

Reflection From a Sharply Bounded Ionosphere for VLF Propagation Perpendicular to the Magnetic Meridian

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There is experimental evidence that VLF signals propagating from west to east suffer less attenuation than from east to west. Earlier work treating the case of nonreciprocal propagation along the magnetic equator is extended in latitude. The nonreciprocity shown by $||R||$ for highly oblique propagation along the magnetic equator persists when the reflection point moves towards a magnetic pole, but at the pole itself $||R||$ is reciprocal. To a first approximation ${}_{\perp}R_{\perp}$ is reciprocal at all magnetic latitudes. The conversion coefficients $||R_{\perp}, {}_{\perp}R||$, are greater for east-to-west propagation than for propagation in the opposite direction, except at a magnetic pole where they are equal.

1. Introduction

There is now a considerable amount of experimental evidence [Crombie, 1958; Taylor 1960] showing that VLF propagation is nonreciprocal, since VLF signals received from a westerly quarter appear to suffer less attenuation than those received from the east. A qualitative explanation of such behavior in terms of interactions between the longitudinal movements of ionospheric electrons in the plane of incidence, and the transverse component of the earth's magnetic field was given by Crombie [1958].

Subsequently, Barber and Crombie [1959] calculated the reflection coefficient of a sharply bounded ionosphere and showed that this explanation was correct for the special case of waves polarized in the plane of incidence, propagating along the magnetic equator. This special case simplified the calculations since the magnetic field is horizontal and transverse to the plane of incidence.

It has also been shown [Crombie, 1960] that the nonreciprocity which could be expected from the lack of reciprocity exhibited by the reflection coefficient does occur when mode theory is used. Wait [1960, 1961] and Wait and Spies [1960], using some coefficients derived from the numerical data of Jöhler [1961], have shown that mode propagation depends on the direction of propagation. More recently, Dobrott and Ishimaru [1961] have also concluded that VLF propagation along the magnetic equator may be nonreciprocal.

It is the purpose of this paper to derive formulas for the reflection coefficient of a sharply bounded ionosphere in the rather more general case where plane waves are incident from the west, or from the east at any magnetic latitude.

2. Introductory Theory

The starting point of this work is a paper by Yabroff [1957a and b] in which, following an outline by Bremmer [1949], he derives perfectly general relationships between the fields in a sharply bounded ionosphere, and obtains the appropriate boundary equations. Yabroff considered the coordinate system shown in figure 1. The vertical direction is x and the earth's magnetic field H_e is contained in the xz plane. The angle of incidence of the waves is β_i , and the magnetic field of the earth is inclined to the vertical at an angle β_e . The incident, reflected and transmitted waves are described in the coordinate system x', y', z' in which for a wave

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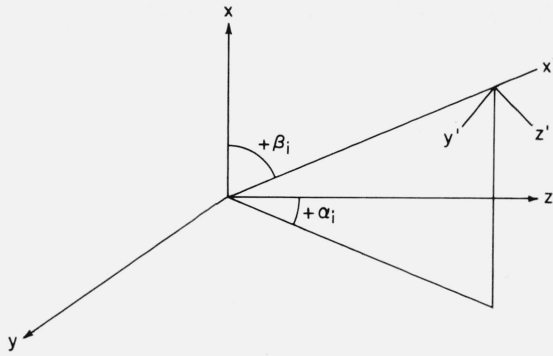


FIGURE 1. Coordinate system.

The positive values of β_i and α_i are measured clockwise from the positive x and z axes respectively.

propagating in the α_i, β_i direction, z' is in the $\alpha_i, \beta_i + 90^\circ$ direction; y' is directed in the $\alpha_i + 90^\circ, 90^\circ$ direction and x' is in the direction α_i, β_i . The lower boundary of the supposedly uniform ionosphere is the xy plane. If z is magnetic north, then y is magnetic east.

