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Weight Calibration Schemes for Two Knife-Edge Direct-Reading Balances

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Direct-reading, two knife-edge, single-pan balances of high quality are shown to be well suited for most laboratory weight calibrations. The requirements for the design of good calibration series on such balances include:

1. Series are based on external mass standards rather the dial-operated weights.
2. Balance sensitivity is measured.
3. Substitution differences, not single balance indications, enter into mass determinations.
4. Tests for inadvertent fluctuations in balance indications are included.
5. Every indication entering into the calibration is subject to some check.
6. Known standard weights are calibrated side by side with unknown weights.
7. Series must have sufficient redundancy to permit a study of errors.
8. Buoyancy corrections, when not negligible, are applied.

Three rapid series are described in which drift of the balance indication is liable to introduce some uncertainty. Three other series are described which require more individual weighings, but nearly eliminate the effects of such slow drifts.

When unknown weights are compared with standards of the same denomination simplification is achieved usually associated with a loss of accuracy. Finally, methods for tolerance testing on single-pan balances are given.

1. Introduction

1.1. Basis for Use of Single-Pan Damped Balances

In recent years a number of well-known balance manufacturers have designed and marketed damped, direct-reading, high-precision laboratory balances. The work here described was carried out with balances which have a single pan, a beam with only two knife-edges, and internal weights that are applied to the pan suspension so as to bring the total applied load to a constant value. This technique of using the same load for all weighings effectively minimizes an important cause of changes in sensitivity. Although the quick-weighing features of such balances currently available may place a limitation on weighing precision somewhat lower than can be obtained by more elaborate weighing methods on the best equal-arm types of balances, the precision attainable under suitable weighing conditions is good enough so that weighings meaningful to a few parts in 10^7 of the balance capacity can be regularly attained. With a set of suitable balances each used for the calibration of about a decade of weights the largest of which is not much smaller than the balance capacity, it is possible to obtain about 1 part in 10^6 or better over a wide range of loads to be weighed. Weights adjusted on the customary bulk-buoyancy basis [1, p. 673][1] rather than the true-mass basis are still subject to small variations of bouyancy due to changes in ambient-air density. The errors introduced by neglecting such effects are about 1 part in 10^6. Consequently higher precisions are attainable only through more detailed bouyancy corrections. Thus the single-pan balances discussed here can provide a precision of weighing that is entirely adequate for the great majority of weight calibrations. The gain in simplicity and speed by their use should dispel any apprehension that weight calibration is too difficult and tedious to be undertaken even in laboratories where accurate weighings are attempted.

[1]Figures in brackets indicate the literature references at the end of this paper.

1.2. Requirements for an Individual Balance for Use in Weight Calibration

The suitability of a balance in weight calibration is easily judged. The sum of the errors due to the imprecision of the balance and the inaccuracy of the standard reference weight must not exceed the acceptable error. The relevant term for describing the imprecision of the balance is its standard deviation for a single observation divided by an appropriate factor to give a limit for the effect of random errors of the measurements. Lashof and Macurdy [2] have described a searching test suitable for single-pan damped balances from which a value of the standard deviation of a single observation is obtained. The other and quite independent uncertainty arises from the inaccuracy of the reference weight. The portion applicable to an unknown weight is the standard deviation of the reference-weight value multiplied by the ratio of the mass of the unknown to that of the standard weight.

1.3. General Requirements for the Weighing Series

Having established that a balance is suitable for a weight calibration the following precautions must still be taken.

1. The mass values must be determined by comparison with externally applied standard weights (not necessarily of the same nominal value) and must not depend on the accuracy of counterpoise or other built-in balance weights.

2. The sensitivity of the balance must be determined (if only as check of direct-reading characteristics of the balance) by addition of sensitivity weights of known value on the pan and must not be assumed to be constant from day to day or equal to the designed value.

3. The calibration series must be entirely based on substitution differences so that it is not necessary to adjust the balance indication to read any particular value. A weight of zero correction need not produce a zero indication.

4. Tests for the presence of fluctuations in the balance indication by extraneous disturbances must be included in the weighing schemes.

5. There must be some check on each individual balance indication.

6. Each series must include one or more standards, having a known mass value, which are experimentally evaluated with the unknown weights.

7. The series must have sufficient redundancy so that the desired precision is achieved by least squares or similar averaging of independent observations.

8. Buoyancy and standard-weight corrections, when not negligible, are applied as for all precise weight calibrations.

2. Pienkowsky Type Weighing Series

Figures 1, 2, and 3,[2] give the actual figures for calibrations by substitution series essentially evolved by A. T. Pienkowsky (unpublished). The aim of these series is to provide simple but sound calibrations with few observations.

In all three series the method of differential weighing is used, in which the indication given by the balance is not used as a measure of the mass of the load on the pan, but in which the difference between indications is taken as the difference in the masses of the loads. In the cases illustrated the sensitivity weighings verify that the scale readings are in milligrams.

In figure 1 the weighings are made in the order shown in the load column. The loads and errors of weights are indicated by a nomenclature which is best illustrated by a few examples:

(50) is the designation of the weight to be calibrated having a nominal value of 50 units.

NH 10_1 is one of the standard weights whose nominal value is 10 units and which is part of the series NH; there is more than one 10-unit weight in that series and the subscript is a distinguishing mark. All mass standards are identified by the set designation preceding the denomination. A broken line before the denomination calls for the designation of the standard.

