

# A Bolometer Mount Efficiency Measurement Technique\*

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In the measurement of microwave power by means of the bolometric technique, the efficiency of the bolometer mount must be measured and applied as a correction in order to meet many of the accuracy requirements of today's technology.

An impedance technique of determining this efficiency was proposed over ten years ago, but has found but little use to date because of the rather severe performance requirements imposed on the attendant instrumentation. This paper describes an improved method of implementing the technique which is based on the reflectometer concept.

A particularly attractive feature of the new method is its substantial independence of connector discontinuity which has been a particularly troublesome source of error in coaxial systems. In further contrast with some of the earlier proposals for implementing this impedance technique, the new method is readily applicable to both matched and unmatched mounts and does not require mathematical approximations (in the first order theory).

A comprehensive error analysis indicates that an accuracy of 0.5 percent is possible in the existing state of the art.

## 1. Background

The "impedance" method of measuring bolometer mount efficiency devised by Kerns [1]<sup>1</sup> is one of the few basic techniques developed thus far for determining this parameter. As originally outlined, however, the accuracy which could be achieved was rather severely limited by the state of the impedance measuring art, and this led in turn to the development of a number of modifications of the technique with the objective of reducing the overall error. Beatty [2], for example, proposed a modification based on certain mathematical approximations and restrictions in generality which provided improved accuracy, but the associated operating procedures proved to be nonetheless time consuming and exacting, and the accessory instrumentation was never developed or refined to the point where one was, on a routine basis, able to place a great deal of confidence in the results. Additional refinements or modifications of the technique have also been suggested by Weinschel [3], Ginzton [4], Lane [5], and perhaps others, but it is probably safe to say that none of these proposals has, as yet, come into widespread use.

At the Boulder Laboratories of the National Bureau of Standards another variation of the impedance method has been developed which provides improved accuracy and simplified operational procedures but, unlike the earlier modifications, requires neither mathematical approximations (in the first order theory), nor restrictions in the generality of the method. A particularly attractive feature of this new version is its substantial independence of connector discontinuity, which has been an especially

troublesome source of error in coaxial systems. This technique constitutes an improved method of making the measurements implicit in the "impedance" method formulated by Kerns, and the subsequent discussion and error analysis will be limited to the procedures to be described. For a discussion of the more basic postulates upon which the procedure is based the reader is referred to the original paper [1]. In particular it should be noted that the technique is, in its present form, applicable to barretter but not to thermistor type bolometers.<sup>2</sup>

## 2. Introduction

It was shown by Kerns [1] that the efficiency of a bolometer mount may, under suitable conditions, be determined from three impedance measurements at the bolometer mount input terminals corresponding to three different values of bolometer resistance. If one of these resistance values is chosen to coincide with the value of resistance for which the efficiency is desired, this result may be expressed in terms of the input reflection coefficients thus [2]:

$$\eta = K \frac{(\Gamma_3 - \Gamma_2)(\Gamma_1 - \Gamma_2)}{|\Gamma_3 - \Gamma_1|(1 - |\Gamma_2|^2)} \quad (1)$$

where the  $\Gamma_1$ ,  $\Gamma_2$ ,  $\Gamma_3$  are the reflection coefficients corresponding to bolometer resistances  $R_1$ ,  $R_2$ ,  $R_3$ , respectively,  $\eta$  is the efficiency when the bolometer

\*A preliminary report of this technique was given at a joint meeting of the International Scientific Radio Union and the Institute of Radio Engineers, April 26, 1958, Washington, D.C.

<sup>1</sup> Figures in brackets indicate the literature references at the end of this paper.

<sup>2</sup> In order to apply the technique to a given bolometer mount, it is a *sufficient* (but not necessary) condition that it be possible to determine the microwave impedance of the bolometer element from measurements of its d-c resistance. A somewhat different sufficient condition is that the resistive component of the microwave impedance be related to the d-c value by means of a real proportionality factor (whose magnitude need not be known), and that the reactive component be constant or independent of the d-c resistance. Necessary conditions have not been determined, but it has not been possible, thus far, to relate the microwave impedance to the d-c resistance in an adequate fashion in order to utilize this technique with thermistors.

resistance has the value  $R_2$  (typically 200 ohms), and  $K = \left| \frac{2R_2(R_3 - R_1)}{(R_2 - R_1)(R_3 - R_2)} \right|$ . For the special case of a "matched" mount ( $\Gamma_2 = 0$ ) the expression becomes

$$\eta = K \left| \frac{\Gamma_3 \Gamma_1}{\Gamma_3 - \Gamma_1} \right|. \quad (2)$$

The determination of the factor  $K$ , while presenting a number of practical problems, does not pose the difficulties encountered in measuring the factor containing the  $\Gamma$ 's. It will be noted that this latter factor is characterized by differences or changes in the reflection coefficient values as the bolometer resistance is varied. Because the resistance variation which can be achieved with the available barretter elements is at best rather limited, the corresponding changes in the reflection coefficient are not large, and if the individual  $\Gamma$ 's are measured (by means of a slotted line for example) a rather severe requirement on the accessory instrumentation results. Beatty [2] has shown that an error of  $\pm 1$  percent in the VSWR measurements may lead to an error as large as  $\pm 6$  percent in the determined efficiency.

More recently, the reflectometer technique has found increased usage in the measurement of reflection coefficient magnitudes. This device, shown in figure 1, ideally yields a response of the form:

$$\left| \frac{b_3}{b_4} \right| = k |\Gamma| \quad (3)$$

where  $b_3$  and  $b_4$  are the wave amplitudes of the signals at the respective detectors,  $\Gamma$  is the reflection coefficient of the bolometer mount or other load terminating arm 2, and  $k$  is a (real) constant whose value may be determined by observing the detector outputs with arm 2 terminated by a load of known reflection—a fixed short for example.

While the usual reflectometer fails to produce this type of response because of imperfect directivity and other deviations from ideal behavior of the directional couplers employed, it is possible to compensate for these imperfections at a single frequency by means of auxiliary tuners [6], as shown in figure 2. Procedures for the adjustment of these tuners, such that the response of eq (3) may be realized, were given in the cited reference [6]. In essence the procedure is to adjust the tuning transformer  $T_x$  such that the directivity of the associated directional coupler becomes infinite, while  $T_y$  is adjusted

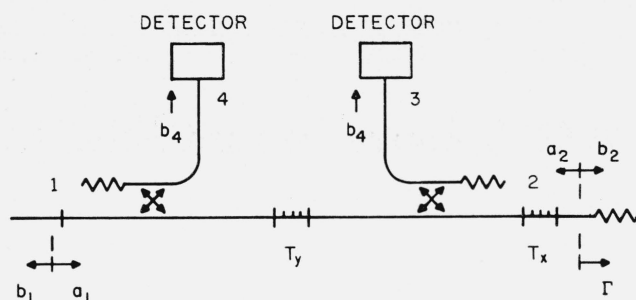


FIGURE 2. Reflectometer with auxiliary tuners.

such that the reflection coefficient of the equivalent generator, at the terminals where the bolometer mount is connected, vanishes.

The reflectometer does not, in this form, provide a determination of the argument or phase angle of the reflection coefficient, however Beatty [2] has noted that for matched mounts of high efficiency the reflection coefficient vectors  $\Gamma_1$  and  $\Gamma_3$  are nearly colinear so that to a good approximation  $|\Gamma_3 - \Gamma_1| = |\Gamma_3| \pm |\Gamma_1|$ . (The proof of a more general counterpart of this statement will be given later in this paper.) To the extent of the validity of this approximation, the cited reflectometer technique may thus be employed to determine the efficiency of matched bolometer mounts by means of eq (2). The application of the reflectometer method to the more general problem of unmatched mounts is, however, by no means obvious.

