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Excitation of VLF and ELF Radio Waves by a Horizontal Magnetic Dipole[1, 2]

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The VLF and ELF modes excited by a horizontal magnetic dipole (vertical loop) in the spherical shell between a finitely conducting earth and an isotropic sharply bounded ionosphere are shown to have a nearly transverse magnetic character. The modes are similar to those of a vertical electric dipole. With the exception of the zero order mode, the propagating modes excited by the magnetic dipole are of slightly higher amplitudes, provided that the far fields of the horizontal magnetic and vertical electric dipoles are equal over flat earth in the absence of ionosphere.

The transient fields generated by a current step in the magnetic dipole are in the first approximation similar to the fields generated by a current impulse in a vertical electric dipole. Response of the zero order mode of the magnetic dipole has been calculated.

1. Introduction

The mode theory of VLF transmissions has been developed by Watson [1919], Budden [1953], Schumann [1954], and Wait [1957, 1960a, 1960b] for excitation by vertical and horizontal electric dipoles and also by vertical magnetic dipoles (horizontal loops). Additional references on this subject have been listed by Wait [1960a, 1960b].

The reciprocity theorem has been found to be useful for relating the fields of a horizontal electric dipole to fields of vertical electric and magnetic dipoles [Wait, 1960a]. The reciprocity relations may be also applied to relate the fields due to horizontal magnetic dipole excitation to known fields due to vertical electric and magnetic dipoles.

2. Reciprocity Relations

The reciprocity theorem states that the voltage V_2 induced in antenna 2 by current I_1 of antenna 1 is the same as the voltage V_1 induced in antenna 1 by an identical current I_2 flowing in antenna 2.

A vertical electric (*VE*) dipole of length ds at $z_0'=z_t$ and a horizontal magnetic (*HM*) dipole parallel to the x-axis (or a vertical loop of area da in the y, z plane) at $z_0=z_r$ are shown in figure 1, where subscripts r and t refer to receiver and transmitter coordinates respectively. The only nonzero H component generated by the *VE* dipole is $H_{\phi'}$. The magnitude of the voltage induced in the *HM* dipole V^{hm} is maximum if $H_{\phi'}$ is parallel to the x-axis, which occurs for $\phi=\frac{\pi}{2}$ and $\frac{3\pi}{2}$. For $e^{i\omega t}$ variation of the fields.

$$V^{hm}=-i\omega\mu_0 H^{ve}_{\phi'}(z_0'=z_t,\ z_0=z_n)\sin\phi\, da, \tag{1}$$

where the superscripts *hm* and *ve* refer to horizontal magnetic and vertical electric dipoles respectively. With the same current applied to the *HM* dipole only the vertical electric field E_z will contribute to V^{ve} and

$$V^{ve}=E_z^{hm}(z_0=z_t, z_0'=z_r)ds. \tag{2}$$

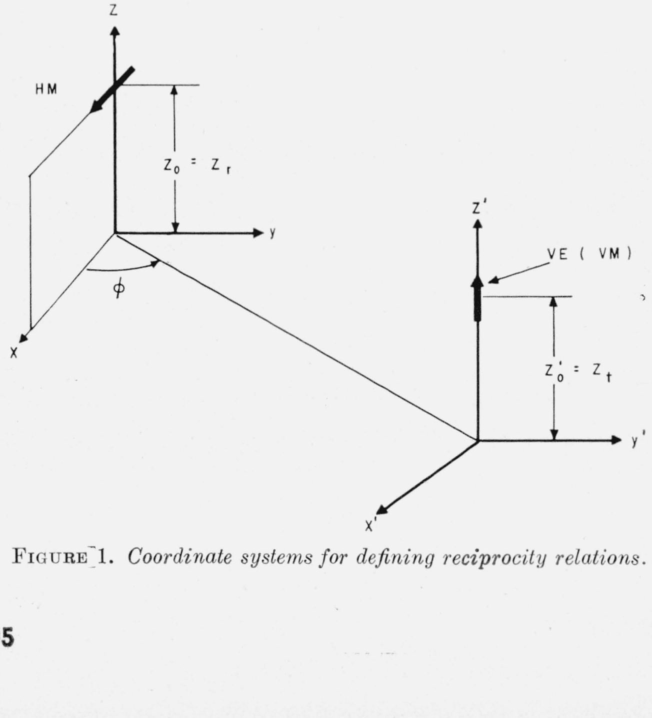

FIGURE 1. *Coordinate systems for defining reciprocity relations.*

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[1] Contribution from Applied Research Laboratory of Sylvania Electronic Systems, Waltham, Mass.