$\Sigma(50)$ is a specific group of weights the sum of whose nominal values is 50 units. On this form $\Sigma(50)=(20)+(20)+(10)$

Cr(50) is the "correction" of the (50) weight. The "correction" of a weight or group of weights is by magnitude and sign equal to the difference: actual minus nominal mass values.

The difference between the appropriate scale readings, together with the correction for the sum $(50)+\Sigma(50)$ carried forward from comparisons in the next higher decade are used to compute the corrections for (50) and $\Sigma(50)$ by the sum and difference method. One-half the sum of equations marked (1) and (2) in the margins of figure 1 is the correction for (50) and one-half the difference is the correction for $\Sigma(50)$ denoted K. The difference between the second and third scale readings is added to the Cr $\Sigma(50)$ to compute the value of the standard, NH 50, as determined by this calibration. The agreement of this value of the standard with the accepted value previously determined by a more precise method is a measure of the accuracy of the calibration. The third and fourth scale readings are used to verify the sensitivity adjustment of the balance. Their difference D_s should equal the mass M_s of the small added weight within the reproducibility of the balance.

The last eight scale readings together with the Cr $\Sigma(50)$ are used to compute the corrections for $(20)_1$, $(20)_2$, (10), $\Sigma(10)$, and standard 10. The scale readings are taken in the order shown and the indicated computations performed, the end result being the corrections of the weights (the numbers by the arrowheads in the margin). The three differences, denoted Z_i, in the right-hand column at the middle of the figure, are three measurements of the difference between $(20)_1$ and $(20)_2$ and their agreement (within 0.03 mg under these particular weighing conditions) is a measure of the precision of the calibration as is the difference between a and h (h is a repetition of a). The latter figure is a convenient measure of the net balance drift. The agreement of the value of the standard 10 as determined by this calibration with its accepted value, is a measure of the accuracy of the calibration. The computations indicated in the shaded areas are computational checks and the figures within each box must agree except for errors due to rounding-off. The double framed boxes on the form are observational checks as indicated above.

The sensitivity of the balance was not determined with that series because it had been determined with the calibration of the (50 g) and $\Sigma(50\text{ g})$, just prior to this series.

A very approximate estimate of the standard deviation s_F of the final values is obtained from the following formula

$$s_F=\frac{1}{3}[\text{Range in } Z+(a-h)+s_K]$$

[2] The forms shown in these and later figures are in regular use for weight calibrations at the National Bureau of Standards.

Temperature	FORM NBS-345.03 (REV. 8-31-60) — U.S. DEPARTMENT OF COMMERCE, NATIONAL BUREAU OF STANDARDS	Sheet 2
Humidity	**SUBSTITUTION CALIBRATION (5, 2, 2, 1, Σ1 SERIES)**	Unit (*Wts.*) g
Barometer	**Single Pan Damped Balances**	Unit (*Cr. and Diff*) mg
ρ =	Σ(50) = (20)$_1$ + (20)$_2$ + (10)	

Observer	Balance	Date	Set	NBS Test No.
HSP	M-1	10-7-59	A	158575

COMPUTATIONAL CHECKS IN SHADED AREAS

OBSERVATIONAL CHECKS IN FRAMED BOXES

Load	Dial Setting	Scale Reading	Substitution Differences
(50)	50.0	0.50	(50) − NH 50 = + .03
NH 50	"	0.47	(50) − Σ(50) = − .00
Σ(50)	"	0.50	NH 50 − Σ(50) = − .03
" + L 10 mg	"	10.49	Obs Ds = + 9.99; Ms = 10.00
(10) Σ(10) NH 10$_1$	30.0	a .54	a+h = A = 1.07
(20)$_1$ NH 10$_1$	"	b .55	
(20)$_2$ NH 10$_1$	"	c .54	b+c = B = 1.09
(20)$_2$ (10)	"	d .58	d+e = D = 1.15
(20)$_1$ (10)	"	e .57	
(20)$_1$ Σ(10)	"	f .56	f+g = F = 1.12
(20)$_2$ Σ(10)	"	g .56	A+B+D+F = 4.43; a+b+c+d+e+f+g+h = 4.43
(10) Σ(10) NH 10$_1$	"	h .53	
" +	"		Obs Ds = Ms = −

From Sheet 1 CrΣ(100)
=Cr{(50) +Σ(50)} = + .04 (1)
(50) −Σ(50) = − .00 (2)

1/2 Sum = Cr (50) = + .02

1/2 Diff. = CrΣ(50) = + .02 = K

NH 50 −Σ(50) = − .03

NH 50 −Σ(50) +K = − .01 =Cr NH 50

Acc. Cr NH 50 = − .027

A−B+D−F
Sum = + .01 =C

B−D = − .06 =E

F−D = − .03 =G

C+E+G = − .08
A−D = − .08

K = + .02
Cr{(2)$_1$+(2)$_2$+(1)} = + .03

1/2 E = − .03
NH 10 − (10) = − .03

1/2 G = − .02
Σ(10) − (10) = − .02

Note: Σ(10) = (5)+(2)$_1$+(2)$_2$+(1)

b−c = Z_1 = + .01
e−d = Z_2 = − .01
f−g = Z_3 = − .00

Mean Z = − .00 = Z'

Total Range in Z's = .02
Net Balance Drift
a−h = + .01

Z' = − .00
(20)$_1$ − (20)$_2$ = − .00

Cr NH 10$_1$
5E+10 Cr (10)
0.1 Sum = − .02
Acc Cr = − .011

Cr (20)$_1$	Cr (20)$_2$	Cr (10)	Cr Σ(10)
−C+5Z'+4K; 0.1 Sum = + .01	−C−5Z'+4K; 0.1 Sum = + .01	2C + 2K; 0.1 Sum = + .01	5G+10Cr (10); 0.1 Sum = − .01

USCOMM-DC 24061-P60

FIGURE 1.