The incident plane wave F_i can be written as

$$F_i = \text{Re} \left\{ F_{oi} \exp \left[j \frac{\omega}{c} (ct - x \cos \beta_i - y \sin \beta_i \sin \alpha_i - z \sin \beta_i \cos \alpha_i) \right] \right\}$$

in which ω = wave angular frequency, c is the velocity of light and t is time. Since the boundary conditions must be satisfied it follows that the wave in the ionosphere F_m , arising from this incident wave, must be of the form

$$F_m = \text{Re} \left\{ F_{om} \exp \left[j \frac{\omega}{c} (ct - Dx - y \sin \beta_i \sin \alpha_i - z \sin \beta_i \cos \alpha_i) \right] \right\}$$

D may be regarded as the cosine of the (complex) angle of refraction in the ionosphere.

Maxwell's equations and the equations of motion of an electron in a magnetic field can be written as

$$\left. \begin{aligned} \nabla \times \underline{E} &= -\mu_0 \frac{\partial \underline{H}}{\partial t} \\ \nabla \times \underline{H} &= -Ne \underline{V} + \epsilon_0 \frac{\partial \underline{E}}{\partial t} \\ m \frac{d\underline{v}}{dt} &= -e \underline{E} - m\nu \underline{V} - \mu_0 (\underline{V} \times \underline{H}_e) \end{aligned} \right\} \quad (1)$$

in which, $-e, m$ are the charge and mass of an electron,

\underline{V} is the electron velocity,

ν is the collision frequency,

ϵ_0, μ_0 are the permittivity and permeability of free space.

From these three equations the three components of \underline{H} and \underline{V} may be eliminated giving three equations relating the components of \underline{E} . In matrix form these equations are:

$$\begin{bmatrix} 1 - a^2 - \frac{s^2 - h_L^2}{s(s^2 - h^2)} & a_T D - \frac{j h_T}{s^2 - h^2} & a_L D + \frac{h_T h_L}{s(s^2 - h^2)} \\ a_T D + \frac{j h_T}{s^2 - h^2} & 1 - a_L^2 - D^2 - \frac{s}{s^2 - h^2} & a_L a_T - \frac{j h_L}{s^2 - h^2} \\ a_L D + \frac{h_T h_L}{s(s^2 - h^2)} & a_T a_L + \frac{j h_L}{s^2 - h^2} & 1 - D^2 - a_T^2 - \frac{s^2 - h_T^2}{s(s^2 - h^2)} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \quad (2)$$

where

$$\begin{aligned}
 s &= (1-jZ)/X & h &= Y/X, \quad h_L = h \cos \beta_e, \quad h_T = h \sin \beta_e \\
 X &= Ne^2/m\epsilon_0\omega^2 = (\text{normalized plasma frequency})^2. \\
 Y &= \mu_0 e H_e / \omega m = \text{normalized gyro frequency.} \\
 Z &= \nu / \omega = \text{normalized collision frequency.} \\
 a_L &= \sin \beta_i \cos \alpha_i, \quad a_T = \sin \beta_i \sin \alpha_i, \quad a = \sin \beta_i.
 \end{aligned}$$

In order that \tilde{E} be finite the determinant of (2) must be zero. Thus

$$b_4 D^4 + b_3 D^3 + b_2 D^2 + b_1 D + b_0 = 0 \quad (3)$$

where

$$\begin{aligned}
 b_4 &= s(s^2 - h^2) - s^2 + h_L^2 \\
 b_3 &= 2a_L h_L h_T \\
 b_2 &= 2s[(1-a^2)h^2 - (s-1)(s-a^2s-1)] - h_T^2(1-a_L^2) - (2-a^2)h_L^2 \\
 b_1 &= -2(1-a^2)a_L h_L h_T \\
 b_0 &= (s-1)\{[(1-a^2)s-1]^2 - (1-a^2)^2 h_T^2 \cos^2 \alpha_i\} \\
 &\quad - (1-a^2)[(1-a^2)s-1](h_T^2 \sin^2 \alpha_i + h_L^2). \quad (3a)
 \end{aligned}$$

The four complex values of D obtained from this quartic represent the two upgoing and two downgoing characteristic waves which can propagate independently in the medium. Values of D with positive real and negative imaginary parts represent upgoing waves.