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Equating (1) and (2) results in

$$E_z^{hm}(z_0=z_t,z_0'=z_r)$$

$$=-i\omega\mu_0 \sin \phi H^{ve}_{\phi'}(z_0'=z_t,z_0=z_r)da/ds. \quad (3)$$

A vertical magnetic (VM) dipole (or a horizontal loop of area da^{vm} parallel to the x', y' plane) at $z_0'=z_t$ and a horizontal magnetic (HM) dipole parallel to the x-axis (or a vertical loop of area da^{hm} in the y, z plane) at $z_0=z_r$ may be considered in the geometry shown in figure 1. The only nonzero H component generated by the VM dipole is $H_{\rho'}$. The magnitude of the voltage induced in the HM dipole V^{hm} is maximum, if $H_{\rho'}$ is parallel to the x-axis, which occurs for $\phi=0$ or π. Hence

$$V^{hm}=i\omega\mu_0 H^{vm}_{\rho'}(z_0'=z_t,\ z_0=z_r) \cos \phi da^{hm} \quad (4)$$

where the superscript vm refers to the vertical magnetic dipole. With the same current applied to the HM dipole only the vertical magnetic field H_z will contribute to V^{vm} and

$$V^{vm}=-i\omega\mu_0 H_z^{hm}(z_0=z_t,z_0'=z_r)da^{vm}. \quad (5)$$

Equating (4) and (5) gives for $da^{vm}=da^{hm}$

$$H_z^{hm}(z_0=z_t,z_0'=z_r)=-\cos \phi H^{vm}_{\rho'}(z_0'=z_t,z_0=z_r). \quad (6)$$

For spherical coordinates the z component is replaced by r component and the ρ component by the θ component of the fields.

The reciprocity relations apply to dipoles in the presence of any linear media. The fields of dipoles above plane earth derived by solving the wave equation should also satisfy (3) and (6). Thus (27) and (116) of Norton [1937], which have been derived for unit dipoles, check the validity of (3). Some algebra is involved in verifying (6) from (35.1) of Sommerfeld [1949] and from (2) to (6), (17) to (19), and (30) of Norton [1937].

3. Fields of the Horizontal Magnetic Dipole

The dipole fields will be examined with the aid of reciprocity relations in an idealized spherical shell that is bounded by a homogeneous earth of radius a, conductivity σ_g and dielectric constant ϵ_g and by a homogeneous ionosphere of radius $(a+h)$, conductivity σ_i and dielectric constant ϵ_i. The above model neglects the effects of the earth's magnetic field.

The vertical electric field E_r^{hm} is computed by (3) applying (5.1), (6.16) of Wait [1960a] and using the relation

$$\frac{d}{d\theta} P_\nu(-\cos \theta)$$

$$=\nu[\cos \theta P_\nu(-\cos \theta)+P_{\nu-1}(-\cos \theta)]/\sin \theta. \quad (7)$$

This results in

$$E_r^{hm}=\frac{I(da)i\omega\mu_0 \sin \phi}{2hr_r} \sum_{n=0}^{\infty} f_n(z_t)f_n(z_r) \cdot \frac{\delta_n}{\sin \nu\pi} \frac{\nu}{\sin \theta}$$

$$[\cos \theta P_\nu(-\cos \theta)+P_{\nu-1}(-\cos \theta)]. \quad (8)$$

The vertical magnetic field H_r^{hm} is computed by (6) applying (9.1), (9.2) of Wait [1960a] and using the relation (7) as

$$H_r^{hm}=-\frac{I(da)\omega\mu_0 \cos \phi}{2khr_t r_r \eta} \sum_{m=1}^{\infty} \frac{\delta_m f_m(z_r)\mu}{\sin \mu\pi \sin \theta}$$

$$\left[f_m(z_t)+r_t \frac{d}{dz_t} f_m(z_t)\right]$$

$$[\cos \theta P_\mu(-\cos \theta)+P_{\mu-1}(-\cos \theta)]. \quad (9)$$

Using the second order representation of the radial functions, applying (6.9), (6.17), (9.3), and (9.4) of Wait [1960a] yields

$$2f_q(z)=e^{ikC_q z}[R_g^q(C_q)]^{-1/2}+e^{-ikC_q z}[R_g^q(C_q)]^{1/2} \quad (10)$$

$$\delta_q=\left(1\pm\frac{\sin 2khC_q}{2khC_q}\right)^{-1} \quad (11)$$

where q is an integer equal to either n or m and where

$$R_p^n(C_n)=\frac{n_pC_n-C_p}{n_pC_n+C_p} \quad (12)$$

$$R_p^m(C_m)=\frac{C_m-n_pC_p}{C_m+n_pC_p} \quad (13)$$

$$C_q=\sqrt{1-S_q^2}, \quad (14)$$

$$C_p=\sqrt{1-(S_q/n_p)^2}, \quad (15)$$

$$S_q=\frac{\rho+0.5}{ka}. \quad (16)$$

The plus sign of (11) should be used with $q=n$, while the minus sign is appropriate for $q=m$. In (16) ρ is a complex number, which is equal to ν of (8) or equal to μ of (9), $k=\omega/c$, c=velocity of light. The refractive index n of the boundary medium characterized by subscript p is

$$n_p^2=(\sigma_p+i\omega\epsilon_p)/\epsilon_0 \quad (17)$$

where σ_p and ϵ_p are the conductivity and permittivity of the boundary medium and where ϵ_0 is the permittivity of free space ($\epsilon_0=(36\pi)^{-1}\ 10^{-9}$ farad/meter). The boundary medium can be either ground ($p=g$) or ionosphere ($p=i$). The refractive index of ground is

$$n_g^2\approx\sigma_g/(i\omega\epsilon_0) \quad (18)$$

where σ_g is the ground conductivity. The refractive index of the ionosphere is

$$n_i^2 = 1 + \sigma_i/(i\omega\epsilon_0). \tag{19}$$

The ionospheric conductivity σ_i is defined by

$$\sigma_i = \frac{\epsilon_0 \omega_p^2}{\underline{\nu}} \tag{20}$$

where $\underline{\nu}$ is the electron collision frequency in the ionosphere and ω_p is the plasma frequency ($\omega_p^2 = 3180$ N, N=number of electrons per meter^3). C_n and C_m are roots of the modal equations

$$R_i^q(C_q) R_g^q(C_q) = \exp\,(+2ikhC_q). \tag{21}$$

S_q that is related to C_q by (14) has a magnitude of approximately unity and a small imaginary part for propagating modes of low attenuation. With S_n and S_m determined from (14), ν of (8) and μ of (9) follow from (16). This completes the formal specification of the fields E_r^{hm} and H_r^{hm}.

