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A Method for the Self-Calibration of Attenuation-Measuring Systems*

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The theory and experimental procedures are given for the self-calibration of insertion loss or attenuation-measuring systems. Four circuit configurations are developed. The calibrations may be obtained by simple graphical means or by an analytical solution. Experimental results are given which demonstrate that, by using the techniques outlined, attenuation calibrations of high accuracy may be made without reference to any previously calibrated attenuator.

1. Introduction

The feasibility of self-calibrating attenuators has been demonstrated previously by Allred [1, 2] [1] and by Laverick [3]. They have shown that it is not necessary to use any reference standard and that only a signal source, a signal null indicator, and a suitable arrangement of transmission-line components are required.

Allred has described a three-branch system for obtaining the phase as well as the magnitude of the attenuation coefficients of two piston attenuators operating with the TE_{11} evanescent mode. This is done by obtaining a set of dial readings from the two attenuators, one in series with a fixed phase shifter constituting one branch and the other constituting the second branch. The third branch contains an adjustable phase shifter and a two-position step attenuator. With any initial setting of the adjustable phase shifter, the two attenuators are adjusted for a null. The step attenuator is then changed, and the attenuators are again adjusted for a null. This operation is repeated as the adjustable phase shifter is set successively to different values. From the knowledge that in each case the step attenuator has undergone the same change in attenuation and phase for each set of measurements, a series of equalities of the form

$$K=\frac{e^{-\gamma_y Y_a}+We^{-\gamma_z Z_a}}{e^{-\gamma_y Y_b}+We^{-\gamma_z Z_b}}$$

is obtained, where K is the complex insertion ratio of the step attenuator, γ_y and γ_z are the propagation constants of each of the two piston attenuators, W is a separately evaluated constant expressing the relative magnitudes and phase angles of the output voltages of the two attenuators, and Y_a, Y_b, Z_a, Z_b are dial readings of the attenuator. These expressions can then be solved for the unknown parameters of the two attenuators.

Laverick's method uses a three-branch system with switches for opening two of the branches. Two of the branches contain phase shifters to set these branches 180° out of phase with the third. The three branches are then adjusted for equal output by successively nulling one branch against one of the other two. All three branches are then connected together, and one of the attenuators is reset for a new null. This corresponds to a 6.02 db change in attenuation. Repeating the operation yields attenuation steps of 3.52 db, 2.50 db, etc.

A new method for the self-calibration of the change in insertion loss of continuously variable attenuation-measuring systems is described in this paper. This method requires two basic operations. The first operation yields the law of attenuation, $A=af(l)$, where a is the attenuation coefficient and $f(l)$ is a function of the adjustment parameter, l, which makes a a constant. The second operation yields the attenuation coefficient, a. The method is applicable to four different circuit configurations which are described in the order of their decreasing dependency upon component stabilities. Circuit 1 (fig. 1) is a single-channel system using the conventional series-substitution circuit. It requires a reasonably stable generator and monitor. The monitor can be of the differential-voltage type, but must be linear over the range to be used.

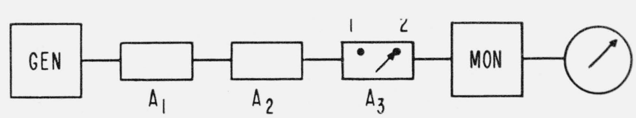

FIGURE 1. *Simple series substitution circuit.*

*Contribution from the Radio Standards Laboratory, National Bureau of Standards, Boulder, Colo.

[1] Figures in brackets indicate the literature references at the end of this paper.

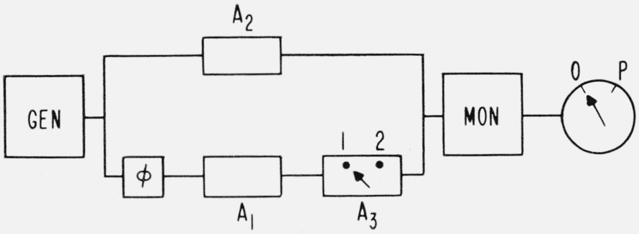

FIGURE 2. *Parallel substitution circuit.*

The second circuit (fig. 2) is the common parallel substitution circuit. It requires a reasonably stable generator and monitor, but the monitor does not need to be linear.