2.1. Boundary Equations

Since the total tangential fields E_y , E_z , H_y , and H_z must be continuous across the boundary, the following boundary equations are obtained:

From E_y :

$$E_{y'i} \cos \alpha_i + E_{z'i} \cos \beta_i \sin \alpha_i + E_{y'r} \cos \alpha_i - E_{z'r} \cos \beta_i \sin \alpha_i = P_1 E_{z1} + P_2 E_{z2} \quad (4a)$$

From E_z :

$$-E_{y'i} \sin \alpha_i + E_{z'i} \cos \beta_i \cos \alpha_i - E_{y'r} \sin \alpha_i - E_{z'r} \cos \beta_i \cos \alpha_i = E_{z1} + E_{z2} \quad (4b)$$

From H_y :

$$E_{y'i} \cos \beta_i \sin \alpha_i - E_{z'i} \cos \alpha_i - E_{y'r} \cos \beta_i \sin \alpha_i - E_{z'r} \cos \alpha_i = S_1 E_{z1} + S_2 E_{z2} \quad (4c)$$

From H_z :

$$E_{y'i} \cos \beta_i \cos \alpha_i + E_{z'i} \sin \alpha_i - E_{y'r} \cos \beta_i \cos \alpha_i + E_{z'r} \sin \alpha_i = T_1 E_{z1} + T_2 E_{z2} \quad (4d)$$

In these equations

$$P = E_y/E_z = M^{-1} \left\{ - \left[1 - a^2 - \frac{s^2 - h_L^2}{s(s^2 - h^2)} \right] \left[a_L a_T - \frac{j h_L}{s^2 - h^2} \right] + \left[a_T D + \frac{j h_T}{s^2 - h^2} \right] \left[a_L D + \frac{h_L h_T}{s(s^2 - h^2)} \right] \right\} \quad (5)$$

$$Q = E_x/E_z = M^{-1} \left\{ - \left[a_L D + \frac{h_T h_L}{s(s^2 - h^2)} \right] \left[1 - a_L^2 - D^2 - \frac{s}{s^2 - h^2} \right] + \left[a_L a_T - \frac{j h_L}{s^2 - h^2} \right] \left[a_T D - \frac{j h_T}{s^2 - h^2} \right] \right\} \quad (6)$$

where M is given by

$$M = \left[1 - a^2 - \frac{s^2 - h_L^2}{s(s^2 - h^2)} \right] \left[1 - a_L^2 - D^2 - \frac{s}{s^2 - h^2} \right] - \left[a_T D - \frac{j h_T}{s^2 - h^2} \right] \left[a_T D + \frac{j h_T}{s^2 - h^2} \right] \quad (7)$$

and

$$R = \eta_0 H_x / E_z = a_T - a_L P \quad (8a)$$

$$S = \eta_0 H_y / E_z = a_L Q - D \quad (8b)$$

$$T = \eta_0 H_z / E_z = DP - a_T Q. \quad (8c)$$

In which

$$\eta_0 = \sqrt{\mu_0 / \epsilon_0}.$$

2.2. Reflection Coefficient R

This can be written in matrix form as

$$R = \begin{bmatrix} \|R\| & \perp R\| \\ \|R\perp & \perp R\perp \end{bmatrix}. \quad (9)$$

The first subscript denotes whether the electric field is parallel ($\|$) or perpendicular (\perp) to the plane of incidence, and the second subscript refers in the same way to the electric field of the reflected wave. The components of R are defined as follows:

$$\|R\| = E_{z'\tau} / E_{z'i} \quad (10a)$$

$$\perp R\| = E_{z'\tau} / E_{y'i}. \quad (10c)$$

$$\|R\perp = E_{y'\tau} / E_{z'i} \quad (10b)$$

$$\perp R\perp = E_{y'\tau} / E_{y'i}. \quad (10d)$$

The components of R are obtained by solution of the boundary eqs (4) for the appropriate pairs of electric fields.