Within the limitations discussed, the described method is practical in rectangular waveguide systems. In coaxial systems, however, the procedure leaves a great deal to be desired, and even in rectangular waveguide, as will be subsequently demonstrated, the procedure, as given, is unnecessarily complex for the application.

The problem in coaxial measurements centers around the impedance discontinuity which is part of the coaxial connector. In order to provide a useful result, an overall accuracy in the efficiency determination of 1 percent would appear to be a reasonable goal, and since this figure will include contributions from a number of different sources it is desirable to reduce the individual contributions to the order of 0.1 percent. Applying this criterion to the amount of mismatch permissible in the equivalent generator impedance implies a generator match with a VSWR of 1.002 or less. While this value may be realized in rectangular waveguide systems by means of sufficiently refined techniques, the prospect of achieving such a result in a coaxial system, where connector discontinuities with VSWR's of the order of 1.1 are typical, is certainly far from encouraging.

It may be noted, in passing, that this problem has appeared so regularly in attempts to refine this technique for use with coaxial type bolometer mounts that one is led to wonder if any significant improvement in the technique is possible with the existing coaxial connectors. For example in Beatty's modification, which was based on a variation of the slotted line technique, the measured efficiency included the

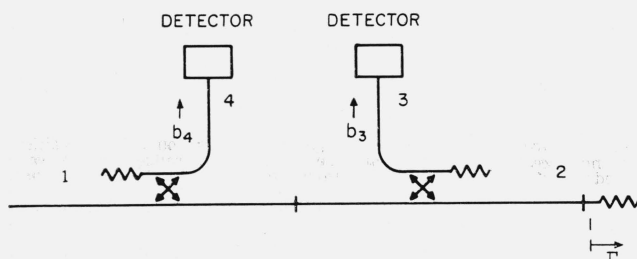


FIGURE 1. Ideal reflectometer.

losses in that portion of the slotted section between the probe position and bolometer mount, leaving one with the awkward additional problem of measuring and applying as a correction the losses in this portion of the slotted line.

### 3. General Theory

The contributions of the new method to be described in the following paragraphs include an "exact" procedure for determining the vector difference between two reflection coefficient values, and a tuning procedure which virtually eliminates the potential error from connector discontinuity.

The magnitude of the vector difference between two reflection coefficient values may be determined with the help of the circuit arrangement shown in figure 3. For convenience an ideal reflectometer will be assumed. Terms of the form  $|\Gamma_3 - \Gamma_1|$  may be determined as follows: The barretter resistance is first adjusted such that the bolometer mount has the input reflection coefficient  $\Gamma_1$ . The signal at the detector 3 is then "nulled" or balanced out by means of the auxiliary signal channel. Under these circumstances the signal provided by the auxiliary channel is proportional to  $-\Gamma_1$ . If the barretter resistance is now adjusted to produce  $\Gamma_3$ , and if suitable precautions (to be described later) have been taken to isolate the two channels, the detector 3 signal will now be proportional to  $|\Gamma_3 - \Gamma_1|$  as required.

In order to illustrate the improved tuning procedure it will prove helpful to consider a special case. If a perfectly matched bolometer mount is assumed, the tuning of the reflectometer may be effected as follows. First, with the bolometer mount connected and the bolometer resistance adjusted to the value corresponding to the perfect match, the transformer  $T_x$  is adjusted for a null in arm 3, corresponding to infinite directivity for the associated coupler. Next the bolometer mount is replaced by a sliding short and the transformer  $T_y$  adjusted such that the ratio  $\frac{b_3}{b_4}$  is constant for all positions of the short. As was explained in the reference [6], this adjustment pro-

duces an equivalent generator match (provided that the previous adjustment has been correctly made), and having completed these two adjustments, the measurement of the mount efficiency may be carried out using eq (2) as already described. (In practice additional tuning elements, whose use will be described later, are required to isolate the auxiliary channel.)

It should be noted that to the extent that the (coaxial) connectors may be considered dissipation free, the attendant impedance discontinuity has no effect upon the sliding short adjustment since an ideal sliding short preceded by a lossless discontinuity still presents a reflection coefficient of unit magnitude and (except for trivial cases) of variable phase angle. The place where the connector discontinuity is potentially important is in the initially assumed impedance match for the bolometer mount. But, as will be proved, *the assumed impedance match is entirely unnecessary*. To be sure, if the mount is matched, the measurement may be carried out as described; while if the mount is not matched, the identical series of measurements and operations described above will yield the factor

$$\frac{(\Gamma_3 - \Gamma_2)(\Gamma_1 - \Gamma_2)}{|\Gamma_3 - \Gamma_1|(1 - |\Gamma_2|^2)}$$

as required in the more general expression for mount efficiency.

In other words, the reflectometer adjustment and measurement proceeds on the basis of an assumed perfect impedance match for the bolometer mount (but employing only the specific procedures mentioned). It then develops that the method gives the correct value of efficiency regardless of whether the mount is matched or not! The proof for these assertions will be found in the following paragraphs.

It is a general property of a four-arm junction (of which the reflectometer of figure 2 may be regarded as a special case) that the ratio of the emergent wave amplitudes in arms 3 and 4 may be written in the form:

$$\frac{b_3}{b_4} = \frac{A\Gamma + B}{C\Gamma + D} \quad (4)$$

where the  $A, B, C, D$  are functions of the parameters of the four-arm junction and the detectors terminating arms 3 and 4, and  $\Gamma$  is the reflection coefficient of the load terminating arm 2.

If the reflection coefficient of the bolometer mount is designated by  $\Gamma_2$  when the bolometer is at its nominal operating value  $R_2$  (the value at which the mount efficiency is to be determined) and the junction has been adjusted (by means of  $T_x$ ) such that the signal  $b_3$  vanishes when arm 2 is terminated by this load ( $\Gamma_2$ ), then  $B = -A\Gamma_2$ . A solution of eq (4), subject to the condition that the ratio  $\frac{b_3}{b_4}$  remain constant (as provided by adjustment of  $T_y$ ) while the phase of  $\Gamma$  varies, yields  $\frac{B}{A} = \left(\frac{C}{D}\right)^* |\Gamma|^2$  where the asterisk (\*) denotes the complex conjugate.

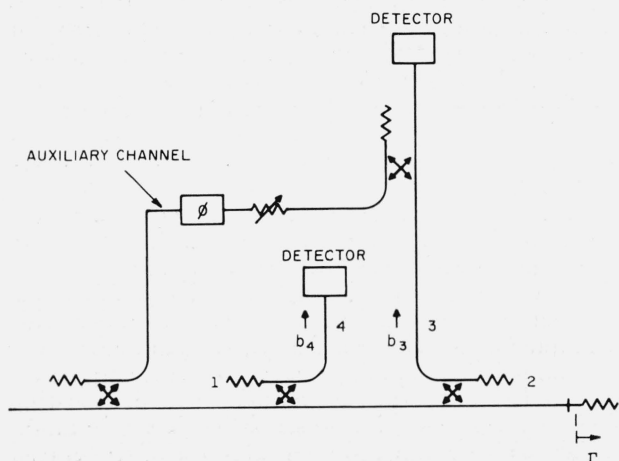


FIGURE 3. Circuit for determination of  $|\Gamma_3 - \Gamma_1|$ .