The expressions for E_r^{hm} and H_r^{hm} may be simplified by introducing further approximations. The consideration will be restricted to cases where

$$r_r \approx r_t \approx a, \tag{22}$$

$$kC_q z_r \ll 1, \tag{23}$$

$$kC_q z_t \ll 1, \tag{24}$$

$$n_g \gg 1. \tag{25}$$

This gives

$$R_g^n(C_n) \approx 1 - \frac{2}{n_g C_n} \approx 1, \tag{26}$$

$$R_g^m(C_m) \approx -1 + \frac{2C_m}{n_g C_g} \approx -1, \tag{27}$$

$$f_n(z) \approx 1 + \frac{1}{8 n_g^2 C_n^2} \approx 1, \tag{28}$$

$$f_m(z) \approx C_m/(i n_g C_g) \approx C_m/(i n_g), \tag{29}$$

$$df_m(z)/dz = ikn_g C_g f_m(z) \approx ikn_g f_m(z). \tag{30}$$

Approximating the Legendre functions by [Watson, 1919; Bremmer, 1949]

$$P_\rho(-\cos\theta) \approx (2\pi\rho \sin\theta)^{-0.5} \exp\left[i\left(\rho + \frac{1}{2}\right)(\pi - \theta) - i\pi/4\right] \tag{31}$$

results in

$$E_r^{hm} = -\sin\phi E_0^{hm} \sqrt{\frac{d/a}{\sin\,(d/a)}}\,\frac{\sqrt{d/\lambda}}{h/\lambda}\, e^{i\frac{2\pi d}{\lambda} - i\frac{\pi}{4}} \cdot \sum_{n=0}^{\infty} \delta_n S_n^{0.5} e^{-i\frac{2\pi d}{\lambda} S_n}, \tag{32}$$

$$H_r^{hm} = -\cos\phi \frac{E_0^{hm}}{\eta n_g} \sqrt{\frac{d/a}{\sin\,(d/a)}}\,\frac{\sqrt{d/\lambda}}{h/\lambda}\, e^{i\frac{2\pi d}{\lambda} - i\frac{\pi}{4}} \cdot \sum_{m=1}^{\infty} \delta_m (1 - S_m^2) S_m^{0.5} e^{-i\frac{2\pi d}{\lambda} S_m}, \tag{33}$$

where the distance along the curved earth is

$$d = a\theta, \tag{34}$$

and where

$$E_0^{hm} = \frac{2\pi I(da)\eta}{\lambda^2 d}\, e^{-i\frac{2\pi d}{\lambda}} \tag{35}$$

is the vertical electric field of the source at a distance d in the direction of maximum intensity on a perfectly conducting plane earth.

4. Comparison of Harmonically Excited Dipole Fields

The fields of the horizontal magnetic dipole E_r^{hm} and H_r^{hm} will be compared first with the corresponding field components of the horizontal electric dipole. It follows from (9.37) and (9.42) of Wait [1960a] that for the n^{th} and the m^{th} mode

$$\left.\frac{E_r^{hm}}{E_0^{hm}}\right|_n = -n_g \tan\phi \left.\frac{E_r^{he}}{E_0^{ve}}\right|_n \tag{36}$$

$$\left.\frac{H_r^{hm}}{E_0^{hm}}\right|_m = n_g \cot\phi \left.\frac{H_r^{he}}{E_0^{ve}}\right|_m \tag{37}$$

where the superscripts *he* and *ve* designate field components of horizontal electric and of vertical electric dipoles respectively. E_0^{hm} is defined in (35), E_0^{ve} is the corresponding expression of the vertical electric dipole defined by (6.25) or (9.38) of Wait [1960a]. The approximation $C_g \approx 1$ was used in the expressions for E_r^{he} and H_r^{he} in order to make them consistent with the derivation leading to (32) and (33). $C_g \approx 1$ constitutes a better approximation than $f_n(z) \approx 1$ and the assumption $f_n(z) \approx 1$ [Wait, 1960a] should be followed by $C_g \approx 1$.

Comparison of (32) and (33) shows that ηH_r^{hm} is several orders of magnitude smaller than E_r^{hm} because

$$n_g^{-1} \ll 1 \tag{38}$$

and because

$$1 - S_m^2 \ll 1 \tag{39}$$

for propagating modes of low attenuation. The H_r^{hm} fields of a given mode are proportional to $n_g^{-1} \sim \sqrt{\omega/\sigma_g}$. The fields are decreased with decreasing frequency ω or increasing ground conductivity σ_g. Similar comments apply to the transverse electric (*TE*) field components that may be derived from H_r. The *TE* fields will not be considered in more detail.

