric charge, Q. A symbolic quantity, Q_s, which will correspond to Q is then postulated, as satisfying the quantity equation analogous to (8.1) and a numerical factor ϵ which would depend on an intervening isotropic medium. A further step is to postulate an electric field as a symbolic vector quantity $\mathbf{E}$ defined by the equation

$$\mathbf{F}_1 = K_2 Q_1 \mathbf{E} \tag{8.2}$$

where $\mathbf{F}_1$ is the force on charge Q_1 in field E. Maxwell also conceived of another symbolic vector quantity $\mathbf{D}_2$ which corresponded to a postulated outward displacement caused by the presence of the concentrated charge Q_2 and was related to it by

$$\int \mathbf{D}_2 \cdot \mathbf{n} dA = K_3 Q_2 \tag{8.3}$$

where the integral is taken over a closed surface surrounding Q_2. On a spherical surface of radius r, centered on Q_2

$$\mathbf{D}_2 = K_3 Q_2 \mathbf{r}_1/4\pi r^2. \tag{8.4}$$

Maxwell showed that there must be the further relation

$$\mathbf{D} = K_4 \epsilon \mathbf{E} \tag{8.5}$$

for isotropic media. Regarding (8.1) as a quantity equation it can be factored and combined with the others to give

$$K_2\mathbf{E} = \mathbf{F}_1/Q_1 = K_1 Q_2 \mathbf{r}_1/\epsilon r^2 = 4\pi K_1 \mathbf{D}_2/K_3\epsilon = 4\pi K_1 K_4 \mathbf{E}/K_3 \tag{8.6}$$

whence

$$K_3 = 4\pi K_1 K_4/K_2. \tag{8.7}$$

With 3 new symbolic quantities Q, $\mathbf{E}$, $\mathbf{D}$ to define, the Synthetiker is free to assign any values he desires to 3 of the K's, the fourth then being fixed by (8.7). The classic choice as Maxwell himself wrote "unless an absurd and useless coefficient be introduced" was to make $K_1=1$. Also K_2 is universally taken as 1. Most classical writers also chose $K_4=1$ and hence $K_3=4\pi$ in both the electrostatic and the analogous magnetostatic equations. Maxwell himself wrote $\mathbf{B}=\mu\mathbf{H}$ like the others but wrote $\mathbf{D}=\epsilon\mathbf{E}/4\pi$, thus introducing a partial rationalization in the equations in his treatise.

As can be seen from columns 4, 5, and 6 of table 2 this classic choice of the K's leads to the appearance of an explicit factor "4π" in many equations where it would not be expected, such as the field equations 13 and 16 and the capacitance of a rectangular plate capacitor (eq (22)). On the other hand, 4π does not appear in the formulas such as that for the capacitance between concentric spheres (eq 23) where it would be expected from the spherical symmetry. Heaviside in the 1880's called attention to this "disease" which he called an "eruption of 4π's" and vigorously urged the use of the alternative choices $K_3=1$ and $K_1=1/4\pi$ as the basis for devising a more "rational" set of "units." His continual reference to a change of units indicates that, although a theoretician par excellence, he had the habit of a Realist in thinking of electromagnetic equations as measure equations, the coefficients of which can be changed by a new choice of physical units. The changes in the physical units chosen by him for Q and for magnetic pole strength propagate throughout the rest of the rationalized system of measurement so that practically all the physical units are affected. Column 7 of table 4 indicates the physical units Heaviside proposed and their relation to those of the classic systems listed in columns 5 and 6. The insertion of the parameters listed in row 6 of table 3 into the equations of column 2 of table 2 will give the rationalized equations which he preferred. This system was used by Lorentz [52] and other theoretical writers but the concrete physical units of the "practical" system had become embodied in so many standard instruments that a shift to the Heaviside system was quite impractical.

Although the expression "rationalized units" has been used almost universally in the literature when referring to the Heaviside-Lorentz system, it would have been much more logical to consider the rationalized equations as being also quantity equations. As such they serve to define a new set of rationalized symbolic quantities. It is mathematical elements thus defined with which Lorentz constructed his mathematical model of the electron. In columns 12 and 14 of table 4 the symbols with subscript h denote the symbolic quantities rationalized in accordance with Heaviside's equations which correspond to the physical quantity listed in the same row of columns 1 and 2. Column 13 or 15 gives the coherent symbolic unit appropriate to each of the rationalized symbolic quantities in column 12 or 14.

The greatest inconvenience from the 4π's occurs in magnetic measurements. Remedial changes in the definition of magnetomotive force, and of H were suggested by Perry [113] in 1891 and Baily [114] in 1895. A more complete system to which Kennelly has given the name "subrationalization" was proposed by Fessenden [116] in 1900. These ideas were incorporated by Giorgi [53] in his proposals of 1901 and similar choices of coefficient were urged later by Karapetoff [54, 55], Dellinger [56], Darrieus, and others.

The Fessenden scheme when combined with the use of 4 basic symbolic quantities involves changing the electric and magnetic constants so that

$${}_r\Gamma_e = {}_n\Gamma_e/4\pi \tag{8.8}$$

and

$${}_r\Gamma_m = 4\pi {}_n\Gamma_m \tag{8.9}$$

but keeps

$$K_2 = K_3 = 1.$$

In electrostatics one writes

$$K_1 = 1/4\pi_r\Gamma_e = 1/_n\Gamma_e \qquad (8.10)$$

and sets

$$K_4 = {}_r\Gamma_e = {}_n\Gamma_e/4\pi. \qquad (8.11)$$

Here the net effect is to make no net change in Coulomb's law and thus to leave the symbolic quantities Q_s, I_s and many other quantities unaltered. However, the change in K_4 reduces D, Γ_e, and ψ by a factor of 4π. In magnetostatics one writes

$$K_1 = 1/4\pi_r\Gamma_m \qquad (8.12)$$

and sets

$$K_4 = {}_r\Gamma_m = 4\pi_n\Gamma_m. \qquad (8.13)$$

This increases the denominator of the Coulomb's law expression by $(4\pi)^2$ and defines a new rationalized magnetic pole larger by 4π than the classic. To retain $K_2 = 1$ the definitions of H and $\mathscr{F}$ must be changed also, giving

$${}_rH = {}_nH/4\pi \text{ and } {}_r\mathscr{F} = {}_n\mathscr{F}/4\pi \qquad (8.14)$$

and making corresponding changes in J and $\mathfrak{R}$. However, the changed value of Γ_m has a compensating effect so that B, Φ, and M are not affected.

If the Fessenden rationalized choice of coefficients is considered as leading to a change in the germane physical units used to measure certain physical quantities, its effect is seen by reference to column 4 of table 4. Here are listed the unrationalized physical units for the 6 physical quantities affected out of the total list of 26 for which the rationalized units are listed in column 3. Alternatively from the Synthetiker's point of view, the Fessenden subrationalization has changed the definitions of the 6 symbolic quantities indicated in column 8 of table 4. Here the subprefixes n and r denote the unrationalized and rationalized symbolic quantities respectively. The corresponding symbolic units in the 4-dimensional electromagnetic system are listed in columns 9 and 10 and in the 4-dimensional electrostatic system in column 11.

This change in the symbolic quantities is also tabulated in table 7 which is in a form to be used when translating a quantity equation in an unrationalized system to the corresponding equation in a subrationalized system (i.e., Fessenden or Giorgi rationalization) or vice versa.

The fact that subrationalization affected only a fraction of the various quantities and the more important fact that the quantities affected and their physical units were not such as are usually embodied in physical standards, made its introduction far more practicable than Heaviside's earlier proposal. All the quantities listed in table 7 are of the nature of auxiliary concepts to some extent removed from direct experimental operations and their measures are always postulated or computed from those of other more tangible quantities. These facts doubtless account for the gradually increasing acceptance of subrationalization.

The further fact that the adoption of rationalization by the IEC in 1950 occurred soon after the renaissance of "quantity calculus" has led Synthetikers to regard the process of rationalization merely as a change in the coefficients of certain equations without changing any dimensional exponents. They thus conclude, logically, that the coherent symbolic units are not affected by the change which is therefore to be considered to be merely the use of a new set of rationalized symbolic quantities. Apparently, it was on such a basis that the SUN Committee of the IUPAP voted that rationalization should be regarded as a change only of quantities and not of units. This action completely ignores the other side of the coin and the fact that the Realist usually prefers to use changed germane physical units to measure unchanged physical quantities. In the councils of the IEC, both points of view are represented but until recently the protagonists of each have failed to appreciate the advantages of the alternative approach.

Konig [88] was one of the first to realize the existence of the two points of view of the "Realist" and the "Synthetiker" and to distinguish between two "levels of abstraction," experimental and dimensional, which correspond to physical and symbolic quantities respectively. He has also made a valiant attempt to develop a complete new specialized algebra designed to handle mathematically the relations between physical quantities considered as mathematical variables. In this modified quantity calculus, the Realist finds preserved his fond tradition that the quantity remains invariant even though the equations are rationalized. The required departures from the rules of ordinary algebra, however, are so serious as to probably discourage the typical Realist, who is normally content to be limited to measure equations. Hence, there seems little to be gained by creating still another mathematical model intermediate between those here called physical and symbolic.

Most writers have followed the historical sequence in which the science was confronted with a *change*. They describe rationalization as a process by which an older system of measurement is changed to a newer one. The Realist sees it as a change in units and the Synthetiker as a change in quantities. Both consider that the other's process must lead either to noncoherence or to a situation where the manner of describing a physical situation changes the situation itself. Either is anathema.

If, on the other hand, one considers that the science is confronted with a *choice* between two alternative systems each of which is internally logical and consistent, the appearance of paradox is largely avoided. In any single complete system, either rationalized or unrationalized, there exist both (1) a pair of sets of physical and of symbolic quantities, the members of which correspond in a manner dependent on the chosen equations of the particular system and also (2) a pair of sets of germane physical and of coherent symbolic units which also correspond in accordance with the same equations as well as with the choice of basic units. In either system the correspondence between the two sets of each pair is complete and self-consistent. The 1950 decision of the IEC was merely to recommend the future use of a particular measurement system with its particular correspondences. The chosen correspondences are neither more nor less self-consistent than those previously used.

9. Summary and Conclusions

It will be seen from the foregoing that there has been a long evolution of the systems of units and measurements in the electrical field in which an initial very excellent start has been successively improved upon but unavoidably at the cost of accumulating complexity and confusion in the literature. This evolution is perhaps nearing its end, and the complexity may be reduced shortly to the "peaceful coexistence" of two systems. The MKSA system in its rationalized form has now won almost universal acceptance in electrical engineering and its use seems to be spreading in physics and in other branches of engineering. The older CGS system still holds undisputed sway in many other branches of science. In electrophysics it is still widely used either in the symmetrical Gaussian form, or by the practically equivalent process of using the CGS electrostatic system for electrostatic problems and the CGS electromagnetic system for magnetic problems. It seems questionable whether the 4-dimensional CGS systems will be much used as alternatives during an interim period.

The practical line of development which leads to the experimental definition, establishment, maintenance, and dissemination of the physical electrical units seems to be in very satisfactory shape. The national standardizing laboratories, coordinated by the services of the International Bureau of Weights and Measures, are continually gaining in the scope and accuracy of their facilities. The lower echelons of the hierarchy of standardizing laboratories are rapidly increasing in numbers and in their recognition by industry and commerce as essential links in the interdependent network of modern manufacturing. The next step in the series of adjustments of the maintained electrical units closer to their ideal value will surely amount to only a very few parts per million and may not be needed for a long time.

The theoretical line of development of measurement systems and nomenclature is temporarily bogged down in the discussions of various international organizations by what superficially seem to be semantic difficulties, i.e., the use of words like "unit" and "quantity" each with two different meanings. However, this is merely a symptom of the still deeper difference in the habits of thought of the two classes of workers in the electrical field. These difficulties can be largely avoided by the careful explicit recognition, as exemplified in this paper, of the two distinct ways of looking at the systems of measurement and their equations. The results of this distinction can be seen by the following summarization.