The third circuit (fig. 3) is similar to that used by Laverick, with the exception that only one switch is used. This circuit minimizes the effect of generator-power and monitor-gain variations. The upper branch must be isolated from the other two branches to prevent an impedance change at their junction when the switch is opened or closed (40 db isolation is generally sufficient). The fourth circuit is similar to that in figure 3, but with the switch removed and the attenuator A_4 replaced by a step attenuator. It then resembles the circuit used by Allred. The accuracy obtainable with this circuit is limited by the resolution and reproducibility of the attenuator dial, by the sharpness of the voltage null obtainable on the monitor, and by the time stability of components. It is essentially independent of generator and monitor variations. If the phase shift of the unknown attenuator depends upon its setting, the variations in insertion loss of the phase shifters must be known.

The methods of Allred and Laverick have limitations as well as advantages compared to the method described in this paper. Allred's method yields the complex attenuation coefficient, but the equations involved are quite difficult to solve. Laverick's method yields fixed 6.02 db and other smaller steps without calculations. The method described here yields only the magnitude of the attenuation coefficient, but it is easy to apply and is not affected by fixed-impedance mismatches in the system.

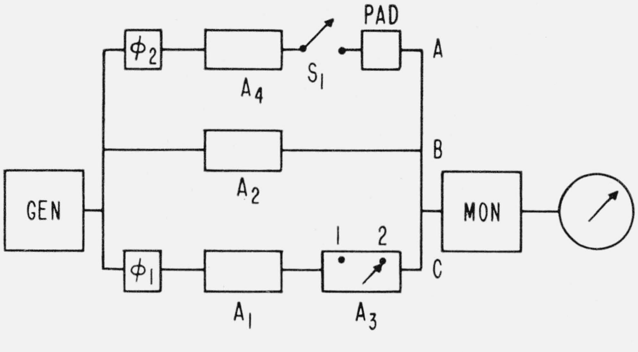

FIGURE 3. *Three-leg substitution circuit.*

It is to be noted that in all three systems the calibration is good only as long as the attenuators are connected to constant-impedance lines. If the impedances are varied, the calibration changes. Also, in the use of any calibrated system for calibrating other attenuators, the unknown attenuator must be matched to its characteristic impedance within the system.

2. Theory

The change in attenuation of an adjustable attenuator in a matched system may be defined as $A=20 \log |V_{01}|/|V_{02}|$, where A is in decibels, V_{01} is the output voltage of the network for the initial dial setting l_0, and V_{02} is the output voltage corresponding to the dial setting l_1. Since the attenuation (or insertion loss) is determined by a ratio, it is dimensionless, and hence no absolute units of voltage or other absolute units of measurement are required for its determination.

The simplest method of calibrating an attenuator would be to connect the attenuator to a voltage source of the proper frequency and to use a monitor which has an output reading proportional to the input voltage for indicating the voltage output of the attenuator. The monitor reading would be recorded as a function of the dial reading, l, of the attenuator. In this case the change in attenuation corresponding to two attenuator dial readings would be

$$\Delta A=20 \log \left|\frac{GV_1}{GV_2}\right|=20 \log \left|\frac{V_1}{V_2}\right| \tag{1}$$

where V_1 is the monitor reading for one attenuator dial setting, l_1; V_2 is the monitor reading for another dial setting, l_2; and G is the unknown proportionality factor of the monitor. This method, however, is impractical, since it is difficult to obtain a high-resolution, high-gain linear monitor without a residual voltage output.

One practical monitor, which could be used with the circuit of figure 1, is a linear voltmeter. This type of voltmeter normally has an expanded scale for high resolution, and its indicated voltage, V, is equal to $G(V_{in}+V_0)$ where G is the unknown gain, V_{in} is the input voltage, and V_0 some constant but unknown equivalent input voltage. Because of the unknown term, V_0, the attenuation cannot be calculated directly as in eq (1). However, it will be shown that V_0 need not be known.

