JOURNAL OF RESEARCH of the National Bureau of Standards—D. Radio Propagation
Vol. 65D, No. 1, January–February 1961

Magneto-Ionic Propagation Phenomena in Low- and Very-Low-Radiofrequency Waves Reflected by the Ionosphere[1]

J. Ralph Johler

(July 8, 1960)

LF–VLF ionosphere reflection coefficients which illustrate the dependence of the amplitude and phase of the reflected wave upon the direction of propagation relative to the direction of the earth's magnetic field are presented. The calculations are based on a plane, sharply bounded, model ionosphere with plane wave excitation, but employ full use of the magneto-ionic formulas for complex directions of propagation in the ionosphere such that the influence of the earth's magnetic field in the different directions of propagation is demonstrated. A special table of values applicable to VLF is presented.

1. Introduction

The classical magneto-ionic theory implies a dependence of the electromagnetic waves on the direction of propagation relative to the direction of the earth's magnetic field.

The application of the classical magneto-ionic theory to the evaluation of low- and very-low-radiofrequency reflection coefficients for a sharply bounded model ionosphere by using an assumed electron equation of motion, together with Maxwell's equations was investigated in a previous paper [1].[2] The importance of the orientation of the earth's magnetic field vector, $\overline{H}_m$, was discussed in considerable detail, and the effect of various orientations of the vector (relative to the direction of propagation) on the precise value of the reflection coefficient was noted. This paper further pursues the subject of "directional effects" at low- and very-low-radiofrequencies. The forces acting on the electron in the ionosphere excited by an electromagnetic field are described by an electron equation of motion, Johler et al. [1] (1960). A particular term of this equation $\mu_o e(\overline{V}\times\overline{H}_m)$ called the "Lorentz force," Lorentz [2] (1906) relates the action of the earth's magnetic induction, $\mu_o\overline{H}_m$, or an electron (charge e) traveling at a velocity, $\overline{V}$, in a medium of permeability μ_o, to the direction of propagation. This dependence upon direction of propagation causes some interesting and predictable phenomena in the low and very low part of the radio spectrum.

The effect of the collision frequency, ν, on the magnitude of abnormal components (i.e., electric-magnetic coupling coefficients, T_{em}, T_{me}) is also investigated. A large angle of incidence, $\phi_i=80°$, $82°$, is considered and the resultant intensities of both normal and abnormal components are noted at various directions of propagation.

2. Theory of the Reflection Coefficient

The essential nature of low- and very-low-radiofrequency waves propagated via the ionosphere can be described in terms of a reflection coefficient. Reflection coefficients or propagation constants (complex index of refraction of the ionosphere) for complex angles of incidence, ϕ_i, figure 1, such as would be required for "mode-type" calculations, Wait [3] (1960), can be readily evaluated from the

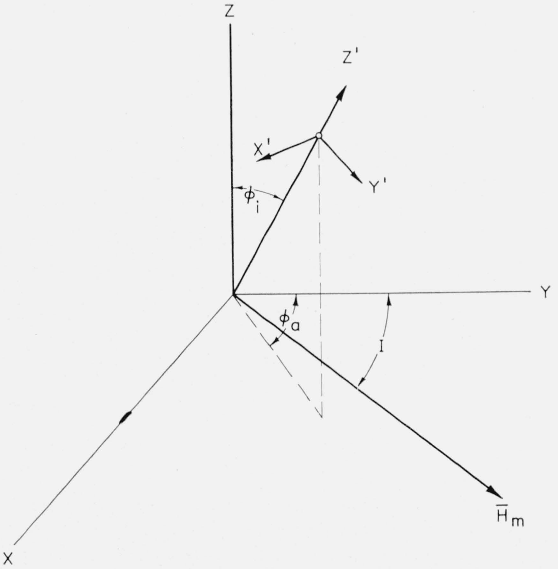

FIGURE 1. *Coordinate systems.*

---

[1] Contribution from Central Radio Propagation Laboratory, National Bureau of Standards, Boulder, Colo.

[2] Figures in brackets indicate the literature references at the end of this paper.

results of the previous paper, Johler et al. [1] (1960) without analytic modification of the formulas. However, for naive discussion purposes in this paper, a real angle of incidence, ϕ_i, will be employed. It should be noted, however, that the wave normal direction, ψ, relative to the direction of the earth's magnetic field vector, $\overline{H}_m$, in the ionosphere is nevertheless complex [1]. The direction, θ, relative to the vertical, z direction, figure 1, is also a complex number as a consequence of Snell's law: $\sin\phi_i=\eta \sin\theta$. The propagation downward from the ionosphere (i.e., wave traveling from the ionosphere into the negative z, figure 1, region) is defined by four reflection coefficients, T_{ee}, T_{em}, T_{me}, T_{mm}, which relate the reflected radiation to the primary or incident radiation. The reflection coefficient, T_{ee}, refers to the vertical electric polarization of the incident plane wave and a similar vertical electric polarization of the reflected wave. The coefficient, T_{em}, describes the generation of the abnormal component by the incident vertical polarization (vertical electric- magnetic coupling). Similarly, T_{mm}, refers to the incident horizontal electric polarization and the corresponding reflected horizontal electric polarization. Also, the abnormal component generated by horizontal electric polarization (vertical magnetic-electric coupling) is described by the coefficient, T_{me}. The reflection coefficients can therefore be defined (see fig. 1),

$$T_{ee}=\frac{E_{y'r}}{E_{y'i}}, \qquad T_{me}=\frac{E_{y'r}}{E_{x'i}},$$

$$T_{em}=\frac{E_{x'r}}{E_{y'i}}, \qquad T_{mm}=\frac{E_{x'r}}{E_{x'i}},$$

where subscripts i and r refer to incident and reflected waves, respectively. Expressions for these coefficients were developed in the previous paper [1]. In addition to the geometric parameters, figure 1, the reflection coefficient is completely defined by the magnetic intensity $H_m=|\overline{H}_m|$ gauss, the electron density N(el/cm^3), and the collision frequency ν(c/s).

3. Discussion

A comparison of the reflection coefficients of the model ionosphere, $|T_{ee}|$, $|T_{em}|$ and phase, arg T_{ee}, arg T_{em}, figure 2, and amplitude, $|T_{mm}|$, $|T_{me}|$ and phase, arg T_{mm}, arg T_{me}, figure 3, for north-east propagation, magnetic azimuth, $\phi_a=45°$, with the corresponding reflection coefficients for south-west propagation, magnetic azimuth, $\phi_a=225°$, is illustrated. It is quite evident that a nonreciprocity exists in the propagation (i.e., the expected value of field intensity would not be the same if transmitter and receiver were interchanged), since neither normal nor abnormal components are precisely the same in both directions except at zero frequency. Indeed, the discrepancy is appreciable not only at LF($<$300 kc/s) but also at VLF($<$30 kc/s).

