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Resonance of the Space Between Earth and Ionosphere

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When noise radiation of roughly one or a few kilocycles per second is emitted in the higher atmosphere, part of it (an extraordinary wave) is propagated downward into the space between earth and ionosphere. Reflection at the earth and ionosphere leads then to a standing wave in this space, whose intensity for a given incident power flux varies very much with frequency. Maximum field strength of the standing wave is derived for the resonance frequencies of the space. The incident wave fronts are assumed to be horizontal. Only clearly defined wave fronts being of a sufficiently wide extension and showing no noticeable irregularities will lead to a definite resonance. The space between earth and ionosphere is comparable to an air gap between two parallel plane reflectors. The lower ionosphere is however only a partial reflector, allowing radiation to enter the resonance space and causing at the same time some loss of energy out of the resonance space (leakage). Stratification of the lower ionosphere has some influence on the resonance phenomena. With a D layer, an additional resonance at a lower frequency is obtained.

It seems too early to decide whether in any observed noises the resonance spectrum of the space between earth and ionosphere becomes apparent, but it is expected that noise spectra observed on the ground are modified by the resonances. In case of monochromatic emissions, the received intensity depends on the position in the resonance spectrum. The resonance spectrum should be received in case of emission of a white-noise spectrum, provided the wave fronts are appropriate.

1. Introduction

1.1. Cavity Between Two Plates

In a space between two infinite parallel plates resonance is observed when half a wavelength or multiples of a half-wavelength fit into the space. The electromagnetic field in this case of resonance is that of a standing wave whose wave planes are parallel to the plates. If one of the plates is made partially transparent, a model of the earth and ionosphere is obtained. The semitransparent plate like the ionosphere lets waves enter from outside—or from the higher atmosphere—and still permits resonance oscillations. The phenomena encountered in the case of two semitransparent plates (wire gratings) or one conductor together with a semitransparent plate (grating or dielectric plate) have been described in the literature [Casey and Lewis, 1952; Wait, 1954; von Trentini, 1956].

The field intensity in the resonance space is strongly dependent on the frequency of the incident waves, showing peaks at the resonance frequencies. The partial transparency of one of the reflectors entails imperfect reflection and contributes thus to a limitation of the resonance peaks.

1.2. Fundamentals

The ground is a good reflector for frequencies of the order of 1 kc/s. With a conductivity of 10^{-3} mho/m, for example, the amount of the complex refractive index of the ground for 1 kc/s is $|n|=134$, a value sufficient for good (almost perfect) reflection in vertical incidence, which is considered exclusively.

The ionosphere is only a partial reflector. The extraordinary wave at very (or extremely) low frequencies finds in the ionosphere (except its lowest part) conditions of free propagation, as is known from the audiofrequency whistlers. The refractive index of the extraordinary wave under these conditions is in general high ($\gg 1$). When the increase to these high values is fairly rapid in terms of wavelengths, we may expect that part of the extraordinary wave passes through the height interval of this rapid increase and part is reflected there. Thus we have for extraordinary waves the combination of an almost perfect reflector (the ground) with a semitransparent reflector that allows waves to enter the resonance space.

The descending wave fronts and consequently also the wave fronts in the resonance space will be assumed as horizontal. An eventual slight inclination of the wave fronts in the ionosphere would, because of the refraction in leaving the ionosphere, cause a considerable inclination of the wave fronts in the resonance space with the result of an essential modification of the resonance spectrum. Random distortion of the wave fronts would correspond to a mixture of propagation directions and consequently lead to a smearing-out of the resonance spectrum. The lateral extension of the wave fronts must be wide enough to have the simple plane-wave theory applicable.

There is some relationship between the resonance effect under consideration and the "tweeks", sequences of pulses caused by multiple reflection of sferics in propagation to a distant location [Potter, 1951]. The lightning flash from which a sferic originates is nearly a point source. The radiation thus is omnidirectional and the tweek spectrum may not be interpreted as a resonance spectrum corresponding to horizontal (or otherwise simply defined) wave fronts. The asymptotic limit frequency of the tweek spectrum, however, is supposed to relate to horizontal wave fronts and should consequently be identical with our fundamental resonance frequency, at least in case of a well-defined reflection level in the ionosphere. In other respects, the behavior of tweeks and that of our resonance phenomenon are not expected to be the same.

The energy of the resonance oscillations is limited by losses. In our case the losses are due to partial transparency of the lower ionosphere (leakage), absorption on the path in the ionosphere below the reflection level, and imperfect ground reflection. Only the partial transparency is included in the theoretical treatment to follow. At the rather low frequencies the partial transparency is thought of as being the main cause of loss under most conditions. It was noted before that the ground is an almost perfect reflector. An estimate of the absorption in ionospheric propagation is made in appendix 1. The absorption coefficient found for normal daytime conditions may not prevent the resonance oscillations, but we have to be aware that absorption tends to depress the quality factor of the resonances and to smooth the resonance curves. This may preferably happen in daytime and at the higher resonance frequencies.

