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Measurements of the Spectrum of Radio Noise From 50 to 100 Cycles Per Second [1]

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Experimental spectra of radio noise in the band of about 50 to 100 cycles per second have been obtained by means of digital processing. Due to the long integration times which were used, the statistical uncertainty in the estimates of power was reduced to about 3 percent (0.13 decibel). It was hoped in this way to observe maximums in the spectrum due to excitation of higher resonant modes of the earth-ionosphere cavity (for the accuracy of these data, such peaks should be observed if the Q of the cavity were 10 or greater at these frequencies). No statistically significant evidence of these cavity effects was found.

1. Introduction

It has been conjectured for some time that physical effects should be observable due to the resonant modes of the cavity formed by the earth and the *D*-layer of the ionosphere. The fundamental mode of a cavity of these dimensions has a frequency, f_1 (principally determined by the circumference of the earth rather than the height of the ionosphere) of about 10.6 cps [1].[2] (This estimate depends somewhat on the assumed effective conductivity of the ionosphere, as does the frequency response, or Q of the cavity.) Higher modes obey the relation

$$f_n=f_1\sqrt{\frac{n(n+1)}{2}} \approx \frac{f_1}{\sqrt{2}}\left(n+\frac{1}{2}\right) \quad \text{for } n>3 \text{ or so.} \tag{1}$$

If we assume that the radio noise in this part of the spectrum is the result of excitation of the cavity by sources whose spectrum changes smoothly over intervals of several cycles, then we may expect the response of the cavity to produce bumps in the received noise spectrum, the maximums occurring at the mode frequencies, f_n, and the effective width of the bumps or, alternatively, the peak-to-valley ratio, being determined by the Q of the cavity.

This paper is a description of a preliminary attempt to observe these cavity effects. As will be explained in the next section, this attempt was made in the frequency range above 50 cps, due to equipment limitations, which implies (eq (1)) a measurement of higher modes (at least the seventh), hence at best an indirect attack on the basic problem. Another advantage of operating at these higher frequencies is that the measured fields are still principally due to radiation from sferics. At lower frequencies, probably including the lowest couple of modes, the noise is largely due to motions of local atmospheric charges [2] and to that extent would not exhibit cavity resonances. Despite this disadvantage, the integration technique used here is capable of displaying small power differences which would result from only a part of the noise exhibiting cavity effects, and future work will be attempted at the lower frequencies to avoid the ambiguities arising at the higher ones.

2. Experimental Procedure

It was decided for this first attempt to record some radio noise on tape so that it could then be digitalized and processed on a large scale digital computer (the IBM 709 machine at Lincoln Lab.). Early observations of the low portion of the audio spectrum showed, as might be expected, that most of the power was contained in narrow bands at 60 cps, 180 cps, etc. To avoid this problem, portable equipment was assembled so that the recording could be made far from hum-producing powerlines. Such a spot was found on the Kankamagus Highway between Conway and Lincoln, N.H. One tape was taken on 23 October 1959 (in a light rain) and four more tapes (as well as a test tape of 100 cps) were recorded on a return trip on 11 November 1959.

A block diagram of the equipment is shown in figure 1. The noise signals were received by a 24-ft vertical whip. (The feature which all the cavity modes share, regardless of mode number and direction of the sources, is a vertical electric field, so a vertical whip was chosen as the antenna.) The antenna fed into a low-pass filter which cut off at about 75 kc to prevent the input stages from being overloaded by strong radio signals at HF, resulting in nonlinearities which would put their modulation into the measurement region. Also at the base of the antenna was a cathode follower which fed the signal to a low-pass filter with a 500-cps cutoff and then through 250 ft of cable to the battery-powered audio amplifier. A further filter cut the signal off at about 200 cps so that the signal put on tape at a fixed re-

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[2] Figures in brackets indicate the literature references at the end of this paper.

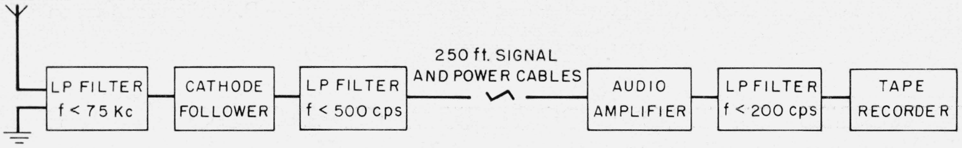

FIGURE 1. *Recording equipment.*

cording level would have a larger power per unit bandwidth. The signal was finally recorded on a battery-operated tape recorder with a spring-driven motor, whose low-frequency response fell off around 50 cps.