(A second solution,  $A/C=B/D$  is trivial since it gives for  $\frac{b_3}{b_4}$  a value which is independent of  $\Gamma$ .) For a sliding short the above result becomes  $\frac{B}{A}=\left(\frac{C}{D}\right)^*$ . Substituting the first and last of these results in eq (4) gives:

$$\frac{b_3}{b_4}=\frac{A(\Gamma-\Gamma_2)}{D(1-\Gamma\Gamma_2^*)} \quad (5)$$

Substituting  $\Gamma_1$  and  $\Gamma_3$  for  $\Gamma$  and taking the absolute value of the ratio of the product to the difference gives:

$$\frac{\left|\frac{A}{D}\right|^2 \left|\frac{\Gamma_3-\Gamma_2}{1-\Gamma_3\Gamma_2^*}\right| \left|\frac{\Gamma_1-\Gamma_2}{1-\Gamma_1\Gamma_2^*}\right|}{\left|\frac{A}{D}\right| \left|\frac{\Gamma_3-\Gamma_2}{1-\Gamma_3\Gamma_2^*}-\frac{\Gamma_1-\Gamma_2}{1-\Gamma_1\Gamma_2^*}\right|} = \left|\frac{A}{D}\right| \frac{|\Gamma_3-\Gamma_2||\Gamma_1-\Gamma_2|}{|\Gamma_3-\Gamma_1|(1-|\Gamma_2|^2)} \quad (6)$$

The term on the right is just the one required in the general expression for mount efficiency, while the factor  $\left|\frac{A}{D}\right|$  may be determined by observing the system response with the bolometer mount replaced by a short or other termination for which  $|\Gamma|=1$ . This may be demonstrated by letting  $\Gamma=e^{i\theta}$ . Then:

$$\left|\frac{b_3}{b_4}\right| = \left|\frac{A}{D}\right| \cdot \left|\frac{e^{i\theta}-\Gamma_2}{1-e^{i\theta}\Gamma_2^*}\right| = \left|\frac{A}{D}\right| \cdot |e^{i\theta}| \cdot \left|\frac{1-\Gamma_2e^{-i\theta}}{1-(\Gamma_2e^{-i\theta})^*}\right| = \left|\frac{A}{D}\right| \quad (7)$$

as required.

Comparison of eqs (5) and (6) with (3) and (2) respectively, indicates that the measurement may be effected as outlined with  $\left|\frac{A}{D}\right|$  taking the part of  $k$ , and

$\frac{\Gamma-\Gamma_2}{1-\Gamma\Gamma_2^*}$  being the more general counterpart of  $\Gamma$ .

#### 4. Circuit Arrangement and Tuning Procedure

A practical waveguide system utilizing these concepts is shown in figure 4. For convenience the directional couplers will be designated by  $P$ ,  $Q$ ,  $R$ , and  $S$ . The auxiliary arm between couplers  $R$  and  $S$  contains an on-off switch, phase shifter, and attenuator as required to balance out the signals from coupler  $P$ . As a matter of operating convenience it is generally helpful (but not essential) to adjust the system in such a way that the signal at detector 4 is nominally constant or independent of  $\Gamma$ . This is achieved by an adjustment of  $T_z$  to be described.

In order to obtain the vector difference between two reflection coefficients by the procedure described earlier, it is important to investigate the possible effect of variations in  $\Gamma$  upon the signal delivered to the detector 3 via the auxiliary arm. It is intuitively evident (and proof will be omitted) that the desired operation will be achieved if the following criteria are satisfied.

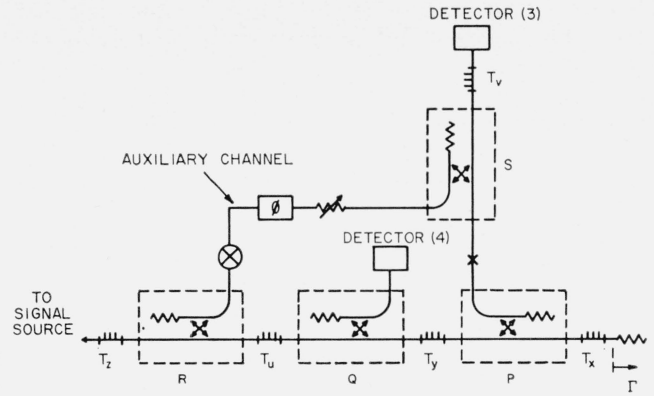


FIGURE 4. Waveguide system for measuring bolometer mount efficiency.

1. The signal coupled into the sidearm of coupler  $R$  is independent of  $\Gamma$ , or more specifically since it is the ratio of the signal in detector 3 to detector 4 which is observed, this ratio must be independent of  $\Gamma$ .

2. The impedance "looking into" coupler  $S$  at its sidearm is independent of  $\Gamma$ .

As a consequence of criterion 2, it will also be true that the impedance looking into this coupler and detector from the main arm will be independent of the operation of the phase shifter, attenuator, or switch in the auxiliary arm.

The adjustment of the tuning transformers to achieve these conditions may be done by the following steps:

(1) Directional coupler  $P$  and tuner  $T_v$  are temporarily removed from the system. In their place a sliding short is connected to coupler  $Q$ , and an auxiliary detector connected at the point  $X$ . The microwave signal is then delivered to the detector 3 via the auxiliary arm, and tuner  $T_v$  is adjusted for a null in the auxiliary detector connected at point  $X$ . This satisfies criterion 2 above.

(2) Tuner  $T_u$  is now adjusted such that the ratio  $\left|\frac{b_3}{b_4}\right|$  is constant for all positions of the sliding short, and  $T_z$  is similarly adjusted such that  $|b_4|$  remains constant as the position of the short is varied. If, as is often the case, the type of instrumentation employed to measure  $|b_4|$  and  $\left|\frac{b_3}{b_4}\right|$  is of such a nature

that the measurement of  $\left|\frac{b_3}{b_4}\right|$  is awkward in the presence of large variations in  $|b_4|$ ,  $T_z$  may be first adjusted such that  $|b_3|$  remains constant as the sliding short is varied, and  $T_u$  then adjusted for a constant value of  $|b_4|$ . Ideally, the two procedures will yield identical results, while in practice, depending upon the degree of precision toward which one is working, it may prove desirable to check the result for conformity to the criteria stated earlier.

If attention is centered on the four-arm junction comprised of couplers  $R$ ,  $Q$ , and tuner  $T_u$ , it is of interest to note that the adjustment of  $T_u$  to make



the ratio  $\left| \frac{b_3}{b_4} \right|$  a constant corresponds to the "trivial" solution  $A/C=B/D$  rejected in the earlier discussion of a reflectometer. Although a sliding short has been specified, other types of variable loads, for example, a fixed short preceded by a variable attenuator, might also be employed in this particular adjustment. For practical purposes, the method of connecting the directional couplers will determine which of the solutions  $A/C=B/D$  or  $B/A=(C/D)^*$  is achieved by the sliding short adjustment, but should the question ever arise as to which of the two conditions has been realized by the given procedure it is sufficient to note that the former solution makes  $\left| \frac{b_3}{b_4} \right|$  completely independent of  $\Gamma$  while this is not true of the latter one.

(3) The coupler  $P$  and tuner  $T_y$  are next replaced, the bolometer mount connected and  $T_z$  adjusted for a null at detector 3 with the bolometer resistance at its nominal value. The switch in the auxiliary arm is in the "off" position for this and the subsequent adjustments.

(4) The bolometer mount is then replaced by the sliding short and  $T_y$  adjusted for a constant ratio  $\left| \frac{b_3}{b_4} \right|$  as the position of the short is varied. This completes the alinement procedure.