The refractive index of the ionosphere for our frequencies is in most heights fairly well represented by the "quasi-longitudinal" approximation (eq (14) of appendix 1, slightly simplified)

$$n^2 = 1 \mp \frac{f_0^2}{f f_H |\cos \alpha|}. \qquad (1)$$

The two signs refer to the ordinary (−) and the extraordinary (+) wave.[1] f_0 is the plasma frequency, f_H the gyrofrequency of the electrons in the terrestrial magnetic field, and α the angle between the wave normal and the direction of the terrestrial magnetic field. Equation (1), sometimes further simplified by neglecting unity, has frequently been used to describe whistler propagation [Storey, 1953], (more references given by Poeverlein [1959a]). The equation is based on the assumptions: $f_0 \gg f$, $f \ll f_H$, α not near $\pi/2$, and negligible collision frequency. In the lowest ionosphere there is, of course, a height interval in which f_0 is comparable to f. At our frequencies this height interval is found in a rather low altitude and is obviously narrow in terms of wavelengths (or decay distances). It has therefore no remarkable influence on the propagation and eq (1) may be used throughout. At higher frequencies, at which f_0 is comparable to f in a significant height interval, the refractive index formula has to be more rigorous and coupling or transition between ordinary and extraordinary waves has to be taken into consideration. This is known to be the case at frequencies of the order of 10 or 20 kc/s.

Contributions of positive ions to the refractive index become noticeable near the gyrofrequency of the ions (approximately 50 c/s for O^+). All derivations from eq (1) are therefore useful only for frequencies well above the ion gyrofrequency.

[1] This choice of signs complies with Storey's [1953] classical paper on whistlers and with the conventional definition of ordinary and extraordinary waves on basis of the Appleton-Hartree formula. The opposite notation, however, is occasionally found.

1.3. Objective

This theoretical study is concerned with the modification of noise signals originating in the higher atmosphere by resonance of the space between earth and ionosphere. Frequency response curves of this space will be derived, which show for the frequencies under consideration the amplification or reduction of the signal intensity on the ground compared with the intensity of the incident wave.

First some simplified models of the ionosphere are assumed, exhibiting one or two reflecting boundaries of homogeneous media. Later, samples of nighttime and daytime stratifications of the lower ionosphere are considered. With a simple reflecting boundary, resonance is obviously obtained when multiples of half-wavelengths fit into the resonance space.

The study applies to wide horizontal wave fronts only. A noise emission coherent over a wide area is therefore required. Such an emission may arise from large-scale geophysical processes (e.g., incident corpuscular radiation) rather than from local events.

White noise or short pulses of a sufficient coherence over a wide area would lead to an immediate observation of the derived frequency response curves. Monochromatic noise undergoes an enhancement or reduction of intensity depending on the position of the frequency in the frequency spectrum (response curve). Generally the noise spectrum observable on the ground is the result of the noise emission, eventual propagation effects on the path through the ionosphere, and the response of the resonance space.

Processes of noise generation in the higher atmosphere have been treated to some extent in the literature [Gallet and Helliwell, 1959] (some more references are given by Gustafsson, Egeland, and Aarons [1960]). A survey of any amount of observational material as for indications of our resonance effect has not yet been made.

This study does not show much of a relationship with familiar theories of ELF or VLF propagation. This is mainly due to the consideration of waves that come down from high altitudes and travel vertically. Other investigations are concerned with propagation from a source to a receiver, both between earth and ionosphere. Some remarks on absorption and on a wave theory of propagation that might be of more general use appear in the appendixes.

2. Simplified Models

2.1. Earth and Sharply Bounded Ionosphere

The earth will be considered as a perfect reflector in all cases that will be discussed. The simplest model of the ionosphere is a homogeneous medium with a sharp boundary at the bottom. The refractive index of the ionosphere according to eq (1) has a high positive value for the extraordinary wave and in general an imaginary value for the ordinary wave. The extraordinary wave will be dealt with now.

The simple model of the resonance space and the ionosphere is shown in figure 1. The electric field in the resonance space is drawn in for various resonance cases. The field may originate from some noise emission in the ionosphere. A descending wave generated in the ionosphere will reach the bottom of the ionosphere and lead to a standing wave in the resonance space below and to a reflected wave traveling back within the ionosphere. The standing wave in the resonance space may be interpreted as the superposition of a descending and an ascending wave traveling in a vertical direction.

Ascending and descending waves will be denoted by F and G respectively. Strictly speaking, the symbols will denote the horizontal electric field strength components of these waves. In the anisotropic ionosphere an additional vertical (longitudinal) field strength component may exist.

Figure 2 is a representation of the horizontal electric field strengths F and G in the complex plane. The vectors indicate amplitudes and phases of ascending waves (F) and descending waves (G). The polarization of the waves (disregarding vertical E components in the ionosphere) is assumed constant over the entire propagation path. In case of a reflecting artificial dielectric, for example, the polarization may be linear in a fixed direction. In case of the extraordinary wave which is determined by the ionospheric propagation characteristics, the polarization is circular in the quasi-longitudinal approximation that is being used now [Ratcliffe, 1959]. The sense of the rotation is clockwise for an observer looking in the direction of the terrestrial magnetic field.