3. Application to Propagation Along a Line of Magnetic Latitude

The work of Yabroff given above is perfectly general and expressions for the reflection coefficient could be derived for any direction of propagation, and for any orientation of the magnetic field. The labor involved would be excessive, however, and the resulting expressions cumbersome. The more practical approach to this general case is to use a computer for the solution of the boundary eqs (4). This has been done by Johler and Walters [1960] for propagation in the magnetic meridian. Johler [1961] has also calculated numerically, reflection coefficients for propagation at angles of 0° , 60° , 120° , 180° , 240° , 300° to the magnetic meridian, omitting the directions 90° and 270° , with which this paper is concerned.

However, in the case of propagation along a line of magnetic latitude the expressions obtained for the reflection coefficient are relatively compact as will be shown below, although even in this case the algebra is very tedious.

For magnetic east-to-west or west-to-east propagation $\alpha_i = 90^\circ$ or 270° . Thus, $a_L = 0$. Making this substitution in eqs (4) gives for the boundary equations:

$$Ey: E_{z'i} C - E_{z'\tau} C = P_1 E_{z1} + P_2 E_{z2} \quad (11a)$$

$$Hz: E_{z'i} + E_{z'\tau} = T_1 E_{z1} + T_2 E_{z2} \quad (11b)$$

$$Ez: -E_{y'i} - E_{y'\tau} = E_{z1} + E_{z2} \quad (11c)$$

$$Hy: E_{y'i} C - E_{y'\tau} C = S_1 E_{z1} + S_2 E_{z2} \quad (11d)$$

where $C = \cos \beta_i$.

$$(8b) \text{ and } (8c) \text{ reduce to } T = DP - a_T Q \quad (12)$$

$$S = -D \quad (13)$$

while (5), (6), and (7) reduce to

$$P=P'M^{-1}=M^{-1}\left\{j\left[1-a^2-\frac{s^2-h_L^2}{s(s^2-h^2)}\right]+\frac{h_T}{s}\left(a_T D+\frac{j h_T}{s^2-h^2}\right)\right\}\frac{h_L}{s^2-h^2} \quad (14)$$

$$Q=Q'M^{-1}=M^{-1}\left\{-j\left(a_T D-\frac{j h_T}{s^2-h^2}\right)-\frac{h_T}{s}\left(1-D^2-\frac{s}{s^2-h^2}\right)\right\}\frac{h_L}{s^2-h^2} \quad (15)$$

$$M=\left[1-a^2-\frac{s^2-h_L^2}{s(s^2-h^2)}\right]\left[1-D^2-\frac{s}{s^2-h^2}\right]-\left[a_T D^2+\frac{h_T^2}{(s^2-h^2)^2}\right] \quad (16)$$

The quantities M , P , Q , S , T , carry the same subscripts as the D which they contain.

3.1. Solution of Boundary Equations

The boundary eqs (11) can be solved in the following way for the components of R . $\parallel R_{\parallel}$ is taken as an example. From (11a) and (11b)

$$\parallel R_{\parallel}=E_z'/E_z'=\frac{T_1 C-P_1+(E_{z2}/E_{z1})(T_2 C-P_2)}{T_1 C+P_1+(E_{z2}/E_{z1})(T_2 C+P_2)}. \quad (17)$$

From (11c) and (11d), on putting $E_y'_i=0$ it follows that

$$E_{z2}/E_{z1}=-\frac{(C+D_1)}{(C+D_2)}. \quad (18)$$

Substitution of (18) in (17) gives

$$\parallel R_{\parallel}=\Delta^{-1}[(T_1 C-P_1)(C+D_2)-(T_2 C-P_2)(C+D_1)], \quad (19)$$

$$\Delta=(T_1 C+P_1)(C+D_2)-(T_2 C+P_2)(C+D_1). \quad (20)$$

Similarly

$$\parallel R_{\perp}=\Delta^{-1}[2C(D_1-D_2)], \quad (21)$$

$$\perp R_{\parallel}=\Delta^{-1}[2C(T_1 P_2-T_2 P_1)], \quad (22)$$

$$\perp R_{\perp}=\Delta^{-1}[(T_1 C+P_1)(C-D_2)-(T_2 C+P_2)(C-D_1)]. \quad (23)$$