If the ratios  $\left| \frac{b_3}{b_4} \right|$  are designated by subscripts 1, 3, and  $S$ , corresponding to values for the bolometer resistance of  $R_1$ ,  $R_3$ , and the response when the bolometer mount is replaced by a short, the desired result may be expressed in terms of the measured values:

$$\frac{\left| \frac{b_3}{b_4} \right|_1 \cdot \left| \frac{b_3}{b_4} \right|_3}{\left( \frac{b_3}{b_4} \right)_3 - \left( \frac{b_3}{b_4} \right)_1} \cdot \left| \frac{b_3}{b_4} \right|_S = \frac{|\Gamma_3 - \Gamma_2| |\Gamma_1 - \Gamma_2|}{|\Gamma_3 - \Gamma_1| (1 - |\Gamma_2|^2)} \quad (8)$$

where the first factor in the denominator of the left side is determined in the manner described at the beginning of section 3.

## 5. Practical Considerations

By means of the techniques described in the preceding sections, the microwave impedance measurements required for a determination of mount efficiency have been reduced to a set of tuning adjustments and the measurement or determination of the ratios  $\left| \frac{b_3}{b_4} \right|$ ,  $\left| \frac{b_3}{b_4} \right|_3$ , etc., for which a variety of techniques are available. Depending upon the type of instrumentation employed, the ratio  $\left| \frac{b_3}{b_4} \right|$  may be indicated directly, or the terms  $|b_3|$  and  $|b_4|$  may be obtained individually. If the adjustment of  $T_z$  has been made with sufficient care, the signal  $|b_4|$  will depend only upon the output level of the signal source; and if the latter is sufficiently stable,  $|b_4|$

will be constant and thus may be cancelled out of eq (7), leaving only the values of  $|b_3|$  to be determined. Indeed, as discussed in the reference [6], if a sufficiently stable signal source is used the directional coupler  $Q$ , detector 4 and tuner  $T_u$  may be omitted from the system. Under these conditions, step (2) of the tuning procedure described above consists of adjusting  $T_z$  such that  $|b_3|$  is constant, while the other steps follow as given with  $\left| \frac{b_3}{b_4} \right|$  replaced by  $|b_3|$ .

The detection techniques which may be employed include audio modulation and detection, heterodyne detection, and bolometric power detection. Although it is not within the scope of this paper to examine each of these methods in detail, it may prove useful to call attention to some of their more prominent features.

Perhaps the most convenient, but one of the least accurate, in the existing state of the art, of the cited methods is that of conventional audio modulation and detection. The detection equipment may take the form of a ratio-meter with barretter detectors

such that  $\left| \frac{b_3}{b_4} \right|$  is indicated directly. Barretter detectors are specified in order to ensure that the detector impedance will be at least nominally constant as required above. Alternatively, a standing-wave type amplifier may be used, and the signal  $|b_4|$  applied to the automatic gain control channel (if available). It should be noted that such an instrument is actually a ratio-meter although the permissible excursion of the input signal applied to the AGC channel is usually quite limited. In practice these excursions may be held to a small value by adjustment of  $T_z$ , and once again, if the signal source is sufficiently stable and the adjustment of  $T_z$  is made with sufficient care, it is only necessary to measure  $|b_3|$ , and the AGC channel is not required. The chief problem with conventional audio techniques appears to be in the difficulty of accurately measuring changes in the level of an audio signal. In addition, because the measurements must be made over a nominal 20 db dynamic range, the deviations of the barretter detector from true square law response would be ultimately a source for concern.

Another objection to the conventional audio technique is that the barretter resistance (in the mount to be measured) tends to follow the modulation envelope, in the same manner as occurs in the detectors. In order to avoid this difficulty it is necessary to hold the microwave power dissipated in the barretter to a small fraction, nominally 1 percent or so, of the total bias power. In addition, the resistance excursions which may be realized with the commercially available barretters are such that the reflected signal is down by another nominal 20 db, and thus even if the modification (to be described below) which permits tight coupling to this reflected signal is employed, the signal power available is only of the order of a few microwatts, which taxes the ability of the best audio detection equipment. In spite of these objections, however,

the audio technique does provide a convenient method of developing a familiarity with the overall system behavior, and as this is being written, some of these objections are being overcome [7]. When used in conjunction with some of the additional refinements to be discussed below, the conventional audio technique would probably permit an efficiency determination to an accuracy of a few percent or better.

The IF substitution technique of attenuation measurement is immediately applicable to the measurement of relative levels of  $|b_3|$ , while a possible scheme for the measurement of  $|b_3/b_4|$  is shown in

figure 5. In figure 5, the 30 Mc/s signal derived from  $b_4$  is adjusted by means of the piston attenuator to equal in amplitude that obtained from  $b_3$ , while phase balance is obtained from the phase shifter as shown. The changes in  $|b_3|$  with respect to  $|b_4|$  may thus be read directly from the piston attenuator. The measurement procedure thus consists of a series of nulling adjustments and is independent of fluctuations in the amplitude of the signal source, or the gain of the 30 Mc/s null detector. For these reasons the scheme appears attractive (on paper at least) for the proposed application.

A preliminary investigation of this technique has shown, however, the existence of a number of attendant practical problems. First, the 30 Mc/s circuits are frequency sensitive, requiring high stability of the intermediate frequency. Second, for proper operation the conversion efficiency of the crystal mixer should be independent of the changes in level of the local oscillator signal which inevitably accompany the operation of the phase shifter, but the conversion gain proved to be more sensitive to the local oscillator amplitude than anticipated. Third, the fact that the null is phase sensitive complicates the alignment procedure somewhat, particularly the adjustment of  $T_v$ . Undoubtedly, a more careful examination of the technique will uncover additional problems; this technique has not been investigated further because the method described below proved to be more convenient with the auxiliary apparatus on hand.

The method referred to above consists of power detection by the bolometric technique. These techniques have been refined to the point [8] where

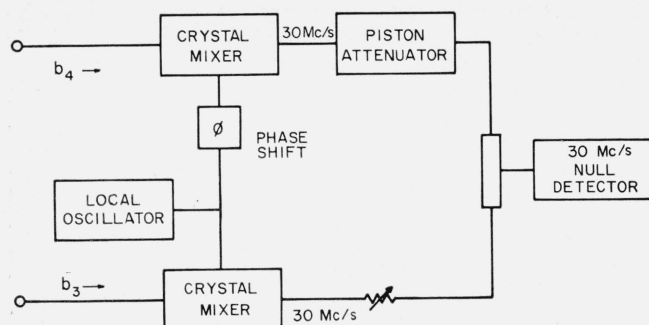


FIGURE 5. Possible heterodyne detection system.

accuracies of 0.1 to 0.2 percent may be realized at the required power levels. (It will be noted that questions as to bolometer mount efficiency of the detectors, etc., are unimportant here, since only power ratios are required.) It will prove useful to consider certain aspects of this procedure in detail since they are also applicable to the techniques described earlier.

The techniques by which the power is measured were discussed in detail in the cited reference [8] and need not be described here. In order to increase the signal at the detectors to the level where power detection is feasible, the configuration shown in figure 6 is employed. With the exception of the reversal of the connections to the coupler  $P$ , the system and tuning procedure are the same as that given for figure 4. A convenient choice of coupling ratio for the couplers  $P, Q, R, S$ , is 10 db. Under these conditions, approximately 80 percent of the power reflected from the termination is delivered to the detector 3. A further increase in this figure may be realized by other choices of coupling ratios, for example a figure of 90 percent may be realized by changing  $R$  and  $S$  to 3 and 20 db respectively, while a value of 98 percent would obtain with the values 20, 20, 3, and 20 db for  $P, Q, R$ , and  $S$ , respectively. These two alternatives would require an additional 3 and 10 db of power from the generator, however, and thus the first set of values appears to be a reasonable compromise when the available power is a potential problem.