The construction of figure 2 starts at the earth's surface. The field strengths F_1 and G_1 there are opposite to each other in consequence of the assumed perfect reflection. In traveling upward the two vectors rotate in opposite senses. The rotation in the air space amounts to kh and is, with a time factor exp $(i\omega t)$, clockwise for the ascending wave (F). In traversing the bottom of the ionosphere there is a sudden change from F_2 and G_2 to F_3 and G_3. The total horizontal field strength components

$$\left.\begin{aligned} E_{\text{hor}} &= F+G \\ H_{\text{hor}} &= H = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n(F-G) \end{aligned}\right\} \qquad (2)$$

must be continuous at the boundary. E_{hor} there is imaginary, H_{hor} real (see fig. 2). In passing the boundary, therefore, the end points of the vectors in figure 2 are shifted symmetrically along a horizontal line with a reduction of the real components by a factor equal to the refractive index ratio (n of the ionosphere in our case). It may be recalled that n of the extraordinary wave is assumed to be real and greater than unity.

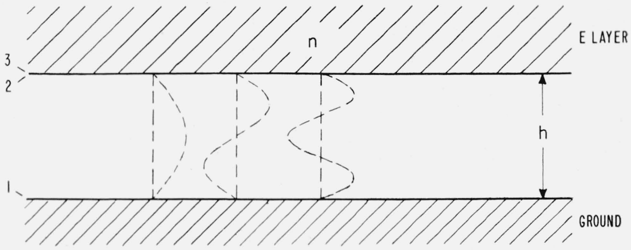

FIGURE 1. *Earth and sharply bounded ionosphere.*

The broken lines represent the distribution of the electric field strength in various resonance cases. The numbers denote the levels for which field strength vectors are shown in figure 2.

It is superfluous to consider the longitudinal electric field strength components of the waves at the boundary. Continuity of a resulting normal D component at any boundary is guaranteed by requiring continuity of H_{hor}.

From our construction it is easily seen that the reduction of the field vectors F and G in going up through the boundary is maximum if the vectors at the boundary are real or

$$kh = \pi,\ 2\pi,\ 3\pi,\ \ldots \qquad (3)$$

This is the resonance condition. In all cases given by eq (3), a relatively strong field in the space between earth and ionosphere (F_1 and G_1) arises from a weak wave incident from above (G_3).

Because G_3 is the field of the primary wave, one might like to start the computation or construction of the field strengths with G_3. This would however necessitate the consideration of multiple reflections between earth and ionosphere, which did not become apparent in the method chosen.

The procedure of figure 2 is also usable for more complicated models of the ionosphere. A multiple layer with several boundaries requires an iterative application of the procedure (section 2.2 deals with a double layer). A step approximation allows us to transfer the method to a continuously varying medium (section 3 and appendix 2).

Figure 2, of course, is nothing but a graphical representation of the analytical expressions describing the waves in the present case. From the figure or from the analytical formulation we derive the ratio between the field strengths arriving at the ground (G_1) and incident on the boundary of the ionosphere from above (G_3)

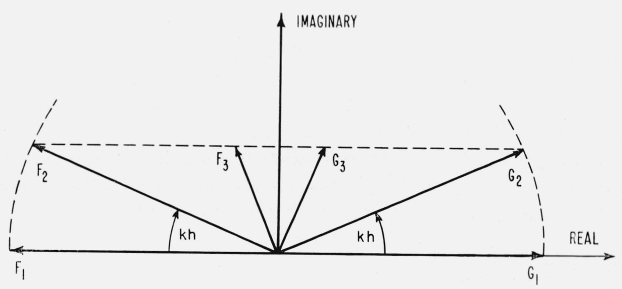

FIGURE 2. *Electric field strengths in the complex plane* (F *electric field strength of the ascending wave,* G *electric field strength of the descending wave*).

$$\left|\frac{G_1}{G_3}\right| = \frac{n}{\sqrt{n^2 \sin^2 kh + \cos^2 kh}}. \tag{4}$$

The ratio of the Poynting vectors may be considered as more significant than the ratio of the field strengths. The horizontal field strength component G of the descending wave in a medium of refractive index n corresponds to a vertical component of the Poynting vector proportional to nG^2 (cf. eqs (2)). An eventual horizontal component of the Poynting vector is not of interest in this study. The "ratio of the Poynting vectors" in the following always refers to the vertical components of the Poynting vectors which are proportional to nG^2. The ratio of the Poynting vectors corresponding to the field strength ratio of eq (4) thus is

$$\frac{1}{n}\left|\frac{G_1}{G_3}\right|^2 = \frac{n}{n^2 \sin^2 kh + \cos^2 kh}. \tag{5}$$

Equations (4) and (5) show that *resonance* is obtained under the condition of eq (3), with an enhancement of the Poynting vector

$$\frac{1}{n}\left|\frac{G_1}{G_3}\right|^2 = n. \tag{6}$$

Maximum reduction of the Poynting vector is obtained for

$$kh = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \ldots \tag{7}$$

with a reduction ratio

$$\frac{1}{n}\left|\frac{G_1}{G_3}\right|^2 = \frac{1}{n}. \tag{8}$$

For comparison the results of the Fresnel reflection theory for waves traversing a boundary may be noted: If the waves come from the medium with high-refractive index, a fraction $2n/(n+1)$ of the electric field strength or a fraction $4n/(n+1)^2$ of the Poynting vector appears in the second medium. Equation (6) shows that the resonance effect in our case seems to improve the transparency of the boundary (in terms of Poynting vectors) by a factor of approximately $n^2/4$. The reduction in the minimum case amounts to approximately 1/4 (eq 8).