The component E_r^{hm} that is associated with transverse magnetic (*TM*) fields may be compared with the corresponding component E_r^{ve} of a vertical electric dipole using an expression of the E_r^{ve} fields derived under similar assumptions [Wait, 1957]. Thus

$$E_r^{hm}\Big|_n = -\frac{\sin\phi E_0^{hm}}{S_n E_0^{ve}} E_r^{ve}\Big|_n = \frac{ikI^{hm}da}{S_n I^{ve} ds} \sin\phi E_r^{ve}\Big|_n. \tag{40}$$

Provided that the fields of the two antennas are the same in the given direction over a perfectly conducting plane earth ($\sin\phi E_0^{hm} = E_0^{ve}$), (40) may be simplified to

$$\frac{E_r^{ve}}{E_r^{hm}}\Big|_n = -S_n. \tag{41}$$

Substituting the reciprocity relation (3) in (40) follows that $(E_r^{ve})/(\eta H_\phi^{ve})$ also satisfy (41). The latter ratio, obtained from (13), (15), and (16) of Wait [1960b], when used in conjunction with (3) provides an alternate way of obtaining (40) and subsequently (41).

A more exact treatment of the horizontal magnetic dipole fields cannot be expected to lessen the importance of *TM* relative to *TE* field components. A better approximation to the *TM* fields of the horizontal magnetic dipole may be worked out from the exact ratio E_r^{ve}/E_r^{hm} derived in the appendix. As long as the source and observation point are at equal radii r the ratio involves only the azimuthal θ functions explicitly. However, the accuracy of the roots S_n and of E_r^{ve} depends on the approximations to the azimuthal and radial functions.

Numerical values of S_n are available from mode calculations of the vertical electric dipole [Howe and Wait, 1957]. For $\sigma_g = \infty$ the values are as follows:

h/λ $\pi\underline{\nu}c/(\omega_p^2 h)$	3.5 0.1	3.5 0.01	7 0.1
1	$0.9975-i\,3\times10^{-4}$	$0.997-i\,10^{-3}$	$0.995-i\,6\times10^{-5}$
2	$.98-i\,3\times10^{-3}$	$.97-i\,8\times10^{-3}$	$.992-i\,4\times10^{-4}$
3	$.93-i\,0.01$	$.91-i\,9\times10^{-3}$	$.98-i\,10^{-3}$
4	$.87-i\,0.02$	$.85-i\,0.01$	$.985-i\,2\times10^{-3}$

The $n=0$ mode is severely attenuated for frequencies above a few kilocycles [Wait, 1957]. In the lower frequency range the first order perturbation solution of the modal equation may be used [Schumann, 1954], which gives

$$C_n = \frac{\omega_n}{\omega} + \frac{ic}{h\omega C_n}\left[\frac{1}{n_g}\sqrt{1-\frac{1-C_n^2}{n_g^2}} + \frac{1}{n_i}\sqrt{1-\frac{1-C_n^2}{n_i^2}}\right]. \tag{42}$$

The cutoff frequency of the nth mode is

$$\omega_n = \frac{n\pi c}{h}. \tag{43}$$

The perturbation method applies for $n \neq 0$ if $C_n \approx \omega_n/\omega$. Substituting this for C_n^{-1} in the right-hand side of (42) and expanding the square root expressions

$$C_n \approx \frac{\frac{\omega}{\omega} + \frac{i_n c}{h\omega_n}\left[\left(\frac{1}{n_g}+\frac{1}{n_i}\right) - \frac{1}{2}\left(\frac{1}{n_g^3}+\frac{1}{n_i^3}\right)\right]}{1 - \frac{ic}{2h\omega}\left(\frac{1}{n_g^3}+\frac{1}{n_i^3}\right)} \tag{44}$$

The latter expression may be also obtained by solving (42) as a quadratic and by subsequently ignoring the second order perturbation. Applying (14), (42) simplifies for $n=0$ and $|n_g|, |n_i| \gg 1$ to

$$S_0 = \sqrt{1 + \frac{c}{hi\omega}\left[\frac{1}{n_g} + \frac{1}{n_i}\right]}. \tag{45}$$

For $n_g \gg n_i$ and $\sigma_i = 1.2\times10^{-6}(\omega_p^2/\underline{\nu} = 1.35\times10^5)$ S_0 has the following values

ω	10^4	10^3	10^2
S_0	$1.04\,e^{-i2^\circ}$	$1.14\,e^{-i7^\circ}$	$1.46\,e^{-i6^\circ}$

The above tabulations show that $|S_n|$ or the ratio $|E_r^{ve}/E_r^{hm}|$ are decreased with increasing mode index n and, with the exception of S_0, also with decreasing frequency. The phase angle between the propagating modes of the two dipoles is negligible. The largest difference between the fields of the two dipoles may be expected for the lowest propagating frequencies or near the mode cutoff of $n \neq 0$ modes. The values of S_n near the mode cutoff may be determined from an approximate solution of the modal equation. The first order perturbation solution is applicable only to the lower order modes. A different approximate solution is obtained by observing that S_n is small near the cutoff ($S_n = 1/(2\,ka)$ at the mode cutoff for perfectly conducting ground and ionosphere according to Schumann [1954]). Hence one may look for solutions of the modal equation of the form

$$C_n = \sqrt{1-S_n^2} = 1 - \Delta_n \tag{46}$$

where $|\Delta_n| \ll 1$. Substituting (46) in (12), letting $p=i$ and defining

$$L = \frac{\epsilon_0\omega}{\sigma_i} = \frac{\omega\underline{\nu}}{\omega_p^2}, \tag{47}$$

results after neglecting terms with Δ_n^2 and higher powers of Δ_n in

$$R_i^n = i(2L - i - 2\sqrt{L}\sqrt{L-i})\left(1 - \frac{2\Delta_n\sqrt{L}}{\sqrt{L-i}}\right) = L_1 + \Delta_n L_2. \tag{48}$$