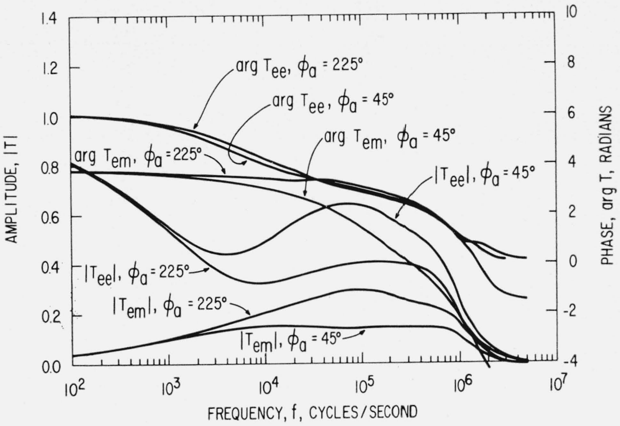

FIGURE 2. *Comparison of north-east propagation with south-west propagation (vertical polarization) illustrating nonreciprocity with the aid of the model ionosphere reflection coefficients, amplitude,* |T|, *and phase, arg* T.

N=870, ν=4 (10^6), ϕ_i=75.08°, H_m=0.5, I=60°, ϕ_a=45°, 225°.

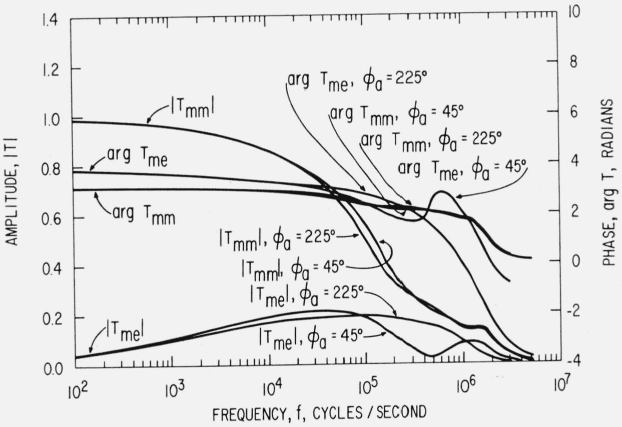

FIGURE 3. *Comparison of north-east propagation with south-west propagation (horizontal polarization) illustrating nonreciprocity with the aid of the model ionosphere reflection coefficients, amplitude,* |T|, *and phase, arg* T.

N=870, ν=4 (10^6), ϕ_i=75.08°, H_m=0.5, I=60°, ϕ_a=45°, 225°.

The effect of the magnetic azimuth, ϕ_a, (reckoned as clockwise angular measurement from magnetic north, fig. 1) is illustrated at a "temperate" magnetic latitude, $I=45°$, figures 4, 5. Both amplitude, $|T|$, and phase, arg T, are presented, $\phi_a=0-360°$, at frequencies 10 and 100 kc/s. Note that the component, T_{ee}, is symmetrical about the eastward, $\phi_a=90°$, and the westward, $\phi_a=270°$, propagation. Note also that the direction of maximum intensity, $\phi_a=90°$ (eastward propagation) is considerably greater than the corresponding reverse direction, $\phi_a=270°$ (westward propagation).

Propagation via the ionosphere at the magnetic equator, $I=0$, tables [3] 1, 5, 9, 13, 17, exhibit small

---

[3] The integer to the right of each table entry, if present, indicates the power of the factor ten (10) by which the entry is to be multiplied. Thus, 6.4307−1=0.64307.

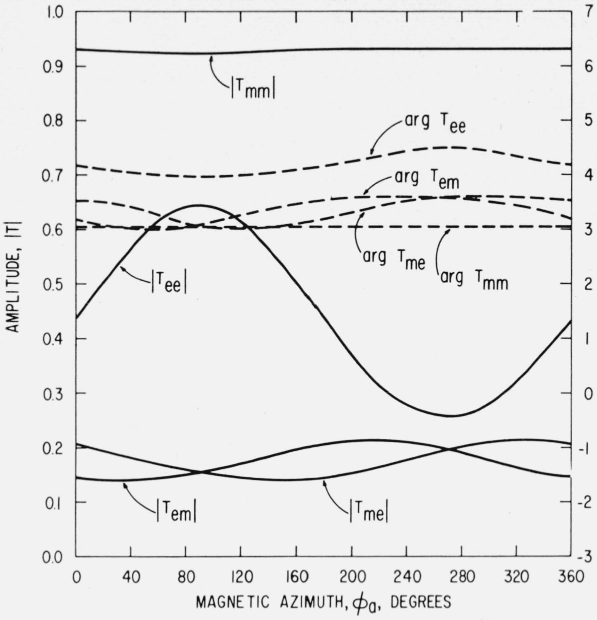

FIGURE 4. *Model ionosphere reflection coefficients, amplitude,* |T|, *and phase, arg* T, *illustrating a dependence of the reflected field upon the direction of propagation relative to the direction of the earth's magnetic field vector.*

$N=1.2\ (10^3)$, $\nu=10^6$, $\phi_i=80.397°$, $H_m=0.5$, $I=45°$, $f=10$ kc/s.

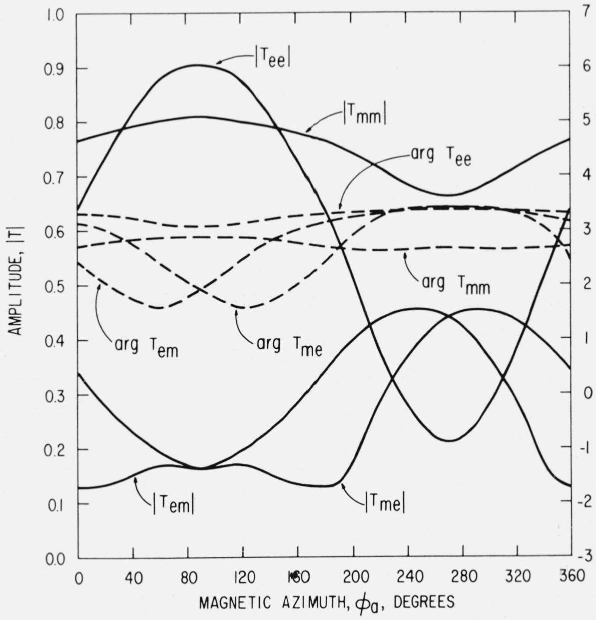

FIGURE 5. *Model ionosphere reflection coefficients, amplitude,* |T|, *and phase, arg* T, *illustrating a dependence of the reflected field upon the direction of propagation relative to the direction of the earth's magnetic field vector.*

$N=1.2\ (10^3)$, $\nu=10^6$, $\phi_i=80.397°$, $H_m=0.5$, $I=45°$, $f=100$ kc/s.

and substantially constant abnormal components ($|T_{em}|$ and $|T_{me}|$ are between 0.01 and 0.04) in all directions, ϕ_a, for a large limiting value of the angle of incidence, $\phi_i=82°$, and a large collision frequency, $\nu=2\ (10^7)$. It is interesting to note that the components differ in phase by π radians; i.e., arg $T_{em}-$arg $T_{me}=\pm\pi p$, where p is an odd integer, $p=1, 3, 5\ .\ .\ .\ .$ The abnormal components, T_{em} and T_{me}, appear to interchange values abruptly at $\phi_a=90°$ and $\phi_a=270°$ at the magnetic equator. The abruptness of this interchange of values decreases as the magnetic inclination, I, increases. It is of interest to note, tables 9, 13, for example, that north-south propagation, $\phi_a=180°$, and south-north propagation $\phi_a=0$, do not exhibit a precise reciprocity at the magnetic equator, $I=0$. Note (tables 9, 13):

$$\phi_a=0 \qquad \phi_a=180^\circ$$

$$|T_{ee}| \quad =|T_{ee}|,$$

$$\arg T_{ee} =\arg T_{ee},$$

$$|T_{em}| \quad =|T_{em}|,$$

$$\arg T_{em} =\arg T_{em}-\pi,$$

$$|T_{mm}| \quad =|T_{mm}|,$$

$$\arg T_{mm}=\arg T_{mm},$$

$$|T_{me}| \quad =|T_{me}|,$$

$$\arg T_{me} =\arg T_{me}+\pi.$$

Thus, at the "magnetic equator", $I=0$, the only nonreciprocity in the propagation along the magnetic meridian is as might be expected, a phase shift of π in the abnormal components as each abnormal component vanishes at precisely eastward or westward propagation.