The enhancement of the Poynting vector in the resonance case does of course not mean a violation of the law of conservation of energy. With perfectly reflecting earth, there is no net power flux in the space between earth and ionosphere. The Poynting vector considered in this space is the Poynting vector of one of the two progressive waves of which the standing wave is supposed to be composed.

2.2. Earth and Double-Layer Ionosphere

The single-layer model (fig. 1) may be a fairly good approximation of the lower ionosphere at night.

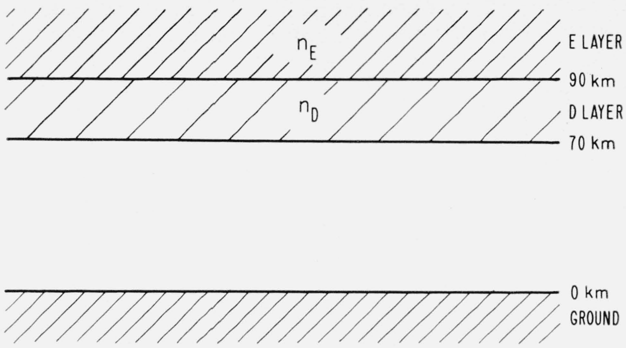

FIGURE 3. *Earth and double-layer ionosphere.*

The daytime ionosphere in lower heights is better approached by two adjacent dielectrics, representing the D and the E layers.[2] This model is shown in figure 3. The heights are chosen in accordance with normal daytime conditions. The refractive indices for the extraordinary wave at a frequency around 500 c/s can be assumed as

$$\left.\begin{aligned} n_D &= 8 \\ n_E &= 50 \end{aligned}\right\}. \tag{9}$$

The vector diagram, figure 2, must now be complemented by a rotation of the vectors corresponding to the phase shift in traveling up through the D layer and one more horizontal shift of the vector end points corresponding to the transition from the D to the E layer. In passing through the boundary between the D and E layers, the horizontal components of the field strength vectors in the vector diagram are reduced by the refractive index ratio n_E/n_D.

In the present case two kinds of resonance may be distinguished, the one resulting from maximum reduction of the electric field strengths F and G (fig. 2) at the bottom of the D layer, the other from maximum reduction at the boundary between D and E layers. The first kind of resonance requires that the field vectors F and G in reaching the D layer are real, or in other words, that an integral number of half-wavelengths fits into the space between ground and D layer. With a height of 70 km for the bottom of the D layer (fig. 3) this yields the resonance frequencies

$$\left.\begin{aligned} f &= 2.14 \times 10^3 \times m \text{ (c/s)} \\ m &= 1, 2, 3, \ldots \end{aligned}\right\}. \tag{10}$$

The minimums between the resonances are found at $m = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$.

The second kind of resonance, corresponding to maximum reduction at the boundary between the D and the E layer requires real field vectors at this boundary. With the model of figure 3 and the refractive indices of eqs (9) resonance frequencies of 509, 1386, . . . c/s and minimums at 151, 945, . . . c/s were derived.

[2] A brief account of a resonance case in the two-layer model was given in an earlier paper [Poeverlein, 1959b].

A simple approximation for the second kind of resonance is obtained by assuming $n_D \gg 1$. With large n_D the horizontal components of F and G (fig. 2) in passing the bottom of the D layer are reduced to almost zero. This means that F and G immediately above the bottom of the D layer become imaginary. A phase shift of $\frac{1}{2}\pi$, $\frac{3}{2}\pi$, $\frac{5}{2}\pi$, . . . within the D layer makes then the vectors F and G real at the top of the D layer. The corresponding resonance frequencies under our conditions are 469, 1406, 2340, . . . c/s.

3. Realistic Cases

3.1. E Layer, Extraordinary Wave

A nighttime E layer profile given by Nertney ([Poeverlein, 1959a] fig. 18) may now be used for the study of resonance of the extraordinary wave. The profile and an approximative step function are shown in figure 4.

The refractive index in the ionosphere (eq 1) depends essentially on the plasma frequency f_0. Values of f_0 following from

$$f_0^2 = 80.7 \times 10^6 \times N \tag{11}$$

are noted in figure 4 (f_0 in c/s, the electron concentration N per cm^3). In the refractive index formula,

$$f_H |\cos \alpha| = 1.316 \times 10^6 \text{ c/s} \tag{12}$$

may be assumed (corresponding to some temperate latitude) and hence

$$f_0^2/(f_H |\cos \alpha|) = 61.3N. \tag{13}$$

The step function of figure 4 similarly to a combination of several dielectrics (sections 2.1 and 2.2) allows the iterative application of the graphical method (fig. 2). The step-by-step construction of the field vectors was begun at the perfectly reflecting ground and conducted up to 104 km height. It appeared that not much contribution to the reflection of the waves comes from heights beyond 104 km, though some inaccuracy may arise from stopping at this somewhat low level and from the large width of the individual steps.