Considering $n_g^2 \gg 1$, substituting (46) in (12) and letting $p=g$ gives

$$R_g^n \approx 1 - 2\,n_g^{-1}\,e^{i\pi/4}(1+\Delta_n) = G_1 + \Delta_n G_2. \tag{49}$$

The solution of the modal equation (21) becomes

$$\Delta_n = \frac{2i(kh-\pi n) - lnL_1G_1}{2ikh + \frac{L_2}{L_1} + \frac{G_2}{G_1}}. \tag{50}$$

It is simplified at the cutoff of the respective mode by applying (43) to

$$\Delta_n = -\frac{lnL_1G_1}{2in\pi + \frac{L_2}{L_1} + \frac{G_2}{G_1}}. \tag{51}$$

The calculated curves of $|S_n|$ and of its argument are depicted in figures 2 and 3 for infinite ground conductivity. $|S_n|$ is smallest for small values of $I = \pi c \underline{\nu} / (h\omega_p^2)$ (or for large ionospheric conductivities) and for the higher modes (higher cutoff frequencies $\omega_n = n\pi c/h$). The argument of S_n exhibits a relatively small variation with n. The increase of $|S_n|$ and of arg S_n with increasing I signifies the increase of mode attenuation with decreasing ionospheric conductivity. The increase of $|S_n|$ from the $\sigma_g = \infty$ to the $\sigma_g = 0.07/h_{km}$ curve ($\sigma_g = 10^{-3}$ mho/m for $h = 70$ km) is less than 20 percent for the $n = 1$ mode in figure 4. This relative increase is even smaller for the higher n values.

The amplitude ratio between electric and magnetic dipole fields in (41) will be larger for propagating modes than near the mode cutoff. The cutoff ratio is larger than 0.25 for the $n = 1$ mode and larger than 0.15 for the first 8 modes with $I > 0.01$ (or for $\sigma_i \geq 1.2 \times 10^{-5}$ with $h = 70$ km). It may be noted that this comparison is made for dipoles that exhibit equal far fields above a perfectly conducting plane ground in the absence of ionosphere. The knowledge of the above amplitude ratios would enter in a comparison of antenna radiation efficiencies. However, no such calculations have been attempted in this paper.

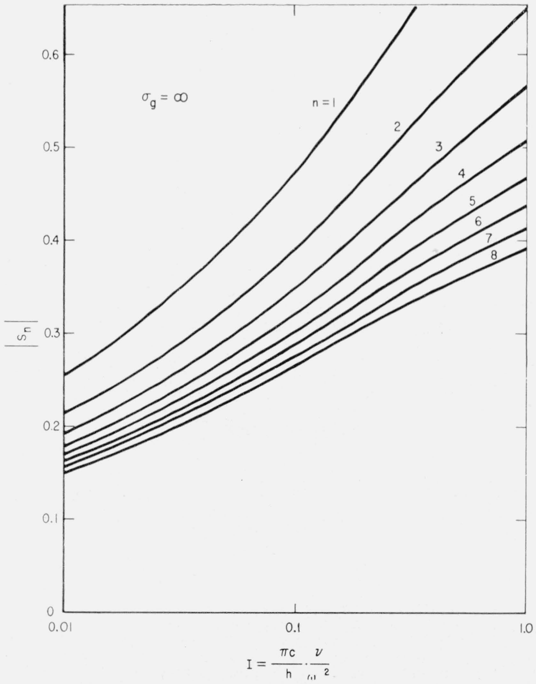

FIGURE 2. *Magnitude of the constant* S_n *near mode cutoff for perfectly conducting ground.*

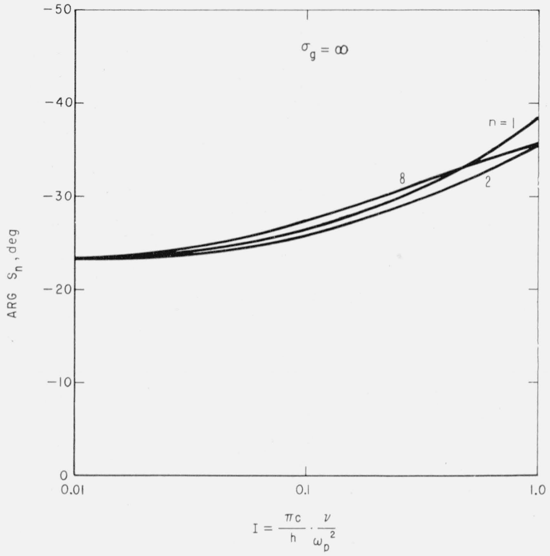

FIGURE 3. *Cutoff arguments of the constant* S_n *for perfectly conducting ground.*