The monitor reading V can be graphed as a function of the dial reading of the attenuator over any desired range. This plot of voltage as a function of l is then compared to a corresponding plot of attenuation arbitrary units as a function of l over the same range (fig. 4). This curve can be obtained by stepping off some unknown fixed attenuation, ΔA, and by obtaining corresponding dial readings as in section 3. This process yields a series of dial readings with equal but unknown attenuation steps, ΔA, between each set of readings. These readings can be plotted and a smoothed curve drawn.

From the A scale in figure 4, two equal attenuation steps are chosen. For simplicity two adjacent steps

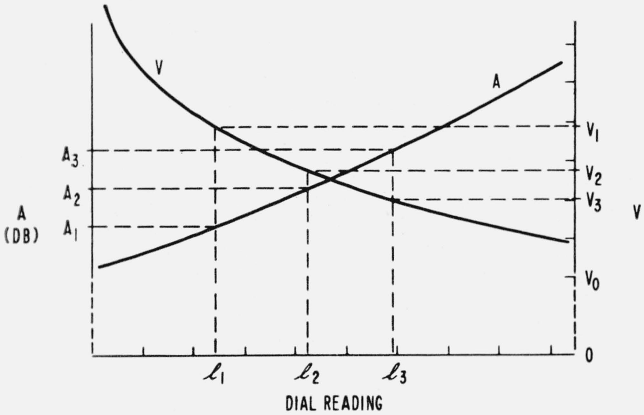

FIGURE 4. *Voltage and attenuation curves.*

are used such that

$$A_3-A_2=A_2-A_1.$$

To A_1 there corresponds the dial reading l_1, to A_2 the dial reading l_2, and to A_3 the dial reading l_3. For each of these dial readings there are corresponding points on the V scale, V_1, V_2, and V_3. Since the two attenuation steps are equal,

$$A_3-A_2=20 \log \left[\frac{G(V_3+V_0)}{G(V_2+V_0)}\right]$$

$$=A_2-A_1=20 \log \left[\frac{G(V_2+V_0)}{G(V_1+V_0)}\right]. \quad (2)$$

Assuming G constant, eliminating V_0, and substituting into the expression for A_3-A_2 or A_2-A_1 gives

$$A_2-A_1=A_3-A_2=20 \log \left(\frac{V_2-V_3}{V_1-V_2}\right). \quad (3)$$

Thus the chosen increments of attenuation can be expressed in terms of the corresponding voltage increments, without regard to the equivalent input voltage, V_0, of the monitor. This process may be repeated to give the calibration of the attenuator, over the desired portion of its range, to an accuracy limited only by the accuracy of the experimental data points and the precision with which the graphical analysis is made.

Increased accuracy of data may be obtained by use of the circuits of figures 2 or 3 where A_2 and/or A_1 are to be calibrated. The plot of attenuation in arbitrary units as a function of dial setting is obtained essentially as before. However, the voltage curve is obtained in a different manner, and the monitor is used only as a null indicator. The principle can be understood by referring to figures 3 and 5. In figure 3, A_1 and A_2 are continuously variable attenuators, and A_4 is a fixed attenuator, the output voltage of which determines the size of the incremental voltage step, ΔV. A_3 is a two-position attenuator used only for stepping off equal ΔA steps for obtaining the A curve. ϕ_1 and ϕ_2 are phase shifters, set so that the voltages from branches A and C are