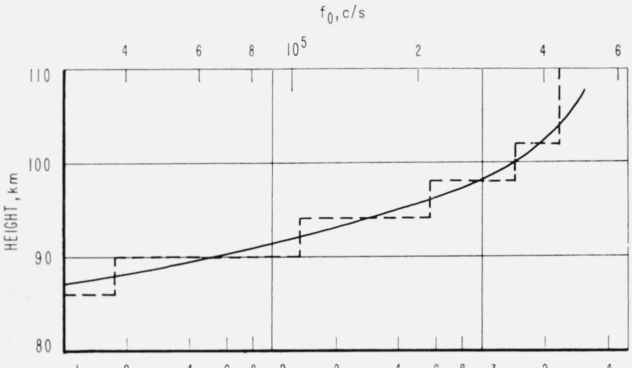

FIGURE 4. *E layer profile.*

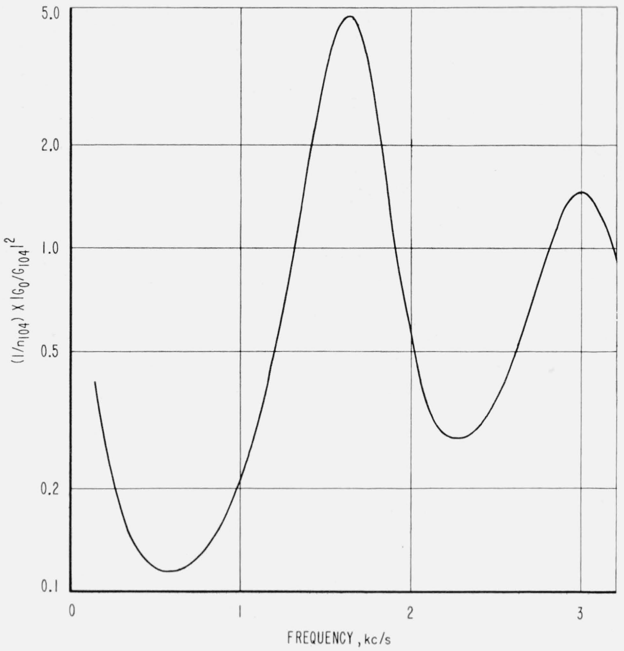

FIGURE 5. *Ratio of the Poynting vectors on the ground and in the ionosphere for the profile of figure 4.*

The graphical construction was used to derive the intensity variation of the descending wave. The ratio of the Poynting vectors on the ground and in 104 km height, $(1/n_{104}) \times |G_0/G_{104}|^2$, is plotted versus frequency in figure 5. The positions of the maximums and minimums are not much different from those for a sharply bounded ionosphere (section 2.1), in 90 or 100 km height. The gradient of the electron concentration apparently is too steep to have much influence. A shallower gradient doubtless has some effect and must lead to diminished resonance frequencies as does a D layer below the E reflection level (next section). The greatest enhancement of the Poynting vector (fig. 5) is seen to amount to 5 (at 1.63 kc/s), the strongest reduction to 0.11 (at 0.60 kc/s).

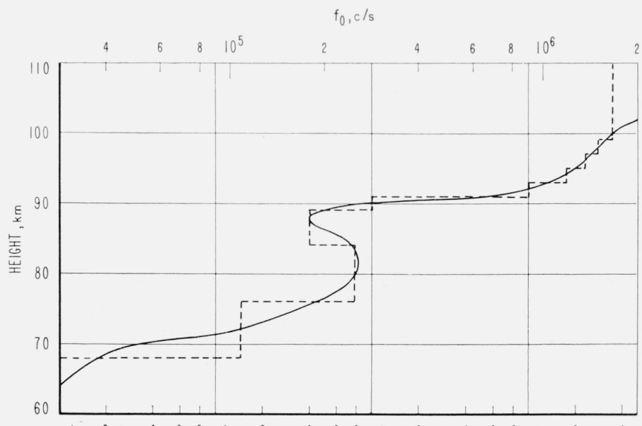

FIGURE 6. *Profile of* D *layer and* E *layer.*

3.2. D Layer and E Layer, Extraordinary Wave

In daytime, with presence of the D layer, the profile of the ionosphere may according to Pfister be that of figure 6 ([Poeverlein, 1959a], fig. 17, curve (b)). The step approximation of this profile leads to the Poynting vector ratio, $(1/n_{100})\times|G_0/G_{100}|^2$, represented in figure 7. The maximum enhancement is 1.02, the strongest reduction 0.06.

The sequence of the resonance peaks seems to be irregular. In fact, the stratification of the ionosphere resembles the simplified model of section 2.2 for which two series of resonances were found. The first resonance peak in the present case (at 440 c/s) corresponds to the first peak of the simplified model (at 509 or 469 c/s) which was explained by the behavior of the field at the bottom of the E layer and the phase shift in passing through the D layer. In the further course of the resonance curve (fig. 7) it seems impossible to separate the two series of resonance peaks of the simplified model.

There is no pretension to accuracy of figure 7. At the higher frequencies, for which the subdivision of the step function (fig. 6) is not fine enough, some estimates were used in the values plotted.

3.3. Ordinary Wave

For the ordinary wave, which is obtained with the minus sign in eq (1), the refractive index is imaginary in most parts of the ionosphere. The graphical method of figure 2 is still usable for imaginary n; however, the electric field strengths F and G become purely imaginary and have unequal amounts. The two waves (F and G) are attenuated, each in progression in its direction of propagation.