The root mean square radio noise voltage in the band below 200 cps was found by comparison with the calibrated 100-cps source to be between ½ and 1 mv. The recording of this same sine wave indicated fluctuations in the tape recorder speed of the order of 1 percent.

3. Digital Processing

Sections of tape somewhat over 10 min long were selected from each of the tapes already mentioned and from a tape of vacuum tube noise recorded by the same system. These were sampled at a rate of 468¾ cps, and the samples were converted to 6-bit numbers (32 levels positive and negative) and recorded on tape which could be read directly into the IBM 709 computer.

A program was written for the 709 to perform the digital equivalent of the analog operations shown in figure 2. The impulse response of the 1-cps filter is taken to be

$$h(t)=\cos \omega t \qquad 0<t<1. \tag{2}$$

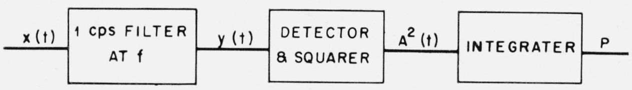

FIGURE 2. *Analog of digital processing.*

The output of such a filter for an input signal $x(t)$ is

$$\begin{aligned} y(t)&=\int_{t-1}^{t} x(\tau)\cos\omega(t-\tau)\,d\tau \qquad (3)\\ &=\left(\int_{t-1}^{t} x(\tau)\cos\omega\tau\,d\tau\right)\cos\omega t\\ &\quad+\left(\int_{t-1}^{t} x(\tau)\sin\omega\tau\,d\tau\right)\sin\omega t\\ &\equiv u(t)\cos\omega t+v(t)\sin\omega t \end{aligned}$$

where u and v are defined by the previous line. The narrow band noise represented by eq (3) may be rewritten

$$y(t)=A(t)\cos(\omega t+\phi(t)) \tag{4}$$

where the (low-pass) amplitude $A(t)$ is, from (3),

$$A(t)=(u^2(t)+v^2(t))^{1/2}. \tag{5}$$

Finally, the power at $\omega=2\pi f$ is estimated from

$$P=\int_0^T A^2(t)\,dt \tag{6}$$

(constant factors may be neglected since we consider only the power at one frequency relative to another). The program finds u and v from the input tape, $x(t)$, squares and adds them and finally integrates to find P. All the integrals are, of course, taken to be sums over the sample points.

Since $x(t)$ is a sample of (we assume) a stationary random process the derived quantity P is a random variable. Its mean value, $E(P)$, is the quantity we seek. It is shown in the appendix that for gaussian noise the standard deviation, $\sigma(P)$, of this estimate of the power is related to its mean by

$$\frac{\sigma(P)}{E(P)}=\sqrt{\frac{2}{3TW}} \tag{7}$$

where W is the 1-cps bandwidth of the filter and T the integration time. For our parameters, $TW=600$, so $\sigma/E(P)\approx .03(\pm 0.13$ db). (It should be pointed out that the noise was not quite gaussian, due apparently to impulsive peaks. These short peaks, which occurred 3 or 4 percent of the time were clipped so as to allow the normal noise a wider range of quantized values. Analysis shows that this should cause an insignificant change in the spectrum.)

As a test of the digital system (conversion and machine processing) the program was run first on the 100-cps sine wave. The broken curve in figure 3 is the spectrum of the filter defined in eq (2) and centered at 100 cps. (As the reader may observe, the separation between nulls, which is the same as the separation between the frequencies f at which P was estimated is slightly smaller than 1 cps. This was done for programing convenience.) If the input signal were a pure sine wave, the samples P would fall on this curve. As previously noted, the recorded test sine wave varied in frequency. The solid curve in figure 3 shows the response to be expected if the frequency of the input wave were uniformly distributed over a 2-cycle range, or about ±1 percent.

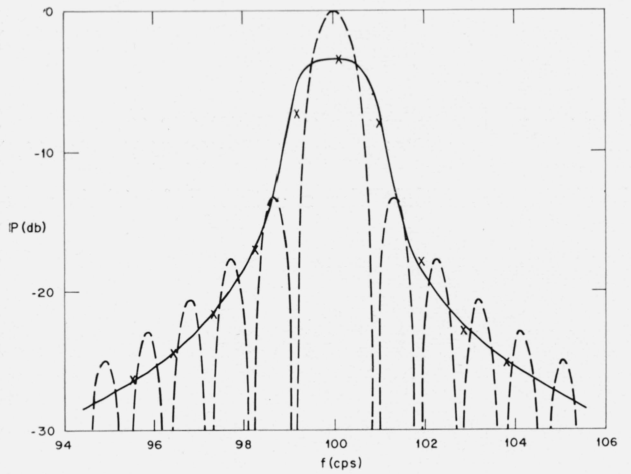

FIGURE 3. *Comparison of computed and theoretical power for test sine wave.*

(This is undoubtedly not precisely the case, but is representative of the averaging behavior of such variations.) The program outputs, P, are shown by crosses. It would appear that the digital system operated properly.