The Realist deals only with the concepts of physical quantities which are characterized qualitatively by "kind" and quantitatively by "magnitude," which he regards as fixed by nature and as independent of the units in terms of which they are measured and the equations used to relate the results of such measurements. He uses only physical units, i.e., specified samples of each kind of physical quantity to which he has assigned the measure "1". He deals only with measure equations in which the literal symbols represent the numerical measures of his physical quantities. He commonly, but by no means universally, prefers to use a set of physical units defined by a choice of (a) a small number of basic units, (b) a set of equations with generally recognized simple coefficients, and (c) a set of derived physical units which are germane both to the basic units and to the equations. However he often for convenience uses other nongermane units, defined as numerical multiples of the normal germane unit and simultaneously he modifies accordingly the coefficient in the equations concerned to restore germaneness. He is therefore constantly aware that his equations are true only in a set of consistent (i.e., germane) units. Hence he frequently writes "in ——— units this equation becomes ———." In *all* operations he trusts the principle that the measure of a given quantity varies inversely as the unit used to measure it, regardless of whether the change in the unit is the result of a change in a basic unit of the system, of a change in the coefficient in an equation (e.g., rationalization) or of the use of a nongermane unit. Hence the conversion factors for *measures* given in table 8 are the reciprocals of the ratios of his corresponding *physical* units. He therefore, for example, writes as quoted at (b) on p. 137 when comparing the measures of a particular magnetic field in terms of two alternative physical units "the number of ampere-turns per meter$=1000/4\pi$ times the number of oersteds."

On the other hand, the Synthetiker deals only with symbolic quantities (i.e., mathematical elements) which are defined by a set of quantity equations. Symbolic quantities are characterized qualitatively by "dimensionality" and quantitatively by "magnitude." His equations are identical in form to the systematic measure equations of the Realist, but the letter symbols in the Synthetiker's quantity equation represent the complete concept of symbolic quantity both qualitative and quantitative. From the parallel between his quantity equations and the Realist's measure equations, he sets up a correspondence between his symbolic quantities and the Realist's physical quantities, giving them the same name.

During the evolution of the science different coefficients have been used in certain equations. Each of the resulting sets of equations has in general constituted a new and different mathematical model with new and different correspondences between the symbolic and the physical quantities of the same name. Hence two symbolic quantities which in different models correspond to the same physical quantity may be of different magnitude and even of different dimensionality. The Synthetiker's quantity equations are true regardless of units and in his operations he has very little use for the concepts of unit and of measure. However, for completeness he conceives of a symbolic unit for each symbolic quantity. This is symbolized by writing the product of a set of basic symbolic units each raised to the same dimensional exponent as in the expression for the dimension of the symbolic quantity of which it is the unit. Changes in the coefficients in the equations do not change the dimensional exponents and therefore do not change the coherent symbolic units of any quantity. The Synthetiker has little use for noncoherent units hence his units are invariant to changes in the coefficients in the equations. They are changed only by changes in the sizes of his basic units. Hence the conversion factors in table 8 are *not* the reciprocals of the corresponding *symbolic* units except when the change in measures results solely from a change in the basic units. The Synthetiker for example therefore writes as quoted at (a) on p. 137 when comparing the symbolic units of two alternative measurement systems "1 oersted=1000 ampere-turns per meter."

A person working only in one system of measurement can continue to think sometimes as a Realist and at other times as a Synthetiker. He can use the same words as names for both kinds of quantities and units, but would be wise to be aware at all times which role he is playing. The writer who is concerned with the relatively rare paper which involves the comparison or discussion of more than one measurement system has a much greater need to be constantly alert as to his role and should for clarity indicate to his reader by the appropriate use of adjectives, such as "physical" or "symbolic" or their equivalents, just what level and type of concept he is discussing in any particular paragraph. A material help could be secured by the consistent use of the unit names as listed in column 3, 5, 6, and 7 of table 4 and combinations of these names when designating physical units only; and in contrast the use of the symbols such as those listed in column 9, 10, 11, 13, and 15 of table 4 and other combinations of basic unit names when designating symbolic units.

Further problems confronting the national and international standardizing bodies include the following. Shall a second quantity "electrization" be recognized to correspond to "magnetization"? Shall two types of magnetic moment (and of electric moment also) be recognized and provided with names, units and symbols or will one suffice? How can a more satisfactory system of names and symbols be invented to denote the different aspects of permeability and of permittivity (i.e., relative versus absolute, a-c versus d-c, initial versus cyclic, differential versus normal). Here a plethora of electric terms (specific inductive capacity, dielectric constant, electric constant, real component of phasor dielectric constant, permittivity, capacitivity) contrasts with a paucity of magnetic terms (permeability, inductivity).

A major cause of the present impasse in international standardization in the field of electrical systems of measurement has been the failure of many disputants to recognize the equal validity of the two habits of thought set forth in this paper. Energy has been wasted in attempts either to decide in favor of one as against the other, or failing this, to formulate some particular, and necessarily ambiguous, wording which would receive the formal approval of both groups, because the two groups gave two different meanings to certain key words. Instead let us hope that steps will be taken soon to officially recognize both habits of thought as equally valid. Each is to be preferred in its own field but the Realist and the Synthetiker should tolerate the usage and appreciate the effectiveness of the other's concepts for particular purposes.

In some distant future, a single measurement system may win universal acceptance. Then there automatically will be one, and only one, correspondence between each symbolic and its corresponding physical quantity or unit. Until that utopia is reached, and the literature of the past has been forgotten, the coexistence of the two habits of thought must be recognized.

The writer expresses his gratitude to the many fellow members of standardizing committees and to his colleagues at the National Bureau of Standards whose patience during protracted discussions of this elusive subject have contributed so much to the clarification of the concepts. In addition to C. C. Murdock and C. H. Page who have so many times corrected my erring logic, and F. L. Hermach, who so meticulously scrutinized and improved the equations and tables, I cannot refrain from also listing gratefully F. Avcin, C. C. Chambers, F. K. Harris, E. I. Hawthorne, H. König, F. R. Kotter, M. Landolt, C. Peterson, S. A. Schelkunoff, J. J. Smith, C. Stansbury, U. Stille, and S. R. Warren, Jr.

10. Appendixes

10.1. Tables

TABLE 1. *Some equations having the same coefficients in all usual systems of measurement*

No.	Relation	Equation
1	uniform velocity	$\mathbf{v}=\mathbf{s}/t$
2	constant acceleration from rest	$\mathbf{a}=\mathbf{v}/t=2\mathbf{s}/t^2$
3	force to accelerate mass	$\mathbf{F}=m\mathbf{a}$
4	work, energy	$W=\int\mathbf{F}\cdot d\mathbf{s}=mv^2/2$ $=\int eidt$
5	power	$P=dW/dt$
6	voltage for given power and current (d-c)	$V=P/I$
7	resistance (d-c)	$R=V/I$
8	electric charge	$Q=\int Idt$
9	capacitance	$C=Q/V$
10	inductance	$V=LdI/dt$
11	electric flux	$\psi=\int\int\mathbf{D}\cdot\mathbf{n}dA$
12	magnetic flux	$\phi=\int\int\mathbf{B}\cdot\mathbf{n}dA$
13	electric current	$I=\int\int\mathbf{i}\cdot\mathbf{n}dA$
14	conductance	$G=1/R=I/V$
15	resistivity	$\rho=RA/l$
16	conductivity	$\gamma=l/RA$
17	force on charge in electric field	$\mathbf{F}=Q\mathbf{E}$
18	force on pole in magnetic field	$\mathbf{F}=m\mathbf{H}$
19	flux of B through closed surface	$\int\int\mathbf{B}\cdot\mathbf{n}dA=0$
20	potential difference	$V_{ab}=\int_a^b\mathbf{E}\cdot d\mathbf{s}$
21	magnetomotive force	$\mathscr{F}=\oint\mathbf{H}\cdot d\mathbf{s}$
22	reluctance	$\mathcal{R}=\mathscr{F}/\phi$

Notes for Table 1

This table lists a number of the equations frequently used in electrical engineering in which the coefficient is the same (and usually 1) in all the various systems of measurement thus far proposed. From the point of view of the Realist, each symbol should have been enclosed in { }'s, since it represents to him the measure of a physical quantity. The Realist uses some of the equations in column 2 to fix the size of the germane physical units for the quantities listed in column 1. From the point of view of the Synthetiker, column 2 lists quantity equations, which he uses to define the symbolic quantities listed in column 1. The sequence of listing is convenient for either purpose but is not important. The sequence is not logically continuous because in any system some steps involve equations of table 2 (e.g., equation 12, table 2) in which the coefficient is different in different systems.

TABLE 2. *Some equations having different coefficients in different systems of measurement*