Propagation from the model ionosphere at the magnetic north pole, $I=90°$, tables 4, 8, 12, 16, as might be expected, is independent of the magnetic azimuth, ϕ_a. A value, $\phi_a=0$, was computed at all frequencies, $f=10$–22 kc/s and a value, $\phi_a=240°$ at $f=10$ kc/s served as a check on the computation. The behavior of the propagation in the vicinity of the magnetic north pole, $I=84.270°$ is illustrated, tables 3, 7, 11, 15.

Propagation from the model ionosphere in "temperate" magnetic latitudes, $I=45°$, tables 2, 6, 10, 14, figures 4, 5, exhibit abnormal components, T_{em}, T_{me}, which in general differ not only along the magnetic meridian, but also at the various values of magnetic azimuth, ϕ_a (0 to 360°). Although the normal components, T_{ee}, T_{mm}, in the north-south, $\phi_a=180°$, and the south-north, $\phi_a=0$, direction (i.e., along a magnetic meridian) are identical, the propagation does not exhibit a precise reciprocity since the abnormal components, T_{em}, T_{me}, are not identical in these directions. However, reciprocity for propagation along a magnetic meridian can be considered to be an approximately valid concept if the collision frequency, ν, and angle of incidence, ϕ_i, are sufficiently great such that the abnormal components, T_{em}, T_{me}, become quite small. Indeed, the large values of the angle of incidence, $\phi_i=82°$, employed in the calculations for the tables tended, in conjunction with the high collision frequency, $\nu=2\ (10^7)$, to reduce greatly the abnormal components.

The intensity of the abnormal component propagated from the ionosphere excited by a vertically polarized transmitter, $|T_{em}|$, has special interest at LF, $f=100$ kc/s, figure 5. The eastward propagation, $\phi_a\sim 90°$, from a vertically polarized transmitter provides a dominant vertically polarized field, $|T_{ee}|$, at the receiver; whereas, westward propagation, $\phi_a\sim 270°$, from a vertically polarized transmitter can under certain conditions (see fig. 5 specifications) provide a dominant horizontally polarized field, $|T_{em}|$ (abnormal component) at the receiver. In practice, it is necessary to consider the influence of the displacement and conduction currents in the ground which can indeed discriminate between horizontal and vertical polarization in such a manner as to obscure the ionosphere phenomena described.

4. Conclusions

The propagation of low- and very-low-radio-frequency waves reflected by the ionosphere is indeed dependent upon the "Lorentz force" on the electron, and hence is dependent upon the direction of propagation relative to the direction of the earth's magnetic field vector in the region of the ionosphere in which the reflection occurs. The intensity of the abnormal components is dependent upon the values of the various parameters (such as N, ν, H_m, ϕ_a, I, ϕ_i) and in particular, the collision frequency, ν, and the angle of incidence (distance) ϕ_i are especially important in the determination of the precise value of the abnormal components. Precise reciprocity in the LF and VLF propagation (interchange of transmitter and receiver) can occur only at the magnetic north or south pole, $I=\pm 90°$. Otherwise, the propagation is dependent upon the direction, ϕ_a, and in general the intensities become greatest in an eastward direction, $\phi_a\sim 90°$. Furthermore, it is conceivable at low frequencies, if the influence on the ground can be neglected or separated from this consideration, that the horizontally polarized component excited by a vertically polarized transmitter can approach in magnitude and even exceed the vertically polarized component in the case of westward propagation, whereas it would appear that the corresponding eastward propagation would exhibit a dominant vertical polarization.

Table 1.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^{\circ}$ $H_m = 0.5$ $I = 0$

f, kc	ϕ_a, degrees	$\lvert T_{ee} \rvert$		arg T_{ee}	ϕ_a, degrees	$\lvert T_{em} \rvert$		arg T_{em}		ϕ_a, degrees	$\lvert T_{em} \rvert$		arg T_{em}	
10	0, 180	6.4307	- 1	3.3383	0	2.9182	- 2	7.8223	- 1	180	2.9182	- 2	3.9238	
12		6.5235	- 1	3.2969		3.1976	- 2	7.5758	- 1		3.1976	- 2	3.8992	
14		6.5863	- 1	3.2632		3.4477	- 2	7.3621	- 1		3.4477	- 2	3.8778	
16		6.6287	- 1	3.2346		3.6746	- 2	7.1727	- 1		3.6746	- 2	3.8589	
18		6.6568	- 1	3.2098		3.8826	- 2	7.0021	- 1		3.8826	- 2	3.8418	
20		6.6745	- 1	3.1879		4.0750	- 2	6.8464	- 1		4.0750	- 2	3.8262	
22		6.6843	- 1	3.1682		4.2540	- 2	6.7028	- 1		4.2540	- 2	3.8119	
10	60, 120	7.1140	- 1	3.3275	60	1.5209	- 2	7.7949	- 1	240	1.3872	- 2	3.9089	
12		7.2128	- 1	3.2863		1.6655	- 2	7.5346	- 1		1.5171	- 2	3.8837	
14		7.2785	- 1	3.2526		1.7945	- 2	7.3100	- 1		1.6328	- 2	3.8619	
16		7.3217	- 1	3.2240		1.9111	- 2	7.1121	- 1		1.7374	- 2	3.8427	
18		7.3492	- 1	3.1991		2.0176	- 2	6.9350	- 1		1.8329	- 2	3.8255	
20		7.3653	- 1	3.1770		2.1159	- 2	6.7743	- 1		1.9210	- 2	3.8099	
22		7.3729	- 1	3.1572		2.2072	- 2	6.6272	- 1		2.0029	- 2	3.7955	
10	240, 300	5.7167	- 1	3.3394	120	1.5209	- 2	3.9211		300	1.3872	- 2	7.6730	- 1
12		5.7951	- 1	3.2965		1.6656	- 2	3.8950			1.5170	- 2	7.4211	- 1
14		5.8472	- 1	3.2616		1.7945	- 2	3.8726			1.6328	- 2	7.2033	- 1
16		5.8816	- 1	3.2321		1.9111	- 2	3.8528			1.7374	- 2	7.0112	- 1
18		5.9034	- 1	3.2066		2.0176	- 2	3.8351			1.8329	- 2	6.8391	- 1
20		5.9161	- 1	3.1840		2.1159	- 2	3.8190			1.9210	- 2	6.6828	- 1
22		5.9220	- 1	3.1638		2.2072	- 2	3.8043			2.0029	- 2	6.5396	- 1