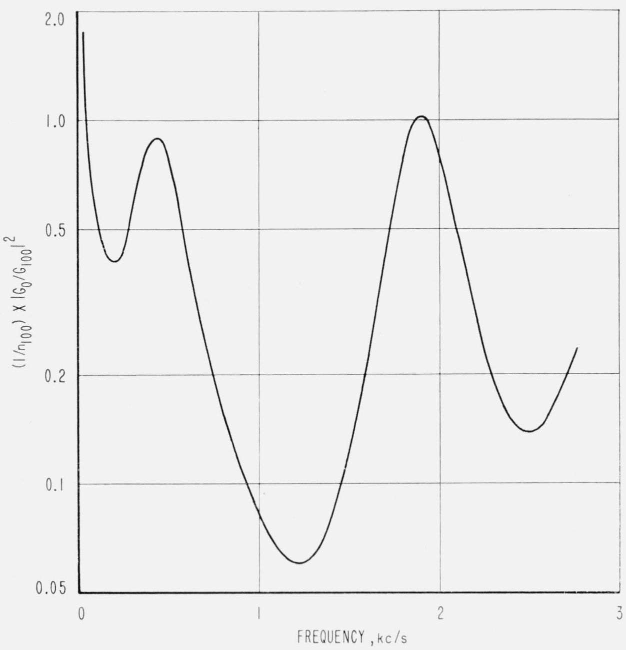

FIGURE 7. *Ratio of the Poynting vectors on the ground and in the ionosphere for the profile of figure 6.*

The resonance case now is the case of disappearance of G in higher altitudes, strictly speaking in the altitude in which the signal is generated. In this case the enhancement of the Poynting vector is infinite, because no initial Poynting vector is present. A finite enhancement would result from considering electron collisions in the refractive index formula. The present study in which collision losses are disregarded allows therefore only the computation of the resonance frequencies. The first resonance frequency in the case without D layer (fig. 4) is found to be 1.52 kc/s, nearly the same as for a sharply bounded ionosphere.

Whereas resonance of extraordinary waves may be excited by waves coming down from any altitude within the ionosphere, the ordinary-wave resonance requires emission of radiation not too far beyond the reflection level, in order to have the ordinary waves only slightly attenuated in traveling down. The radiation source may be somewhat above the reflection level of the ordinary waves or in the part of the ionosphere below the reflection level (lower tail of E, or D layer). The polarizations of ordinary and extraordinary waves show opposite senses of rotation.

4. Comparison With Observations

As noted in section 1.3, only an emission of white noise or short pulses in the ionosphere, fulfilling the coherence requirement, would lead to an observation of the derived resonance functions (figs. 5 and 7). In case of a given noise frequency the resonance function determines the enhancement or diminution of the signal on the ground compared with the noise intensity descending from higher altitudes.

The field observable on the ground should be a horizontal H. On a perfectly reflecting ground there should be no horizontal E. Imperfectness of the ground reflection, however, makes a horizontal E possible. Circular polarization as expected from extraordinary (or ordinary) waves would correspond to two horizontal E (or H) components with 90-deg phase shift.

Noise emissions exhibiting two or sometimes three peaks in the spectrum were observed in the auroral zone in connection with geomagnetic disturbances [Aarons, Gustafsson, and Egeland, 1960; Gustafsson, Egeland, and Aarons, 1960]. The observed spectrum resembles at least roughly the resonance functions. The noise peaks in most cases were near 750 c/s and between 2 and 3.5 kc/s. The peak near 750 c/s is rather stable in frequency. In one event of noise emission, for example, it is reported to have varied as little as ± 150 c/s. At other occasions there were, however, larger deviations. The observers suggest gyroresonance of protons as cause of the 750-c/s peak.

The second peak in the observed spectrum is more variable. It may be that this peak is caused by resonance enhancement of practically white noise and the first peak by gyroresonance radiation, possibly in near coincidence with our first resonance peak under the prevailing conditions. A clue to the nature of the noises could be obtained from observation of their polarization. In proton gyroradiation ordinary waves are emitted. Our standing-wave resonance, on the other hand, may preferrably be excited by extraordinary waves, as the remarks in section 3.3 indicate. Simultaneous noise observations on the ground and in the ionosphere might also allow a decision about the character of the noises.

For a discussion of VLF and ELF noises with respect to characteristics and possible origin one may refer to Gustafsson, Egeland, and Aarons [1960].

5. Additional Remarks

From a *pulse-type signal* emitted in the ionosphere, a series of pulses will be received on the ground. The first pulse reaches the ground immediately after leaving the ionosphere and traveling down through the air space. The others traveled back and forth between earth and ionosphere once or several times.

As previously, only noise signals that have extensive horizontal wave fronts are considered. The spectrum of the received series of pulses is then necessarily identical with the resonance spectrum obtained from white noise emissions in the ionosphere, because a pulse (provided it is short) has the same spectrum as white noise. In the simplified model of a sharply bounded homogeneous ionosphere with a refractive index independent of frequency, there is no distortion of the pulse in the ionospheric reflection, only a reduction of the field strength by a factor $(n-1)/(n+1)$. The geometric series of pulses thus obtained has a Fourier spectrum corresponding to eq (4) of the above resonance theory. This proves the equivalence of the two considerations in this special case.

The question arises whether the *whistlers* observed in the audiofrequency band are also affected by resonance of the space between earth and ionosphere. Whistlers in coming down from high altitudes may have extensive wave fronts, but presumably not in a horizontal position. A possible variation of the resonance phenomena resulting from this inclination is not so much of interest now. The modification of the whistler by the resonance effect means nothing but a multiple repetition of the whistler signal, just the same as in the case of pulses. It will not be discussed now whether this repetition may or may not be recognizable in recordings. An important feature of the resonance spectrum is the first minimum, which appears in figure 5 at 600 c/s and will be slightly shifted towards higher frequencies in case of inclined wave fronts. This minimum may contribute to a low-frequency cutoff of the observable whistler spectrum.