No.	1 Relation	2 Equations (parametric form)	3 MKSA (Giorgi) rationalized	4 Electrostatic	5 Gaussian	6 Electromagnetic
1	Electric flux from charge Q	$\Psi=\Gamma_r Q$	$\Psi=Q$	$\Psi=4\pi Q$	$\Psi=4\pi Q$	$\Psi=4\pi Q$
2	Magnetic flux from pole m	$\Phi=\Gamma_r m$	$\Phi=m$	$\Phi=4\pi m$	$\Phi=4\pi m$	$\Phi=4\pi m$
3	Permittivity, $\epsilon\Gamma_e$	$\mathbf{D}=\epsilon\Gamma_e\mathbf{E}$	$\mathbf{D}=\epsilon_r\Gamma_e\mathbf{E}$	$\mathbf{D}=\epsilon\mathbf{E}$	$\mathbf{D}=\epsilon\mathbf{E}$	$\mathbf{D}=\epsilon\mathbf{E}/c^2$
4	Electric polarization, $\mathbf{P}_e$	$\mathbf{D}=\Gamma_e\mathbf{E}+\Gamma_r\mathbf{P}_e$	$\mathbf{D}={}_r\Gamma_e\mathbf{E}+\mathbf{P}_e$	$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}_e$	$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}_e$	$\mathbf{D}=\mathbf{E}/c^2+4\pi\mathbf{P}_e$
5	Permeability, $\mu\Gamma_m$	$\mathbf{B}=\mu\Gamma_m\mathbf{H}=\Gamma_m\mathbf{H}+\mathbf{B}_i$	$\mathbf{B}=\mu_r\Gamma_m\mathbf{H}={}_r\Gamma_m\mathbf{H}+\mathbf{B}_i$	$\mathbf{B}=\mu\mathbf{H}/c^2=\mathbf{H}/c^2+\mathbf{B}_i$	$\mathbf{B}=\mu\mathbf{H}=\mathbf{H}+\mathbf{B}_i$	$\mathbf{B}=\mu\mathbf{H}=\mathbf{H}+\mathbf{B}_i$
6	Magnetic polarization, $\mathbf{J}$	$\mathbf{B}=\Gamma_m\mathbf{H}+\Gamma_r\mathbf{J}$	$\mathbf{B}={}_r\Gamma_m\mathbf{H}+\mathbf{J}$	$\mathbf{B}=\mathbf{H}/c^2+4\pi\mathbf{J}$	$\mathbf{B}=\mathbf{H}+4\pi\mathbf{J}$	$\mathbf{B}=\mathbf{H}+4\pi\mathbf{J}$
7	Magnetization, $\mathbf{M}$	$\mathbf{B}=\Gamma_m(\mathbf{H}+\Gamma_r\mathbf{M})$	$\mathbf{B}={}_r\Gamma_m(\mathbf{H}+\mathbf{M})$	$\mathbf{B}=(\mathbf{H}+4\pi\mathbf{M})/c^2$	$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$	$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$
8	Electric susceptibility, χ_e	$\chi_e=(\epsilon-1)/\Gamma_r=\frac{\mathbf{P}_e}{\Gamma_e\mathbf{E}}$	$\chi_e=\epsilon-1=\frac{\mathbf{P}_e}{{}_r\Gamma_e\mathbf{E}}$	$\chi_e=(\epsilon-1)/4\pi=\mathbf{P}_e/\mathbf{E}$	$\chi_e=(\epsilon-1)4\pi=\mathbf{P}_e/\mathbf{E}$	$\chi_e=(\epsilon-1)/4\pi=\mathbf{P}_e c^2/\mathbf{E}$
9	Magnetic susceptibility, χ_m	$\chi_m=(\mu-1)/\Gamma_r=\frac{\mathbf{M}}{\mathbf{H}}=\frac{\mathbf{J}}{\Gamma_m\mathbf{H}}$	$\chi_m=\mu-1=\mathbf{M}/\mathbf{H}$	$\chi_m=(\mu-1)/4\pi=\mathbf{M}/\mathbf{H}$	$\chi_m=(\mu-1)/4\pi=\mathbf{M}/\mathbf{H}$	$\chi_m=(\mu-1)/4\pi=\mathbf{M}/\mathbf{H}$
10	Coulomb's Law	$F=\frac{\Gamma_r Q_1Q_2}{4\pi\epsilon\Gamma_e r^2}$	$F=\frac{Q_1Q_2}{4\pi\epsilon_r\Gamma_e r^2}$	$F=\frac{Q_1Q_2}{\epsilon r^2}$	$F=\frac{Q_1Q_2}{\epsilon r^2}$	$F=\frac{c^2Q_1Q_2}{\epsilon r^2}$
11	Coulomb's Law (magnetostatic)	$F=\frac{\Gamma_r m_1m_2}{4\pi\mu\Gamma_m r^2}$	$F=\frac{m_1m_2}{4\pi\mu_r\Gamma_m r^2}$	$F=\frac{c^2m_1m_2}{\mu r^2}$	$F=\frac{m_1m_2}{\mu r^2}$	$F=\frac{m_1m_2}{\mu r^2}$
12	Force on length l of long parallel current-carrying wires at separation r	$F=\frac{\mu\Gamma_m\Gamma_r I_1I_2l}{2\pi\Gamma_s^2r}$	$F=\frac{\mu_r\Gamma_m I_1I_2l}{2\pi r}$	$F=\frac{2\mu I_1I_2l}{c^2r}$	$F=\frac{2\mu I_1I_2l}{c^2r}$	$F=\frac{2\mu I_1I_2l}{r}$
13	Magnetic field from currents	$\Gamma_s\oint\mathbf{H}\cdot d\mathbf{s}=\Gamma_r NI+\frac{d\Psi}{dt}$	$\oint\mathbf{H}\cdot d\mathbf{s}=NI+\frac{d\Psi}{dt}$	$\oint\mathbf{H}\cdot d\mathbf{s}=4\pi NI+\frac{d\Psi}{dt}$	$c\oint\mathbf{H}\cdot d\mathbf{s}=4\pi NI+\frac{d\Psi}{dt}$	$\oint\mathbf{H}\cdot d\mathbf{s}=4\pi NI+\frac{d\Psi}{dt}$
14	Electric field from flux change	$\Gamma_s\oint\mathbf{E}\cdot d\mathbf{s}=-\frac{d\Phi}{dt}$	$\oint\mathbf{E}\cdot d\mathbf{s}=\frac{-d\Phi}{dt}$	$\oint\mathbf{E}\cdot d\mathbf{s}=\frac{-d\Phi}{dt}$	$c\oint\mathbf{E}\cdot d\mathbf{s}=\frac{-d\Phi}{dt}$	$\oint\mathbf{E}\cdot d\mathbf{s}=\frac{-d\Phi}{dt}$
15	Speed of propagation, v	$\epsilon\mu v^2=\Gamma_s^2/\Gamma_e\Gamma_m$	$\epsilon\mu v^2=c^2$	$\epsilon\mu v^2=c^2$	$\epsilon\mu v^2=c^2$	$\epsilon\mu v^2=c^2$
16	Divergence of displacement	$\text{div }\mathbf{D}=\Gamma_r q$	$\text{div }\mathbf{D}=q$	$\text{div }\mathbf{D}=4\pi q$	$\text{div }\mathbf{D}=4\pi q$	$\text{div }\mathbf{D}=4\pi q$
17	Poynting vector	$\mathbf{S}=\Gamma_s\mathbf{E}\times\mathbf{H}/\Gamma_r$	$\mathbf{S}=\mathbf{E}\times\mathbf{H}$	$\mathbf{S}=\mathbf{E}\times\mathbf{H}/4\pi$	$\mathbf{S}=c\mathbf{E}\times\mathbf{H}/4\pi$	$\mathbf{S}=\mathbf{E}\times\mathbf{H}/4\pi$
18	Energy density, electric	$W_e=\mathbf{E}\cdot\mathbf{D}/2\Gamma_r=\Gamma_e\epsilon\mathbf{E}^2/2\Gamma_r$	$W_e=\mathbf{E}\cdot\mathbf{D}/2={}_r\Gamma_e\epsilon\mathbf{E}^2/2$	$W_e=\mathbf{E}\cdot\mathbf{D}/8\pi=\epsilon\mathbf{E}^2/8\pi$	$W_e=\mathbf{E}\cdot\mathbf{D}/8\pi=\epsilon\mathbf{E}^2/8\pi$	$W_e=\mathbf{E}\cdot\mathbf{D}/8\pi=\epsilon\mathbf{E}^2/8\pi c^2$
19	Energy density, magnetic	$W_m=\mathbf{H}\cdot\mathbf{B}/2\Gamma_r$ $=\Gamma_m\mu\mathbf{H}^2/2\Gamma_r$	$W_m=\mathbf{H}\cdot\mathbf{B}/2=\mu_r\Gamma_m\mathbf{H}^2/2$	$W_m=\mathbf{H}\cdot\mathbf{B}/8\pi$ $=\mu\mathbf{H}^2/8\pi c^2$	$W_m=\mathbf{H}\cdot\mathbf{B}/8\pi=\mu\mathbf{H}^2/8\pi$	$W_m=\mathbf{H}\cdot\mathbf{B}/8\pi=\mu\mathbf{H}^2/8\pi$
20	Magnetic field of current element	$d\mathbf{H}=\frac{\Gamma_r Id\mathbf{s}\sin\alpha}{4\pi r^2\Gamma_s}$	$d\mathbf{H}=\frac{Id\mathbf{s}\sin\alpha}{4\pi r^2}$	$d\mathbf{H}=Id\mathbf{s}\sin\alpha/r^2$	$d\mathbf{H}=Id\mathbf{s}\sin\alpha/cr^2$	$d\mathbf{H}=Id\mathbf{s}\sin\alpha/r^2$
21	Inductance of slender toroid	$L=\Gamma_r\Gamma_m\mu N^2A/s$	$L={}_r\Gamma_m\mu N^2A/s$	$L=4\pi\mu N^2A/c^2s$	${}^mL=4\pi\mu N^2A/s$	$L=4\pi\mu N^2A/s$
22	Capacitance of plate capacitor	$C=\Gamma_e\epsilon A/\Gamma_r d$	$C={}_r\Gamma_e\epsilon A/d$	$C=\epsilon A/4\pi d$	$C=\epsilon A/4\pi d$	$C=\epsilon A/4\pi c^2d$
23	Capacitance of spherical capacitors	$C=\frac{4\pi\epsilon\Gamma_e a\cdot b}{(b-a)\Gamma_r}$	$C=\frac{4\pi\epsilon_r\Gamma_e a\cdot b}{b-a}$	$C=\frac{\epsilon a\cdot b}{b-a}$	$C=\frac{\epsilon a\cdot b}{b-a}$	$C=\frac{\epsilon a\cdot b}{(b-a)c^2}$
24	Force on current element	$\mathbf{F}=Id\mathbf{s}\times\mathbf{B}/\Gamma_s$	$\mathbf{F}=Id\mathbf{l}\times\mathbf{B}$	$\mathbf{F}=Id\mathbf{l}\times\mathbf{B}$	$\mathbf{F}=Id\mathbf{l}\times\mathbf{B}/c$	$\mathbf{F}=Id\mathbf{l}\times\mathbf{B}$
25	Force on moving charge	$\mathbf{F}=Q\mathbf{v}\times\mathbf{B}/\Gamma_s$	$\mathbf{F}=Q\mathbf{v}\times\mathbf{B}$	$\mathbf{F}=Q\mathbf{v}\times\mathbf{B}$	$\mathbf{F}=Q\mathbf{v}\times\mathbf{B}/c$	$\mathbf{F}=Q\mathbf{v}\times\mathbf{B}$

Notes for Table 2

In columns 3, 4, 5, and 6 are given for the four most commonly used systems of measurement some of the equations in which some of the coefficients are different in the different systems.

Column 2 gives the same equations in a more generalized parametric form. To obtain the equations appropriate to still other measurement systems the parameters as listed in the appropriate row of table 3 can be substituted in the equations of column 2.

It should be noted that a Realist would write all of these as measure equations enclosing each letter symbol in { }'s. To economize on space these have been omitted and the equations appear only in the form of the Synthetiker's quantity equations.

Table 3

		Parameters for equations, table 2, column 2				Postulated units for basic quantities							
	1	2	3	4	5	6	7	8	9	10	11	12	13
Row	Measurement system	Γ_m	Γ_e	Γ_s	Γ_r	Length	Mass	Time	Permeability	Current	Charge	Resistance	Symbols in table 4 for symbolic quantity
[a] 1a	MKSA (rationalized) (2nd interpretation)	${}_r\Gamma_m$	${}_r\Gamma_e$	1	1	meter	kilogram	second	[e] henry/meter	----------	----------	----------	${}_rX_f$
1b	MKSA (rationalized) (3rd interpretation)	${}_r\Gamma_m$	${}_r\Gamma_e$	1	1	meter	kilogram	second	----------	ampere	----------	----------	${}_rX_f$
1c	MKSA (rationalized) (1st interpretation	$4\pi\cdot10^{-7}$	$10^7/4\pi c^2$	1	1	meter	kilogram	second	----------	----------	----------	----------	
2	MKSA (Unrationalized) (2nd interpretation)	${}_n\Gamma_m$	${}_n\Gamma_e$	1	4π	meter	kilogram	second	[e] henry/meter	----------	----------	----------	${}_nX_f$
3	CGS electrostatic	$1/c^2$	1	1	4π	centimeter	gram	second	----------	----------	----------	----------	X_s
4	CGS electromagnetic	1	$1/c^2$	1	4π	centimeter	gram	second	----------	----------	----------	----------	X_m
5	Gaussian	1	1	c	4π	centimeter	gram	second	----------	----------	----------	----------	X_g
6	Heaviside-Lorentz	1	1	c	1	centimeter	gram	second	----------	----------	----------	----------	X_h
7	CGS-Franklin (unrationalized)	${}_n\Gamma_m$	${}_n\Gamma_e$	1	[d] 4π	centimeter	gram	second	----------	----------	franklin	----------	${}_nX_f$
8	CGS-Biot (unrationalized)	${}_n\Gamma_m$	${}_n\Gamma_e$	1	[d] 4π	centimeter	gram	second	----------	biot	----------	----------	${}_nX_f$
9	[b] Practical	1	$1/c^2$	1	4π	10^7 meter	10^{-11} gram	second	----------	----------	----------	----------	
10	[c] International	[c] 1	[c] $1/c^2$	1	4π	centimeter	----------	second	----------	International ampere	----------	International ohm	
11	Definitive (Campbell 1933)	${}_r\Gamma_m$	${}_r\Gamma_e$	1	1	meter	kilogram	second	----------	----------	----------	Definitive ohm	
12	Ampere-Ohm (Karapetoff 1911)	$4\pi.10^9$	$10^9/4\pi c^2$	1	1	centimeter	----------	second	----------	International ampere	----------	International ohm	

Notes for Table 3

General: The spaces left blank in columns 7, 9, 10, 11, and 12 correspond to *derived*, not basic, quantities or units. The speed of light denoted by c has the value $2.997925\cdot10^8$ meter/second in row 1c; $2.997925\cdot10^{10}$ centimeter/second in rows 3, 4, 5, 6, 7, 8, 10, and 12; and 30 quadrants/second in row 9.