Substitution for T_1 , P_1 , T_2 and P_2 in these equations gives, after much work, the following relatively compact expressions for $\parallel R_{\parallel}$ and $\perp R_{\perp}$

$$\parallel R_{\parallel}=\Delta^{-1}\left[\left\{j\left[\left(1-\frac{1}{s}\right)CD_1D_2-D_0^2(C+D_1+D_2)\right]+\frac{h_T a_T}{s}[C(C+D_1+D_2)-D_1D_2]\right\}\left[\frac{s^2-h_L^2}{s(s^2-h^2)}\right]-1\right](D_1+D_2)+\left[j\left(2D_0^2+\frac{a^2}{s}\right)-\frac{2Ca_T h_T}{s}\right]\left[\left(\frac{s}{s^2-h^2}-1\right)(D_0^2+D_1D_2)+\frac{h_T^2-h_L^2 D_1D_2}{s(s^2-h^2)}\right] \quad (24)$$

$$\perp R_{\perp}=\Delta^{-1}\left[\left\{j\left[D_0^2(C-D_1-D_2)-D_1D_2\left(1-\frac{1}{s}\right)C\right]-\frac{h_T a_T}{s}[D_1D_2-C(C+D_1+D_2)]\right\}\left[\frac{s^2-h_L^2}{s(s^2-h^2)}\right]-1\right](D_1+D_2)+\left[j\left(2D_0^2+\frac{a^2}{s}\right)+\frac{2Ch_T a_T}{s}\right]\left[\left(\frac{s}{s^2-h^2}-1\right)(D_0^2+D_1D_2)+\frac{h_T^2-h_L^2 D_1D_2}{s(s^2-h^2)}\right] \quad (25)$$

in which

$$\Delta=\Delta\left[\frac{h_L}{s^2-h^2}(D_1-D_2)\right]=\left\{j\left[\left(1-\frac{1}{s}\right)CD_1D_2+D_0^2(C+D_1+D_2)\right]+\frac{h_T a_T}{s}[C(C+D_1+D_2)+D_1D_2]\right\}\left[\frac{s^2-h_L^2}{s(s^2-h^2)}-1\right](D_1+D_2)+j\frac{a^2}{s}\left[\left(\frac{s}{s^2-h^2}-1\right)(D_0^2+D_1D_2)+\frac{h_T^2-h_L^2 D_1D_2}{s(s^2-h^2)}\right] \quad (26)$$

and $D_0^2 = 1 - a^2 - 1/s$, the value of D^2 when there is no magnetic field present.

Equation (22) reduces to

$$\perp R_{\parallel} = -\Delta_1^{-1} \frac{2h_L}{s^2 - h^2} \left[\left(1 - \frac{1}{s}\right) D_0^2 + \frac{h_T^2 a_T^2}{s^2} \right] C. \quad (27)$$

Equation (21) reduces to

$$\parallel R_{\perp} = \Delta_1^{-1} \cdot 2CM_1M_2 \left(\frac{s^2 - h_L^2}{h_L} \right). \quad (28)$$

When M_1M_2 is expanded, it is found to contain terms involving $D_1^2 + D_2^2$ and $D_1^2D_2^2$. These are respectively equal to $-b_2/b_0$ and b_4/b_0 in eq (3). Substitution of these in (28) gives, after considerable reduction,

$$\parallel R_{\perp} = -\Delta_1^{-1} \frac{2h_L}{s^2 - h^2} \left[\left(1 - \frac{1}{s}\right) D_0^2 + \frac{h_T^2 a_T^2}{s^2} \right] C \quad (29)$$

which is identical to $\perp R_{\parallel}$.

4. Discussion of Reflection Coefficient Formulas

These formulas have been checked under various limiting conditions to ensure that they conform with other published results.