The resonance effect this paper deals with refers to standing waves between two parallel reflectors. The *curvature* of the actual reflectors (earth and ionosphere) may necessitate some correction of the theory at the lowest frequencies.[3] The lateral extension of the field must be wide in terms of wavelengths, but does not have to include the entire spherical shell.

[3] For the general theory of VLF propagation between curved earth and ionosphere, refer to Wait [1960].

A resonance effect caused by the spherical shape of the earth is the *resonance of the zero-order mode propagated between earth and ionosphere* which results from winding-up of the waveguide to a spherical shell [Schumann, 1957; König, 1959; Wait, 1960]. The fundamental frequency of this resonance effect corresponds to a phase variation of 2π along a path around the earth and amounts according to Schumann [1957] and König [1959] to approximately 9 c/s.

The fundamental frequency of our resonance effect in case of no leakage into the ionosphere is identical with the *cutoff frequency* of the first-order *mode* between earth and ionosphere, because this cutoff relates to vertically traveling waves. The higher-order resonance frequencies similarly represent the cutoff frequencies of the higher-order modes in this case.

Appendix 1. Effect of Collisions

The quasi-longitudinal approximation of the refractive index derived from the Appleton-Hartree formula with inclusion of the collision frequency ν of the electrons is[4] [Ratcliffe, 1959]

$$n^2=1-\frac{\omega_0^2}{\omega(\omega-i\nu\pm\omega_H|\cos\alpha|)}. \tag{14}$$

This approximation is usable at sufficiently high frequencies and at very low frequencies, provided the angle α between wave normal and terrestrial magnetic field is not near 90 deg. In both cases the approximative formula can be further simplified.

The high-frequency approximation of n with the assumption of $\omega\gg\omega_0$, $\omega\gg\omega_H$, and $\nu\ll\omega$ is

$$n=1-\frac{1}{2}\frac{\omega_0^2}{\omega^2}-i\cdot\frac{1}{2}\frac{\omega_0^2\nu}{\omega^3}. \tag{15}$$

At sufficiently low frequencies we may assume $\omega_0\gg\omega, \omega_0^2\gg\omega\omega_H, \nu\ll\omega_H$ and obtain then for the extraordinary wave

$$n=\left(\frac{\omega_0^2}{\omega\omega_H\,|\cos\alpha|}\right)^{\frac{1}{2}}\left(1-i\cdot\frac{1}{2}\frac{\nu}{\omega_H\,|\cos\alpha|}\right). \tag{16}$$

The absorption in ionospheric propagation is given by $\exp\left(\int\frac{\omega}{c}\,\mathrm{Imag}\,(n)\cdot ds\right)$. The absorption coefficient thus is at high frequencies

$$\frac{\omega}{c}\,|\mathrm{Imag}\,(n)|=\frac{1}{2}\frac{\omega_0^2}{\omega^2}\frac{\nu}{c} \tag{17}$$

and at sufficiently low frequencies

$$\frac{\omega}{c}\,|\mathrm{Imag}\,(n)|=\frac{1}{2}\frac{\omega_0\omega^{\frac{1}{2}}}{(\omega_H\,|\cos\alpha|)^{3/2}}\frac{\nu}{c}. \tag{18}$$

The absorption coefficient for high frequencies, eq (17), shows the familiar frequency dependence of

[4] Angular frequencies are written in place of the frequency quantities of eq (1).

the "nondeviative" absorption (with neglect of ω_H compared with ω). The absorption coefficient for very low frequencies, however, decreases with decreasing frequency. This has the consequence that at our rather low frequencies the absorption in the lower ionosphere is only small.

A numerical evaluation based on eq (14) and daytime data of the D layer (fig. 6 and ν as noted) gave the following amounts of the absorption coefficient of the extraordinary wave at 1 kc/s (for $f_H|\cos \alpha|=1.2\cdot 10^6$ c/s):

0.021 per km in 72 km with $\nu=10^7$ per sec,

0.019 per km in 80 km with $\nu=2\cdot 10^6$ per sec.

These values should lead to a considerable absorption on paths of 50 km length, but the absorption coefficient decreases rapidly above 80 km and the decisive vertical path lengths are shorter.

A closer investigation of the absorption and its influence on the resonance curve are still desirable, but the above estimate suggests that the absorption is small and may not deteriorate badly the resonance curves, particularly not at the lower frequencies. At night the absorption coefficients are much smaller than the ones derived for daytime and in daytime the lowest resonance frequency is below 1 kc/s, as figure 7 shows.

It may be pointed out that the absorption coefficients for high and very low frequencies are different not only in their variation with frequency but also in their dependence on ω_0. The absorption coefficient, of course, is not in an immediate connection with the reflection coefficient of the lower ionosphere at very low frequencies.

Appendix 2. Wave Theory Based on the Vector Method

The construction of the complex field strength vectors (fig. 2) can be used for a rigorous wave-theoretical treatment of propagation in a medium that varies continuously with one coordinate. It was seen that the method of constructing field vectors is applicable to a step approximation of a continuously varying medium. The start of the graphical procedure was at a perfect reflector, in our case the ground. The solution consequently is a standing-wave solution.