Specific Notes:

[a] Items in row 1a indicate the basis of the rationalized MKSA system as recognized by the IEC in 1938. This corresponds to the "2nd interpretation" (see p. 158), namely that space constitutes a prototype standard of magnetic permeability to which is assigned the conventional value of $4\pi\cdot10^{-7}$. This interpretation is satisfactory both to the Realist who is thereby given an experimentally realizable fourth basic physical unit and to the Synthetiker who is given a fourth independent symbolic quantity, permeability, on which to base his set of dimensions.

Items in row 1b indicate the basis implied by the IEC in 1950 that the "ampere" be regarded as the fourth unit. This corresponds to the 3rd interpretation (see p. 158). This is satisfactory to the Synthetiker, to whom it is immaterial which of the mutually coherent units of current and of permeability is regarded as the basic one. It is unsatisfactory to the Realist because no prototype standard is currently recognized for defining the ampere independently as a physical unit except by first defining something equivalent to a physical unit of permeability.

The items in row 1c correspond to the 1st interpretation (see p. 158) of Γ_m as a numerical coefficient. This is satisfactory to the Realist, who derives the same set of physical units from row 1c as from row 1a. It is unsatisfactory to the Synthetiker because it, like rows 3, 4, 5, and 6, yields a set of only 3-dimensional symbolic quantities and units.

[b] The physical units for most electrical quantities derived on the system listed in row 9 are identical with those of rows 1a, 1b, and 1c and differ by only a few parts in 10,000 from those in row 10.

[c] In the International system Γ_m and Γ_e were experimentally measured constants of nature equal to 0.99951 and $1.00049/c^2$ respectively. In practice these departures from 1.0000 were usually ignored.

[d] The 4-dimensional CGS systems are sometimes used with the equations rationalized by setting Γ_r equal to 1.

[e] The "henry/meter" is a convenient equivalent of the more logica "kilogram·meter²/ampere² second²" as a name for the unit of permeability.

Table 4. *Quantities, units, and their correspondences*

Physical quantities, and their germane physical units

Row	Quantity	Abbreviation	Rationalized (Fessenden) MKSA	Unrationalized MKSA	Unrationalized CGS–ESU	Unrationalized CGS–EMU	Rationalized (Heaviside) Heaviside-Lorentz	Row
			Names of the physical units					
	1	2	3	4	5	6	7	
1	Length	l	meter (m)		centimeter	centimeter	centimeter	1
2	Mass	M	kilogram (kg)		gram	gram	gram	2
3	Time	T	second (s)		second	second	second	3
4	Force	$\mathbf{F}$	newton (n)		dyne	dyne	dyne	4
5	Work	W	joule (j)		erg	erg	erg	5
6	Power	P	watt (w)		erg/second	erg/second	erg/second	6
7	Current	I	ampere (a)		statampere, esu (gaussian)[c]	abampere (or biot), emu	statampere/$\sqrt{4\pi}$	7
8	Voltage	V	volt (v)		statvolt esu, (gaussian)	abvolt, emu	$\sqrt{4\pi}$ statvolt	8
9	Electric gradient	$\mathbf{E}$	volt/meter		statvolt/cm, esu (gaussian).	abvolt/cm, emu	$\sqrt{4\pi}$ statvolt/cm	9
10	Charge	Q	coulomb (c)		statcoulomb (or franklin), esu (gaussian).	abcoulomb, emu	statcoulomb/4 $\sqrt{4\pi}$	10
11	Electric flux	Ψ	coulomb (c)	coulomb/4π	statcoulomb/4π, esu (gaussian).	abcoulomb/4π, emu	statcoulomb/$\sqrt{4\pi}$	11
12	Electric displacement	$\mathbf{D}$	coulomb/meter^2	coulomb/ 4π meter^2	statcoulomb/4πcm^2, esu (gaussian).	abcoulomb/4π cm^2, emu	statcoulomb/ $\sqrt{4\pi}$ cm^2.	12
13	Electric polarization	$\mathbf{P}_e$	coulomb/meter^2		statcoulomb/cm^2, esu (gaussian).	abcoulomb/cm^2, emu	statcoulomb/ $\sqrt{4\pi}$ cm^2.	13
14	Capacitance	C	farad (f)		statfarad, esu (gaussian)	abfarad, emu	statfarad/4π	14
15	Resistance [a]	R	ohm (Ω)		statohm, esu (gaussian)	abohm, emu	4π statohm	15
16	Conductance [b]	G	mho (siemens)		statmho, esu (gaussian)	abmho, emu	statmho/4π	16
17	Resistivity	ρ	ohm-m (Ω-m)		statohm-cm, esu (gaussian)	abohm-cm, emu	4π statohm-cm	17
18	Conductivity	γ	mho/m		statmho/cm, esu (gaussian)	abmho/cm, emu	statmho/4π cm	18
19	Inductance	L	henry (h)		stathenry,[d] esu (gaussian)	abhenry,[d] emu (gaussian)	4π abhenry, 4π stathenry[d].	19
20	Magnetic flux	Φ	weber (wb)		statweber, esu	maxwell, line, emu (gaussian).[c]	$\sqrt{4\pi}$ maxwell	20
21	Magnetic induction	$\mathbf{B}$	tesla (t)		stattesla, esu	gauss, emu (gaussian)	$\sqrt{4\pi}$ gauss	21
22	Magnetic polarization	$\mathbf{J}$	tesla (t)	4π tesla	4π stattesla, esu	4π gauss, emu (gaussian)	$\sqrt{4\pi}$ gauss	22
23	Magnetic field strength.	$\mathbf{H}$	ampere-turn/ meter.	ampere-turn/ 4π meter.	statampere-turn/4π cm, esu	oersted, emu (gaussian)	$\sqrt{4\pi}$ oersted	23
24	Magnetization	$\mathbf{M}$	ampere-turn/ meter.					24
25	Magnetomotive force	$\mathscr{F}$	ampere-turn	ampere-turn/ 4π.	statampere turn/4π, esu	gilbert, emu (gaussian)	$\sqrt{4\pi}$ gilbert	25
26	Reluctance	$\mathscr{R}$	ampere-turn/ weber.	ampere-turn/ 4π weber.		maxwell/gilbert, emu (gaussian).	maxwell/gilbert	26

Notes on Table 4

General: Table 4 shows in each row for some particular physical quantity the correspondences between it, with its physical units used by the Realist, and the symbolic quantities and units used by the Synthetiker. Column 1 contains the name of the physical quantity and column 2 the abbreviation for the quantity used by the Realist when for example he writes "{**H**}" for the measure of magnetic field strength. Columns 3, 4, 5, 6, and 7 contain merely the names of the corresponding germane physical units used in the 6 different measurement systems by the Realist to measure the quantities listed in column 1. In column 4 only those germane physical units have been listed which are different from the corresponding physical units of the rationalized system. Unfortunately the complete definition of any one of these physical units is impossibly long to use in a Table. Even a name such as "ampere-turn/4π meter" in column 4 row 23 should be considered merely as an abbreviation for "that sample of magnetic field strength present in a long slender solenoid when the excitation is caused by a current sheet having 1 ampere for each 4π meters of axial length." The name should not be considered a quotient obtained by dividing separate factors.

In contrast the letter symbols in columns 8, 12, and 14 are those used by the Synthetiker to designate the symbolic quantities which he uses in the six systems and the entries in columns 9, 10, 11, 13, and 15 are his symbolic units. The subscripts $n—f$, $r—f$, s, m, g, h for the symbolic quantities denote that they are used respectively in the unrationalized (${}_nX_f$), and rationalized (${}_rX_f$) 4-dimensional systems, the classic 3-dimensional CGS electrostatic (X_s), electromagnetic (X_m), Gaussian (X_g) and Heaviside-Lorentz (X_h) systems (see also column 13, table 3).

The correspondences can be seen by following any row. Thus in row 23 to the Realist's single physical quantity **H** of column 2, the Synthetiker may set up a correspondence with either ${}_n\mathbf{H}_f$ or ${}_r\mathbf{H}_f$ of column 8 or $\mathbf{H}_s$ of column 12 or $\mathbf{H}_m$, or $\mathbf{H}_h$ of column 14 ($\mathbf{H}_g$ is identical with $\mathbf{H}_m$) depending upon which measurement system and set of equtaions he prefers to use. However the Synthetiker can use the single symbolic unit cm$^{-1/2}$g$^{1/2}$s^{-1} in column 15 to measure either H_m in the unrationalized CGS electromagnetic system or H_h in the Heaviside-Lorentz system. The Realist uses the oersted (column 6) in the former and a nameless unit (column 7) larger by $\sqrt{4\pi}$ in the latter system.