Temperature	FORM NBS-345.02 (1-12-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 3
Humidity	**SUBSTITUTION CALIBRATION (5, 2, 2, 1, SERIES)**	Unit (Wts.) g
Barometer	**Single Pan Damped Balances**	Unit (Cr. and Diff.) mg
ρ =	$\Sigma(5) = (2)_1 + (2)_2 + (1)$	

Observer	Balance	Date	Set	NBS Test No.
HSa	M-2	10-28-59	I	158783

COMPUTATIONAL CHECKS IN SHADED AREAS

OBSERVATIONAL CHECKS IN FRAMED BOXES

Load	Dial Setting	Scale Reading	Substitution Differences
(5)	5.00	.129	(5) − NH 5 = + .029
NH 5	" .	.100	(5) − Σ(5) = + .030
Σ(5)	" .	.099	NH 5 − Σ(5) = + .001
" + 5 mg a	" .	5.099	Obs D_s = 5.000; M_s = 5.008
(1) C1 NH 1	3.00	a .081	a+h=A = .166
(2)$_1$ NH 1	" .	b .084	
(2)$_2$ NH 1	" .	c .100	b+c=B = .184
(2)$_2$ (1)	" .	d .109	d+e=D = .198
(2)$_1$ (1)	" .	e .089	
(2)$_1$ C1	" .	f .060	f+g=F = .144
(2)$_2$ C1	" .	g .084	A+B+D+F = .692
(1) C1 NH 1	" .	h .085	a+b+c+d+e+f+g+h = .692
" +	.	.	

From Sheet 2 CrΣ(10) =Cr{(5) + Σ(5 .)} = + .033

(5) − Σ(5) = + .030

1/2 Sum = Cr (5) = + .032

1/2 Diff. = Cr Σ(5) = + .002 = K

NH 5 − Σ(5) = + .001

NH 5 − Σ(5) + K = + .003 =Cr NH 5

Acc. Cr NH 5 = + .003

A−B+D−F Sum = + .036 = C	b−c=Z_1 = − .016 e−d=Z_2 = − .020 f−g=Z_3 = − .024
B−D = − .014 = E	Mean Z = Z′ = − .020
C−E = + .022 A−F = + .022	Total Range in Z′s = .008
K = + .002 Cr{(2)$_1$+(2)$_2$+(1)} = + .002	Net Balance Drift a − h = − .004
1/2 E = − .007 NH 1 − (1) = − .007	Z′ = − .020 (2)$_1$ − (2)$_2$ = − .020

Obs D_s = . M_s = .

Cr (2)$_1$	Cr (2)$_2$	Cr (1)	Cr NH 1
−C+5Z′+4K	−C−5Z′+4K	2C +2K	1/2 E + Cr(1)
0.1 Sum = − .013	0.1 Sum = + .007	0.1 Sum = + .008	Sum = + .001
			Acc Cr = − .002

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FIGURE 2.

Temperature	FORM NBS-345.01 (REV. 8-29-60)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 2
Humidity	**SUBSTITUTION CALIBRATION (5, 3, 2, 1, Σ1 Series)**		Unit (Wts.) g
Barometer	**Single Pan Damped Balances**		Unit (Cr. and Diff.) mg
ρ =	Σ(10) = (5) + (3) + (2)		

Observer	Balance	Date	Set	NBS Test No.
gaw	M-1	2-6-61	B	G28462

COMPUTATIONAL CHECKS IN SHADED AREAS

OBSERVATIONAL CHECKS IN FRAMED BOXES

Load	Dial Setting (g)	Scale Reading	
NH 50	50.0	0.93	a - b = - .01 m - l = - .01 Mean = A = - .01
(50)	" .	a 0.98	
(30) (20)	" .	b 0.99	b - g = B = + .01
(30) NH 20	" .	c 0.94	c - h = C = - .09
(30) (10) Σ(10)	" .	d 1.04	d - i = D = + .12
(20) NH 20 (10)	" .	e 1.08	e - j = E = + .18
(20) NH 20 Σ(10)	" .	f 1.07	f - k = F = + .16
+ (b+c+d+e+f) = +		5.12	B+C+D+E+F
- (g+h+i+j+k) = -		4.74	
Sum = +		0.38	= + 0.38
NH 20 (10) Σ(10) C10	50.0	g 0.98	b + g 1.97
(20) (10) Σ(10) C10	" .	h 1.03	c + h 1.97
(20) NH 20 C10	" .	i 0.92	d + i 1.96
(30) Σ(10) C10	" .	j 0.90	e + j 1.98
(30) (10) C10	" .	k 0.91	f + k 1.98
(30) (20)	" .	l 1.00	l + g 1.98
(50)	" .	m 0.99	Obs D_s = 9.99 M_s = 10.00
" +1.10 mg	"	m′ 10.98	

From sheet 1
K = Cr Σ(100)
= + .05

Computed K:
Cr {(5) + (3) + (2)}
= + .05

2 Cr Σ(50)
K - A = + .06 = P

Q = 4Cr(30) - 6Cr(20)