One frequency far removed from 100 cps was also examined to find the noise level of the system. The power at that frequency was some 40 db below that at the frequency of the sine wave. Since there are somewhat over 100 frequency bands, 1 cycle wide, on the tape, and the sine wave was recorded at roughly the same power level as the radio noise, we may expect that the radio noise power is about 20 db over the system noise in any band.

One more statistical consideration deserves mention. If the (effectively) 1-cps filter of figure 2 (the dashed curve of fig. 3) is compared with one centered at the next sampling frequency, separated from the first by 1 cps, it will be seen that there is some overlap in the two filter responses. Thus when a noise waveform is passed through these two filters, they respond to some extent to the same noise components, and the resulting spectrum estimates are therefore not statistically independent. It can be shown that the correlation is proportional to the integral of the product of the two filter spectra [3]. For neighboring frequencies this correlation turns out to be 0.15, actually a rather low correlation. Estimates of the noise spectrum separated by 2 cps or more are, of course, much less correlated.

4. Results

Some representative results of machine computations are shown in figures 4, 5, and 6. The ordinate in each curve is the power P on a linear scale (arbitrary units) as a function of the frequency. The calculated values are at frequencies separated by approximately one cycle and the points are joined by line segments for easier legibility.

The most obvious feature of the spectra in figures 5 and 6 is the peak due to hum pickup at 60 cps (which is actually quite small, amounting to less than 3-db increase in a 1-cps band). Aside from these, there

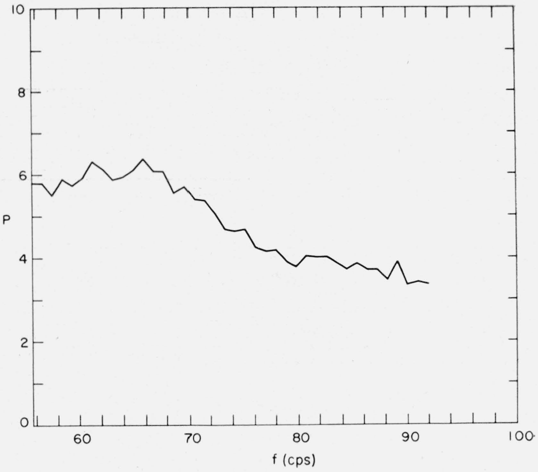

FIGURE 4. *Computed spectrum of vacuum tube noise.*

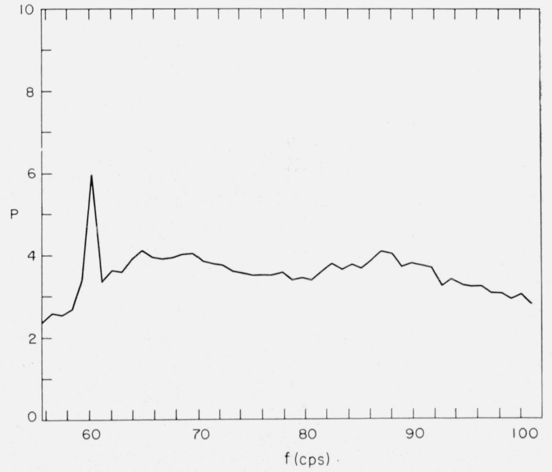

FIGURE 5. *Computed spectrum of tape recorded at 10:45 pm, 11 November 1959.*

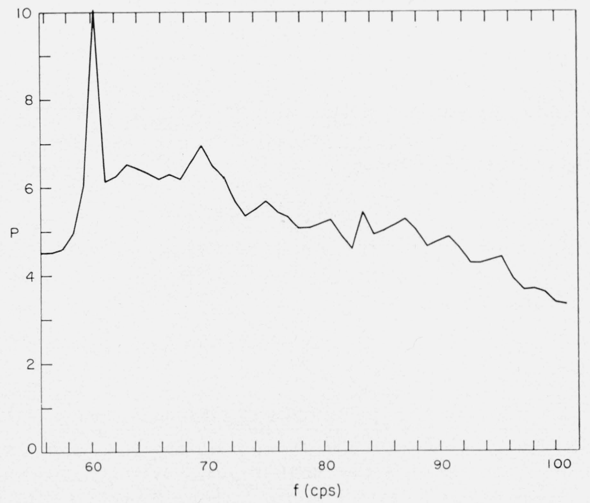

FIGURE 6. *Computed spectrum of tape recorded at 3:30 pm, 11 November 1959.*

are no very pronounced differences in power over distances of a few cycles. The largest peak-to-valley ratio is that near 70 cps in figure 6, which is about 15 percent (0.6 db), while almost all the other fluctuation are of the order of the predicted 3 percent statistical variations (as are, naturally, those of the test noise in fig. 4). We may thus conclude that there is no statistically significant evidence here of cavity effects.