4.1. No Magnetic Field

If the magnetic field is zero then $\parallel R_{\parallel}$ and $\perp R_{\perp}$ reduce to

$$\parallel R_{\parallel} = \frac{(D_0^2 + a^2)C - D_0}{(D_0^2 + a^2)C + D_0}$$

$$\perp R_{\perp} = \frac{C - D_0}{C + D_0}$$

while $\parallel R_{\perp} = \perp R_{\parallel} = 0$. These limiting forms were given by Yabroff [1959b].

4.2. Horizontal Magnetic Field

If the magnetic field is horizontal, $h_L = 0$, $h_T = h$, and (19) reduces to

$$\parallel R_{\parallel} = \frac{\left[jD_2 \left(1 - \frac{1}{s}\right) + \frac{h_T a_T}{s} \right] C - \left(jD_1^2 + \frac{h_T a_T D_2}{s} \right)}{\left[jD_2 \left(1 - \frac{1}{s}\right) + \frac{h_T a_T}{s} \right] C + \left(jD_1^2 + \frac{h_T a_T D_2}{s} \right)}. \quad (30)$$

From figure 1, h_T is negative and for west-to-east propagation a_T is also negative. In Barber and Crombie's paper [1959] their term equivalent to $h_T a_T$ was taken as negative for west-to-east propagation, because of the different coordinate systems used by them. Bearing this in mind it is possible to show that (30) is identical with their earlier result.

If in (25) $h_L = 0$ then (25) reduces again to

$$\perp R_{\perp} = \frac{C - D_1}{C + D_1} = \frac{C - D_0}{C + D_0}$$

confirming the result of Barber and Crombie [1959]. It is easily seen from (27) and (29) that when $h_L = 0$ both $\perp R_{\parallel}$ and $\parallel R_{\perp}$ become zero.

4.3. Vertical Incidence, Vertical Magnetic Field

Here h_T , a_T , a become zero and $C=1$.

Substitution in (24), (25), (27), (29), give

$$\|R_{\parallel}=\perp R_{\perp}=\frac{D_1 D_2 - 1}{(D_1 + 1)(D_2 + 1)}, \quad \perp R_{\parallel}=\|R_{\perp}=j \frac{D_1 - D_2}{(D_1 + 1)(D_2 + 1)}$$

which are results previously obtained by Bremmer [1949].

4.4. Reciprocity

The expressions (24), (25), (27), and (29) all contain $h_T a_T$. This is positive for west-to-east propagation and negative for east-to-west propagation. Thus, provided that h_T is not zero, all the reflection coefficient terms may exhibit nonreciprocity.

5. Numerical Work

In order to evaluate the expressions for the reflection coefficients it is necessary first of all to find D_1 and D_2 from (3). For propagation along a line of magnetic latitude $a_L=0$ and (3) becomes a quadratic in D^2 . Thus

$$b_4 D^4 + b_2 D^2 + b_0 = 0,$$

and

$$D^2 = [-b_2 \pm \sqrt{b_2^2 - 4b_0 b_4}] (2b_4)^{-1}.$$

Substituting $a_L=0$ into eqs (3a) gives

$$\begin{aligned} b_2 &= 2s [(1-a^2)h^2 - s(s-1)D_0^2] - h_T^2 - (2-a^2)h_L^2 \\ b_1 - 4b_0 b_4 &= h^4 - 2(1-a^2)h^2 h_L^2 + 4s(s-1)D_0^2 h_L^2 + (1-a^2)^2 h_L^4 \\ b_4 &= s(s^2 - h^2) - (s^2 - h_L^2). \end{aligned}$$

The values of D_1 , D_2 having positive real and negative imaginary parts are chosen since these represent the upgoing "ordinary" and "extraordinary" waves in the ionosphere.

