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Smooth Earth Diffraction Calculations for Horizontal Polarization

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This paper presents a simplified method of determining the attenuation relative to free space in the so-called far diffraction region for horizontally polarized radio waves diffracted over a smooth spherical earth. A criterion is given which permits use of the method not only for far beyond line-of-sight paths but, in many practical situations, at line-of-sight or even slightly within.

Although numerical procedures have been developed by various authors for the calculation of field intensity arising from the diffraction of radio waves around a smooth spherical earth [Norton, 1941; Bremmer, 1949; Norton, Rice, and Vogler, 1955], the actual computation is often laborious and time consuming. Because of the number of independent parameters associated with the problem and the mathematical complexity of the solution, many intermediate calculations and graph readings are necessary for a given propagation path. However, for certain particular cases of importance a simplified method, explained in this paper, may be used to good advantage; furthermore, an improved criterion is given for determining the ranges of the distance, heights, frequency, and effective earth's radius factor, k, for which this method is applicable.

In the so-called "far diffraction" region field intensity is determined by the first term of the Van der Pol-Bremmer residue series [Bremmer, 1949]. This term consists of four factors containing, essentially, the dependences on the distance, two antenna heights, and the electromagnetic ground constants and polarization of the transmitted wave. The residue series is strictly valid only for the case of a smooth "airless" earth of radius ka where a is the actual earth's radius; however, recent investigations [Millington, 1958; Norton, 1959; Wait, 1959; Bremmer, 1960] have shown that for certain models of the atmosphere, atmospheric effects may be satisfactorily included by introducing corrections to the antenna heights and defining the factor k as:

$$k=\left[1+\left(\frac{a}{n}\frac{dn}{dh}\right)_{h=0}\right]^{-1} \tag{1}$$

where dn/dh is the gradient of the refractive index, n, with respect to height, h, above the earth's surface. Determinations of the proper height corrections for various model atmospheres are contained in the above references.

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By defining the quantities:

$$x_{0,1,2}=\beta_0 C_0^2 f_{\mathrm{mc}}^{1/3} d_{0,1,2}, \tag{2}$$

where β_0 is a function of the polarization and ground constants, f_{mc} denotes the radio frequency in Mc/s, $C_0=(4/3k)^{1/3}$, and d_0, d_1, d_2 are indicated in the geometry of figure 1, the attenuation relative to free space A, (in decibels *below* free space) may be expressed as:

$$A=G(x_0)-F(x_1)-F(x_2)-C_1 \tag{3}$$

The distance dependence is thus contained in the function $G(x_0)$. The height dependences are contained in $F(x_1)$ and $F(x_2)$ through the relationship $d_{1,2}\,(\mathrm{mi})\cong\sqrt{(3k/2)\,h_{1,2}\,(\mathrm{ft})}$.

In the particular case of horizontal polarization, and the further restriction that:

$$K\equiv 0.017774\, C_0 f_{mc}^{-1/3}[\{\epsilon-1\}^2 + \{1.8\times10^4\sigma(\mathrm{mhos/m})/f_{\mathrm{mc}}\}^2]^{-1/4}<0.01 \tag{4}$$

where ϵ is the dielectric constant of the ground referred to air as unity and σ is the ground conductivity in mhos per meter, the term C_1 in (3) is very nearly constant as is the parameter β_0 appearing in the definition of x in (2): $C_1=20.67$ and $\beta_0=1.607$ Thus where the above restrictions hold, the attenuation relative to free space is given by:

$$A=G(x_0)-F(x_1)-F(x_2)-20.67 \tag{5}$$

where:

$$x_{0,1,2}=1.607\ C_0^2 f_{\mathrm{mc}}^{1/3} d_{0,1,2}(\mathrm{mi}), \tag{5a}$$