Table 2.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^{\circ}$ $H_m = 0.5$ $I = 45^{\circ}$

f, kc	ϕ_a, degrees	$\lvert T_{ee} \rvert$		arg T_{ee}	ϕ_a, degrees	$\lvert T_{em} \rvert$		arg T_{em}	ϕ_a, degrees	$\lvert T_{em} \rvert$		arg T_{em}
10	0, 180	6.4706	- 1	3.3361	0	2.2933	- 2	2.4742	180	4.9217	- 2	3.4406
12		6.5630	- 1	3.2953		2.2771	- 2	2.3673		5.1006	- 2	3.4343
14		6.6256	- 1	3.2620		2.2720	- 2	2.2697		5.2563	- 2	3.4285
16		6.6679	- 1	3.2339		2.2764	- 2	2.1802		5.3945	- 2	3.4232
18		6.6959	- 1	3.2095		2.2888	- 2	2.0977		5.5189	- 2	3.4181
20		6.7136	- 1	3.1879		2.3079	- 2	2.0216		5.6322	- 2	3.4132
22		6.7234	- 1	3.1685		2.3324	- 2	1.9511		5.7363	- 2	3.4085
10	60, 120	6.9474	- 1	3.3295	60	2.4697	- 2	2.6841	240	4.3756	- 2	3.3903
12		7.0449	- 1	3.2890		2.4266	- 2	2.6032		4.5073	- 2	3.3830
14		7.1102	- 1	3.2558		2.3900	- 2	2.5284		4.6210	- 2	3.3765
16		7.1538	- 1	3.2277		2.3597	- 2	2.4584		4.7212	- 2	3.3705
18		7.1821	- 1	3.2033		2.3352	- 2	2.3925		4.8108	- 2	3.3648
20		7.1993	- 1	3.1816		2.3161	- 2	2.3300		4.8919	- 2	3.3594
22		7.2082	- 1	3.1622		2.3017	- 2	2.2705		4.9659	- 2	3.3542
10	240, 300	5.9784	- 1	3.3379	120	3.7306	- 2	3.2457	300	2.8422	- 2	3.0368
12		6.0618	- 1	3.2962		3.7759	- 2	3.2299		2.8186	- 2	3.0038
14		6.1178	- 1	3.2623		3.8104	- 2	3.2163		2.7927	- 2	2.9744
16		6.1552	- 1	3.2337		3.8375	- 2	3.2042		2.7661	- 2	2.9478
18		6.1796	- 1	3.2088		3.8590	- 2	3.1933		2.7394	- 2	2.9231
20		6.1944	- 1	3.1869		3.8762	- 2	3.1832		2.7130	- 2	2.9001
22		6.2021	- 1	3.1672		3.8900	- 2	3.1739		2.6870	- 2	2.8783

Table 3.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^o$ $H_m = 0.5$ $I = 84.27^o$

f, kc	ϕ_a, degrees	$\lvert T_{ee}\rvert$		arg T_{ee}	ϕ_a, degrees	$\lvert T_{em}\rvert$		arg T_{em}	ϕ_a, degrees	$\lvert T_{em}\rvert$		arg T_{em}
10	0, 180	6.5072	- 1	3.3342	0	4.2815	- 2	3.0972	180	4.6885	- 2	3.1864
12		6.5995	- 1	3.2940		4.2670	- 2	3.0695		4.7133	- 2	3.1669
14		6.6619	- 1	3.2611		4.2459	- 2	3.0451		4.7274	- 2	3.1501
16		6.7041	- 1	3.2334		4.2210	- 2	3.0232		4.7345	- 2	3.1353
18		6.7321	- 1	3.2093		4.1940	- 2	3.0033		4.7368	- 2	3.1219
20		6.7497	- 1	3.1880		4.1657	- 2	2.9848		4.7355	- 2	3.1097
22		6.7596	- 1	3.1689		4.1368	- 2	2.9676		4.7317	- 2	3.0984
10	60, 120	6.5743	- 1	3.3336	60	4.3267	- 2	3.1040	240	4.6400	- 2	3.1808
12		6.6674	- 1	3.2934		4.3149	- 2	3.0768		4.6617	- 2	3.1610
14		6.7304	- 1	3.2606		4.2961	- 2	3.0529		4.6732	- 2	3.1440
16		6.7730	- 1	3.2329		4.2731	- 2	3.0314		4.6780	- 2	3.1289
18		6.8013	- 1	3.2088		4.2478	- 2	3.0119		4.6782	- 2	3.1153
20		6.8190	- 1	3.1876		4.2209	- 2	2.9938		4.6752	- 2	3.1028
22		6.8289	- 1	3.1685		4.1933	- 2	2.9770		4.6698	- 2	3.0913
10	240, 300	6.4399	- 1	3.3347	120	4.5278	- 2	3.1501	300	4.4339	- 2	3.1378
12		6.5311	- 1	3.2944		4.5351	- 2	3.1273		4.4354	- 2	3.1142
14		6.5929	- 1	3.2615		4.5334	- 2	3.1074		4.4287	- 2	3.0936
16		6.6346	- 1	3.2337		4.5259	- 2	3.0898		4.4170	- 2	3.0753
18		6.6623	- 1	3.2097		4.5146	- 2	3.0738		4.4019	- 2	3.0587
20		6.6798	- 1	3.1884		4.5008	- 2	3.0592		4.3848	- 2	3.0435
22		6.6895	- 1	3.1692		4.4851	- 2	3.0456		4.3663	- 2	3.0294

Table 4.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^o$ $H_m = 0.5$ $I = 90^o$

f, kc	ϕ_a	$\lvert T_{ee}\rvert$		arg T_{ee}	$\lvert T_{em}\rvert$		arg T_{em}
10	All Values	6.5080	- 1	3.3342	4.5011	- 2	3.1438
12		6.6002	- 1	3.2939	4.5054	- 2	3.1206
14		6.6626	- 1	3.2611	4.5011	- 2	3.1004
16		6.7048	- 1	3.2334	4.4913	- 2	3.0824
18		6.7328	- 1	3.2093	4.4780	- 2	3.0661
20		6.7504	- 1	3.1880	4.4623	- 2	3.0512
22		6.7603	- 1	3.1689	4.4450	- 2	3.0373