Some special cases are of interest in obtaining the results of section 4.

$$\text{For } h=0; D_1^2 = D_2^2 = D_0^2 = 1 - a^2 - 1/s.$$

$$\text{For } h_L=0; h_T=h \text{ and } D_1^2 = D_0^2, \quad D_2^2 = 1 - a^2 - \frac{s-1}{s^2 - h^2 - s}.$$

$$\text{For } h_T=0; h_L=h \text{ and } a=0, \quad D_{1,2}^2 = 1 - \frac{1}{s \pm h}.$$

Some approximate calculations have been made for daytime conditions to show the effect of the angle of dip of the earth's magnetic field on the four reflection coefficient terms using the formulas (24), (25), (27), and (29) developed above. The calculations are based on the following parameters:

$$\omega_0 = 10^6 \text{ corresponding to an electron density } \approx 310/cc,$$

$$\nu = 10^7/\text{sec}, \quad \omega = 10^5/\text{sec}, \quad \alpha_i \approx 78^\circ 80', \quad C=0.2.$$

The earth's magnetic field has been taken as 0.5 gauss and the inclination of the field to the vertical $\beta_e = 90^\circ, 60^\circ, 30^\circ$, and 0° . The results of these calculations are shown in table 1.

TABLE 1

	$\parallel R_{\parallel}$		$\perp R_{\perp}$		$\parallel R_{\perp} = \perp R_{\parallel}^*$		
	West to east	East to west	West to east	East to west	West to east	East to west	
90°-----	0.72/175°	0.33/175°	0.76/171°	0.76/171°	0.0	0.0	Horizontal field.
60°-----	.69/176°	.38/176°	.76/162°	.76/162°	.042/136°	.081/192°	
30°-----	.67/179°	.42/183°	.68/162°	.68/160°	.075/145°	.122/177°	Vertical field.
0°-----	.59/180°	.59/180°	.66/150°	.66/150°	.083/153°	.083/153°	

*For propagation in the Northern Hemisphere the phase angle of $\parallel R_{\perp}$ and $\perp R_{\parallel}$ should be reduced by 180°.

It is clearly seen from this table that $\parallel R_{\parallel}$, $\parallel R_{\perp}$ and $\perp R_{\parallel}$ all show nonreciprocal properties, whereas $\perp R_{\perp}$ does not. In the case of $\parallel R_{\parallel}$ the difference between east-to-west and west-to-east propagation is greatest at the magnetic equator where the magnetic field is horizontal, and gradually decreases as the reflection point moves nearer a magnetic pole. Nevertheless, the difference is very appreciable even when the angle of dip is 60° ($\beta_e=30^\circ$). At the pole, where the magnetic field is vertical the two reflection coefficients are equal, since there is then no transverse field.

At the magnetic equator $\parallel R_{\perp}$ and $\perp R_{\parallel}$ are zero. When the magnetic field is inclined $\parallel R_{\perp}$ and $\perp R_{\parallel}$ become finite, but again their values for east-to-west and west-to-east propagation differ. This difference again disappears at the magnetic pole.

Despite the fact that eq (25) for $\perp R_{\perp}$ contains $a_T h_T$, it seems from table 1 that, for the values of the parameters used, $\perp R_{\perp}$ is essentially reciprocal. Thus in (25) the variations due to the sign of $a_T h_T$ must, to a first approximation cancel. This presumably will not necessarily occur for other values of the parameters.

The above numerical calculations have been made for highly oblique incidence, since long distance VLF propagation is of interest at present. However, the formulas are applicable at any angle of incidence. Thus the predictions of the formulas could be tested by making observations of $\parallel R_{\parallel}$ and/or $\parallel R_{\perp}$ at points situated at equal distances to the west and east of a VLF transmitter. Alternatively, observation of the ground-interference pattern to the west and east of the transmitter could be made with airborne equipment.

6. Conclusions

This paper extends the earlier work of Barber and Crombie [1959] to the case of west-to-east and east-to-west propagation at any magnetic latitude. The nonreciprocity shown by $\parallel R_{\parallel}$ for highly oblique propagation along the magnetic equator persists when the reflection point moves towards a magnetic pole, but at the pole itself $\parallel R_{\parallel}$ is reciprocal. To a first approximation at least, $\perp R_{\perp}$ is reciprocal at all magnetic latitudes. However, the conversion coefficients $\parallel R_{\perp}$, $\perp R_{\parallel}$, which are shown to be equal, are greater for east-to-west propagation than for propagation in the opposite direction, except at a magnetic pole where they are equal.

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