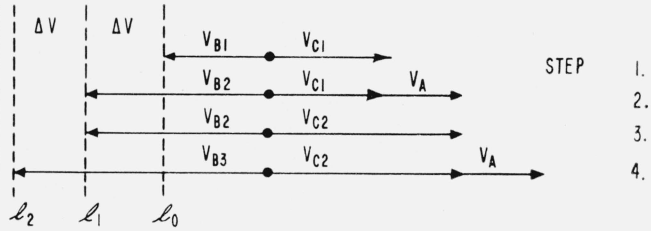

FIGURE 5. *Addition of equal voltage increments.*

continuously in phase but 180° out of phase with the voltage from branch B. The process by which the voltage curve is obtained may be clarified by reference to figure 5, which depicts the voltage outputs of the various branches of the circuit. Here, for simplicity, the letters denoting the particular branch are used to denote the voltage output of that branch. Initially branches B and C are nulled with the switch open, and the initial dial reading l_0 of A_2 is recorded. The switch is then closed, adding the voltage V_A to V_{C1}. A new null is achieved by adjusting ϕ_1 and advancing the dial of A_2 to l_1 so that the voltage from branch B equals $V_{C1}+V_A$. The voltage increment, ΔV, in the output of branch B, is then equal to V_A. The switch is then opened, and a null is again obtained by adjusting ϕ_1 and advancing A_1 so that $V_{C2}=V_{B2}$. The switch is closed, ϕ_2 adjusted, and the dial of A_2 advanced to l_3, so that again $V_{B3}=V_{C2}+V_A$. Thus another voltage increment, ΔV, has been added to the output of A_2. The operation is continued until the desired range of A_2 has been covered.

From the data thus obtained, the voltage curve (curve V, figure 4) can be plotted. The unit of voltage is the arbitrary voltage step ΔV, and the initial voltage, V_0, corresponding to the dial setting l_0 of A_2 is unknown. However, eq (3) and the discussion of it still apply. Thus, a monitor indicating only a null can be used to obtain voltage ratios, and the calibration of the attenuator, A_2, can be obtained as before.

The preceding discussion gave a graphical procedure for calibrating an unknown attenuator. An analysis is now presented in which it is shown that, after the law of attenuation of the attenuator has been obtained, the attenuator can be calibrated by using three dial readings corresponding to two equal steps in the output voltage and by consulting a table which has been computed from a derived equation.

The law of attenuation, $A=af(l)$, can be obtained from comparisons of equal attenuation steps and the corresponding dial readings of the dial of the attenuator to obtain a function of l such that $dA/df(l)=a$, a constant. Integrating this expression gives

$$\Delta A=a\,[f(l_i)-f(l_j)], \quad (4)$$

where a is the attenuation coefficient yet to be determined. This function of l will then yield a straight line when plotted against A as in figure 4. The function $f(l)$ depends upon the type of attenuator used. For piston-type attenuators ideally matched, $f(l)=l$ and for unmatched piston-type attenuators with normal insertion losses $f(l)$ closely approaches l. For rotary-vane attenuators in an ideally matched system, $f(l)=\log \cos^2 l$.

When the attenuator dial setting is changed from l_i to l_j, the change in attenuation in then given by eq (4). Further, if V_i and V_j are the output voltages of the attenuator corresponding to the dial settings l_i and l_j, then also

$$\Delta A=20 \log \left|\frac{V_i}{V_j}\right|. \tag{5}$$

If l_o is the dial setting corresponding to the output voltage V_0, l_1 is that corresponding to $V_0+\Delta V$, and l_2 is that corresponding to $V_0+2\Delta V$, then the following equations hold:

$$\Delta A_1=\alpha[f(l_0)-f(l_2)]=20 \log \frac{|V_0|}{|V_0+2\Delta V|}$$

$$\Delta A_2=\alpha[f(l_1)-f(l_2)]=20 \log \frac{|V_0+\Delta V|}{|V_0+2\Delta V|}.$$

(It may be noted that ΔA_1 and ΔA_2 are not equal, as was true in the graphical procedure.) If for convenience V_0 is written as some multiple, k, of the voltage increment ($V_0=k\Delta V$), the equations can be rewritten as

$$\Delta A_1=\alpha[f(l_0)-f(l_2)]=20 \log \frac{k}{k+2}, \tag{6}$$

$$\Delta A_2=\alpha[f(l_1)-f(l_2)]=20 \log \frac{k+1}{k+2}. \tag{7}$$

Dividing the second equation by the first yields

$$\frac{f(l_1)-f(l_2)}{f(l_0)-f(l_2)}=\frac{\log \dfrac{k+1}{k+2}}{\log \dfrac{k}{k+2}}. \tag{8}$$

The term on the left side of (8) is the experimentally measured constant, determined by the law of attenuation, the chosen initial dial setting, l_0, and the magnitude of ΔV.

A series-approximation method may be used to find the value of k to any desired accuracy, or a table may be obtained from National Bureau of Standards, Electronic Calibration Center, Boulder, Colo., for the direct determination of k to 6-place accuracy from experimental data.

Once k is known, eq (6) may be used to determine ΔA_1 in decibels and also α. This step results in the final calibration of the attenuator.