Table 5.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^\circ$ $H_m = 0.5$ $I = 0$

f, kc	ϕ_a, degrees	$\|T_{mm}\|$		arg T_{mm}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}		ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}	
10	0, 180	8.8429	- 1	3.0158	0	2.9182	- 2	3.9238		180	2.9182	- 2	7.8223	- 1
12		8.7395	- 1	3.0034		3.1976	- 2	3.8992			3.1976	- 2	7.5758	- 1
14		8.6452	- 1	2.9919		3.4477	- 2	3.8778			3.4477	- 2	7.3621	- 1
16		8.5579	- 1	2.9812		3.6746	- 2	3.8589			3.6746	- 2	7.1727	- 1
18		8.4764	- 1	2.9711		3.8826	- 2	3.8418			3.8826	- 2	7.0021	- 1
20		8.3997	- 1	2.9616		4.0750	- 2	3.8262			4.0750	- 2	6.8464	- 1
22		8.3271	- 1	2.9525		4.2540	- 2	3.8119			4.2540	- 2	6.7028	- 1
10	60, 120	8.8384	- 1	3.0174	60	1.5209	- 2	3.9211		240	1.3872	- 2	7.6731	- 1
12		8.7348	- 1	3.0054		1.6655	- 2	3.8950			1.5171	- 2	7.4211	- 1
14		8.6405	- 1	2.9944		1.7945	- 2	3.8726			1.6328	- 2	7.2033	- 1
16		8.5536	- 1	2.9841		1.9111	- 2	3.8528			1.7374	- 2	7.0112	- 1
18		8.4727	- 1	2.9745		2.0176	- 2	3.8351			1.8329	- 2	6.8391	- 1
20		8.3969	- 1	2.9653		2.1159	- 2	3.8190			1.9210	- 2	6.6829	- 1
22		8.3252	- 1	2.9566		2.2072	- 2	3.8043			2.0029	- 2	6.5395	- 1
10	240, 300	8.8384	- 1	3.0174	120	1.5209	- 2	7.7949	- 1	300	1.3872	- 2	3.9089	
12		8.7349	- 1	3.0054		1.6656	- 2	7.5346	- 1		1.5171	- 2	3.8837	
14		8.6406	- 1	2.9944		1.7945	- 2	7.3100	- 1		1.6328	- 2	3.8619	
16		8.5537	- 1	2.9841		1.9111	- 2	7.1121	- 1		1.7374	- 2	3.8427	
18		8.4728	- 1	2.9745		2.0176	- 2	6.9350	- 1		1.8329	- 2	3.8255	
20		8.3970	- 1	2.9654		2.1159	- 2	6.7743	- 1		1.9210	- 2	3.8099	
22		8.3254	- 1	2.9567		2.2072	- 2	6.6272	- 1		2.0029	- 2	3.7956	

Table 6.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^\circ$ $H_m = 0.5$ $I = 45^\circ$

f, kc	ϕ_a, degrees	$\|T_{mm}\|$		arg T_{mm}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}
10	0, 180	8.8103	- 1	3.0119	0	4.9217	- 2	3.4406	180	2.2933	- 2	2.4742
12		8.7037	- 1	2.9992		5.1006	- 2	3.4343		2.2771	- 2	2.3673
14		8.6064	- 1	2.9874		5.2563	- 2	3.4285		2.2720	- 2	2.2697
16		8.5165	- 1	2.9764		5.3945	- 2	3.4232		2.2764	- 2	2.1802
18		8.4325	- 1	2.9661		5.5189	- 2	3.4181		2.2888	- 2	2.0977
20		8.3534	- 1	2.9563		5.6322	- 2	3.4132		2.3079	- 2	2.0216
22		8.2786	- 1	2.9470		5.7363	- 2	3.4085		2.3324	- 2	1.9511
10	60, 120	8.8089	- 1	3.0131	60	3.7306	- 2	3.2457	240	2.8422	- 2	3.0368
12		8.7028	- 1	3.0006		3.7759	- 2	3.2299		2.8186	- 2	3.0038
14		8.6061	- 1	2.9891		3.8104	- 2	3.2163		2.7927	- 2	2.9744
16		8.5169	- 1	2.9784		3.8375	- 2	3.2042		2.7661	- 2	2.9478
18		8.4338	- 1	2.9683		3.8590	- 2	3.1933		2.7394	- 2	2.9231
20		8.3557	- 1	2.9588		3.8762	- 2	3.1832		2.7130	- 2	2.9001
22		8.2819	- 1	2.9496		3.8900	- 2	3.1739		2.6870	- 2	2.8783
10	240, 300	8.8071	- 1	3.0123	120	2.4697	- 2	2.6841	300	4.3756	- 2	3.3903
12		8.6998	- 1	2.9997		2.4267	- 2	2.6032		4.5073	- 2	3.3830
14		8.6019	- 1	2.9882		2.3900	- 2	2.5284		4.6210	- 2	3.3765
16		8.5115	- 1	2.9774		2.3597	- 2	2.4584		4.7212	- 2	3.3705
18		8.4272	- 1	2.9672		2.3352	- 2	2.3925		4.8108	- 2	3.3648
20		8.3480	- 1	2.9577		2.3161	- 2	2.3300		4.8919	- 2	3.3594
22		8.2732	- 1	2.9486		2.3017	- 2	2.2705		4.9659	- 2	3.3542

Table 7.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^\circ$ $H_m = 0.5$ $I = 84.27^\circ$

f, kc	ϕ_a, degrees	$\|T_{mm}\|$		arg T_{mm}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}
10	0, 180	8.7806	- 1	3.0082	0	4.6885	- 2	3.1864	180	4.2815	- 2	3.0972
12		8.6710	- 1	2.9952		4.7133	- 2	3.1669		4.2670	- 2	3.0695
14		8.5711	- 1	2.9831		4.7274	- 2	3.1501		4.2459	- 2	3.0451
16		8.4786	- 1	2.9718		4.7345	- 2	3.1353		4.2210	- 2	3.0232
18		8.3923	- 1	2.9612		4.7368	- 2	3.1219		4.1940	- 2	3.0033
20		8.3110	- 1	2.9512		4.7355	- 2	3.1097		4.1657	- 2	2.9848
22		8.2342	- 1	2.9417		4.7317	- 2	3.0984		4.1368	- 2	2.9676
10	60, 120	8.7808	- 1	3.0084	60	4.5278	- 2	3.1501	240	4.4339	- 2	3.1378
12		8.6714	- 1	2.9953		4.5351	- 2	3.1273		4.4354	- 2	3.1142
14		8.5716	- 1	2.9833		4.5334	- 2	3.1074		4.4287	- 2	3.0936
16		8.4793	- 1	2.9720		4.5259	- 2	3.0898		4.4169	- 2	3.0753
18		8.3931	- 1	2.9614		4.5146	- 2	3.0738		4.4019	- 2	3.0587
20		8.3120	- 1	2.9514		4.5008	- 2	3.0592		4.3848	- 2	3.0435
22		8.2353	- 1	2.9419		4.4851	- 2	3.0456		4.3663	- 2	3.0294
10	240, 300	8.7803	- 1	3.0081	120	4.3267	- 2	3.1040	300	4.6400	- 2	3.1808
12		8.6706	- 1	2.9951		4.3149	- 2	3.0768		4.6617	- 2	3.1610
14		8.5705	- 1	2.9830		4.2961	- 2	3.0529		4.6732	- 2	3.1440
16		8.4778	- 1	2.9717		4.2731	- 2	3.0314		4.6780	- 2	3.1289
18		8.3913	- 1	2.9611		4.2478	- 2	3.0119		4.6782	- 2	3.1153
20		8.3100	- 1	2.9511		4.2209	- 2	2.9938		4.6752	- 2	3.1028
22		8.2330	- 1	2.9416		4.1933	- 2	2.9770		4.6698	- 2	3.0913

Table 8.