The power delivered to the detector 4 is stabilized or held constant by means of techniques developed in this laboratory, [9] and thus only the values of power at the detector 3 are required.

In order that the detector signal may be as large as possible, it is desirable to operate the barretter, in the mount to be calibrated, over as wide a resistance excursion as possible. The commercially available barretters are not all equally suitable in

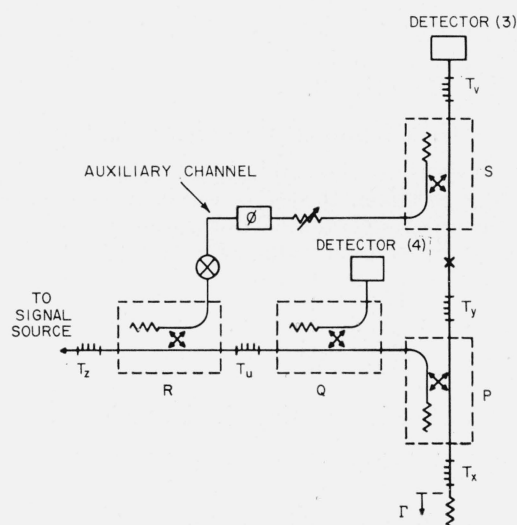


FIGURE 6. Alternative waveguide system for measuring bolometer mount efficiency.

this regard, but the type selected for this application may be operated over the range 140 to 260 ohms (nominal operating resistance 200 ohms). In operation the barretter forms one arm of a self-balancing bridge [8], another arm of which is varied to produce the required changes in resistance.

The bolometer mount under measurement is provided with a nominal 10 mw of microwave power, resulting in reflected signals of approximately 310 and 170  $\mu$ w at the resistance values of 140 and 260 ohms. A practical problem exists with reference to the measurement at 140 ohms, since the 10 mw of microwave power alone, without any additional d-c power, will bias the element at perhaps 180 ohms. This difficulty is avoided by an indirect measurement technique. In order to obtain the measurement corresponding to the 140-ohm resistance value the microwave power is first reduced to such a value that the element may be operated at 140 ohms. The signal in the detector 3 is then nulled or balanced out by means of the auxiliary signal channel. Assuming linearity of the system, this null will be independent of power level. The bolometer resistance is then returned to 200 ohms, and the microwave power increased to its nominal 10 mw value. Under these conditions, the only signal delivered to detector 3 will come from the auxiliary channel and will be equal in amplitude (but opposite in phase) to the signal which would be obtained from the bolometer mount were it possible to reduce its resistance to the 140 ohm value in the presence of the 10 mw signal. This signal is then measured and used in the equations where the 140 ohm value is required.

With the bolometer mount replaced by a short, the reflected signal has, of course, a power level equal to that in the incident wave, nominally 10 mw. The procedure thus requires measurements over the nominal 20 db range between the 10mw and 100  $\mu$ w power levels. In addition to the measurement of these signal levels, it is also important to be able to recognize the presence of signal levels in the region 60 to 80 db below these values during the course of the tuning operations. For this purpose it has proved convenient to split off a portion of the signal from arm 3 by means of a directional coupler and employ an auxiliary heterodyne detector. A waveguide switch in the secondary arm of this coupler is kept in the nontransmitting position when the adjustment of tuner  $T_e$  is made and when the signal amplitudes are being measured by the bolometric detector. In other words, the switch is in the transmitting position only when the high sensitivity for tuning adjustments is required. This isolates the remainder of the system from possible variations in the impedance of this heterodyne detector.

Although little has been said concerning the details of the power measurement, this aspect of the procedure is not without its share of practical problems. In order to make power measurements to an accuracy of a few tenths of a percent at the 100 $\mu$ w level, temperature control of the bolometer mount to about 0.001 ° C is required. A second

problem concerns the barretter element itself. In practice it is frequently observed that even though the ambient temperature is held constant, the bias power required to maintain a given operating resistance may still be subject to considerable variation or drift. Certain types of elements have been observed to be much worse than others in this respect, but even among the better ones it is usually necessary to apply the bias power continuously for several days or even a week before stable operation is achieved.

Although the foregoing discussion has been concerned primarily with the barretter element, the measurement also imposes stringent stability requirements on the associated apparatus as well. This will be easily recognized when attention is called to the fact that the bolometric measurement is in reality a differential power measurement. Thus if it is required to measure a one percent change to an accuracy of a part in  $10^3$ , a stability or resolution in the total d-c bias power of a part in  $10^5$  is implied.

## 6. Approximate Methods

The foregoing procedure is "exact" in the sense that the only mathematical approximations which have been introduced thus far have been in assuming that eqs (1) and (4) adequately describe the bolometer mount and reflectometer behavior. In a large percentage of the cases of practical interest a considerable simplification in the technique may be effected with little loss in accuracy by the introduction of suitable approximations. As a matter of fact, the accuracy achieved in practice is actually improved in certain instances by one of these approximations which will be described.

The first of these approximations relates to the substitution of  $||\Gamma_3| \pm |\Gamma_1||$  for  $|\Gamma_3 - \Gamma_1|$  in eq (2), or more generally, the substitution of

$$\left| \frac{\Gamma_3 - \Gamma_2}{1 - \Gamma_3 \Gamma_2^*} \right| \pm \left| \frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*} \right| \text{ for } \left| \frac{\Gamma_3 - \Gamma_2}{1 - \Gamma_3 \Gamma_2^*} - \frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*} \right|$$

in eq (6). (The minus sign is used if both  $R_1$  and  $R_3$  are either greater than or less than  $R_2$ , while the positive sign is used when, as in the following example, one resistance value is above and the other below  $R_2$ .)

In order to examine the validity of this approximation it is convenient to obtain a general expression for this error in terms of the magnitude and arguments of the complex numbers  $\Gamma_1$ ,  $\Gamma_3$ , or their more

general counterparts  $\frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*}$ , etc. Let  $A = |A|$  and

$B = |B|e^{i(\pi - \phi)}$  represent two complex numbers. Then for small values of  $\phi$ , the error in replacing the vector difference by the sum of the magnitudes is given by the expression:

$$\text{Error} = \frac{|AB|}{(|A| + |B|)^2} (1 - \cos \phi), \quad (9)$$

The input reflection coefficient of the bolometer mount is given by the equation [2]:

$$\Gamma = S_{11} + \frac{S_{12}^2 \Gamma_l}{1 - S_{22} \Gamma_l} \quad (10)$$

where  $\Gamma_l$  is the load reflection coefficient, and the  $S_{m,n}$  are the scattering coefficients of the bolometer mount (see fig. 7).