If propagation of plane waves in a continuously varying medium is to be studied, we replace the medium by an approximative step function and add a perfect reflector somewhere outside, thus creating conditions for the vector method and a standing-wave solution. A traveling-wave solution or any more general solution follows from superposition of two independent standing-wave solutions. These two standing-wave solutions may either be obtained by assuming the perfect reflector at two different locations or by assuming two alternative reflectors, one that does and one that does not reverse the electric field strength in reflection of the incident wave. The only approximation introduced is the step function. By making the steps differentials the method becomes rigorous. Nevertheless, the method has certain limitations. Before showing them, a brief analytical formulation will be given.

The medium will be assumed to vary with the coordinate z. For waves propagated in the direction of the gradient, the field strengths may then be written

$$\left.\begin{aligned} E_x &= C(z)\sin\left[\int n(z)k_0dz+\phi(z)\right] \\ H_y &= i\sqrt{\frac{\epsilon_0}{\mu_0}}\,n(z)C(z)\cos\left[\int n(z)k_0dz+\phi(z)\right] \end{aligned}\right\}. \tag{19}$$

The time variation factor exp $(i\omega t)$ has been omitted. The amplitude function $C(z)$ and the phase function $\phi(z)$ are sought. E_x and H_y represent transverse field strengths only. An additional longitudinal electric field component E_z may however exist, due to the anisotropy of the medium. The polarization of the wave in the x,y plane must be constant in order to have the continuity considerations of the next paragraph valid. The polarization may be linear (with E_x and H_y only) or circular (for example with $E_y=iE_x$, $H_x=-iH_y$).

The continuity of E_x and H_y at boundaries requires that at the steps of the step function

$$\left.\begin{aligned} \Delta(Cs)&=0 \\ \Delta(nCc)&=0 \end{aligned}\right\}. \tag{20}$$

Herein s and c are abbreviations for the sine and cosine appearing in eqs (19):

$$\left.\begin{aligned} s&=\sin\left[\int nk_0dz+\phi\right] \\ c&=\cos\left[\int nk_0dz+\phi\right] \end{aligned}\right\}. \tag{21}$$

The integral term in the argument of these functions varies continuously. Only ϕ is supposed to jump at the steps of the step function. The differences of s and c needed in eqs (20) are therefore

$$\left.\begin{aligned} \Delta s&=c\Delta\phi \\ \Delta c&=-s\Delta\phi \end{aligned}\right\}. \tag{22}$$

Equations (20) together with eqs (22) yield

$$\left.\begin{aligned} \frac{\Delta C}{C}&=-\frac{\Delta n}{n}c^2 \\ \Delta\phi&=\frac{\Delta n}{n}sc \end{aligned}\right\}. \tag{23}$$

Making the steps differentials leads to the rigorous wave-theoretical differential equations for C and ϕ

$$\left.\begin{aligned} \frac{dC}{C}&=-\frac{dn}{n}c^2 \\ d\phi&=\frac{dn}{n}sc \end{aligned}\right\}. \tag{24}$$

It has to be noticed that the quantities s and c in eqs (24) are functions of ϕ (eqs (21)). The last of eqs (24) allows the computation of the phase function ϕ without considering the amplitude function, whereas the first equation involves both the amplitude function and the phase function. Equations (24) are also readily obtained from Maxwell's equations.

A set of equations closely related to eqs (24) has been derived by van Cittert [1939] [Kofink, 1947]. Van Cittert considered oblique incidence on a one-dimensionally stratified medium. His equations therefore are more general in this respect. The medium he assumes as isotropic. The equations are necessarily different for the two cases of (linear) polarization.

Slow variation of the refractive index suggests averaging the factors c^2 and sc over many periods of c and s. Equations (24) then become approximately

$$\left.\begin{aligned} \frac{dC}{C} &= -\frac{1}{2}\frac{dn}{n} \\ d\phi &= 0 \end{aligned}\right\}, \tag{25}$$

whence we obtain

$$\left.\begin{aligned} C &\propto \frac{1}{\sqrt{n}} \\ \phi &= \text{const} \end{aligned}\right\}. \tag{26}$$

This is the behavior of amplitude and phase in the geometric-optical approximation.

The above study was limited to *normal* incidence on a one-dimensionally stratified medium. A generalization for *oblique incidence* (on a one-dimensionally stratified medium) may not be difficult, as van Cittert's formulation shows. The *more severe limitation* arises from the demand of constant polarization. In case of oblique incidence the polarization in the planes of the stratification has to stay constant. This is easily achieved in isotropic media. In anisotropic media constant polarization is obtained in normal incidence if the polarization can be assumed as circular, independent of the location in the layer. In more general cases of propagation in anisotropic media (oblique incidence or elliptic polarization) the polarization in the planes of the stratification varies. The continuity at the steps of our step function can then be preserved only by simultaneous consideration of ordinary and extraordinary waves with variable amplitude factors. This indicates the well-known phenomenon of coupling between the two types of waves.

The step method is not applicable near a *zero of the refractive index*. In approaching the zero, the required step length decreases towards zero. However, a rigorous expression for the refractive index that includes the collision frequency has no zero at any location.

Complex refractive index leaves the analytical formulations valid, but makes the graphical procedure of figure 2 more intricate.

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