D - B + 2C - E - F

Sum = - .41 = Q

Optional Computational Aids

C - B = - .10
E - C = + .27
2Q = - .82
P = + .06
F - C = + .25

Total Range in Sums
= .02

Net balance drift
a - m = - .01

- 1/2 (C - B) = + .05
(20) - NH 20
= + .05

1/2 (D - B) = + .06
(10) + Σ(10) - (20)
= + .06

1/2 (E - F) = + .01
(10) - Σ(10)
= + .01

Cr (50)
1/2 (K + A) = + .02

Cr NH 50
NH 50 - (50) = - .05
+ Cr(50) = + .02
Cr NH 50 = - .03

Acc Cr NH 50 = - .03

Cr (30)
3P+Q
0.1 Sum = - .02

Cr (20)
2P-Q
0.1 Sum = + .05

Cr NH 20
1/2 (C-B)+Cr(20)
Sum = - .00
Acc Cr = + .01

Cr (10)
5 (E-C)+P+2Q
0.1 Sum = + .06

Cr Σ(10)
5 (F-C)+P+2Q
0.1 Sum = + .05

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FIGURE 3.

where s_K is the estimated standard deviation of K.

The series illustrated in figure 2 is the same as that in figure 1, except that there is no $\Sigma(1)$ and that in the example given the weights involved are only one-tenth as large. The weighings are taken in the same order. The checks and the computations are the same except that those needed to find the correction for the summation are not made. C 1, a "constant weight," appears where $\Sigma(10)$ appeared in figure 1. The value of this "constant weight" is not computed. Its presence in the series introduces the desired redundancy. Apart from the requirement of adjustment to nominal value within the on-scale range of the balance, the "constant weight" need only remain constant while the observations in one series are taken.

The nomenclature indicating the loads and errors of the weights in figure 3 is the same as in figures 1 and 2. The weighings are made in the order shown. The first scale reading and those designated a through m together with the correction for the $\Sigma(100)$ (from another sheet, not shown, on which the comparisons in the next higher decade, corrected for air buoyancy, are given) are used to determine the corrections for the weights (50), (30), (20), (10), $\Sigma(10)$, NH 50, and NH 20. $\Sigma(50)$ on this form signifies $(30)+(20)$. To obtain the corrections for the weights the indicated computations are performed. Again the checks on computation are in the shaded areas and the figures within each box must agree except for rounding-off errors. The double-framed boxes on the form are observational checks. The agreement of the sums $b+g$, $c+h$, etc., in the lower half of the fourth column from the left, corresponds to the agreement between Z's in figures 1 and 2, and is an indication of the precision of the calibration, as is the agreement between a and m (m is a repetition of a). This latter figure is used principally as a measure of the balance drift. The agreement of the values of the standards, NH 50 and NH 20, determined by this calibration with their accepted values provides valuable overall checks.

The last weighings, m' and m, are used to determine the sensitivity of the balance. Their difference should equal the mass of the small added weight within the reproducibility of the balance.

Discussion of Pienkowsky-type series would not be complete without reference to the possible adverse effects on precision resulting from drift in balance indication. Over the entire time of completing balance readings at any one load in a series (fig. 1 to 3) such balance drift is liable to introduce an uncertainty in the ultimate mass values. "Trend elimination" therefore becomes an important feature in the design of such series.

The reader can readily convince himself by inspection of figure 3, for example, that the series is so designed that for a steady balance drift proportional to time, under a uniform rate of recording observations, the effects of balance drift on the mass comparison are entirely eliminated. The stated conditions of operation, however, are in practice difficult to achieve within the required limits; nor is the $(a-m)$ test any measure of the nonuniformity of drift. A low value for the $(a-m)$ difference in balance indication may hide a maximum in the extent of drift during the observational period. It is true that some knowledge of the individual characteristics of a balance might help; some designs for instance appear to produce drift periods of the order of an hour. On such instruments an experienced observer will try to complete observations within a quarter of that period. The reader should here notice that a higher value of $(a-m)$ may then actually be indicative of a better set of observations than would be characteristic of a lower $(a-m)$ value! Another feature of balances (except some high-capacity types) is that they are influenced by the thermal disturbance caused by an operator sitting within several feet of the balance. The light illuminating the scale may cause a similar disturbance. Drifts for the first half-hour of observations tend therefore to be far greater than later. Because of these effects an observer should not interrupt his observations within a series.

It is of course true that better tests for drifts could be introduced, leading to superior methods of trend elimination. Such tests achieved by additional observations would, however, detract from the basic aim of the Pienkowsky type of series, to provide extremely simple but sound mass calibration with very few observations. The use of these series should be discouraged for all but experienced observers working with well tested balances in favorable environments.

The examples given in figures 1 to 3 will provide appropriate weighing series for the majority of weight sets. The proper use of these series will usually suggest itself even when the weight set differs slightly from those used for the illustrations. For instance, if in a terminating series involving the smallest weights of a set having the 5321 sequence an extra 1 weight is provided, it will take the place of the weight group $\Sigma 1$ used in figure 3. If, however, the set does not contain an extra weight 1 an additional mass standard _ _ _ 1 can be introduced giving another valuable check. An optional but less desirable alternative is to substitute a "constant weight," as defined above, the correction for which is not computed.

A less straightforward example of the use of the series given in the illustrations concerns the 5211 $\Sigma 1$ weight sequences sometimes encountered, for which the 5221 $\Sigma 1$ series (fig. 1) can be applied. A standard 2 weight replaces the second (2) weight to be calibrated and the additional (1) weight in the 5211 $\Sigma 1$ set replaces the standard 1 weight in the illustration. It should be noted that the $\Sigma(10)$ obtained from the next higher decade must have been made up as follows: $(5)+(2)+$ _ _ _ $2+(1)$. If $\Sigma(10)=(5)+(2)+(1)_1+(1)_2+\Sigma(1)$ is preferred, the algebraic solutions for all the weights must be correspondingly amended. As is discussed below, least squares solutions must be used. The $(5)+(2)+$ _ _ _ $2+(1)$ summation will probably commend itself, firstly because the form given in the illustration can be employed without much change and, secondly, because the $(5)+(2)+(1)_1+(1)_2+\Sigma(1)$ summation includes an inconveniently large number of weights. There are circumstances, however, under which the additional handling of the ___ 2 weight in the summation will not be considered justifiable.