A substantial simplification in the analysis results from the fact that it is permissible to choose the impedance normalizing parameters in such a way that  $\Gamma_l = 0$  when the bolometer is at its nominal operating value  $R_2$ . For any other value of  $R$ ,  $\Gamma_l$  has the value

$$\Gamma_l = \frac{R - R_2}{R + R_2}$$

Utilizing these results, the term

$$\frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*}$$

becomes:

$$\frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*} = \frac{\frac{S_{12}^2 \Gamma_{l1}}{1 - |S_{11}|^2}}{1 - \left( S_{22} + \frac{S_{11}^* S_{12}^2}{1 - |S_{11}|^2} \right) \Gamma_{l1}} \quad (11)$$

Let  $\Psi_{12}$  denote the argument of  $S_{12}$  and let

$$F = \left( S_{22} + \frac{S_{11}^* S_{12}^2}{1 - |S_{11}|^2} \right); \quad (12)$$

then, since the efficiency  $\eta$  of the bolometer mount when terminated in  $R_2$  is given by the expression [2]:

$$\eta = \frac{|S_{12}|^2}{1 - |S_{11}|^2}, \quad (13)$$

eq (11) may be written:

$$\frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*} = \frac{\eta \Gamma_{l1} e^{i2\Psi_{12}}}{1 - F \Gamma_{l1}}, \quad (14)$$

and

$$\arg \frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*} = 2\Psi_{12} - \tan^{-1} \frac{-\text{Im}(F) \Gamma_{l1}}{1 - \text{Re}(F) \Gamma_{l1}}. \quad (15)$$

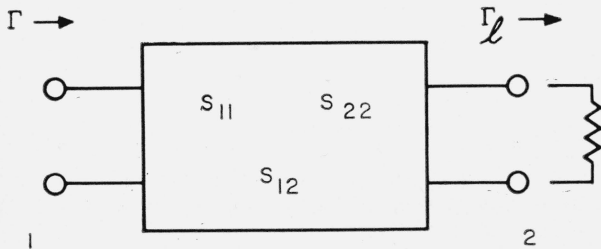


FIGURE 7. Scattering coefficient representation of bolometer mount.

It is shown in the appendix that  $|F|$  satisfies the inequality

$$|F| \leq 1 - \eta. \quad (16)$$

In a large percentage of the cases of practical interest the efficiency will be in excess of 90 percent, while  $|\Gamma_l|$  is ordinarily limited to the range 0.1 to 0.2. Substituting these results in eq (9) yields the approximate result:

$$\text{Error}_{(\max)} \approx \frac{1}{2} |\Gamma_{l1} \Gamma_{l3}| (1 - \eta)^2. \quad (17)$$

This is the approximate maximum limit of error

which obtains when  $\arg F = \pm \frac{\pi}{2}$ . For other values of  $\arg F$  the error will be smaller, becoming zero for  $\arg F = 0$  or  $\pi$ .

In a typical case where the bolometer is operated at the resistance values 140–200–260 ohms and for a mount efficiency of 90 percent the error is approximately  $\leq 0.012$  percent. The uncertainty introduced by replacing the vector difference by the sum of the magnitudes in this example is thus wholly negligible. Under these conditions it will thus be recognized that the auxiliary waveguide channel, which was introduced to permit a determination of this vector difference, contributes virtually nothing to the accuracy of the final result as far as this aspect of the operation is concerned.

The auxiliary loop is still a practical necessity, however, where power detection is employed because of the problem with the 140-ohm measurement as outlined above. In addition, this auxiliary loop may contribute to the accuracy of the result as follows:

Let the resistance excursion of the bolometer element be chosen in such a way that  $\Gamma_{l1} = -\Gamma_{l3}$ ; then it can be shown that:<sup>3</sup>

$$\frac{1}{4} \left| \frac{\Gamma_3 - \Gamma_2}{1 - \Gamma_3 \Gamma_2^*} - \frac{\Gamma_1 - \Gamma_2}{1 - \Gamma_1 \Gamma_2^*} \right| \approx \left| \frac{(\Gamma_3 - \Gamma_2)(\Gamma_1 - \Gamma_2)}{(\Gamma_3 - \Gamma_1)(1 - |\Gamma_2|^2)} \right| \quad (18)$$

with an error

$$\text{Error} \leq \Gamma_{l1}^2 (1 - \eta)^2 \quad (19)$$

which is approximately twice the value obtained in eq (17).

One of the substantial sources of error attending this technique occurs in practice in the measurement of the signal amplitudes (corresponding to the various resistance values) at the detector 3. As outlined above, a total of four measurements are required, while use of the first approximation described reduces this number to three. Use of the approximation (18) however, reduces the number of required measurements to two, the vector difference and the measurement with the bolometer mount replaced by the

<sup>3</sup> The proof may be effected by substituting eq (10), (12), and (16) in (18) and simplifying the resultant expressions.



short. In addition the power level which obtains at detector 3 in measuring this vector difference is four times that obtained from  $\Gamma_{11}$  alone which reduces the dynamic range and improves the accuracy with which these measurements can be made. In the existing state of the art this latter technique is the most accurate, for mounts of high efficiency, of those described.

## 7. Measurement of Attenuation

The foregoing presentation has been directed primarily at the problem of measuring bolometer mount efficiencies. The efficiency, however, determines the dissipative component of attenuation, and a modification of the techniques described should also prove useful in other applications, such as measuring the efficiency of waveguide to coax adaptors, short sections of waveguide, etc. A complete discussion of these potential applications and the requisite modifications in the method is not, however, within the scope of this paper.

## 8. Error Analysis

In any technique or method with potential applications in the field of standards work, a thorough investigation of the attendant sources of error is an important part of the description. The sources of error to be discussed are the following: (1) Error due to misadjustment of tuning transformers, (2) error due to dissipation in reference short, (3) error due to uncertainty in the values of bolometer resistance, and (4) error in the measurement of the signal amplitudes. These will be treated in the order listed.

The error analysis or study which has been made in connection with this technique, is, however, too lengthy to be presented in its entirety. For this reason the discussion to follow will be largely of a summary nature, and devoted primarily to the waveguide system of figure 6. The extension of these results to other variations of the method, however, should prove straightforward. The procedure to be followed in the analysis is that of assuming ideal operation of the system except for the source of error being considered. This technique yields the first order correction terms to the ideal theory for individual sources of error.

### 8.1. Error Due to Misadjustment of Tuning Transformers

#### a. Misadjustment of Tuner $T_v$

The adjustment of  $T_v$  and  $T_u$  is important in obtaining the vector difference ( $\Gamma_1 - \Gamma_3$ ) etc. As a consequence of the adjustment of tuner  $T_u$  the signals delivered to detector 3 via arms 1 and 3 of directional coupler  $S$  (see fig. 6) may be considered as originating from two independent sources (of constant amplitude and frequency). If tuner  $T_v$  is not properly adjusted, the signal delivered via arm 3 will vary with changes in  $\Gamma$  (as produced by changes in bolometer resistance). If the "null" observed at  $X$  in step 1 of the

tuning procedure is down from the signal at detector 3 by at least 60 db, then the voltage amplitude of the signal emerging from arm 1 will be down by a factor of  $10^3$ . During the course of the measurement,  $\Gamma$  varies between the approximate limits of  $\pm 1/7$ , and the maximum anticipated variation in the signal delivered to detector 3 via the auxiliary channel due to this variation is approximately 3 parts in  $10^4$ . Finally, this is approximately one-half the total signal being measured, so the maximum anticipated error is one-half this value and may be neglected.

#### b. Misadjustment of Tuner $T_u$

The problem once again is one of attempting to insure that the signal delivered to detector 3 via the auxiliary loop is independent of  $\Gamma$ . This time the possible interaction is through coupler  $R$  instead of coupler  $S$ . In practice the variation in reflection coefficient observed "looking in" arm 3 of coupler  $P$  during the course of the measurement will be approximately  $\pm 0.015$ . If with the sliding short connected to the output of coupler  $Q$  (tuning operation 2),  $T_u$  is adjusted so that the maximum variations in  $\left| \frac{b_3}{b_4} \right|$  are reduced to 1 percent (0.1 percent may be realized in practice without too much difficulty), the maximum variation in the auxiliary channel signal will be of the order of a part or two in  $10^4$  and again may be neglected.