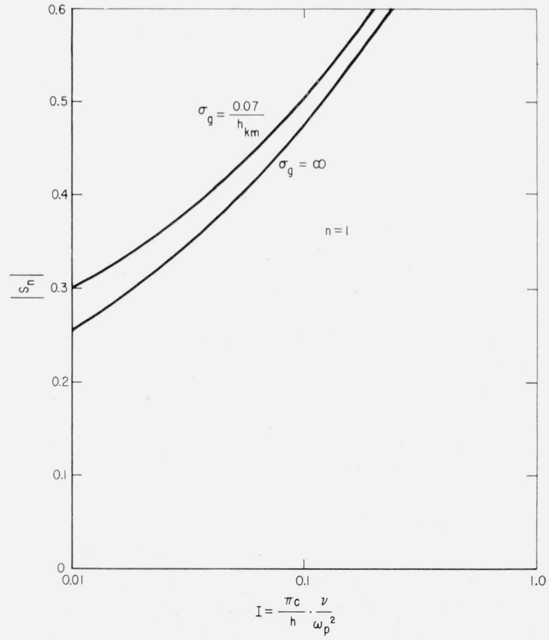

FIGURE 4. *Magnitude of the constant* S_n *near cutoff for the* n=1 *mode.*

5. Transient Fields

The transient fields of the horizontal magnetic dipole may be computed as the Fourier integral of (32) for a specified spectrum of the antenna current. For a step of current the far field approximation (32) gives

$$E_r^{hm}(\omega)=K\sqrt{i\omega}\sum_{n=0}^{\infty}\delta_n S_n^{0.5}\,e^{-i\frac{\omega d}{c}S_n} \tag{52}$$

with

$$K=\frac{\sin\phi\, I_0\, da\eta}{2hc\sqrt{2\pi ac}\,\sin\,(d/a)} \tag{53}$$

and with $\delta_0=0.5$ and $\delta_{n\neq 0}=1$.

The response of the $n=0$ mode may be reduced to tabulated Fourier integrals after approximating S_0 of (52) by a power series. With $n_g\gg n_i\gg 1$ (45) reduces to

$$S_0\approx\sqrt{1+\frac{1}{h\sqrt{i\omega\mu\sigma_i}}}. \tag{54}$$

Expanding $\sqrt{S_0}$ and S_0 in powers of $(1/\sqrt{i\omega})$ and ending the expansions with the quadratic terms results in

$$E_r^{hm}(\omega)=Ke^{d/(2c\beta^2)}\left[\sqrt{i\omega}+\frac{1}{2\beta}-\frac{3}{8\beta^2\sqrt{i\omega}}\right]e^{-\frac{i\omega d}{c}-\frac{\sqrt{i\omega}d}{c\beta}}, \tag{55}$$

where

$$\beta=2h\sqrt{\mu\sigma_i}. \tag{56}$$

Applying the transform pairs (806), (801), and (807) of Campbell and Foster [1948] gives

$$E_{ro}^{hm}(\tau)\approx\frac{Ke^{1/u-0.25/x}}{2\alpha^{1.5}\sqrt{\pi x}}\left[\frac{1}{2x^2}-\frac{1}{x}\left(1-\frac{1}{u}\right)-\frac{3}{u^2}\right] \tag{57}$$

where

$$x=\tau/\alpha, \tag{58}$$

$$\tau=(t-d/c)>0, \tag{59}$$

$$\alpha=\left(\frac{d}{\beta}\right)^2=\frac{\epsilon_0 d^2}{4\sigma_i h^2}=\frac{\nu d^2}{4\omega_p^2 h^2}, \tag{60}$$

$$u=\frac{2c\beta^2}{d}=\frac{8\sigma_i h^2}{d}\sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{61}$$

The power series expansion of (54) is applicable to frequencies where $\omega>\omega_0=(h^2\mu\sigma_i)^{-1}$. Hence (57) will be most accurate for $\tau<<2\pi/\omega_0$ or for

$$x<<x_0=2\pi\left[\frac{2h^2\sigma_i}{d}\sqrt{\frac{\mu_0}{\epsilon_0}}\right]^2=\pi u^2/8. \tag{62}$$

Considering $h=90$ km, $d=10^4$ km, $\sigma_i=10^{-6}$ mho/m as an example, $x_0=2.3$ with $u=2.4$. The representation of the transient peak in figure 5 will be accurate even for this pessimistically low σ_i value at $d=10^4$ km. Higher σ_i and smaller d result in larger x_0 (and u), which improves the accuracy of the transient tail representations.

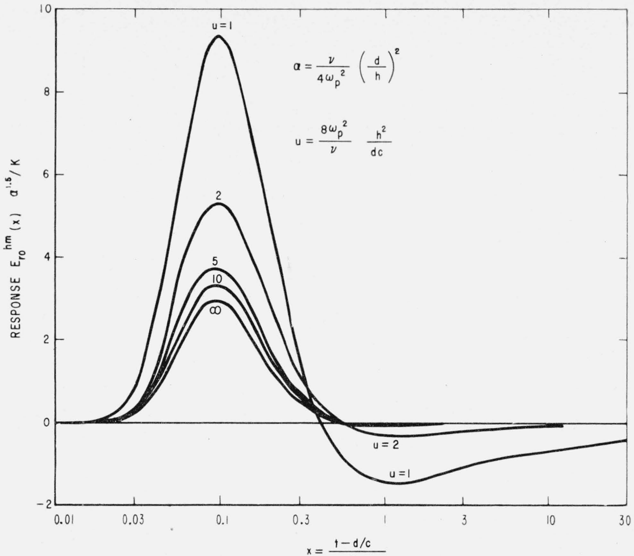

FIGURE 5. *Response to a current step of the* n=0 *mode of a horizontal magnetic dipole.*