For very unusual weight sequences in weight sets the authors would be glad to furnish Pienkowsky-type weighing series on request. However, the reader might wish to devise such series for himself. For this purpose an outline of the principal considerations in devising these series follows:

To the list of denominations of unknown weights in one decade are added one or more denominations for standard weights to be intercompared with the the unknown weights. A "constant weight," as defined above, may also be introduced. A larger denomination for the standard provides a better check on the accuracy than a standard weight of smaller denomination, but more important is the provision of the necessary combinations for weighings with a minimum number of nominal values for the total loads. Often there is difficulty in finding a convenient solution, if the (5) and $\Sigma(5)$ are determined by intercomparison with the smaller denominations. Intercomparison of the (5) and $\Sigma(5)$ with a check on a standard 5 separate from the intercomparison of the smaller denominations may then be the best choice.

In the Pienkowsky-type series each independent balance indication after the first at each particular total load provides one independent observation. (The sensitivity check reading is not counted as an independent observation.) A condition or mathematical restraint is established by some sum of unkown denominations, usually $\Sigma(5)$ or $\Sigma(10)$, the correction for which is determined by comparison with a standard or by a series at the next higher decade. The number of statistical degrees of freedom of the series then equals the number of independent observations plus one (for the restraint imposed by the assumed weight or summation weight), diminished by the number of weights evaluated in the series (including the "constant" weight where used). The computation of all the weight corrections must follow by least-square solutions. It is preferable to design the series so that the coefficients in these solutions do not involve recurring decimals.

The details of the series must be planned to eliminate all types of errors as far as possible. The verification of the direct-reading characteristic of the balance by a sensitivity weighing has already been discussed. Similarly, the need for measurement of the precision of the observations will be understood from previous elucidation. All the algebraic solutions for any one weight, any one sum, or any one difference of weights in the series can be used for determining a statistical range for the evaluation of precision. The choice should be determined by the desirability of having a large number of independent values to compare. This number for some sums will be larger than the number of degrees of freedom as defined above. The second consideration to guide the choice is the need for as many observations as possible to be involved in the range determination. In this way a safeguard is provided against misreadings of the balance indication.

The effect of balance drift has also been discussed above. It is here necessary to stress only the need for designing weighing series so that, firstly, a measure of the drift is obtained by at least one repetition of at least one load at as large an interval as possible; and that, secondly, the observations are grouped for optimum statistical trend elimination. However, it is not always practicable for the various checks on precision to fulfil this trend-elimination requirement. It will then be probable that the calibrated values are not worse than is indicated by the checks.

The series should be planned to detect a false placement of a weight in a combination. In the series, therefore, every weight should be placed on and removed from the balance at least twice. In the opinion of the authors, the designer of the series should leave to the observer the judgment when an individual discrepancy should no longer be accepted as within the precision range of observation.

Finally, every computational step should be subject to at least one check on the form, as a safeguard against computational blunders. Except for rounding-off errors, every discrepancy revealed by these checks points to error in the computation or the check itself.

3. Hayford-Benoit Type Weighing Series

The chief weakness of the Pienkowsky-type series lies in the difficulty of recognizing and properly eliminating the effects of slow drifts in balance indications; wherefore the Pienkowsky-type series cannot quite realize the full potentialities of a balance.

Figures 4, 5, and 6, illustrate series which reduce the effect of balance drift by being based on self-consistent pairs of weighings. The figures refer to actual weight sets calibrated by means of a 1-kg balance. The series used are equivalent to those evolved at the National Bureau of Standards for transposition weighings on equal-arm balances [1, p. 661 ff]. The type of weighing series is here named Hayford-Benoit after the two investigators to whom we are indebted for careful analyses of these methods [3, 4]. It should be noted that there is a fundamental distinction between the two types of weighing series: In the Pienkowsky type one additional reading of a balance indication may increase the number of degrees of freedom by one whereas in the Hayford-Benoit type at least two readings are needed.

The nomenclature for the loads and the corrections for the weights is the same as for figures 1, 2, and 3. The differential method of weighing is again used. Therefore, for a given substitution weighing, the dial-operated weights must not be changed. Each of the series is made up of from six to nine equations. Each weighing equation states the observed difference between two weights or groups of weights. These observed differences are denoted a_1, a_2, a_3, etc. First one weight or one group of weights is placed on the balance pan. The dials are then set for the nominal

Temperature	FORM NBS-347.02 (11-7-60) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 2
Humidity	**SUBSTITUTION CALIBRATION (2,1,1,Σ1 Series)**	Unit *(Wts.)* ozt
Barometer	**Single Pan Damped Balances**	Unit *(Cr. and Diff.)* mg
ρ =	~~Σ(1)~~ = Σ(6) = (2)₁ + (2)₂ + (1) + Σ(1)	

