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Propagation at Oblique Incidence Over Cylindrical Obstacles[1]

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Investigations of propagation of short electromagnetic waves at oblique incidence over smooth, perfectly conducting cylindrical obstacles are described. It is shown that the effect of oblique incidence can be considered as a change in the effective radius of curvature of the diffracting obstacle. The power in the shadow region of a cylindrical obstacle decreases with angle of obliqueness for horizontally polarized waves and can decrease, remain constant, or increase with angle of obliqueness for vertically polarized waves depending on the geometry of the propagation link. In all cases, vertical polarization gives a stronger field in the shadow region than horizontal polarization. In addition it is shown that the diffracted field behind an obstruction of uniform radius of curvature is the same as that behind an obstacle of uniformly varying radius of curvature, provided the effective radius is the same.

1. Introduction

Of common occurrence in radio engineering, is the situation where an obstruction lies across the direct line path between transmitting and receiving stations. If the obstacle is smooth, of uniform radius of curvature, and is located normal to the line joining the transmitter and receiver, then its effect has been investigated by several authors [1, 2, 3, 4] [2] who studied diffraction by smooth cylindrical obstacles at grazing angles. In particular the latter two [3, 4] brought their investigations to the point to be useful for numerical calculations. However, if the obstacle is located at an oblique angle to the direct line path and/or is better approximated by a cylinder of varying radius of curvature along its length, then the application of the above analysis is not straightforward. In fact, few investigations [5, 6] have been extended to include the case when the electromagnetic energy is other than normally incident on the obstacle.

One case of field measurements has been reported [7] where a diffracting ridge was inclined under an angle of 60° to the path of radiation. However, at that time, no theory was available to take the obliqueness of the obstacle into account. Furthermore, no experimental evidence of the effect of oblique incidence appears to have been published to date.

In this paper, propagation of electromagnetic energy at oblique incidence over smooth, perfectly conducting cylindrical obstacles of both uniform and linearly varying radius of curvature is investigated. The experimental results are compared to a generalized theory of diffraction by smooth conical obstacles [6].

[1] Contribution from Research Laboratories, RCA Victor Company, Ltd., Montreal, Canada.
[2] Figures in brackets indicate the literature references at the end of this paper.

2. Theory

In figure 1, T and R represent transmitter and receiver locations respectively. Let a cylindrical obstacle of uniform height horizontally, but varying

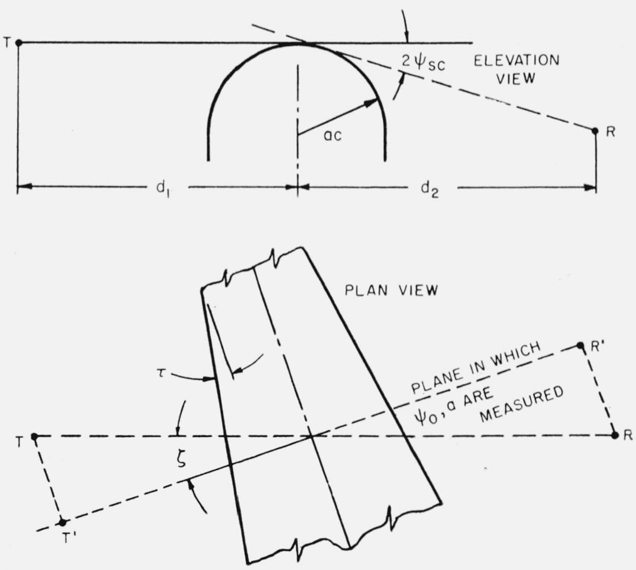

FIGURE 1. *Notation for propagation at oblique incidence over a cylindrical obstacle.*

radius of curvature (cone angle 2τ), cross the propagation path so that the normal to the axis of the cylindrical obstacle is inclined at an angle ζ (angle of obliqueness) to the straight line joining the transmitter-receiver. The scattering angle and radius of curvature in the vertical plane containing the receiver and transmitter are then:

$$2\ \psi_{sc}=2 \cos \zeta \cos \tau\ \psi_0$$

$$a_c=a/(\cos \zeta \cos \tau)$$

where $2\psi_0$, a are the scattering angle and radius of curvature, respectively, measured in the plane perpendicular to the axis of the cylinder and passing through the point where a vertical plane containing the transmitter and receiver would intersect the crest of the obstacle. (In general, the transmitter-receiver does not lie in this plane [6] which defines ψ_0 and a.)

The results for diffraction by cylindrical obstacles [3] at normal incidence have been shown to be directly applicable [6] for conical obstacles at oblique incidence provided the following substitutions are made:

$$2\psi_0 \rightarrow 2\psi_{sc}=2 \cos \zeta \cos \tau\ \psi_0 \tag{1}$$

$$a \rightarrow a_{\mathrm{eff}}=a/\cos^2 \zeta \cos \tau \tag{2}$$

Hence it now becomes convenient, for the case of oblique incidence and/or varying radius of curvature to speak of an effective radius of curvature a_{eff} given by eq (2). The scattering angle ψ_{sc} is the actual scattering angle determined by the geometry of the propagation path.

Equation (2) indicates that the effect of oblique angles of incidence are of much greater influence than a changing radius of curvature of the intervening obstacle. Thus the distinction between a cylindrical obstacle of constant radius of curvature and one of linearly varying radius of curvature (half-cone) will not be significant except for obstructions of large cone angles (>30°) which are seldom found in practice. As can be seen the variations become more pronounced as the angle of obliqueness increases. These predictions will be verified in the following section.