3. Experimental Procedures

In this section the detailed procedures are described for using the four circuits to obtain equal attenuation and voltage steps as a function of the dial setting of an attenuator.

3.1. Circuit 1

In the simple series-substitution circuit of figure 1, A_2 is the continuously variable attenuator to be calibrated. A_3 is a step attenuator which may be changed by a fixed amount ΔA, which need not be known. A_1 is a continuously adjustable attenuator used for power level control and may be an adjustable output control on the power source.

Attenuation as a function of dial setting is obtained as follows:

1. The attenuator to be calibrated, A_2, is set at the lowest desired value of attenuation, with dial reading l_0; A_3 is set at its higher attenuation, position 2, and A_1 is adjusted to give any desired fiducial reading on the output meter.
2. A_3 is then set at its lower attenuation position 1, A_2 is adjusted to bring the meter to the previously chosen fiducial point, and its dial reading l_1 is recorded. The attenuation of A_2 has now been increased by an amount ΔA in advancing the dial from l_0 to l_1.
3. A_3 is reset to position 2, and A_1 is adjusted so that the meter again returns to the fiducial point.
4. Steps 2 and 3 are repeated until the desired range of A_2 is covered.

Each time the dial reading of A_2 is changed, its attenuation is increased by an unknown amount ΔA. The resulting data may be used to plot a curve of attenuation, in unknown units, as a function of dial reading.

Voltage as a function of dial setting is obtained from a monitor which is linear over the range of the output voltage of the attenuator under calibration, or which has an output voltage that can be expressed as $V=G(V_{in}+V_0)$, where G is the constant but unknown gain of the monitor, V_{in} is its input voltage, and GV_0 is the residual output voltage. In this case the indicated voltage of the monitor is simply plotted as a function of the dial reading of the attenuator. This is a simple procedure, but the monitor may impose undesirable limitations on the accuracy of the voltage measurement.

3.2. Circuit 2

Where increased accuracy is required, the parallel-substitution circuit of figure 2 is used. A phase shifter ϕ, having constant loss independent of phase shift, has been added to the components of circuit 1. This phase shifter adjusts the voltage output of one branch to be 180° out of phase with the other. Hence the system operates on the null principle. For obtaining the attenuation of A_2 as a function of dial reading, the following method is used:

1. With the attenuator to be calibrated, A_2, set at the lowest desired value of attenuation l_0, and with A_3 set for its lower attenuation position 1, A_1 and ϕ are adjusted for a null reading on the monitor.
2. A_3 is set to position 2, A_2 is advanced and ϕ is adjusted until there is again a null reading on the monitor, and the dial reading l_1 is recorded.

3. A_3 is returned to position 1, and A_1 and ϕ are adjusted to obtain a null reading.

4. Steps 2 and 3 are repeated until the desired range of the attenuation of A_2 is covered.

For obtaining voltage as a function of dial reading, the following procedure is used:

1. With A_2 set at any convenient initial setting l_0 and A_3 set at either position, a null is achieved by adjusting A_1 and ϕ.

2. A_2 is advanced until some fiducial reading P_1 is obtained and the dial reading l_1 is recorded.

3. A_1 and ϕ are adjusted to obtain a new null.

4. A_2 is advanced until the reading P_1 is observed, and the dial reading l_2 is recorded.

5. Steps 1 through 4 are repeated if additional points are desired.

By following this procedure a series of dial readings l_0, l_1, l_2. . . . is obtained which corresponds to equal but unknown increments in the output voltage of the attenuator under calibration.

3.3. Circuit 3

To eliminate the effects of monitor instabilities, the circuit of figure 3 may be used. This circuit requires the addition of a switch, another phase shifter, ϕ_2, attenuator, A_4, and an isolation pad.

With S_1 open, the attenuation of A_2 as a function of dial reading is obtained exactly as oulined for figure 2. However, voltage as a function of the dial reading is obtained without relying on the constancy of a fiducial point. This is achieved by introducing from the third branch a voltage 180° out of phase with the output of A_2 and of suitable amplitude determined by A_4. The procedure is as follows:

1. With S_1 open, A_2 is set to the highest desired attenuation reading, l_0, and ϕ_1 and A_1 are adjusted until a null reading on the monitor is obtained.