Observer HEa	Balance M-3	Date 10-22-59	Set B	NBS Test No. G24777

COMPUTATIONAL CHECKS IN SHADED AREAS | OBSERVATIONAL CHECKS IN FRAMED BOXES

Load	Dial Setting	Scale Reading	Computations
~~(1)₁~~ (6)	186.6g	25.25	~~(1)₁-(1)₂~~ (6) - d6 = a₁ = -0.35
~~(1)₂~~ d6	"	25.60	Obs-Cal= -.02
~~(1)₁~~ (6)	"	25.30	~~(1)₁-Σ(1)~~ (6) - Σ(6) = a₂ = -0.95
~~Σ(1)~~ Σ(6)	"	26.25	Obs-Cal= -.03
~~(1)₂~~ d6	"	25.80	~~(1)₂-Σ(1)~~ d6 - Σ(6) = a₃ = -0.60
~~Σ(1)~~ Σ(6)	"	26.40	Obs-Cal= -.01
~~(1)₁, (1)₂~~ (6), d6	373.2g	46.70	~~(1)₁+(1)₂-(2)~~ (6) + d6 - 1lbt₁₄ = a₄ = -2.45
~~(2)~~ 1lbt₁₄	"	49.15	Obs-Cal= +.05
~~(1)₂, Σ(1)~~ d6, Σ(6)	"	47.50	(1)₂+Σ(1)-(2) d6 + Σ(6) - 1lbt₁₄ = a₅ = -1.65
~~(2)~~ 1lbt₁₄	"	49.15	Obs-Cal= -.07
~~(1)₁, Σ(1)~~ (6), Σ(6)	"	47.20	~~(1)₁+Σ(1)-(2)~~ (6) + Σ(6) - 1lbt₁₄ = a₆ = -1.90
~~(2)~~ 1lbt₁₄	"	49.10	Obs-Cal= +.01
" + L20 mg₁	"	69.15	

Obs D_s = 20.05

L20 mg₁ = M_s = 20.01

From Sheet 1 or ~~Below:~~ K = +3.98

$Cr\{(2)+(1)_1+(1)_2+\Sigma(1)\}$

= Computed K = +3.98

Cr ~~(2)~~ 1lbt₁₄ $\frac{-a_4-a_5-a_6+2K}{5}$ = +2.79

Cr ~~(1)₁~~ (6) $\frac{+5a_1+5a_2+3a_4-2a_5+3a_6+4K}{20}$ = -0.02

Cr ~~(1)₂~~ d6 $\frac{-5a_1+5a_3+3a_4+3a_5-2a_6+4K}{20}$ = +0.31

AccCr = +0.25

CrΣ(1 6) $\frac{-5a_2-5a_3-2a_4+3a_5+3a_6+4K}{20}$ = +0.90

If Series Based on:

$(2) = \_\_\_2; K = \frac{5Cr\_\_\_2 + a_4 + a_5 + a_6}{2}$

= -

$(1)_2 = \_\_\_1; K = \frac{20Cr\_\_\_1 + 5a_1 - 5a_3 - 3a_4 - 3a_5 + 2a_6}{4}$

= -

Check Sums

$a_1-a_2+a_3$	= 0.00
$a_2-a_4+a_5$	= -.15
$a_3-a_4+a_6$	= -.05
$a_1+a_5-a_6$	= -.10

$s = \sqrt{\frac{\Sigma(\text{Obs-Cal})^2}{3}}$

s = 0.06 mg

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FIGURE 4.

Temperature Humidity Barometer ρ =	FORM NBS-347.03 (11-60) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS **SUBSTITUTION CALIBRATION (2,2,1,Σ1 Series)** **Single Pan Damped Balances** Σ(1) =	Sheet 5 Unit (Wts.) g Unit (Cr. & Diff.) mg

Observer	Balance	Date	Set	~~NBS Test No.~~
K.E.O.	M-3	11-19-59		Corp. No. 3001

COMPUTATIONAL CHECKS IN SHADED AREAS | OBSERVATIONAL CHECKS IN FRAMED BOXES

Load	Dial Setting	Scale Reading	Computations	
~~(1 →~~ [100]*	100.0	2.10	~~(1)-Σ(1)~~ [100] - L100g₁ = -0.25 =a_1	From Sheet 4 or Below: K = -0.10
~~Σ(1 →~~ L100g₁	"	2.35	Obs-Cal= -0.05	Cr (2)$_1$+(2)$_2$+(1) =
~~(1), Σ(1)~~ [100]* L100g₁	200.0	2.30	(1)+Σ(1)-(2)$_1$ =a_2 = +0.20	Computed K = -0.10
~~(2)$_1$~~ V200₁	"	2.10	Obs-Cal= -0.01	S, m, M
~~(1), Σ(1)~~ [100]* L100g₁	"	2.40	(1)+Σ(1)-(2)$_2$ =a_3 = +0.20	$\frac{a_1+a_5+a_6}{3}$ = -0.20 = S
~~(2)$_2$~~ V200₂	"	2.20	Obs-Cal= +0.01	$2S+a_2+a_3$ = -0.00 = m
~~(2)$_1$~~ V200₁	"	2.20	(2)$_1$-(2)$_2$ =a_4 = -0.00	4K - m = -0.40 = M
~~(2)$_2$~~ V200₂	"	2.20	Obs-Cal= +0.02	N
~~(1), (2)$_1$~~ [100]* V200₁	300.0	2.35	(1)+(2)$_1$-Σ(1)-(2)$_2$ =a_5 = -0.20	$-a_2+a_3+2a_4+2a_5-2a_6$
~~Σ(1), (2)$_2$~~ L100g₁ V200₂	"	2.55	Obs-Cal= +0.02	$\frac{5}{7}$ Sum = -0.07 = N
~~(1), (2)$_2$~~ [100]* V200₂	"	2.05	(1)+(2)$_2$-Σ(1)-(2)$_1$ =a_6 = -0.15	Cr ~~(1 →~~ [100]
~~Σ(1), (2)$_1$~~ L100g₁ V200₁	"	2.20	Obs-Cal= +0.03	2K +2m
" + L 20 mg₁	"	22.20	Obs D_s = 20.00 M_s = 20.01	0.1 Sum = -0.02