Symbolic quantities and their coherent symbolic units

Row	4-dimensional				3-dimensional				Row
	Symbolic quantity	MKSA	CGS–Bi	CGS–Fr	CGS–ESU		CGS–EMU		
		Symbolic units			Symbolic quantity	Symbolic unit	Symbolic quantity	Symbolic unit	
	8	9	10	11	12	13	14	15	
1	L	m	cm	cm	L	cm	L	cm	1
2	M	kg	g	g	M	g	M	g	2
3	T	s	s	s	T	s	T	s	3
4	$\mathbf{F}$	$\mathrm{m\,kg\,s^{-2}}$	$\mathrm{cm\,g\,s^{-2}}$	$\mathrm{cm\,g\,s^{-2}}$	$\mathbf{F}$	$\mathrm{cm\,gs^{-2}}$	F	$\mathrm{cm\,gs^{-2}}$	4
5	W	$\mathrm{m^2kg\,s^{-2}}$	$\mathrm{cm^2g\,s^{-2}}$	$\mathrm{cm^2g\,s^{-2}}$	W	$\mathrm{cm^2gs^{-2}}$	W	$\mathrm{cm^2gs^{-2}}$	5
6	P	$\mathrm{m^2kg\,s^{-3}}$	$\mathrm{cm^2g\,s^{-3}}$	$\mathrm{cm\,g\,s^{-3}}$	P	$\mathrm{cm^2gs^{-3}}$	P	$\mathrm{cm^2gs^{-3}}$	6
7	I_f	a	bi	$\mathrm{s^{-1}fr}$	I_s,I_g,I_h	$\mathrm{cm^{3/2}g^{1/2}s^{-2}}$	I_m	$\mathrm{cm^{1/2}g^{1/2}s^{-1}}$	7
8	V	$\mathrm{m^2kg\,s^3a^{-1}}$	$\mathrm{cm^2g\,s^{-3}bi^{-1}}$	$\mathrm{cm^2g\,s^{-2}fr^{-1}}$	V_s,V_g,V_h	$\mathrm{cm^{1/2}g^{1/2}s^{-1}}$	V_m	$\mathrm{cm^{3/2}g^{1/2}s^{-2}}$	8
9	$\mathbf{E}_f$	$\mathrm{m\,kg\,s^{-3}a^{-1}}$	$\mathrm{cm\,g\,s^{-3}bi^{-1}}$	$\mathrm{cm\,gs^{-2}fr^{-1}}$	$\mathbf{E}_s,\mathbf{E}_g,\mathbf{E}_h$	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	E_m	$\mathrm{cm^{1/2}g^{1/2}s^{-2}}$	9
10	Q_f	sa	$\mathrm{s\,bi}$	fr	Q_s,Q_s,Q_h	$\mathrm{cm^{3/2}g^{1/2}s^{-1}}$	Q_m	$\mathrm{cm^{1/2}g^{1/2}}$	10
11	${}_n\Psi_f,\ {}_r\Psi_f$	sa	$\mathrm{s\,bi}$	fr	Ψ_s,Ψ_g,Ψ_h	$\mathrm{cm^{3/2}g^{1/2}s^{-1}}$	Ψ_m	$\mathrm{cm^{1/2}g^{1/2}}$	11
12	${}_n\mathbf{D}_f,\ {}_r\mathbf{D}_f$	$\mathrm{m^{-2}sa}$	$\mathrm{cm^{-2}s\,bi}$	$\mathrm{cm^{-2}fr}$	$\mathbf{D}_s,\mathbf{D}_g,\mathbf{D}_h$	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	D_m	$\mathrm{cm^{-3/2}g^{1/2}}$	12
13	${}_n\mathbf{P}_f,{}_r\mathbf{P}_f$	$\mathrm{m^{-2}sa}$	$\mathrm{cm^{-2}s\,bi}$	$\mathrm{cm^{-2}fr}$	$\mathbf{P}_s,\mathbf{P}_g,\mathbf{P}_h$	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	P_m	$\mathrm{cm^{-3/2}g^{1/2}}$	13
14	C_f	$\mathrm{m^{-2}kg^{-1}s^4a^2}$	$\mathrm{cm^{-2}g^{-1}s^4bi^2}$	$\mathrm{cm^{-2}g^{-1}s^2fr^2}$	C_s,C_g,C_h	cm	C_m	$\mathrm{cm^{-1}s^2}$	14
15	$R_f,X_f,\ Z_f$ [a]	$\mathrm{m^2kg\,s^{-3}a^{-2}}$	$\mathrm{cm^2g\,s^{-3}bi^{-2}}$	$\mathrm{cm^2g\,s^{-1}fr^{-2}}$	R_s,R_g,R_h [a]	$\mathrm{cm^{-1}s}$	R_m [a]	$\mathrm{cm\,s^{-1}}$	15
16	G_f,B_f,Y_f [b]	$\mathrm{m^{-2}kg^{-1}s^3a^2}$	$\mathrm{cm^{-2}g^{-1}s^3bi^2}$	$\mathrm{cm^{-2}g^{-1}s\,fr^2}$	G_s,G_g,G_h [b]	$\mathrm{cm\,s^{-1}}$	G_m [b]	$\mathrm{cm^{-1}s}$	16
17	ρ_f	$\mathrm{m^3kg\,s^{-3}\,a^{-2}}$	$\mathrm{cm^3gs^{-3}bi^{-2}}$	$\mathrm{cm^3gs^{-1}fr^{-2}}$	ρ_s,ρ_g,ρ_h	s	ρ_m	$\mathrm{cm^2s^{-1}}$	17
18	γ_f	$\mathrm{m^{-3}kg^{-1}s^3a^2}$	$\mathrm{cm^{-3}g^{-1}s^3bi^2}$	$\mathrm{cm^{-3}g^{-1}s\,fr^2}$	$\gamma_s,\gamma_g,\gamma_h$	$\mathrm{s^{-1}}$	γ_m	$\mathrm{cm^{-2}s}$	18
19	L_f	$\mathrm{m^2kg\,s^{-2}a^{-2}}$	$\mathrm{cm^2g\,s^{-2}bi^{-2}}$	$\mathrm{cm^2gfr^{-2}}$	$L_s,{}^eL_g,{}^eL_n$	$\mathrm{cm^{-1}s^2}$	$L_m,{}^mL_g,{}^mL_h$ [d]	cm	19
20	Φ_f	$\mathrm{m^2kg\,s^{-2}a^{-1}}$	$\mathrm{cm^2gs^{-2}bi^{-1}}$	$\mathrm{cm^2gs^{-1}fr^{-1}}$	Φ_s	$\mathrm{cm^{1/2}g^{1/2}}$	Φ_m,Φ_g,Φ_h	$\mathrm{cm^{3/2}g^{1/2}s^{-1}}$	20
21	$\mathbf{B}_f$	$\mathrm{kg\,s^{-2}a^{-1}}$	$\mathrm{gs^{-2}bi^{-1}}$	$\mathrm{gs^{-1}fr^{-1}}$	$\mathbf{B}_s$	$\mathrm{cm^{-3/2}g^{1/2}}$	B_m,B_g,B_h	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	21
22	$\mathbf{J}_f$	$\mathrm{kg\,s^{-2}a^{-1}}$	$\mathrm{gs^{-2}bi^{-1}}$	$\mathrm{gs^{-1}fr^{-1}}$	$\mathbf{J}_s$	$\mathrm{cm^{-3/2}g^{1/2}}$	J_m,J_g,J_n	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	22
23	${}_n\mathbf{H}_f,{}_r\mathbf{H}_f$	$\mathrm{m^{-1}a}$	$\mathrm{cm^{-1}bi}$	$\mathrm{cm^{-1}s^{-1}fr}$	$\mathbf{H}_s$	$\mathrm{cm^{1/2}g^{1/2}s^{-2}}$	$H_mH_gH_h$	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	23
24	$\mathbf{M}_f$	$\mathrm{m^{-1}a}$	$\mathrm{cm^{-1}bi}$	$\mathrm{cm^{-1}s^{-1}fr}$	$\mathbf{M}_s$	$\mathrm{cm^{1/2}g^{1/2}s^{-2}}$	M_m	$\mathrm{cm^{-1/2}g^{1/2}s^{-1}}$	24
25	${}_n\mathscr{F}_f,{}_r\mathscr{F}_f$	a	bi	$\mathrm{s^{-1}fr}$	$\mathscr{F}_s$	$\mathrm{cm^{3/2}g^{1/2}s^{-2}}$	$\mathscr{F}_m,\mathscr{F}_g,\mathscr{F}_h$	$\mathrm{cm^{1/2}g^{1/2}s^{-1}}$	25
26	${}_n\mathscr{R}_f,\ {}_r\mathscr{R}_f$	$\mathrm{m^{-2}kg^{-1}s^2a^2}$	$\mathrm{cm^{-2}g^{-1}s^2bi^2}$	$\mathrm{cm^{-2}g^{-1}fr^2}$	$\mathscr{R}_s$	$\mathrm{cm\,s^{-2}}$	$\mathscr{R}_m,\mathscr{R}_s,\mathscr{R}_h$	$\mathrm{cm^{-1}}$	26

Specific Notes:

[a] The names of the units shown for resistance are also used to express reactance and impedance.

[b] The names of the units shown for conductance are also used to express susceptance and admittance.

[c] The notation "(gaussian)" applied to certain unit names in columns 5 and 6 indicates that these constitute the set of physical units used in the symmetrical CGS or Gaussian system.

[d] In the symmetrical systems inductance may be regarded either as an electric quantity (symbol ${}^eL$) or as a magnetic quantity (symbol ${}^mL$). The physical units appropriate to these two cases are the stathenry and the abhenry respectively. In the Heaviside-Lorentz system either unit is greater by a factor of 4π. The corresponding symbolic quantities are listed in column 12 and 14 and are defined by equation 10, table 1, and equation 21, table 2, respectively.

Table 5. *Prefixes for decimal multiples*

Prefix	Abbreviation	Factor
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10^{+1}
hecto	h	10^{+2}
kilo	k	10^{+3}
mega	M	10^{+6}
giga	G	10^{+9}
tera	T	10^{+12}

The prefix "myria" is sometimes used for 10^{+4} and "lakh" for 10^{+5}.

Note.—Table 6 is located on page 165.

Table 7. *Conversion of symbolic quantities in quantity equations*

Relations between unrationalized symbolic quantities "$_nX_f$" and symbolic quantities "$_rX_f$" rationalized in the Fessenden-Giorgi manner used in quantity equations in systems based on four basic symbolic units

1 Quantity	2 3 Rationalized Unrationalized
Displacement density	$4\pi{}_r\mathbf{D}_f={}_n\mathbf{D}_f$
Electric flux	$4\pi{}_r\Psi_f={}_n\Psi_f$
Absolute permittivity	$4\pi\epsilon_r\Gamma_e=\epsilon_n\Gamma_e$
Electric susceptibility	${}_r\chi_e/4\pi={}_n\chi_e$
Magnetic field strength	$4\pi{}_r\mathbf{H}_f={}_n\mathbf{H}_f$
Magnetomotive force	$4\pi{}_r\mathscr{F}_f={}_n\mathscr{F}_f$
Magnetic polarization	${}_r\mathbf{J}_f/4\pi={}_n\mathbf{J}_f$
Reluctance	$4\pi{}_r\mathcal{R}_f={}_n\mathcal{R}_f$
Absolute permeability	$\mu_r\Gamma_m/4\pi=\mu_n\Gamma_m$
Magnetic susceptibility	${}_r\chi_m/4\pi={}_n\chi_m$
Magnetization	${}_r\mathbf{M}_f={}_n\mathbf{M}_f$ (or $\mathbf{I}$)

To change an unrationalized equation to the rationalized form substitute the corresponding item in column 2 for each item in column 3 which appears in the equation; and conversely.

Table 8. *Conversion of measures*

Multiply the measure in germane or coherent units of the system listed at the top of the column by the factor listed in the table to obtain the measure in the MKSA rationalized system. Here c is $2.997925 \cdot 10^{10}$

Row			MKSA	CGS–ESU	CGS–EMU	Gaussian	Heaviside-Lorentz
				Unrationalized			Rationalized
1	Length	L	1	10^{-2}	10^{-2}	10^{-2}	10^{-2}
2	Mass	M	1	10^{-3}	10^{-3}	10^{-3}	10^{-3}
3	Time	T	1	1	1	1	1
4	Force	$\mathbf{F}$	1	10^{-5}	10^{-5}	10^{-5}	10^{-5}
5	Work	W	1	10^{-7}	10^{-7}	10^{-7}	10^{-7}
6	Power	P	1	10^{-7}	10^{-7}	10^{-7}	10^{-7}
7	Current	I	1	$10/c$	10	$10/c$	$10/\sqrt{4\pi}c$
8	Voltage	V	1	$10^{-8}c$	10^{-8}	$10^{-8}c$	$10^{-8}\sqrt{4\pi}c$
9	Elec. gradient	$\mathbf{E}$	1	$10^{-6}c$	10^{-6}	$10^{-6}c$	$10^{-6}\sqrt{4\pi}c$
10	Charge	Q	1	$10/c$	10	$10/c$	$10/\sqrt{4\pi}c$
11	Elec. flux	Ψ	$1/4\pi$	$10/4\pi c$	$10/4\pi$	$10/4\pi c$	$10/\sqrt{4\pi}c$
12	Elec. displacement	$\mathbf{D}$	$1/4\pi$	$10^5/4\pi c$	$10^5/4\pi$	$10^5/4\pi c$	$10^5/\sqrt{4\pi}c$
13	Elec. polarization	$\mathbf{P}_e$	1	10^5c	10^5	10^5c	$10^5/\sqrt{4\pi}c$
14	Capacitance	C	1	$10^9/c^2$	10^9	$10^9/c^2$	$10^9/4\pi c^2$
15	Resistance	R	1	$10^{-9}c^2$	10^{-9}	$10^{-9}c^2$	$10^{-9}\ 4\pi c^2$
16	Conductance	G	1	$10^9/c^2$	10^9	$10^9/c^2$	$10^9/4\pi c^2$
17	Resistivity	ρ	1	$10^{-11}c^2$	10^{-11}	$10^{-11}c^2$	$10^{-11}\ 4\pi c^2$
18	Conductivity	γ	1	$10^{11}/c^2$	10^{11}	$10^{11}/c^2$	$10^{11}/4\pi c^2$
19	Inductance	L	1	$10^{-9}c^2$	10^{-9}	$10^{-9}c^2$, 10^{-9} [a]	$10^{-9}4\pi c^2$, $10^{-9}4\pi$ [a]
20	Mag. flux	Φ	1	$10^{-8}c$	10^{-8}	10^{-8}	$10^{-8}\sqrt{4\pi}$
21	Mag. induction	$\mathbf{J}$	1	$10^{-4}c$	10^{-4}	10^{-4}	$10^{-4}\sqrt{4\pi}$
22	Mag. polarization	J	4π	$4\pi \cdot 10^{-4}c$	$4\pi \cdot 10^{-4}$	$4\pi\ 10^{-4}$	$10^{-4}\sqrt{4\pi}$
23	Mag. field strength	$\mathbf{H}$	$1/4\pi$	$10^3/4\pi c$	$10^3/4\pi$	$10^3/4\pi$	$10^3/\sqrt{4\pi}$
24	Magnetization	$\mathbf{M}$	1	$10^3/c$	10^3	10^3	$10^3/\sqrt{4\pi}$
25	Magnetomotive force	$\mathscr{F}$	$1/4\pi$	$10/4\pi c$	$10/4\pi$	$10/4\pi$	$10/\sqrt{4\pi}$
26	Reluctance	$\mathcal{R}$	$1/4\pi$	$10^9/4\pi c^2$	$10^9/4\pi$	$10^9/4\pi$	$10^9/4\pi$
27	Permittivity	$\epsilon\Gamma_e$	$1/4\pi$	$10^{11}/4\pi c^2$	$10^{11}/4\pi$	$10^{11}/4\pi c^2$	$10^{11}/4\pi c^2$
28	Permeability	$\mu\Gamma_m$	4π	$4\pi \cdot 10^{-7}c^2$	$4\pi \cdot 10^{-7}$	$4\pi \cdot 10^{-7}$	$4\pi \cdot 10^{-7}$

[a] Inductance may be measured either in CGS electrostatic or electromagnetic units in the Gaussian System and in units greater by a factor of 4π in the Heaviside-Lorentz System.