$N = 10^3$ $\nu = 2(10^7)$ $\phi_i = 82^\circ$ $H_m = 0.5$ $I = 90^\circ$

f, kc	ϕ_a	$\|T_{mm}\|$		arg T_{mm}	$\|T_{me}\|$		arg T_{me}
10	All Values	8.7800	- 1	3.0082	4.5011	- 2	3.1438
12		8.6704	- 1	2.9951	4.5054	- 2	3.1206
14		8.5704	- 1	2.9830	4.5011	- 2	3.1004
16		8.4779	- 1	2.9717	4.4913	- 2	3.0824
18		8.3915	- 1	2.9611	4.4779	- 2	3.0661
20		8.3102	- 1	2.9511	4.4623	- 2	3.0512
22		8.2333	- 1	2.9416	4.4450	- 2	3.0373

Table 9.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^o$ $H_m = 0.5$ $I = 0$

f, kc	ϕ_a, degrees	$\lvert T_{ee}\rvert$		arg T_{ee}	ϕ_a, degrees	$\lvert T_{em}\rvert$		arg T_{em}		ϕ_a, degrees	$\lvert T_{em}\rvert$		arg T_{em}	
10	0, 180	5.5106	- 1	3.6472	0	1.5969	- 2	9.2038	- 1	180	1.5969	- 2	4.0620	
12		5.6996	- 1	3.5862		1.7759	- 2	8.9780	- 1		1.7759	- 2	4.0394	
14		5.8509	- 1	3.5384		1.9391	- 2	8.7854	- 1		1.9391	- 2	4.0201	
16		5.9744	- 1	3.4994		2.0897	- 2	8.6170	- 1		2.0897	- 2	4.0033	
18		6.0769	- 1	3.4667		2.2298	- 2	8.4669	- 1		2.2298	- 2	3.9883	
20		6.1629	- 1	3.4386		2.3610	- 2	8.3312	- 1		2.3610	- 2	3.9747	
22		6.2360	- 1	3.4140		2.4847	- 2	8.2070	- 1		2.4847	- 2	3.9623	
10	60, 120	6.0983	- 1	3.6292	60	8.3103	- 3	9.2518	- 1	240	7.6588	- 3	4.0495	
12		6.3099	- 1	3.5702		9.2483	- 3	9.0152	- 1		8.5058	- 3	4.0267	
14		6.4783	- 1	3.5238		1.0103	- 2	8.8127	- 1		9.2769	- 3	4.0072	
16		6.6151	- 1	3.4857		1.0891	- 2	8.6353	- 1		9.9870	- 3	3.9902	
18		6.7280	- 1	3.4536		1.1624	- 2	8.4767	- 1		1.0647	- 2	3.9749	
20		6.8225	- 1	3.4260		1.2310	- 2	8.3332	- 1		1.1264	- 2	3.9611	
22		6.9024	- 1	3.4019		1.2955	- 2	8.2017	- 1		1.1844	- 2	3.9485	
10	240, 300	4.9235	- 1	3.6629	120	8.3103	- 3	4.0668		300	7.6589	- 3	9.0791	- 1
12		5.0865	- 1	3.5988		9.2483	- 3	4.0431			8.5058	- 3	8.8513	- 1
14		5.2173	- 1	3.5486		1.0103	- 2	4.0229			9.2769	- 3	8.6566	- 1
16		5.3243	- 1	3.5077		1.0891	- 2	4.0051			9.9870	- 3	8.4860	- 1
18		5.4129	- 1	3.4735		1.1624	- 2	3.9893			1.0647	- 2	8.3335	- 1
20		5.4873	- 1	3.4442		1.2310	- 2	3.9749			1.1264	- 2	8.1954	- 1
22		5.5503	- 1	3.4187		1.2955	- 2	3.9618			1.1844	- 2	8.0689	- 1

Table 10.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^o$ $H_m = 0.5$ $I = 45^o$

f, kc	ϕ_a, degrees	$\lvert T_{ee}\rvert$		arg T_{ee}	ϕ_a, degrees	$\lvert T_{em}\rvert$		arg T_{em}	ϕ_a, degrees	$\lvert T_{em}\rvert$		arg T_{em}
10	0, 180	5.5512	- 1	3.6405	0	2.4175	- 2	2.9446	180	3.9510	- 2	3.4778
12		5.7404	- 1	3.5804		2.4077	- 2	2.8824		4.1017	- 2	3.4702
14		5.8917	- 1	3.5333		2.3944	- 2	2.8254		4.2318	- 2	3.4641
16		6.0151	- 1	3.4948		2.3799	- 2	2.7725		4.3467	- 2	3.4590
18		6.1174	- 1	3.4626		2.3653	- 2	2.7227		4.4500	- 2	3.4547
20		6.2033	- 1	3.4349		2.3515	- 2	2.6756		4.5441	- 2	3.4508
22		6.2761	- 1	3.4107		2.3387	- 2	2.6306		4.6306	- 2	3.4474
10	60, 120	5.9594	- 1	3.6285	60	2.6130	- 2	3.0448	240	3.6347	- 2	3.4348
12		6.1643	- 1	3.5699		2.6096	- 2	2.9953		3.7537	- 2	3.4256
14		6.3277	- 1	3.5237		2.5998	- 2	2.9506		3.8552	- 2	3.4183
16		6.4605	- 1	3.4860		2.5864	- 2	2.9096		3.9439	- 2	3.4122
18		6.5704	- 1	3.4543		2.5709	- 2	2.8713		4.0230	- 2	3.4070
20		6.6624	- 1	3.4269		2.5545	- 2	2.8352		4.0945	- 2	3.4024
22		6.7403	- 1	3.4030		2.5377	- 2	2.8011		4.1599	- 2	3.3983
10	240, 300	5.1430	- 1	3.6513	120	3.3619	- 2	3.3404	300	2.8191	- 2	3.2099
12		5.3147	- 1	3.5892		3.4347	- 2	3.3234		2.8434	- 2	3.1823
14		5.4523	- 1	3.5406		3.4926	- 2	3.3092		2.8579	- 2	3.1585
16		5.5647	- 1	3.5010		3.5401	- 2	3.2971		2.8659	- 2	3.1374
18		5.6578	- 1	3.4678		3.5799	- 2	3.2865		2.8695	- 2	3.1183
20		5.7360	- 1	3.4394		3.6138	- 2	3.2770		2.8700	- 2	3.1010
22		5.8023	- 1	3.4146		3.6433	- 2	3.2685		2.8681	- 2	3.0849