Certain regularities do appear in figure 6 however, which, despite the small size of the fluctuations, attract some attention. Including the one alluded to in the preceding paragraph, the peaks occur at 63, 69, 75, 81, and 87 cps. (With some self-convincing the reader may see equally unsignificant substantiating evidence at 87 cps in figure 5 and in some of the other curves not shown here.) From the expression (eq (1)) for the mode frequencies we find

$$\Delta f = f_{n+1} - f_n \approx \frac{f_1}{\sqrt{2}} \tag{8}$$

and using the 6 cps here indicated for Δf, we derive $f_1 \approx 8.5$ cps, which seems rather low. The frequencies themselves are consistent with the condition that they be multiples of the quantity Δf by integers plus one-half. If we accept the shape of the comparatively pronounced peaks of figure 6 we may, by a computation involving the superposition of identical, periodically spaced resonance curves, compute the (optimistic) value of $Q \approx 10$ for these "modes." This is just about the limit for these calculations, since the peak-to-valley ratio for a set of modes with Q less than about 9 would be smaller than that due to random changes.

Unfortunately, we can accept these figures only as intriguing possibilities or, more probably, as another demonstration of how easy it is to find nonexistent periodicities in spectra. We cannot overemphasize that only one of these peaks is of a size to be considered statistically significant, and the reader may find other "peaks" which do not fall in the conjectured sequence. The conclusion must be, as previously stated, that to the limit of these measurements, no good evidence of cavity effects has been observed. The decisive tests have still to be conducted, possibly with longer integration time, definitely at frequencies nearer, if not at, the fundamental frequency.

5. Appendix

We wish to determine the mean and standard deviation of the estimate, P, of the power of a narrow band gaussian noise

$$y(t) = u(t)\cos\omega t + v(t)\sin\omega t \tag{A1}$$

where

$$P = \int_0^T (u^2(t) + v^2(t))dt. \tag{A2}$$

u and v are taken to be stationary low-pass gaussian processes, both with mean value $E(u)=0$, mean square $E(u^2)=1$ and autocorrelation $E[u(t)u(t+\tau)] = \rho(\tau)$. Clearly

$$E(P) = \int_0^T [E(u^2) + E(v^2)]\,dt = 2T \tag{A3}$$

$$E(P^2) = \int_0^T \int_0^T [E(u^2(s)u^2(t)) + E(v^2(s)v^2(t)) + E(u^2(s)v^2(t)) + E(u^2(t)v^2(s))]\,dsdt. \tag{A4}$$

Let us assume that the spectrum of $y(t)$ is symmetric about ω, so that $E(u(s)v(t))=0$. Using a standard result about the fourth moment of gaussian noise [4], we obtain

$$E(P^2) = 4T^2 + 4\int_0^T \int_0^T \rho^2(s-t)\,dsdt. \tag{A5}$$

Thus

$$\sigma^2(P) = E(P^2) - E^2(P) = 4\int_0^T \int_0^T \rho^2(s-t)dsdt \tag{A6}$$

$$= 8T\int_0^T \left(1 - \frac{r}{T}\right) \rho^2(r)\,dr.$$

For the filter output we are considering, it can be shown either directly from eq (3) or by Fourier-transforming the spectrum in figure 3 that

$$\rho(r) = 1 - |r|W \qquad 0 < |r| < \frac{1}{W} \tag{A7}$$

where W is the bandwidth of the filter (in this case 1 cps). Since $TW \gg 1$, we may neglect r/T in comparison with unity. Therefore

$$\sigma^2(P) \approx 8T\int_0^{\frac{1}{W}} (1 - rW)^2\,dr = \frac{8T}{3W}, \tag{A8}$$

giving

$$\frac{\sigma(P)}{E(P)} = \sqrt{\frac{2}{3TW}}. \tag{7}$$

We are very grateful to M. Freimer, who wrote the 709 program discussed in section 3. We would also like to thank W. Rutkowski and J. B. Steele, who performed the sampling and digital conversion and recording on IBM tape.

6. References

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