10.2. Chronology—Electrical Units

1791 —Commission on the Meter received by Louis XVI.

1799 —Metric System legal in France by "Law of 18 Germinal, year 3."

1822 —J. B. J. Fourier published his "Theory of Heat" with discussion of physical dimensions.

1822 —A. M. Ampere suggested concepts of "electric tension" and "electric current."

1827 —G. S. Ohm published his "Law."

1833 —K. F. Gauss introduced absolute measurements in terrestrial magnetism.

1840 —W. Weber introduced absolute meas. of current, tangent galvanometer.

1851 —W. Weber introduced absolute meas. of resistance.

1860 —W. Siemens used Hg column as standard of resistance 1 m × 1 sq mm.

1862–67 —Brit. Assoc. Committee on Electrical Standards active, ohm = 10^7 MGS unit, "Weber" = unit of charge.

1867 —BA Comm. 5th Report used "farad" for 10^{-13} MGS electromagnetic unit (i.e., 1 microfarad).

1867–73 —Brit. Assoc. Committee on the Selection and Nomenclature of Dynamical and Electrical Units—CGS system, dyne, erg, prefixes from micro to mega-, horsepower = $7.46 \cdot 10^9$ erg/sec, volt = 10^{+8} CGS em, farad = 10^{-9} CGS em, B. A. ohm = 10^9 CGS em = 104.8 cm × 1 sq mm Hg column.

1872 —Clark Zn–Hg standard cell proposed as 1.457 (!) volt.

1875 —Convention of the Meter established Internat. Committee on Weights and Measures.

1881 1st Internat. Elec. Congress (Paris) 1 ampere=1 volt/1 ohm, 1 farad=1 coulomb/1 volt.

1882 —O. Heaviside first suggested rationalization.

1884 —Committee of 1st Int. Cong. reports 1 "legal ohm"=resist. of 106. cm×1 sq mm Hg.

1887 —Physikalisch-technische Reichsanstalt founded in Berlin.

1889 —2nd Internat. Elec. Cong. (Paris) joule, watt and quadrant (i.e., 10^7 meters) as units of energy, power, and inductance.

1891 —3rd Internat. Elec. Cong. (Frankfort).

1892 —Informal Elec. Cong. (Edinburgh).

1893 —Weston Cd-Hg standard cell=1.018 volt. 4th Internat. Elec. Cong. (Chicago) confirmed decimal multiple CGS basic for joule, watt, volt, coulomb, farad and henry, but offered "equal" alternative ampere—0.00118 g/sec Ag; ohm=106.3 cm Hg, volt=1/1.434×Clark cell.

1894 —Above alternative units made legal in US and UK.

1894 —A.I.E.E. proposed gilbert, weber, oersted, gauss as names for CGS electromagnetic units of magnetomotive force, flux, reluctance, and induction, respectively.

1896 —Alternative units made legal in France.

1898 —Alternative units made legal in Germany.

1899 —Natl. Physical Lab. founded in Teddington, England.

1900 —Fessenden and others suggested subrationalization, 5th Internat Elec. Cong. (Paris) maxwell for unit of flux; gauss for unit of "magnetic intensity" (taken by some as H, by others as B).

1901 —Giorgi suggested subrationalized, 4-dimensional, MKS system, Natl. Bur. of Standards founded at Washington.

1904 —6th Internat. Elec. Congress (St. Louis) set up permanent Internat. Electrotech. Commission.

1905 —Conf. of Nat. Std. Labs. (Berlin) ohm and ampere to be basic, volt derived, Weston cell substituted for Clark cell as reference standard.

1908 —Internat. Conf. on Electrical Units and Standards (London). Distinguished between (1) "practical" decimal multiples of CGS em and (2) "International" ohm and ampere defined by Hg and Ag.

1910–11 —Internat. Technical Comm at NBS set consistent International "Washington Unit" values for coils and cells. Weston normal cell derived at 1.0183 volt.

1911 —7th Internat. Elec. Cong. (Turin) definitions and symbols; I (not C) for current; $R+jX$ for inductive resistor.

1921 —6th Internat. General Conf. on Weights and Measures extended scope of ICWM to cover Electricity and Photometry.

1927 —Consultative Committee on Electricity set up by ICWM.

1928 —IEC (Bellagio) sets up subcommittee on Magnetic Units.

1928 —Int. Bur. of Weights and Measures with enlarged buildings and staff began periodic intercomparison of electrical standards.
AIEE Stds, Comm. urged shift from "International" to "absolute" units.

1930 —IEC (Stockholm and Oslo) voted $B = \mu_0 H$ with μ_0 having dimensions; confirmed CGS units of: flux = maxwell; flux density = gauss; field strength = oersted, magnetomotive force = gilbert. Proposed units of frequency = hertz, of reactive power = var.

1931 —IEC divided TC No. 1 to create Subcommittee on Electric and Magnitudes and Units (EMMU)
IUPAP creates Committee on Symbols, Units, and Nomenclature (SUN)

1932 —EMMU (Paris) proposed "weber = 10^8 maxwells; siemens = mho. 8th Gen. Conf. Weights and Measures authorized change to "absolute" electrical units at discretion of Internat. Committee.

1935 —EMMU (Scheveningen) adopted Giorgi (MKS) system with 4th unit left open; confirmed hertz and siemens, confirmed weber = 10^8 maxwells; Consultative Comm of ICWM advocated μ_0 as basis for "absolute" units.

1938 —EMMU (Torquay) recommended μ_0 as link to mechanics; proposed newton = 10^5 dynes.

1946 —Internat. Comm Weights and Measures set values for new absolute units and date for adoption.

1948 —Jan. 1. absolute electrical units effective.

1950 —U.S. Congress passed Public Law 617 fixing electrical units. EMMU (Paris) selected ampere as 4th unit; confirmed newton; recommended total rationalization of Giorgi (MKSA) System; appointed Comm of Experts to interpret "rationalization."

1951 —IUPAP recommended "that in the case that the equations are rationalized, the rationalization should be effected by the introduction of new quantities."
SUN in Doc. 51–5 proposed 4-dimensional CGS system.

1952 —I.S.O. Tech Comm No. 12 issued draft table of quantities and units.

1954 —EMMU (now Tech Comm No. 24) (Philadelphia) approved rationalized equations; proposed tesla = 1 weber/meter ^{2}. 10th Gen. Conf. on Weights and Measures established "System International" (SI) of units based on meter, kilogram, second, ampere, candela, and degree Kelvin.

1956 —IEC (Munich) confirmed tesla.

1956 —SUN issued Document 56–7 [13] defining susceptibility; questioning current symbols for permeability and permittivity; stating status of symbols for electro-magnetic and dipole moments.

1958 —T. C. 24 (of IEC) (Stockholm) discussed rationalization, revised convention on sign of reactive power.

1959 —T. C. 24 (Madrid) discussed rationalization; proposed lenz = 1 ampere-turn/meter.

1960 —11th Gen. Conf. on Weights and Measures defined meter by wave length of Kr^{86}; defined second by tropical year; confirmed IEC name tesla.

10.3. Notation, Glossary, and Organizations

a. Notation

X	Generalized abbreviation to identify any physical quantity (see column 2, table 4).
X_i	General symbol for a symbolic quantity which serves as an element in a mathematical model.
$\{X\}_a$	Measure of X in system a.
$_aU_x$	Germane physical unit of X in system a.
$\langle X_i \rangle_a$	Coherent symbolic unit of X_i in system a.
$\{X_i\}_a$	Measure of X_i in system a.
$[X_i]$	Dimension of X_i.
ϵ	Relative permittivity.
μ	Relative permeability.
Γ_e, Γ_m, Γ_r, Γ_s	Parameters. See table 3 and "Constant, magnetic . . ." below.

b. Glossary

Absolute—An adjective applied to "method" or "measurement" to designate an operation in which a quantity is measured, usually indirectly and in terms of the ultimate basic units (usually those of length, mass and time) of the measurement system used.

Additivity—That attribute of a physical quantity as a result of which the measure of the conventional resultant combination of two or more examples of the quantity is equal to the sum of the measures of the component physical quantities.

Basic—An adjective applied to "unit" or "quantity" to identify members of the small group of $p=N-n$ units or quantities from which the Realist derives his other physical units and the Synthetiker his other symbolic quantities. Other writers use "fundamental."

Coherent—An adjective applicable to a symbolic unit which indicates that it is related simply and consistently to (1) the basic units of the system and (2) the dimensions of the symbolic quantity of which it is a unit. Note: when coherent symbolic units are used the measure equation in terms of them is identical in form with the quantity equation. (Guggenheim uses "germane" in this sense.)

Constant, electric, Γ_e—The factor of proportionality which relates electric charges to the electrostatic forces they produce. In an electrostatic system it has a conventionally chosen value characteristic of the measurement system. In an electromagnetic system it has a value derived experimentally from a conventionally chosen magnetic constant. Most writers hitherto have called this quantity "permittivity (or capacitivity) of space" with the symbol ϵ_0 or ϵ_v.

Constant, magnetic, Γ_m—The factor of proportionality which relates electric currents to the electrodynamic forces they produce. In an electromagnetic system it has a conventionally chosen value characteristic of the measurement system. In an electrostatic system it has a value derived experimentally from a conventionally chosen electric constant. Most writers hitherto have called this quantity "permeability of space" with the symbol μ_0 or μ_v.

Dimension—A label of convenience which indicates for a derived symbolic quantity the relative rate at which it would vary with virtual variations in the basic symbolic quantities of the system. By extension, to a similar encoded relation to other symbolic quantities not necessarily basic. Dimensions form elements of a multiplicative group. Hence the product of any pair of dimensions is a dimension. The unit element of the group is "numeric" or "pure number" which is therefore a dimension.

Dimensional exponent—The exponent relating the relative rates of change of a derived symbolic quantity and a more basic symbolic quantity in a measurement system. For example if $X=f(Y,Z)$ the dimensional exponent of X relative to Y is $n=\frac{Y}{X}\cdot\frac{\partial X}{\partial Y}$. Other writers use "dimension" (see sec. 7).

Germane—An adjective applicable to a physical unit which indicates that it is related simply and consistently to (1) the basic units of the system and (2) the coefficients in the equations of the system.

Kind—That attribute of a physical quantity which distinguishes it qualitatively in regard to its physical nature, its relation to the phenomena, etc. from quantities of other kinds. Two physical quantities are of the same kind if operational methods are available for the meaningful comparison of their relative magnitudes.