$$C_0=(4/3k)^{1/3}, \tag{5b}$$

$$d_{1,2}(\mathrm{mi})\cong[(3k/2)h_{1,2}(\mathrm{ft})]^{1/2}, \tag{5c}$$

and with the conditions: (a) horizontal polarization,

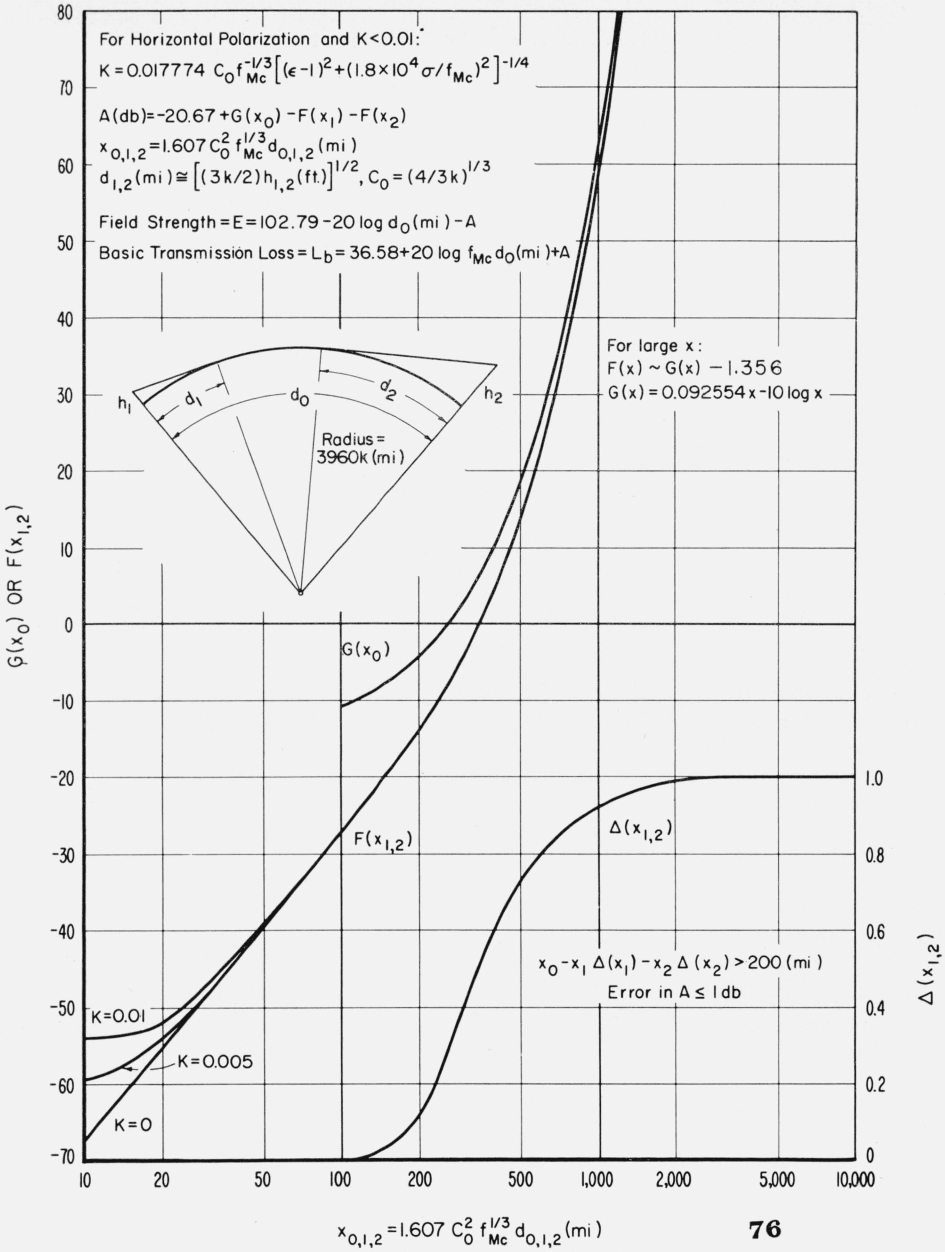

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FIGURE 1. *Curves for the calculation of A, the attenuation relative to free space.*

and (b) $K<0.01$; see (4). Curves of the functions $G(x_0)$ and $F(x_{1,2})$ are plotted versus x in figure 1. Note that $F(x)+1.356$ is asymptotically equal to $G(x)=0.092554x-10\log x$ for large values of x. It might be mentioned that a brief study of the parameter K will show that only for unusual combinations of k, f_{mc}, ϵ and σ will K ever exceed 0.01.

Since (5) is based upon only the first term of the residue series, a fairly good indication of its range of validity may be gained from the ratio of the second term, T_2, to the first term, T_1, of the series Thus, if we require the error in A to be less than some given value, say δ, we have:

$$20\log|1+T_2/T_1|\leq 20\log[1+|T_2/T_1|]\leq\delta$$

This is the procedure used to obtain the $\Delta(x_{1,2})$ curve shown in the lower right hand corner of figure 1. For A to be accurate to within 1 db (approximately) it may be shown that:

$$x_0-x_1\Delta(x_1)-x_2\Delta(x_2)>200 \text{ mi.} \qquad (6)$$

Notice that in certain cases (5) is valid (at least to within 1 db) even for just within line-of-sight paths as long as the condition (6) holds. For instance, with $k=4/3$, $f_{mc}=100$, $d_0=65.38$ mi, and $h_2=32.81$ ft, then for an $h_1=1640.42$ ft, which would correspond to a radio line-of-sight path, (6) becomes:

$$487.67-427.25(0.65)-60.42(0)=210>200 \text{ mi}$$

and from eq (5)

$$A=G(487.67)-F(427.25)-F(60.42)-20.67$$
$$=18.3-7.8+36.2-20.67=26.0$$

The value of A as read from the C.C.I.R. Atlas [Atlas, 1955] (assuming horizontal polarization and for either "sea" or "land") is approximately 27.3.

It may be seen from (6) that the first term of the Van der Pol-Bremmer residue series is applicable not only far beyond the horizon but even in some cases at the horizon; (in this regard it is of interest to note that the lower limit of (6) appears to be approximately the point at which the C.C.I.R. curves depart from linearity). Furthermore, the ease with which the attenuation may be calculated using (5) is apparent from the above example. Curves for the calculation of A by this method for vertical polarization are now being prepared for publication in the near future. Since A is defined as attenuation relative to free space, its relationship to E, the field strength expressed in db above 1 μv/m for 1 kw E.R.P. from a half-wave dipole, is simply:

$$E=102.79-20 \log d_0 \text{ (mi)}-A \tag{7}$$

In terms of basic transmission loss, L_b:

$$L_b=36.58+20 \log f_{mc}d_0\text{(mi)}+A. \tag{8}$$

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