Table 11.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^o$ $H_m = 0.5$ $I = 84.27^o$

f, kc	ϕ_a, degrees	$\lvert T_{ee} \rvert$		arg T_{ee}	ϕ_a, degrees	$\lvert T_{em} \rvert$		arg T_{em}	ϕ_a, degrees	$\lvert T_{em} \rvert$		arg T_{em}
10	0, 180	5.5886	- 1	3.6345	0	4.1384	- 2	3.2478	180	4.3604	- 2	3.2998
12		5.7779	- 1	3.5752		4.1891	- 2	3.2233		4.4359	- 2	3.2802
14		5.9292	- 1	3.5287		4.2240	- 2	3.2022		4.4934	- 2	3.2636
16		6.0525	- 1	3.4908		4.2482	- 2	3.1837		4.5386	- 2	3.2493
18		6.1546	- 1	3.4589		4.2648	- 2	3.1671		4.5747	- 2	3.2367
20		6.2404	- 1	3.4316		4.2759	- 2	3.1520		4.6041	- 2	3.2253
22		6.3131	- 1	3.4077		4.2829	- 2	3.1382		4.6283	- 2	3.2150
10	60, 120	5.6453	- 1	3.6329	60	4.1690	- 2	3.2522	240	4.3284	- 2	3.2958
12		5.8369	- 1	3.5739		4.2218	- 2	3.2280		4.4015	- 2	3.2759
14		5.9899	- 1	3.5275		4.2586	- 2	3.2072		4.4569	- 2	3.2592
16		6.1146	- 1	3.4897		4.2845	- 2	3.1890		4.5002	- 2	3.2447
18		6.2179	- 1	3.4580		4.3027	- 2	3.1726		4.5346	- 2	3.2318
20		6.3045	- 1	3.4307		4.3152	- 2	3.1578		4.5624	- 2	3.2203
22		6.3781	- 1	3.4069		4.3234	- 2	3.1442		4.5851	- 2	3.2098
10	240, 300	5.5319	- 1	3.6360	120	4.2793	- 2	3.2787	300	4.2168	- 2	3.2703
12		5.7189	- 1	3.5765		4.3444	- 2	3.2570		4.2773	- 2	3.2481
14		5.8684	- 1	3.5298		4.3924	- 2	3.2386		4.3212	- 2	3.2292
16		5.9903	- 1	3.4918		4.4286	- 2	3.2226		4.3538	- 2	3.2127
18		6.0913	- 1	3.4598		4.4563	- 2	3.2083		4.3783	- 2	3.1980
20		6.1761	- 1	3.4324		4.4778	- 2	3.1954		4.3967	- 2	3.1847
22		6.2480	- 1	3.4085		4.4944	- 2	3.1837		4.4106	- 2	3.1726

Table 12.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^o$ $H_m = 0.5$ $I = 90^o$

f, kc	ϕ_a	$\lvert T_{ee} \rvert$		arg T_{ee}	$\lvert T_{em} \rvert$		arg T_{em}
10	All Values	5.5893	- 1	3.6344	4.2676	- 2	3.2745
12		5.7787	- 1	3.5751	4.3307	- 2	3.2525
14		5.9299	- 1	3.5286	4.3768	- 2	3.2338
16		6.0532	- 1	3.4907	4.4113	- 2	3.2175
18		6.1554	- 1	3.4589	4.4375	- 2	3.2030
20		6.2411	- 1	3.4315	4.4575	- 2	3.1900
22		6.3138	- 1	3.4076	4.4728	- 2	3.1780

Table 13.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^\circ$ $H_m = 0.5$ $I = 0$

f, kc	ϕ_a, degrees	$\|T_{mm}\|$		arg T_{mm}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}		ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}	
10	0, 180	9.3138	- 1	3.0697	0	1.5969	- 2	4.0620		180	1.5969	- 2	9.2038	- 1
12		9.2513	- 1	3.0627		1.7759	- 2	4.0394			1.7759	- 2	8.9780	- 1
14		9.1942	- 1	3.0563		1.9391	- 2	4.0201			1.9391	- 2	8.7854	- 1
16		9.1415	- 1	3.0502		2.0897	- 2	4.0033			2.0897	- 2	8.6170	- 1
18		9.0923	- 1	3.0446		2.2298	- 2	3.9883			2.2298	- 2	8.4669	- 1
20		9.0460	- 1	3.0392		2.3610	- 2	3.9747			2.3610	- 2	8.3312	- 1
22		9.0023	- 1	3.0341		2.4847	- 2	3.9623			2.4847	- 2	8.2070	- 1
10	60, 120	9.3121	- 1	3.0700	60	8.3103	- 3	4.0668		240	7.6588	- 3	9.0791	- 1
12		9.2493	- 1	3.0631		9.2482	- 3	4.0431			8.5058	- 3	8.8513	- 1
14		9.1919	- 1	3.0568		1.0103	- 2	4.0229			9.2769	- 3	8.6566	- 1
16		9.1388	- 1	3.0509		1.0891	- 2	4.0051			9.9870	- 3	8.4859	- 1
18		9.0892	- 1	3.0453		1.1624	- 2	3.9893			1.0647	- 2	8.3334	- 1
20		9.0427	- 1	3.0401		1.2310	- 2	3.9749			1.1264	- 2	8.1953	- 1
22		8.9986	- 1	3.0351		1.2955	- 2	3.9618			1.1844	- 2	8.0689	- 1
10	240, 300	9.3121	- 1	3.0700	120	8.3103	- 3	9.2518	- 1	300	7.6589	- 3	4.0495	
12		9.2493	- 1	3.0631		9.2483	- 3	9.0151	- 1		8.5058	- 3	4.0267	
14		9.1918	- 1	3.0568		1.0103	- 2	8.8127	- 1		9.2769	- 3	4.0073	
16		9.1388	- 1	3.0509		1.0891	- 2	8.6352	- 1		9.9870	- 3	3.9902	
18		9.0892	- 1	3.0453		1.1624	- 2	8.4767	- 1		1.0647	- 2	3.9749	
20		9.0427	- 1	3.0401		1.2310	- 2	8.3331	- 1		1.1264	- 2	3.9611	
22		8.9986	- 1	3.0351		1.2955	- 2	8.2017	- 1		1.1844	- 2	3.9485	

Table 14.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^\circ$ $H_m = 0.5$ $I = 45^\circ$

f, kc	ϕ_a, degrees	$\|T_{mm}\|$		arg T_{mm}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}	ϕ_a, degrees	$\|T_{me}\|$		arg T_{me}
10	0, 180	9.2963	- 1	3.0674	0	3.9510	- 2	3.4778	180	2.4175	- 2	2.9446
12		9.2318	- 1	3.0602		4.1017	- 2	3.4702		2.4077	- 2	2.8824
14		9.1729	- 1	3.0536		4.2318	- 2	3.4641		2.3944	- 2	2.8254
16		9.1184	- 1	3.0474		4.3467	- 2	3.4590		2.3799	- 2	2.7725
18		9.0676	- 1	3.0415		4.4500	- 2	3.4547		2.3653	- 2	2.7227
20		9.0198	- 1	3.0360		4.5441	- 2	3.4508		2.3515	- 2	2.6756
22		8.9745	- 1	3.0307		4.6306	- 2	3.4474		2.3387	- 2	2.6306
10	60, 120	9.2952	- 1	3.0677	60	3.3619	- 2	3.3404	240	2.8191	- 2	3.2099
12		9.2305	- 1	3.0606		3.4347	- 2	3.3234		2.8434	- 2	3.1823
14		9.1715	- 1	3.0540		3.4926	- 2	3.3092		2.8579	- 2	3.1585
16		9.1169	- 1	3.0479		3.5401	- 2	3.2971		2.8659	- 2	3.1374
18		9.0661	- 1	3.0422		3.5799	- 2	3.2865		2.8695	- 2	3.1183
20		9.0182	- 1	3.0367		3.6138	- 2	3.2770		2.8700	- 2	3.1010
22		8.9730	- 1	3.0315		3.6433	- 2	3.2685		2.8681	- 2	3.0849
10	240, 300	9.2957	- 1	3.0674	120	2.6130	- 2	3.0448	300	3.6347	- 2	3.4348
12		9.2309	- 1	3.0603		2.6096	- 2	2.9953		3.7537	- 2	3.4256
14		9.1718	- 1	3.0536		2.5998	- 2	2.9506		3.8552	- 2	3.4183
16		9.1171	- 1	3.0475		2.5864	- 2	2.9096		3.9439	- 2	3.4122
18		9.0660	- 1	3.0417		2.5709	- 2	2.8713		4.0230	- 2	3.4070
20		9.0179	- 1	3.0362		2.5545	- 2	2.8352		4.0945	- 2	3.4024
22		8.9724	- 1	3.0309		2.5377	- 2	2.8011		4.1599	- 2	3.3983