Magnitude—That attribute of a quantity which distinguishes it quantitatively in regard to size, extent, intensity, etc., relative to other quantities of the same kind.

Measure—The number obtained by either (1) measuring a physical quantity by comparing it, experimentally with a physical unit of the same kind; or (2) by dividing a symbolic quantity by a symbolic unit of the same kind. Other writers have also used "magnitude;" "value," "numerical value."

Quantity, physical—Any example of a "real" physical entity, as conceived by the experimenter for the precise description of a phenomenon and operationally defined so as to be measurable. It is characterized by its kind and magnitude. Other writers have also used "entity," "physical entity," "magnitude," "quantity," "experimental quantity," "concrete quantity" for this concept.

Quantity, symbolic—Any example of an element which, in a mathematical model, corresponds to some physical quantity in nature. Other writers have also used "concrete quantity" (Maxwell), "abstract quantity," "mathematical variable," "magnitude," "idon" for this concept.

Rationalization—A name given by Oliver Heaviside to the use of a (in his opinion) more rational set of coefficients in the electromagnetic equations. He assumed this to be secured by the use of a set of rationalized derived units. In a set of rationalized equations the factor "4π" is made to appear only in those equations involving geometric arrangements having spherical symmetry.

Realist—A fictitious character postulated to perform experimental measurement operations and to use mathematical manipulation on measure equations only. He therefore deals only with physical quantities, physical units, and measure equations.

Standard, physical—A physical system some property of which embodies an example of a physical quantity to which a value has been assigned to indicate its supposed measure in terms of some physical unit.

Standard, prototype—A standard which serves to define a basic physical unit of a measurement system by fixing independently an essential feature of its definition. Some writers (e.g., A. G. McNish [14, 15] limit this adjective to standards which are entirely independent of values assigned to all other prototype standards.

Standard, reference—The standard or group of standards of highest rank in a given laboratory which serve to maintain in that laboratory the unit of some physical quantity.

Synthetiker—A fictitious character postulated to use only quantity equations which express the relations among symbolic quantities. He derives symbolic units in terms of which he can formally write measures for symbolic quantities.

Unit—A particular sample of a quantity either physical or symbolic in terms of which the quantity can be measured or expressed quantitatively.

Unit, physical—A particular sample of a physical quantity of such magnitude that it is assigned the measure "1."

Unit, symbolic—A particular sample of a symbolic quantity of such magnitude that it is assigned the measure "1."

c. Organizations

ICWM —International Committee on Weights and Measures. Pavillon Breteuil, Sêvres, France (French initials CIPM). See footnote 3, p. 139.

IBWM —International Bureau of Weights and Measures (French initials BIPM). See footnote 3, p. 139.

IEC —International Electrotechnical Commission (French initials CEI) founded 1904. Serves as organization of UNESCO in field of electrical engineering.

EMMU—Electric and Magnetic Magnitudes and Units. Former name of IEC Technical Committee TC 24 dealing with this subject.

IUPAP—International Union of Pure and Applied Physics. Serves as organization of UNESCO in field of physics.

SUN —Symbols, Units and Nomenclature. Committee of IUPAP on this subject.

ISO —International Standards Organization. Is the branch of UNESCO for standardization. Its Technical Committee TC 12 cooperates very closely with TC 24 of IEC in field of electrical engineering.

ASA —Am. Standards Association. Coordinates standardization activities of professional societies in the U.S. and internationally. Its Committee C61 cooperates with TC 24 of IEC.

d. National Standardizing Laboratories

NBS —National Bureau of Standards, Washington 25, D.C. and Boulder, Colorado.

NPL —National Physical Laboratory, Teddington, England.

PTB —Physikalisch-technische Bundesanstalt, Braunschweig, West Germany.
IM —Institute of Metrology, Leningrad, USSR.
LCIE —Laboratoire Centrale des Industries Electriques, Fontenay-aux-Roses, France.
BFPM —Bureau Federale des Poids et Measures, Berne, Switzerland.
ETL —Electrotechnical Laboratory, Tokyo, Japan.
NSL —National Standards Laboratory, Chippendale, NSW, Australia.
NRC —National Research Council of Canada, Ottawa, Ontario.

10.4. References

The literature dealing with systems of units and dimensions is extremely voluminous. A valuable and extensive list of such references in all branches of physical measurements is given by Stille. [10]. The references listed below have been selected largely to document particular points stressed in the text, or to refer to items of current or historical interest or to valuable summary papers in English.

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(Paper 66C2–94)

Publications of the National Bureau of Standards*

Selected Abstracts

Correction factors for the calibration of encapsulated radium sources, R. M. Lee and T. P. Loftus, *J. Research NBS* **66A** *(Phys. and Chem.) No. 2, (Mar.–Apr. 1962) 70 cents.*
Several aspects of the procedure and corrections for the calibration of encapsulated radium sources at NBS have recently been investigated. It was found that a chamber equipped with a guard ring type electrode system allowing the use of a vibrating reed electrometer as a current detector provides more versatility and precision than the gold leaf electroscope now in use for routine calibrations. Absorption corrections for the U.S. primary national radium standards have been determined for the NBS chamber: 0.78% for standard 5440 and 1.01% for standard 5437. The Owen-Naylor integral equation for absorption of rays in the walls of cylindrical radium sources has been evaluated by a power series expansion of the integrand. Absorption coefficients and correction factors for platinum and Monel metal (materials commonly used for source capsules) have been computed for the NBS chamber.

Revised standard values for pH measurements from 0 to 95 °C, R. G. Bates, *J. Research NBS* **66A** *(Phys. and Chem.) No. 2 (Mar.–Apr. 1962) 70 cents.*
Seven primary standard solutions serve to fix the NBS conventional activity scale of pH (termed $p\mathrm{H}_s$) from 0 to 95 °C. The original emf data have been re-examined, and the values of the acidity function $p(a\mathrm{H}\gamma_{\mathrm{Cl}})$, from which $p\mathrm{H}_s$ is derived, have been recalculated with the use of a single consistent set of standard potentials and electrochemical constants. The convention proposed recently by Bates and Guggenheim for the numerical evaluation of the individual activity coefficient of chloride ion in the buffer solutions has been adopted, and by this means $p\mathrm{H}_s$ values to the third decimal have been assigned. These "experimental" $p\mathrm{H}_s$ values in the temperature range 0 to 95 °C have been smoothed as a function of temperature by least-squares treatment. The properties and uses of the standards are discussed and directions for the preparation of the solutions are given.

Cross-sectional correction for computing Young's modulus from longitudinal resonance vibrations of square and cylindrical rods, W. E. Tefft and S. Spinner, *J. Research NBS* **66A** *(Phys. and Chem.) No. 2 (Mar.–Apr. 1962) 70 cents.*
The cross-sectional correction involved in the calculation of Young's modulus from the longitudinal resonance vibrations of both square and cylindrical bars has been determined by an empirical method.
On an order of accuracy of 1 part in 1000, Bancroft's correction, developed for traveling waves in cylinders was found to be satisfactory. For this purpose the thickness of the square bars is related to the diameter of an equivalent cylindrical bar by, $3d^2=4t^2$.
For accuracies of 1 part in 10,000, modifications in Bancroft's correction must be applied. These modifications take a different form for the square and cylindrical rods.

Bibliography and index on vacuum and low pressure measurement, W. G. Brombacher, *NBS Mono. 35 (Nov. 10, 1961) 60 cents.*
The bibliography contains 1538 references, of which 52 are on books. About 550 of the periodical references are specifically on pressure measurement including both vacuum gages and micromanometers. The balance are on vacuum technology, including adsorption, degassing, vacuum pumps, controlled gas leaks, valves, seals and vacuum systems, all of which bear on the technique of vacuum measurement. The indices consist of an author index and an index of the subject matter of the listed references.

Effect of mortar properties on strength of masonry, C. C. Fishburn, *NBS Mono. 36 (Nov. 20, 1961) 30 cents.*
The physical properties of mortars, the bond strength of the mortars to masonry units, and the structural strength of concrete masonry and composite masonry walls containing the mortars are discussed and compared. All of the mortars were tempered to as wet a consistency as could be conveniently handled by the mason.
The compressive strength of the walls increased, in general, with the compressive strength of the mortar. The racking and flexural strengths of the walls increased with the bond strength of the mortar. The strength of bond test specimens tended to increase with the compressive strength of the wet consistency mortars that were used. However, bond strength appeared to be the dominant factor affecting the racking and flexural strength of the walls. Increase in both bond strength and wall strength with compressive strength of the mortar was not proportional to the relative compressive strengths of the type N and type S mortars.
The stiffness of walls subjected to compressive and flexural loads increased with the bond and compressive strength of the mortars. However, the stiffness of walls subjected to flexural loads appeared to be more dependent upon the number of bed joints in the tensile face and on their extension in bond than upon the bending strains in the masonry materials.

Tabulation of data on microwave tubes, C. P. Marsden, W. J. Keery, and J. K. Moffitt, *NBS Handb. 70 (Nov. 1, 1961) $1.00.*
A tabulation of microwave electron tubes with characteristics of each type has been arranged in the form of two major listings, a Numerical Listing in which the tubes are arranged by type number, and a Characteristic Listing in which the tubes are arranged by the kind of tube, and further ordered on the basis of minimum frequency and power. The tabulation is accompanied by a listing of similar tube types and other manufacturers of certain types.

Safety rules for the installation and maintenance of electric supply and communications lines. Comprising Part 2, the definitions, and the grounding rules of the sixth edition of the national electrical safety code, *NBS Handb. 81 (1961) $1.75.*
This Handbook consists of definitions, grounding rules, and Part 2 of the sixth edition of the National Electrical Safety Code, dealing with the construction and maintenance of overhead and underground lines, previously published as National Bureau of Standards Handbook H32. The present edition of these rules is the result of a revision which has been carried out by the Sectional Committee in accordance with the procedure of the American Standards Association, and the text has been recognized as an American Standard. This revision serves to aline the rules with new developments and current practice in the industry. It represents the work of five technical subcommittees over a period of about eight years. Changes were made in approximately one hundred and fifty rules and definitions.

Behavior and evaluation of rubber, R. D. Stiehler, *Am. Concrete Pipe Assoc. Tech. Memo. (Oct. 1961).*
This paper was presented at the Short Course of Instruction, American Concrete Pipe Association in St. Louis on November 29, 1960. The elastic behavior of rubber is discussed and the evaluation of rubber with special emphasis on gaskets used to seal joints in concrete pipe is briefly described.

Welded butt joints with fine wires, L. Martz, *Rev. Sci. Instr.* **32,** *No. 8, 990–991 (Aug. 1961).*
This article describes a relatively simple laboratory technique for rapidly hand-producing welded butt junctions with fine wires.

Excess noise in microwave detector diodes, J. J. Faris and J. M. Richardson, *IRE Trans. Microwave Theory and Tech.* **MTT-9,** *No. 4, 312–314* (*July 1961*).
The dependence of available excess noise in type 1N26 microwave crystal diode rectifiers on applied microwave power was measured. This may be approximated by a power law with constants characteristic of the particular crystal. As a consequence of the dependence of both excess noise and d-c rectified power on input power level, there is a level which minimizes the ratio of these quantities. Similarly, in the case of a modulated microwave carrier there is an input level which minimizes the ratio of excess noise to demodulated power, and so provides optimum detection of small modulation.

A simple calibration technique for vibrating sample and coil magnetometers, N. V. Frederick, *Proc. IRE* **49,** *1449* (*Sept. 1961*).
A simple and convenient method for calibrating several modern magnetometers without reference to a "standard sample" is presented with some typical results.

Performance characteristics of turbine flowmeters, M. R. Shafer, *Trans. ASME J. Basic Eng. Paper No. 61–WA–25* (*1961*).
The general performance of turbine-type or propeller flowmeters operating on liquid hydrocarbons in the range 0.5 to 250 gpm is described. Particular characteristics investigated include the effects of flow rate, viscosity, pressure level, entrance flow pattern, and orientation on the performance of these meters. It is shown that metering precision better than 0.2 percent can be attained for selected ranges of flow rate and viscosity when entrance conditions and meter orientation are suitably controlled. Other factors briefly reviewed include dynamic response, totalization considerations, and the readout instrumentation.