It follows from reciprocity relation (3) that for equal current waveforms $I^{hm}(t)$ and $I^{ve}(t)$ in the *HM* and *VE* dipoles

$$E_r^{hm}(t)=-\mu_0 H_\phi^{ve}(t)\,\sin\,\phi da/ds. \tag{63}$$

A current in the *VE* dipole that is proportional to the time derivative of the current in the *HM* dipole

$$I^{ve}(t)=TI^{hm}(t), \tag{64}$$

results in

$$E_r^{hm}(t)=-\mu_0\,\sin\,\phi da H_\phi^{ve}(t)/(Tds). \tag{65}$$

The response $E_r^{hm}(t)$ to a current step $I_0u(t)$ should be proportional to the response $H_\phi^{ve}(t)$ to a current impulse $I_0T\delta(t)$. Assuming that (64) applies and assuming further that

$$S_0\approx 1 \tag{66}$$

(this is obviously incorrect at the lower frequencies), it follows from (40) that

$$E_r^{hm}(t)\approx-\sin\,\phi E_r^{ve}(t)da/(cTds). \tag{67}$$

The impulse response of a vertical electric dipole has been calculated considering factors proportional to finite powers of S_0 equal to unity [Schumann, 1952; Wait, 1960b]. The calculated impulse response of the *VE* dipole [Schumann, 1952; Wait, 1960b] will be compared with the step response of the *HM* dipole (57) by means of (65) and also by means of the approximate relation (67). The leading term of the impulse response (42) of Wait [1960b] (eqs (38), (39), (40), and (42) of Wait [1960b] should be multiplied by $\pi/2$), when substituted in either (65) or (67) results in $E_r^{hm}(t)$ that is equal to (57) with the two terms proportional to u^{-1} and u^{-2} of the square brackets set equal to zero. There is no agreement between the higher response terms because of the S_0 approximation as discussed earlier. The impulse response of Schumann [1952], when substituted in (67) is the same as (57) with $u=\infty$.

The first order perturbation solution of the modal equation becomes inaccurate for the higher frequencies involved in calculating the transient response of the $n\neq 0$ modes. Even the simplest (and inaccurate) expressions of S_n that may be obtained from (44) and (14) for $|n_g|>>|n_i|>>1$ involve integrals similar to those encountered in the transient analysis of lossy rectangular waveguides [Cerillo, 1948]. The transient response of the $n\neq 0$ modes has not been calculated in this paper.

6. Appendix. Radial Electric Fields of Vertical Electric and Horizontal Magnetic Dipoles

The fields of a vertical dipole may be derived from a single scalar function. Applying (29) of Schumann [1954] to (3)

$$\frac{E_r^{ve}}{E_r^{hm}}=\frac{ds_1\dfrac{\partial}{\partial\theta}\left(\sin\theta\,\dfrac{\partial u_1}{\partial\theta}\right)}{rk^2\cos\phi\, da_2\sin\theta\,\dfrac{\partial u_2}{\partial\vartheta}} \tag{68}$$

where for u_2 the coordinates of the source and of the observation points are interchanged relative to u_1. As long as the source and the observation point are approximately at the same radius ($r_t\approx r_r$), both u functions are the same. The ratio of the vertical electric fields of horizontal magnetic and vertical electric dipoles for equal currents on a perfectly conducting plane ground is

$$\frac{E_0^{ve}}{E_0^{hm}}=\frac{ids_1}{kda_2}. \tag{69}$$

With

$$u_1\approx u_2\sim P_\nu\,(-\cos\theta) \tag{70}$$

(68) becomes

$$\frac{E_r^{ve}}{E_r^{hm}}=\frac{E_0^{ve}}{E_0^{hm}\cos\phi}\,\frac{i}{rk}\,\frac{(\nu+1)\sin\theta\,P_\nu\,(-\cos\theta)}{\cos\theta\,P_\nu\,(-\cos\theta)+P_{\nu-1}\,(-\cos\theta)}. \tag{71}$$

The earlier approximations will reduce (71) to (41). However, more accurate ν values [Wait, 1960a, ch. 12] and a better approximation to $P_\nu\,(-\cos\theta)$ may be used in (71) to obtain a more exact E^{ve}/E_r^{hm} ratio.

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Selected Abstracts

Amplitude-probability distributions for atmospheric radio noise, W. Q. Crichlow, A. D. Spaulding, C. J. Roubique, and R. T. Disney, *NBS Mono. 23 (1960), 20 cents.*
Families of amplitude-probability distribution curves are presented in a form such that by using three statistical parameters of atmospheric radio noise, of the type published by the National Bureau of Standards, the corresponding amplitude-probability distribution may be readily chosen. Typical values of these parameters are given.