#### c. Misadjustment of Tuner $T_x$

The function of tuner  $T_x$  is to adjust the four arm junction in such a way that it satisfies the condition  $B = -A\Gamma_2$ . The failure to exactly realize this adjustment may be accounted for by letting  $B = -A(\Gamma_2 + \delta)$ . Substituting this result in the equation for determining the bolometer mount efficiency yields two approximate upper limits for the error due to this source,

$$E \leq \frac{1}{1 - |\Gamma_2|^2} \left[ \frac{1}{\eta} \left| \frac{\Gamma_{11} + \Gamma_{13}}{\Gamma_{11} \Gamma_{13}} \right| + 2|\Gamma_2| + 4 \left( \frac{1}{\eta} - 1 \right) \right] |\delta| \quad (20)$$

and

$$E \leq \frac{1}{1 - |\Gamma_2|^2} \left[ \frac{1}{\eta} \left| \frac{\Gamma_{11} + \Gamma_{13}}{\Gamma_{11} \Gamma_{13}} \right| + 4|\Gamma_2| + 2 \left( \frac{1}{\eta} - 1 \right) \right] |\delta|. \quad (21)$$

The choice of which expression to use obviously depends upon the relative magnitudes of  $\Gamma_2$  and  $\left( \frac{1}{\eta} - 1 \right)$ . It will be noted that the error can be reduced by choosing the resistance variation such that  $\Gamma_{11} = -\Gamma_{13}$ , and if the VSWR ( $\sigma$ ) of the mount is less than 1.02, the coefficient of  $|\delta|$  has the approximate limit of 0.2 for an efficiency greater than 90 percent. On the other hand, for a resistance excursion of 140 to 200 to 260 ohms and  $\sigma = 1.5$  as in an earlier example, this coefficient will have the approximate limit 2.8. The desirability of choosing resistance values such that  $\Gamma_{11} = -\Gamma_{13}$  is readily apparent.

If the approximate method implied by eq (18) is employed, the approximate limit of error is given by the expression

$$E \leq 2 \left[ \frac{|\Gamma_2|}{1-|\Gamma_2|^2} + \eta(1-\eta)\Gamma_{i1}^2 \right] \cdot |\delta|. \quad (22)$$

In this expression the coefficient of  $|\delta|$  has the approximate values of 0.06 and 0.5 for VSWR's of 1.02 and 1.5 respectively. The use of this approximation thus reduces the error.

If the tuner  $T_x$  has been adjusted such that the signal level at detector 3 is down by 60 db from the level which exists with the shorting plate connected in place of the bolometer element,  $|\delta|$  will have the nominal value of  $10^{-3}$ , while an 80 db "null" will yield a value for  $|\delta|$  of  $10^{-4}$ . The error due to this source can thus usually be held to 0.1 percent or less.

#### d. Misadjustment of Tuner $T_y$

The function of tuner  $T_y$  is to adjust the junction such that it satisfies the condition  $D/C = (A/B)^*$ . The failure to exactly realize this condition may be accounted for by letting  $D/C = (A/B)^* + \epsilon$ . Substituting this result into the equation for bolometer mount efficiency yields the approximate expression  $E \approx |\epsilon|$  for the error due to this source. This error arises primarily in the determination of  $|A/D|$  by means of the fixed short.

The failure to realize the proper adjustment of  $T_y$  can result from either failure to entirely eliminate the variations in  $\left| \frac{b_3}{b_4} \right|$  as the sliding short is moved, or from deviations of the sliding short from the assumed ideal behavior. An analytic treatment of this problem yields for  $|\epsilon|$  the approximate limit:

$$|\epsilon| \leq (1+2|\Gamma_2|) \left[ \frac{1}{4} \frac{P_{\max} - P_{\min}}{P_{av}} + (1-|\Gamma_m|^2)|\Gamma_2| + (1-\eta_j) \right] \quad (23)$$

where  $P_{\max}$ ,  $P_{\min}$ , and  $P_{av}$  represent the maximum, minimum, and average values of the ratio  $\left| \frac{b_3}{b_4} \right|^2$  as observed during the course of the sliding short adjustment,  $\Gamma_m$  is the reflection coefficient of the sliding "short", and  $\eta_j$  is the efficiency of the waveguide joint which connects the sliding short to the remainder of the waveguide system. In the above expression the waveguide in which the short slides has been assumed lossless, while in practice, when the correct adjustment has been realized, there will be a gradual and uniform decrease in the ratio  $\left| \frac{b_3}{b_4} \right|$  as the sliding short is withdrawn.

The error due to  $\eta_j$  may be eliminated by including a precision waveguide section in arm 2, as shown in figure 8, into which the sliding short may be inserted for this tuning operation. Other implications of this arrangement will be discussed in subsequent para-

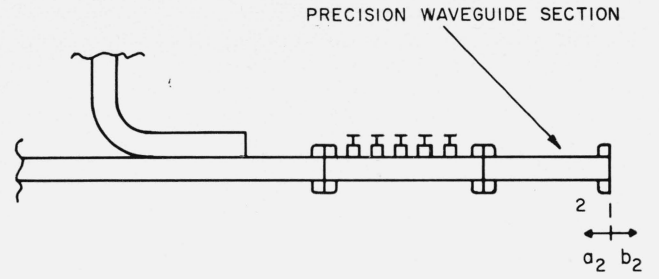


FIGURE 8. Illustration of use of precision waveguide section.

graphs. It will be noted that the error due to the reflection coefficient of the sliding short ( $\Gamma_m$ ) differing from unit magnitude decreases as the impedance match of the bolometer mount is improved. In

practice the variations in  $\left| \frac{b_3}{b_4} \right|^2$  can usually be reduced to the order of several parts in  $10^3$  such that the error from these sources can usually be held to 0.1 to 0.2 percent.

#### e. Error Due to Imperfect Nulling When Using Auxiliary Channel

If this "null" is measured in terms of the signal level which exists in the detector prior to the nulling procedure it will be readily recognized that a 60 db minimum corresponds to a possible difference between the two signals of a part in  $10^3$ , etc. Since this signal is next combined with another one of nominally the same amplitude and phase the error contribution to the final measurement is approximately one-half this value.

### 8.2. Error Due to Dissipation in Reference Short

The error due to this source is closely connected with section (d) and is given by the approximate expression:

$$E \leq (1+2|\Gamma_2|)(1-|\Gamma_s|) \quad (24)$$

where  $\Gamma_s$  is the reflection coefficient of the reference or standard short. In the present state of the art the highest reliable reflection coefficients are provided by "quarter wave" shorts, that is, a quarter wavelength section of waveguide terminated by a soldered shorting plate, or the entire structure may be electroformed in a single piece. The advantage of this construction is in the elimination of the longitudinal component of current flow at the plane of connection, thus tending to minimize flange losses. Measurements and calculations of the reflection coefficient magnitudes of such shorts [10] have yielded values in excess of 0.999.

If such a device is used in conjunction with the uniform waveguide section as in figure 8, the measurement will include, as part of the bolometer mount losses, the dissipation occurring at the bolometer mount input flange. If, on the other hand,

the variations in  $\left| \frac{b_3}{b_4} \right|$  have not been completely eliminated, there is some advantage in taking the average of the results with a quarter wave and conventional "flat" short. This tends to cancel out the error due to imperfect tuning but introduces other uncertainties if there is an appreciable amount of flange loss [11]. The general features of this problem are discussed in greater detail in the cited reference.