Effect of mercury-alloy ratio on the physical properties of amalgams, W. T. Sweeney and C. L. Burns, *J. Am. Dental Assoc.* **63,** *No. 9, 374–381* (*Sept. 1961*).
Some physical properties of amalgams made from four dental alloys of widely different particle size were examined for mercury to alloy ratios between 1:1 and 10:1. The compressive strengths, dimensional changes on setting, flow, and residual mercury contents of the amalgams were determined by standard methods. For mercury-alloy ratios ranging from the manufacturers' recommended values to a ratio of 10:1 there was little observed effect on the compressive strength. Over this range the residual mercury content varied a maximum of 3% for any one alloy. An additional study was made of the effect of strain rate on crushing strength, using 4×8 mm cylindrical specimens. Varying head speed from 0.003 to 0.050 inch per minute produced crushing strengths ranging from 30,000 to 50,000 psi. These data indicate that the physical properties of amalgams are not significantly affected by the mercury-alloy ratio, provided an essential minimum of mercury is present.

Present status of panoramic roentgenography, J. W. Kumpula, *J. Am. Dental Assoc.* **63,** *No. 8, 194–200* (*Aug. 1961*).
Several panoramic techniques have been developed which can image entire dental arches and their associated structures on one film. Illustrations show full mouth roentgenographs. Concentric and eccentric techniques produce a variety of roentgenograms with adequate detail to obtain a diagnosis of the general mouth condition. This paper presents representative pictures using the various techniques.

Thermal conductivity of some commercial iron-nickel alloys, T. W. Watson and H. E. Robinson, *Trans. ASME, Series C. J. Heat Transfer* **83,** *No. 4, 403–408* (*Nov. 1961*).
Results of laboratory determinations of thermal conductivities in the temperature range −150 to 540 deg C are presented for 12 iron-nickel alloys. Six samples are of low nickel content, in the range from 1 to 9 per cent, and six others have nickel contents in the range from 35 to 80 per cent. A sample of AISI 1015 steel is included for comparative purposes.
The determinations were made on bar specimens about 2.54 cm in diameter and 37 cm long, by an absolute steady-state method with heat flowing longitudinally in the bar. Computation of results from observed data was effected by means of a digital computer.

Photographic response to successive exposures of different types of radiation, M. Ehrlich and W. L. McLaughlin, *J. Opt. Soc. Am.* **51,** *No. 11, 1172–1181* (*Nov. 1961*).
Reversal effects occurring as a result of exposure of photographic materials to two different types of radiation in succession, such as the Weinland, Clayden, Villard, or Herschel effects, and their opposites, have been discussed extensively in the literature. Nevertheless, a systematic analysis of the behavior of the photographic latent image under a specified set of exposure conditions is still lacking. The problem may be formulated in the following way: Given the response characteristics of an emulsion for several types of radiation of different wavelengths and intensities; is there a way to predict the characteristic behavior of the emulsion when any two of these types of radiation act upon the emulsion in sequence?
In the present paper, the authors show the results of experiments in which photographic films were given two successive exposures to X- and gamma radiation of different photon energies and intensities, to gamma radiation and visible light, and to visible light and infrared radiation. An analysis of the data leads to the conclusion that the second irradiation changes the shape of the photographic density-versus exposure curve characteristic for the first type of exposure into one closely resembling that characteristic for the second type. Associated with this process are, in some instances, changes in curve shape that suggest transformations of the latent image, which lead to reversal effects and to transitional sensitization and desensitization phenomena. Some of the double-exposure effects found in the literature are discussed in relation to the data presented here.

Multiple biologic recording for digital analysis, H. L. Mason, *Proc. Interdisciplinary Clinic on the Instrumentation Requirements for Psychophysiological Research, FIER Clinic on Psychophysiological Instrumentation, Lafayette Clinic, p. 65–67* (*May 16–17, 1961*).
A description is given of a recording system for digital analysis of a number of psychophysiological variables. Its present setup records the involuntary bodily reactions of a human subject.

Tongs used in testing for radioactive contamination, T. G. Hobbs, *Health Physics* **6,** *No. 3 & 4, 224a, 224b, 225* (*Oct. 1961*).
The principle hazard of smear-testing for radioactive contamination is that some of the contaminating material may get on the hands of the person making the test. The device illustrated considerably reduces this possibility by making it unnecessary for the hands to come near the area to be tested.
Ordinary laboratory tongs have been modified by attaching a ring and an insert at the end. The surfaces of the ring and insert are angled slightly so the smear paper will not drop through the ring when the paper is clamped between the ring and insert. The outer surface of the ring is angled to prevent its contact with the area to be smeared. Good surface contact between the paper and the area is provided by a felt pad or blotter paper glued to the lower face of the insert, which extends below the ring. Coating the tongs with strippable paint aids in decontamination, if necessary.

Calibration of vibration pickups at large amplitudes, E. Jones, S. Edelman, and K. S. Sizemore, *J. Acoust. Soc. Am.* **33,** *No. 11, 1462–1466* (*Nov. 1961*).
Axial resonances of long rods and tubes were used to generate motion for accurate calibration of vibration pickups over the frequency range from below one to above 20 kc at acceleration levels up to 12000g. The resonators were driven by an electromagnetic shaker at low frequencies and by a piezoelectric ceramic stack shaker at high frequencies. Vibration amplitude was measured optically by means of a microscope using stroboscopic light and by means of the interference fringe disappearance technique. Adequate overlap between the two methods was achieved by going up to the 60th dis-

appearance of the fringes. A simple, direct measurement of the phase angle between the pickup signal and the motion is described. Construction details of a small, light pickup which is unaffected by the high acceleration levels are given.

Sound absorption by areas of finite sizes, R. K. Cook, *Proc. 3d Intern. Congress on Acoustics (Elsevier Publ. Co., Amsterdam, The Netherlands, 1961.)*
The absorption coefficient for a small area of an acoustical material is much greater than for a very large (or infinite) area. Data are presented which show that for diffusely incident sound the additional absorption is proportional to $1/\sqrt{A}$, where A=area of the material. Calculations for sound at perpendicular incidence on circular patches of a normal impedance material are presented. These show an appreciable increase for the absorption coefficient when the diameter is reduced from a very large value to a value about three times the wave-length.

Wind resistance of asphalt shingle roofing, W. C. Cullen, *Natl. Acad. Sci.-Natl. Research Council p. 33–42 (1961).*
The factors affecting the wind-resistance of asphalt shingles were investigated. Background information on the composition and construction, the advantages and limitations, and the application of asphalt shingles are discussed. Based on laboratory wind tests, simulated service tests and a field survey, the following conclusions are drawn: (1) The heavier a free-tab shingle the more resistant it was to wind damage. (2) Adhesive systems in current use, both factory and field applied, were effective in preventing wind damage. (3) Laboratory tests were corroborated by field observations. (4) Criteria were developed for evaluating self-sealing shingles: Shingle conditioned 16 hours at 140° F. should withstand winds of 60 mph for 2 hours.

Rate of vaporization of refractory substances, J. J. Diamond, J. Efimenko, R. F. Hampson, and R. F. Walker, Editors, J. H. de Boer et al., **Reactivity of solids,** *Proc. 4th Intern. Symp. Amsterdam, 1960, p. 725–734 (1960).*
The more important factors affecting the rate of vaporization of solid systems are summarized. Techniques for measuring the rates of vaporization of refractory substances at temperatures in the 1600–3000° C range are briefly described. The techniques pertain to measurements both in vacuum and in the presence of foreign gases. Some of the factors and the experimental techniques are illustrated by brief reference to studies of the vaporization of platinum and aluminium oxide.

Crack propagation and the fracture of concrete, M. F. Kaplan, *J. Am. Concrete Inst.* **58,** *No. 5, 591–610 (Nov. 1961).*
The Griffith-crack theory of fracture strength is discussed. Tests were done on concrete beams with crack-simulating notches, and two methods, which have been called the analytical and the direct experimental methods, were used to determine the critical strain energy-release rate G_c associated with the rapid extension of the crack. There was good agreement between G_c values for beams with different notch depths and which were loaded both by the third-point and center-point methods. However 3- by 4- by 16-in. beams gave somewhat larger G_c values than did 6- by 6- by 20-in. beams. Although further research is necessary, the indications are that the Griffith concept of a critical strain energy-release rate being a condition for rapid crack propagation and consequent fracture, is applicable to concrete. The critical strain energy-release rate may be ascertained by suitable analytical and experimental procedures and the fracture strength of concrete containing cracks can thereby be predicted.

Timing potentials of Loran C, R. H. Doherty, G. Hefley, and R. F. Linfield, *Proc. IRE* **49,** *1659–1673 (Nov. 1961).*
The Loran-C navigation system is capable of synchronizing and setting clocks to a relative accuracy of better than one microsecond throughout the system's service area. The East Coast Loran-C chain will be synchronized with the national frequency standards and uniform time source located at Boulder. Time synchronization and time distribution will be demonstrated on the Atlantic Missile Range. Inter-range time synchronization and precise time for large areas of the world could be provided in the future.
A Loran-C receiver functions as a slaved oscillator and a trigger generator. The generated triggers bear a time relationship to the triggers at the master transmitter, which is known to within a microsecond. Clocks operating from these sources are compared with clocks operating from in-independent free-running oscillators.
A fundamental relationship between time and position is considered. Loran-C as a navigation and timing system can provide both position and time simultaneously.

Other NBS Publications

Journal of Research 66A (Phys. and Chem.) No. 2 (Mar.–Apr. 1962) 70 cents.
- Correction factors for the calibration of encapsulated radium sources. R. M. Lee and T. P. Loftus. (See above abstract.)
- Description and analysis of the second spectrum of tantalum, TA II. C. C. Kiess.
- Vibration-rotation bands of carbonyl sulfide. A. G. Maki, E. K. Plyler, and E. D. Tidwell.
- Ionization in the plasma of a copper arc. C. H. Corliss.
- The vapor pressure of palladium. R. F. Hampson and R. F. Walker.
- Revised standard values for pH measurements from 0 to 95 °C. R. G. Bates. (See above abstract.)
- Conductometric determination of sulfhydryl groups in swollen polycaprolactam fibers having disulfide and alkylene sulfide crosslinks. S. D. Bruck and S. M. Bailey.
- Chromatographic analysis of petroleum fractions used in oil-extended rubber. D. J. Termini and A. R. Glasgow.
- Cross-sectional correction for computing Young's modulus from longitudinal resonance vibrations of square and cylindrical rods. W. E. Tefft and S. Spinner. (See above abstract.)

Journal of Research 66B (Math. and Math. Phys.) No. 1 (Jan.–Mar. 1962) 75 cents.
- Error bounds for eigenvectors of self-adjoint operators. N. W. Bazley and D. W. Fox.
- Intermediary equatorial orbits of an artificial satellite. J. P. Vinti.
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- Measurement of wave fronts without a reference standard: Part 2. The wave-front-reversing interferometer. J. B. Saunders.

Journal of Research 66D (Radio Prop.) No. 2 (Mar.–Apr. 1962) 70 cents.
- Atmospheric phenomena, energetic electrons, and the geomagnetic field. J. R. Winckler.
- The summer intensity variations of [OI] 6300 A in the tropics. D. Barbier, F. E. Roach, and W. R. Steiger.
- Generation of radio noise in the vicinity of the earth. P. A. Sturrock.
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- Some problems connected with Rayleigh distributions. M. M. Siddiqui.
- Impedance of a monopole antenna with a radial-wire ground system on an imperfectly conducting half space, part I. S. W. Maley and R. J. King.
- Theory of the infinite cylindrical antenna including the feed-point singularity in antenna current. R. H. Duncan.
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