2. A_2 is set to any lower attenuation reading, l_1, as desired; the switch, S_1, is closed, and A_4 and ϕ_2 are adjusted until a null reading on the monitor is obtained.

3. S_1 is opened, and A_1 and ϕ_1 are adjusted for *nulled* voltage output.

4. S_1 is closed, and A_2 and ϕ_2 are adjusted for a null indication. The dial reading l_2 of A_2 is recorded.

5. Steps 3 through 4 may be repeated if further equal voltage steps are required.

By this procedure, the output voltage of A_2 as a function of the dial reading may be obtained using the monitor only as a null indicating device.

3.4. Circuit 3 (Modified)

A further modification of the circuit may be made to reduce interaction effects when the switch, S_1, is opened and closed.

1. Switch S_1 is removed or closed.

2. Attenuator A_4 is replaced by a step attenuator which may be set to either of two fixed values of attenuation A_{41} or A_{42}.

3. The foregoing procedure is used with the exception that when it is specified that S_1 is to be open, A_4 is set to the higher attenuation level A_{42}, and when it is specified that S_1 is to be closed, the lower attenuation level A_{41} should be used.

4. The total attenuation in branch A with A_4 set at the higher attenuation level, A_{42}, should be adjusted so that a null can be obtained for the preceding second step.

4. Experimental Results

The change in attenuation of an X-band, rotary-vane attenuator corresponding to two chosen dial readings was measured by the use of circuit 1. This value was found to be 2.10 db. The corresponding value obtained by an IF substitution calibration system was 2.0 ± 0.1 db. The linearity of the monitor was not accurately known and may have been the principal cause of error.

A 30 M/cs attenuation-measuring system, using a parallel-branch circuit with a piston attenuator operating in the TE_{11} evanescent mode as the reference standard [2], was calibrated with the circuit of figure 2. To increase resolution of the constant-gain monitor used with this system and to obtain equal voltage steps, the circuit shown in figure 6 was used. In this circuit the d-c output of the monitor, with the switch S_1 closed, is bucked out by the emf developed across P_1. The resulting signal, which consists of noise and time variations of power and gain, is fed into the RC filter network and then amplified. To obtain equal voltage steps, P_2 is adjusted so that the voltage across S_1 corresponds to the desired voltage step and then can be switched into or out of the circuit by opening and closing S_1. For use with circuit 3, P_2 is not used. The d-c amplifier is of high input impedance with variable gain. At the null or zero position the meter reads zero, and when the voltage across P_2 is added to the circuit, the meter reading can be returned to zero by increasing the voltage into the monitor by an amount ΔV.

The measured value of the attenuation coefficient of the 30 Mc/s system was found to be 10.006 db/in. with a standard deviation of 0.003 db/in. The value of the attenuation coefficient computed from theoretical considerations is 10.000 ± 0.002 db/in. over the measured range.

The circuit of figure 3 was applied to the preceding system, with the third branch coupled to the generator through a 35-db directional coupler. A 30-db attenuator was used for the attenuator A_4 and the switch was isolated from the monitor junction by a 20-db attenuator. A correction was necessary for the change in impedance at the junction. This impedance change resulted from the change of

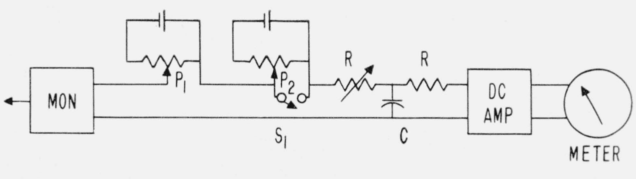

FIGURE 6. *Voltage differential and integrating circuit.*

reflected impedance through the 20-db isolation attenuator to the monitor junction as the switch was opened and closed. The use of a larger attenuator would have effectively eliminated this interaction effect. The measured value of the attenuation coefficient of the system was 10.003, with a standard deviation of 0.003 db/in.

Laverick's method was tried on this setup by inserting a switch in one of the other branches. The measured value of the attenuation coefficient was 10.023 db/in. with a standard deviation of 0.004 db/in. The error is assumed to be due to improper impedance matching of the three branches.

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5. References

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