Cr ~~(2)$_1$~~ V200₁ M + N 0.1 Sum = -0.05	Cr ~~(2)$_2$~~ V200₂ M - N 0.1 Sum = -0.03	Cr ~~Σ(1)~~ L100g₁ Cr (1) - S Sum = +0.18 Acc. Cr. = +0.16

If Series Based on: (2)$_1$=_ _2 and (2)$_2$=_ _2 ; 5{Cr_ _2+Cr_ _2} = = M $\frac{M+m}{4}$ = = K	Check Sums $a_2-a_3+a_4$ = 0.00 $a_1+a_4-a_5$ = -0.05 $a_1-a_4-a_6$ = -0.10 $a_4-\frac{a_5}{2}+\frac{a_6}{2}$ = +0.02	$s = \sqrt{\frac{\Sigma(\text{Obs}-\text{Cal})^2}{3}}$ s = 0.04

*FROM SET E, 2.6 TEST NO. 105

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FIGURE 5.

Temperature	FORM NBS-347.01 (11-23-60)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 3
Humidity	**SUBSTITUTION CALIBRATION** (2, 2, 1, 1, . . 1 Series)		Unit (*Wts.*) lb
Barometer	Single Pan Damped Balances		Unit (*Cr. and Diff.*) mg
ρ =			

Observer HEQ	Balance M-3	Date 10-20-59	Set A	NBS Test No. 62477

COMPUTATIONAL CHECKS IN SHADED AREAS — OBSERVATIONAL CHECKS IN FRAMED BOXES

Load	Dial Setting	Scale Reading	Computations
(1)$_1$	453.6g	1.0	$(1\ )_1-(1\ )_2 = a_1 = -0.70$
B1 lb$_2$	" .	1.7	Obs−Cal= −.04
(1)$_1$	" .	1.1	$(1\ )_1 - \_\_\_1 = a_2 = +3.80$
----1 lb"	" .	−2.7	Obs−Cal= +.01
B1 lb$_2$	" .	1.8	$(1\ )_2 - \_\_\_1 = a_3 = +4.40$
----1 lb"	" .	−2.6	Obs−Cal= −.05
(1)$_1$, B1 lb$_2$	907.1g	97.2	$(1\ )_1+(1\ )_2-(2\ )_1 = a_4 = +4.50$
(2)$_1$	" .	92.7	Obs−Cal= .00
B1 lb$_2$, ---1 lb"	" .	93.4	$(1\ )_2 + \_\_\_1 - (2\ )_1 = a_5 = +0.70$
(2)$_1$	" .	92.7	Obs−Cal= −.01
(1)$_1$, ----1 lb"	" .	92.8	$(1\ )_1 + \_\_\_1 - (2\ )_1 = a_6 = +0.05$
(2)$_1$	" .	92.75	Obs−Cal= +.05
(1)$_1$, B1 lb$_2$	" .	97.0	$(1\ )_1+(1\ )_2-(2\ )_2 = a_7 = +3.50$
(2)$_2$	" .	93.5	Obs−Cal= +.04
B1 lb$_2$, ----1 lb"	" .	93.1	$(1\ )_2 + \_\_\_1 - (2\ )_2 = a_8 = -0.35$
(2)$_2$	" .	93.45	Obs−Cal= −.02
(1)$_1$, ----1 lb"	" .	92.4	$(1\ )_1 + \_\_\_1 - (2\ )_2 = a_9 = -1.0$
(2)$_2$	" .	93.4	Obs−Cal= −.01
" + L10 mg	" .	103.35	Obs D_s = 9.95, M_s = 10.00

From Sheet 2 or Below: K = +7.76

$Cr\ \{(2)_1+(2)_2+(1)_1\}$ =

Computed K = +7.76

$a_1+a_2+a_4+a_6+a_7+a_9 = +10.15 = B_1$

$-a_1+a_3+a_4+a_5+a_7+a_8 = +13.45 = B_2$

$-a_2-a_3+a_5+a_6+a_8+a_9 = -8.80 = B_3$

$-a_4-a_5-a_6 = -5.25 = B_4$

$-a_7-a_8-a_9 = -2.15 = B_5$

$B_1+B_2+B_3+2B_4+2B_5$ = 0 Equal Zero

$Cr\ \_\_\_\_1$ lb" $= \dfrac{-B_1+5B_3+B_4+B_5+5K}{25} = -0.91$

Acc Cr = −0.91

Cr (2 lb)$_1$	Cr (2 lb)$_2$	Cr (1 lb)$_1$	Cr B1 lb$_2$
$\dfrac{-6B_1+11B_4-14B_5+30K}{75}$	$\dfrac{-6B_1-14B_4+11B_5+30K}{75}$	$\dfrac{4B_1+B_4+B_5+5K}{25}$	$\dfrac{-B_1+5B_2+B_4+B_5+5K}{25}$
= +1.92	= +2.96	= +2.88	= +3.54 Acc Cr = +3.6

If Series Based on ____1 ;

$$K = \frac{25Cr\_\_\_\_1 + B_1 - 5B_3 - B_4 - B_5}{5}$$

= −

Check Sums

$a_1-a_2+a_3$ = −0.1

$a_2-a_4+a_5$ = 0.0

$a_1+a_5-a_6$ = −0.05

$a_3-a_4+a_6$ = −0.05

$a_2-a_7+a_8$ = −0.05

$a_3-a_7+a_9$ = −0.1

$a_1+a_8-a_9$ = −0.05

$$s = \sqrt{\frac{\Sigma(\text{Obs}-\text{Cal})^2}{5}}$$

s = 0.04 mg

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FIGURE 6.

value of the load, and the scale reading observed and recorded. Similarly the other weight or group of weights are compared. In each pair of weighings the sign of the difference between the two weights is the sign that the heavier of the weights has in the observation equation. The scale readings and the differences between the weights are expressed in milligrams.