Low- and very low-radiofrequency model ionosphere reflection coefficients, J. R. Johler, L. C. Walters, and J. D. Harper, Jr., *NBS Tech. Note 69 (PB161570), (July 1, 1960) $2.00.*
The results of extensive computations performed during the course of a theoretical investigation of a sharply bounded model ionosphere for low- and very low-radiofrequency wave propagation are presented in the form of graphs and tables. *This Tech. Note supplements the work described in,* **On the theory of reflection of low- and very-low-radio frequency waves from the ionosphere,** *J. Research NBS* **64D,** *No. 3, 269 (May–June 1960).*

The relation of h_{max}F2 to M(3000)F2 and h_pF2, J. W. Wright and R. E. McDuffie, *J. Radio Research Laboratories* **7,** *No. 32, 409 (July 1960).*
Several easy methods of inferring directly from ionograms the height of the F region peak, using the quantities $M(3000)F2$ or h_pF2, are compared with values of $h_{max}F2$ obtained from $N(h)$ profiles. It is found that such methods are free from bias at night at low and medium latitudes, but that systematic differences are significant in the daytime, and at high latitudes at all times. Examples are given illustrating the incorrect diurnal variation of $h_{max}F2$ estimated from these simple parameters, in comparison with simultaneous values of h_{max} obtained from $N(h)$ profiles.

A test of a procedure for easy estimation of representative monthly electron density profiles for the ionosphere, J. W. Wright, *J. Geophys. Research* **65,** *No. 10, 3215–3217 (Oct. 1960).*
A recently proposed method for obtaining a representative monthly electron-density profile for a given hour of the day applies standard $N(h)$ procedures to a single representative virtual-height curve obtained in a special way from the individual daily virtual-height curves for that hour. The method is tested by comparing examples of the resulting profile with the average of the profiles for the individual days. The differences are small compared with the dispersion of the individual profiles about their mean. It is concluded that the method is valuable for obtaining a worldwide $N(h)$ morphology with a minimum expenditure of effort.

Spiral patterns in geophysics, V. Agy, *J. Atmospheric and Terrest. Phys.* **19,** *No. 2, 136–140 (Oct. 1960).*
Recent analyses of magnetic and ionospheric data, notably by A. P. Nikolski, have resulted in spiral "precipitation" patterns which lead the authors to claim support for Störmer's theory of the aurora. Although it may be possible to argue against these claims by attacking the methods of analysis and/or the Störmer theory itself, an entirely different approach is used in the paper presented here: an examination of pertinent points of Störmer's theory shows that the analytical spirals mentioned cannot be Störmer spirals and this conclusion holds regardless of the soundness of the analyses and, indeed, of the validity of Störmer's theory.

Supplementary world maps of F2 critical frequencies and maximum usable frequency factors, Donald H. Zacharisen, *NBS Tech. Note 2–2 (PB151361–2) (Oct. 1960) $3.50.*
This report supplements NBS Tech. Note 2 (April 1959), and completes the basic data required for F2-layer maximum usable frequency predictions. Prediction charts are given for the months of February, April, May, August, October, and November. Auxiliary charts are included to aid in predicting F2-layer MUFs.

The four parameters used for predicting MUFs are foF2 and the 4000 km MUF factor for a twelve-month running average Zurich sunspot number of 50, and the rates of change of foF2 and 4000 km MUF factor with sunspot number. The first three parameters are presented in map form for each even hour of Greenwich Mean Time. The fourth parameter is presented on a chart of geomagnetic latitude and local time.

The height of maximum luminosity in an auroral arc, F. E. Roach, J. G. Moore, E. C. Bruner, Jr., H. Cronin, and S. M. Silverman, *J. Geophys. Research* **65,** *No. 11, 3575–3580 (Nov. 1960).*
The height of maximum luminosity of an auroral arc is estimated from simultaneous observations at three stations in western United States during a night of general auroral activity (November 27–28, 1959). Photometrically this arc is characterized by a selective enhancement of the [OI] 6300 A line. From twenty-four individual measurements the height is found to be 412 km with a standard deviation of ±23 km for one observation and ±5 km for the mean. The geographical position of the arc, its orientation, and its movement during the night are discussed.

FM and SSB radiotelephone tests on a VHF ionospheric scatter link during multipath conditions, J. W. Koch, W. B. Harding, and R. J. Jansen, *IRE Trans. Commun. Systems* **CS–8,** *No. 3, 183–186 (Sept. 1960).*
Experiments have been carried out on an ionospheric-scatter link to observe the effects of long-delayed multipath signals, caused by F_2 propagated back scatter, on the intelligibility of voice communication. Frequency modulation and single-sideband modulation equipments were used for the tests. During periods when the back-scatter signal levels approached the level of the normal ionospheric-scatter signals, the frequency-modulation voice transmissions were unintelligible; however, under the same conditions, single-sideband voice communication intelligibility remained at almost 100 per cent although there was some loss in quality.

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The optical refractive index is known to be determined principally by the temperature and pressure of the atmosphere, whereas the radio refractive index is, in addition, affected by the water content of the atmosphere, the relationship between these quantities being expressed in the following way:

$$N=(n-1)\,10^6=\left(\frac{77.6}{T}\right)(P+[4810e/T]) \qquad (1)$$

where total air pressure P, and water vapor e, are in millibars, and the temperature T is in degrees Kelvin [Smith, 1953]. The quantities P, e, and T have long been routinely measured at the surface of the earth by standard weather bureau stations. For some time they have been measured from the surface up to great heights using balloon-borne radiosonde equipment at a large number of places over the earth's surface. More recently, equipment has been developed to measure rapidly and directly the radio refractive index of the atmosphere using radio techniques. The measurement of the radio refractive index properties of the atmosphere and the application to radio propagation problems is the subject of this paper.

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