### 8.3. Error Due to Uncertainty in the Values of Bolometer Resistance

The bolometer resistances enter the expression for bolometer mount efficiency through the factor

$$K = \left| \frac{2R_2(R_3 - R_1)}{(R_2 - R_1)(R_3 - R_2)} \right|.$$

In practice the bolometer is biased by means of a self-balancing d-c bridge [8]. This device is capable of maintaining the bolometer resistance within 0.01 percent of the value called for by the bridge parameters. Use of NBS type resistors makes it possible to maintain the value of the reference arm within 0.01 percent, while an error in the upper or ratio arms cancels out since the  $R$ 's may be multiplied by an arbitrary factor without changing the value of  $K$ . The  $R$ 's may thus be effectively maintained within the limits  $\pm 0.02$  percent. However, because the expression for  $K$  involves their differences, the total error due to this source is of the order of 0.1 percent. In addition the measurement will be in error by the ratio of the bolometer lead resistance to  $R_2$ .

### 8.4. Error Due to Measurement of Signal Amplitudes

An approximate limit to the error introduced in the measurement of the signal amplitudes is the sum of the errors in the individual measurements. For the approximation method of eq (18), this error can be kept within 0.15 percent if power detection is employed, while for other variations of the procedure and/or other detection procedures the error will in general be larger.

The error due to instability in the frequency of the signal generator has not been considered explicitly but in general will manifest itself in an inability to realize the tuning conditions to the required degree of precision.

In summary, it will be recognized that the ultimate accuracy attainable will depend primarily upon which of the several alternate procedures is employed and upon the degree of refinement employed in the attendant adjustments and measurements. The foregoing results are somewhat arbitrarily summarized in the following table. The minimum values are intended to represent the approximate best values in the existing state of the art, while the larger values are intended to be typical of what may be achieved if the requirements are relaxed somewhat.

Source of error	Nominal error limits in percent	
	$\sigma < 1.02$	$\sigma = 1.5$
Adjustment of $T_g$ -----	0.00-0.015	0.00-0.015
Adjustment of $T_u$ -----	.00- .02	.00- .02
Adjustment of $T_x$ -----	.00- .02	.05- .3
Adjustment of $T_y$ -----	.05- .3	.1 - .5
Imperfect null in use of auxiliary arm-----	.02- .05	.02- .05
Reference short-----	.05- .3	.05- .3
Bolometer resistance-----	.1 - .15	.1 - .15
Measurement of signal am- plitudes-----	.15-1.0	.15-1.0
Total-----	0.4 -1.8%	0.5 -2.3%

## 9. Experimental Results

An X-band waveguide setup employing the concepts developed in this paper is shown in figure 9. The efficiency values obtained utilizing this method have consistently shown agreement with microcalorimetric determinations [12] to within one-half of one percent and usually within a few tenths of a percent. This is of considerable interest since the two methods should, in principle, differ by the amount of the substitution error, and suggests that the substitution error is typically at least an order of magnitude smaller than the previously estimated limits.

## 10. Appendix

The inequality

$$|f| \leq 1 - \eta \quad (16)$$

can be proved in the following manner.

Let the terminals of the 2 arm junction represented in figure 7 be reversed such that the load is connected to arm 1. Then the input reflection coefficient

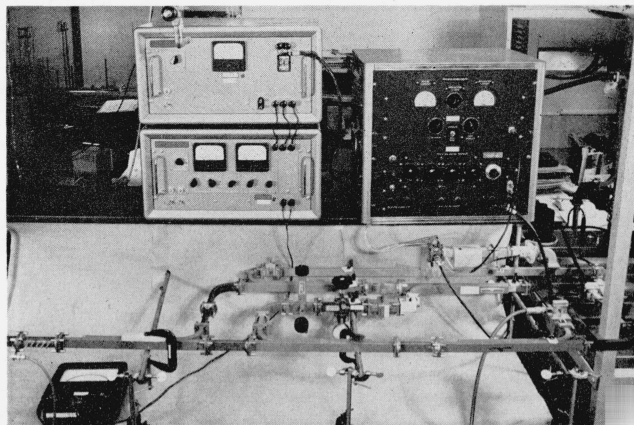


FIGURE 9. Picture of waveguide system and accessory instrumentation employed in the measurement of bolometer mount efficiency.

cient at arm 2 will be given by: (cf. eq. 10)

$$\Gamma_{in} = S_{22} + \frac{S_{12}^2 \Gamma_l}{1 - S_{11} \Gamma_l}$$

This can be written in the form,

$$\Gamma_{in} = \frac{\alpha \Gamma_l + \beta}{\gamma \Gamma_l + \delta} \quad (25)$$

$$\text{where } \alpha = S_{12}^2 - S_{11} S_{22}$$

$$\beta = S_{21}$$

$$\gamma = S_{11}$$

$$\delta = 1. \quad (26)$$

Equation (25) indicates that  $\Gamma_{in}$  is related to  $\Gamma_l$  by means of a linear fractional transformation. This transformation has the well known property<sup>4</sup> of mapping circles into circles with straight lines as limiting cases.

Let arm 1 be terminated by a sliding short. Then the locus of  $\Gamma_l$  is the unit circle centered at the origin. From the above property of the transformation it will be recognized that the locus of  $\Gamma_{in}$  is also a circle, and since a reflection coefficient magnitude can never exceed unity it is evident that this circle must lie completely within or on the unit circle as shown in figure 10. The distance to the center of this circle,  $r_c$ , and the radius,  $r$ , thus satisfy the inequality:

$$r_c \leq 1 - r. \quad (27)$$

In terms of the constants of the transformation,  $r_c$  and  $r$  are given by

$$r_c = \frac{|\beta \delta^* - \alpha \gamma^*|}{|\delta|^2 - |\gamma|^2} \quad (28)$$

and

$$r = \frac{|\alpha \delta - \beta \gamma|}{|\delta|^2 - |\gamma|^2}. \quad (29)$$

Substituting eqs (26) into (28) and (29) and these in turn into (27) and recalling the definition of  $F$ , yields the result:

$$|F| = \left| S_{22} + \frac{S_{11}^* S_{12}^2}{1 - |S_{11}|^2} \right| \leq 1 - \eta$$

where  $\eta = \frac{|S_{12}|^2}{1 - |S_{11}|^2}$ . See eq (13).

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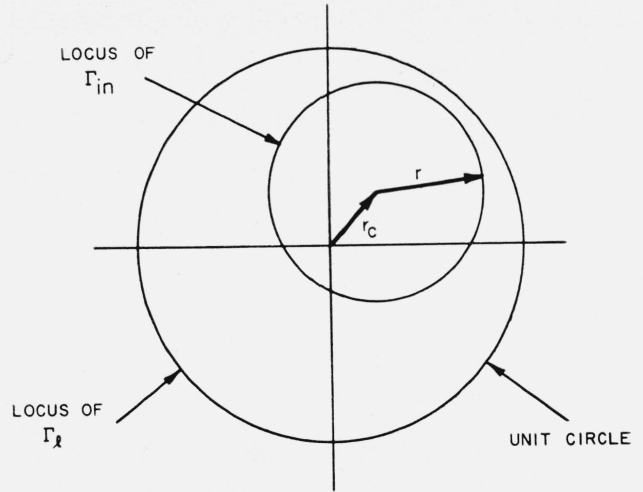


FIGURE 10. Diagram giving locus of  $\Gamma_l$  and  $\Gamma_{in}$ .

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<sup>4</sup> See a text on complex variable theory.