Figures 4, 5, and 6 show the observations and computations for the 2, 1, 1, $\Sigma 1$; the 2, 2, 1, $\Sigma 1$; and the 2, 2, 1, 1, _ _ _1 series respectively. The computations and checks are the same as if the a's had been obtained by transposition weighing. The estimate of the standard deviation of a single observation, s, provided by the particular series of weighings, is a measure of the precision of the individual weighings. The agreement of the value of the standard weight, where one is used, obtained by this calibration with its accepted value is a further valuable indication of the accuracy of the calibration. When a standard weight is not included in the series an additional comparison is made. In this comparison a standard weight is evaluated in terms of a weight whose value was determined in the series. The agreement of the value of the standard weight obtained by this comparison with its accepted value will serve as an indication of the accuracy of the calibration.

The Hayford-Benoit type series, of course, can be applied to other groups of weights; the method for developing such weighing series is simpler than for Pienkowsky-type series where an all-important choice of possible comparisons has to be made. For Hayford-Benoit series, however, it is best to make all possible comparisons at all loads. Moreover, the derivation of these series is well described in the literature [4]. The adaptation to single-pan balances presents no difficulty.

4. Other Weighing Procedures

When a test weight is compared only once with a standard weight of the same denomination, an acceptable but less accurate procedure is illustrated as follows.

Load	Scale reading
NH 10 g_1	$+0.517=I_1$
(10 g)	$+0.559=I_2$
(10 g)$+\beta$ 10 mg	$+10.565=I_3$
NH 10 $g_1+\beta$ 10 mg	$+10.527=I_4$

In this example the balance was adjusted to read about 0.5 mg more than the actual loads so that negative numbers would not be introduced into the record. For the average difference for (10 g)$-$NH 10 g_1, the computation is as follows:

$$(10\text{ g})-\text{NH }10\text{ g}_1=\tfrac{1}{2}\,[(I_2-I_1)+(I_3-I_4)]=+0.040$$

(in scale divisions).

The observed differences in scale divisions are verified to equal corresponding differences in milligrams within the precision of the balance.

$$I_3-I_2=10.006\text{ (in scale divisions)}=10\text{ mg}$$

but β 10 mg$=$10.008 mg (accepted value)

$$\text{hence }(10\text{ g})-\text{NH }10\text{ g}_1=0.040\times\frac{10.008}{10.006}=+0.040\text{ mg}$$

(therefore scale divisions represent milligrams within the accuracy of measurement).
But Cr NH 10 $g_1=-0.011$ mg (accepted value).
Thus Cr (10 g)$=+0.029$ mg.

At the National Bureau of Standards weights whose value is based on this type of weighing are not said to have been "calibrated." That term is reserved for measurements based on at least two mass standards.

In many instances weights for use on less precise balances may be tested on a direct-reading, quick-weighing balance. Weights are accepted if their values fall within some acceptable deviation from nominal value. Such tests may be performed rapidly and without written observations or computations by methods that are practically free from systematic effects.

For such "tolerance tests" a standard weight of the denomination of the test weights is placed on the balance pan; the balance indication is then adjusted to read the actual value of the standard; next a sensitivity weight is added to change the scale reading by a known amount. If the change in scale reading should not agree with the actual value of the sensitivity weight the balance may require adjustment. This should happen only very rarely.

The sensitivity having been proved correct the standard is removed and test weights substituted on the balance pan. The indicated values may then be taken as actual values of test weights for use on balances of lower precision. At the end of the series the standard weight must be replaced on the pan and the reading indicated by the balance must agree within the planned precision of the weighings. If many test weights are involved, the standard must be replaced on the pan after each small group of weights (consisting of say three weights each) has been tested. If the results show that the balance is remaining constant, the size of the group may be increased moderately.

Even with the inclusion of four authors, these developments in weighing technique have not been adequately characterized as the team effort upon which they are based. Every member of the National Bureau of Standards Mass Laboratory has contributed significantly. In addition, we have had the benefit of unusually wide discussion and criticism from interested scientists in other laboratories. Above all, we should single out the help we have received on statistical considerations from C. Eisenhart and J. M. Cameron of the Statistical Engineering Section of the National Bureau of Standards.

5. References

[1] National Bureau of Standards, Precision measurement and calibration, NBS Handb. 77, Vol. III. Optics, metrology, and radiation (1961).

[2] T. W. Lashof and L. B. Macurdy, Testing a quick-weighing balance, Anal. Chem. **26,** 707 (1954).

[3] John A. Hayford, On the least square adjustment of weighings, U.S. Coast and Geodetic Survey Appendix 10, Report for 1892 (1893).

[4] M. J. R. Benoit, Travaux et Mémoirs du Bureau International des Poids et Mesures **13,** 1, (1907).

(Paper 66C1–85)