Table 15.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^{\circ}$ $H_m = 0.5$ $I = 84.27^{\circ}$

f, kc	ϕ_a, degrees	$\lvert T_{mm}\rvert$		arg T_{mm}	ϕ_a, degrees	$\lvert T_{me}\rvert$		arg T_{me}	ϕ_a, degrees	$\lvert T_{me}\rvert$		arg T_{me}
10	0, 180	9.2802	- 1	3.0653	0	4.3604	- 2	3.2998		4.1384	- 2	3.2478
12		9.2138	- 1	3.0579		4.4359	- 2	3.2802	180	4.1891	- 2	3.2233
14		9.1533	- 1	3.0511		4.4934	- 2	3.2636		4.2240	- 2	3.2022
16		9.0972	- 1	3.0447		4.5386	- 2	3.2493		4.2482	- 2	3.1837
18		9.0449	- 1	3.0387		4.5747	- 2	3.2367		4.2648	- 2	3.1671
20		8.9958	- 1	3.0330		4.6041	- 2	3.2253		4.2759	- 2	3.1520
22		8.9492	- 1	3.0276		4.6282	- 2	3.2150		4.2829	- 2	3.1382
10	60, 120	9.2801	- 1	3.0653	60	4.2793	- 2	3.2787		4.2168	- 2	3.2703
12		9.2138	- 1	3.0579		4.3444	- 2	3.2570	240	4.2773	- 2	3.2481
14		9.1532	- 1	3.0511		4.3924	- 2	3.2386		4.3212	- 2	3.2292
16		9.0972	- 1	3.0447		4.4286	- 2	3.2226		4.3538	- 2	3.2127
18		9.0449	- 1	3.0388		4.4563	- 2	3.2083		4.3783	- 2	3.1980
20		8.9958	- 1	3.0331		4.4778	- 2	3.1954		4.3967	- 2	3.1847
22		8.9493	- 1	3.0277		4.4944	- 2	3.1837		4.4106	- 2	3.1726
10	240, 300	9.2803	- 1	3.0652	120	4.1690	- 2	3.2522		4.3284	- 2	3.2958
12		9.2139	- 1	3.0578		4.2218	- 2	3.2280	300	4.4015	- 2	3.2759
14		9.1533	- 1	3.0510		4.2586	- 2	3.2072		4.4569	- 2	3.2592
16		9.0972	- 1	3.0446		4.2845	- 2	3.1890		4.5002	- 2	3.2447
18		9.0449	- 1	3.0386		4.3027	- 2	3.1726		4.5346	- 2	3.2318
20		8.9957	- 1	3.0329		4.3152	- 2	3.1578		4.5624	- 2	3.2203
22		8.9491	- 1	3.0275		4.3234	- 2	3.1442		4.5851	- 2	3.2098

Table 16.

$N = 3(10^3)$ $\nu = 2(10^7)$ $\phi_i = 82^{\circ}$ $H_m = 0.5$ $i = 90^{\circ}$

f, kc	ϕ_a	$\lvert T_{mm}\rvert$		arg T_{mm}	$\lvert T_{me}\rvert$		arg T_{me}
10	All	9.2799	- 1	3.0652	4.2676	- 2	3.2745
12	Values	9.2135	- 1	3.0578	4.3307	- 2	3.2525
14		9.1529	- 1	3.0510	4.3768	- 2	3.2338
16		9.0968	- 1	3.0446	4.4113	- 2	3.2175
18		9.0445	- 1	3.0386	4.4375	- 2	3.2030
20		8.9953	- 1	3.0329	4.4575	- 2	3.1900
22		8.9487	- 1	3.0275	4.4728	- 2	3.1780

Table 17

$N = 10^3$ $\phi_i = 82^{\circ}$ $H_m = 0.5$ $I = 0$ $\phi_a = 0$

f, kc	ν	$\lvert T_{ee}\rvert$		arg T_{ee}	$\lvert T_{em}\rvert$		arg T_{em}	
10	0	9.9841	- 1	5.9879	5.6404	- 2	1.4088	
10	10^6	3.4326	- 1	4.8070	8.0119	- 2	9.3737	- 1
10	$4(10^6)$	4.7778	- 1	3.8325	5.7508	- 2	9.1968	- 1
10	$2(10^7)$	6.4307	- 1	3.3383	2.9182	- 2	7.8223	- 1
10	10^8	6.5471	- 1	3.0070	1.2215	- 2	5.5561	- 1

f, kc	ν	$\lvert T_{mm}\rvert$		arg T_{mm}	$\lvert T_{me}\rvert$		arg T_{me}
10	0	9.9841	- 1	3.1129	5.6404	- 2	4.5504
10	10^6	9.7612	- 1	3.1006	8.0119	- 2	4.0790
10	$4(10^6)$	9.4888	- 1	3.0802	5.7508	- 2	4.0613
10	$2(10^7)$	8.8429	- 1	3.0158	2.9182	- 2	3.9238
10	10^8	7.5707	- 1	2.8663	1.2215	- 2	3.6972

Summary of tables

Table	Polarization	N	ν	ϕ_i	H_m	I	ϕ_a	f
		el/cm^3	*c/s*	*Degrees*	*Gauss*	*Degrees*	*Degrees*	*kc/s*
1	*e*	10^3	$2(10^7)$	82	0.5	0	0–300	10–22
2						45		
3						84.27		
4						90	([a])	
5	*m*					0	0–300	
6						45		
7						84.27		
8						90	([a])	
9	*e*	$3(10^3)$				0	0–300	
10						45		
11						84.27		
12						90	([a])	
13	*m*					0	0–300	
14						45		
15						84.27		
16						90	([a])	
17	*e,m*	10^3	$0–10^8$			0	0	

[a] Independent of ϕ_a.

5. References

[1] J. R. Johler and L. C. Walters, On the theory of reflection of low- and very-low-radiofrequency waves from the ionosphere, J. Research NBS **64D,** 269 (May 1960).

[2] H. A. Lorentz, The theory of electrons (G. E. Stechert and Co., New York, N.Y., 1906).

[3] J. R. Wait, Terrestrial propagation of very low frequency radio waves, a theoretical investigation, J. Research NBS **64D,** 153 (1960); see also J. R. Wait and K. Spies, Influence of earth curvature and the terrestrial magnetic field on VLF propagation, J. Geophys. Research (Aug. 1960).

(Paper 65 D1–103)