3. Experiment

3.1. Apparatus and Technique

Measurements of power diffracted by smooth, perfectly conducting cylinders of both uniform and linearly varying radius of curvature were conducted within the laboratory using model techniques described previously [3, 6]. Uniform cylinders with ka values of 0 (knife edge), 154, 202 and 254 and cylinders of varying radius of curvature (half-cone) with 15° half-cone angle, and ka values of 130, 194 and 254 were used in the experiments.

Measurements of diffracted power as function of angle of obliqueness could be performed by rotating the model about a vertical axis in the vertical plane containing the fixed receiver-transmitter positions and passing through the axis of the model. In this manner, ka_{eff} values greater than 2,000 could be obtained. The sizes of the models permitted investigations for angles of obliqueness of up to 60° from the normal before end effects became important. In addition, power measurements behind obstacles for scattering angle of up to 12° were made.

In all measurements, the top surface of the diffracting cylinder was horizontal, and the incident electric vector orientated so as to be either parallel (horizontal polarization) or perpendicular (vertical polarization) to the grazing surface. In this way, cross-polarization effects were minimized. As well, since the obstacles were perfectly conducting, no cross-polarized effects due to oblique incidence of the radiation should be present [5].

3.2. Power Variation With Angle of Obliqueness

The power variation at different receiver heights with angle of obliqueness (ζ) is shown in figure 2

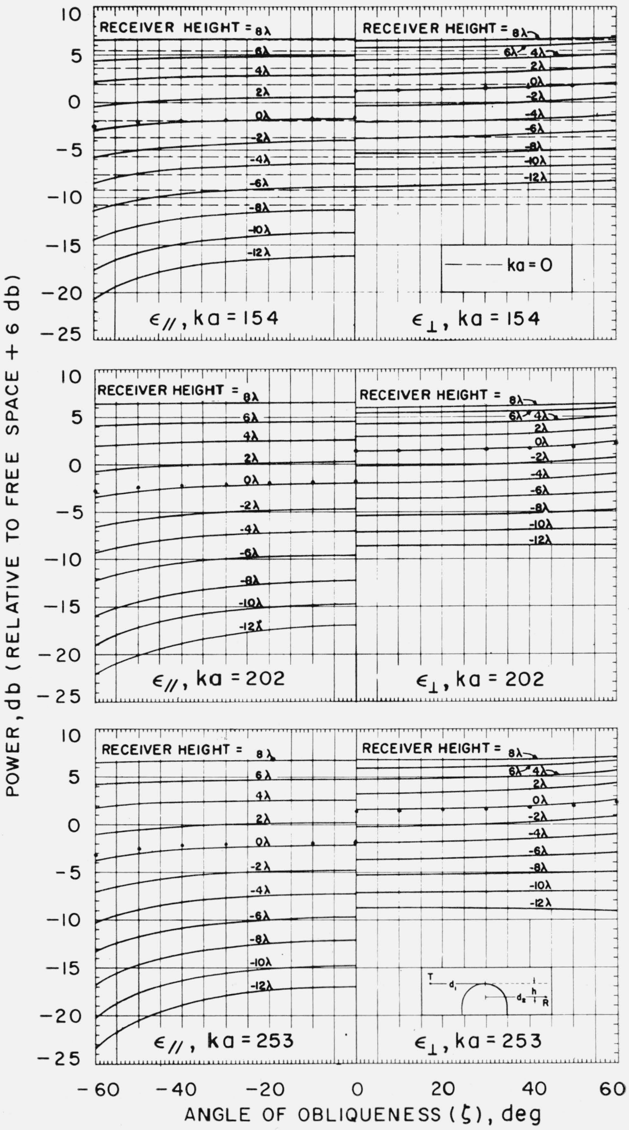

FIGURE 2. *Experimental measurements of power variation with angle of obliqueness for a smooth, perfectly conducting cylindrical obstacle of constant radius of curvature for both horizontal polarization* ($\epsilon\|$) *and vertical polarization* ($\epsilon\perp$).

The dashed lines are for a knife edge: $ka \rightarrow 0$. The dots represent theoretical values. ($d_1=150\lambda$, $d_2=113\lambda$, $\lambda=1.25$ cm.)

(cylindrical obstacles of uniform radius of curvature) and figure 3 (obstacles of linearly varying radius of curvature) for obstacles of various size.

a. Vertical Polarization

For vertical polarization, the diffracted power in the receiving space varies with angle of obliqueness as follows:

1. Illuminated region ($2\psi_{sc}<0$, corresponding to receiver heights $h>0$). The diffracted power increases with angle of obliqueness and increasing ka_{eff}. Small values of ka_{eff} show little variation with angle of obliqueness.
2. Halo region ($2\psi_{sc}\sim 0$–$5°$, $h\sim 0$ to -5λ). Power increased with increasing angle of obliqueness and increasing ka_{eff}.
3. Shadow region ($2\psi_{sc}>5°$, $h<-5\lambda$). The power deep in the shadow depends on the value of ka_{eff}. For small ka_{eff} values, the power continues to increase with increasing angles of obliqueness while for large values of ka_{eff} the diffracted power decreases with angle of obliqueness. Alternately, regions are possible where the diffracted power is independent of angle of obliqueness.

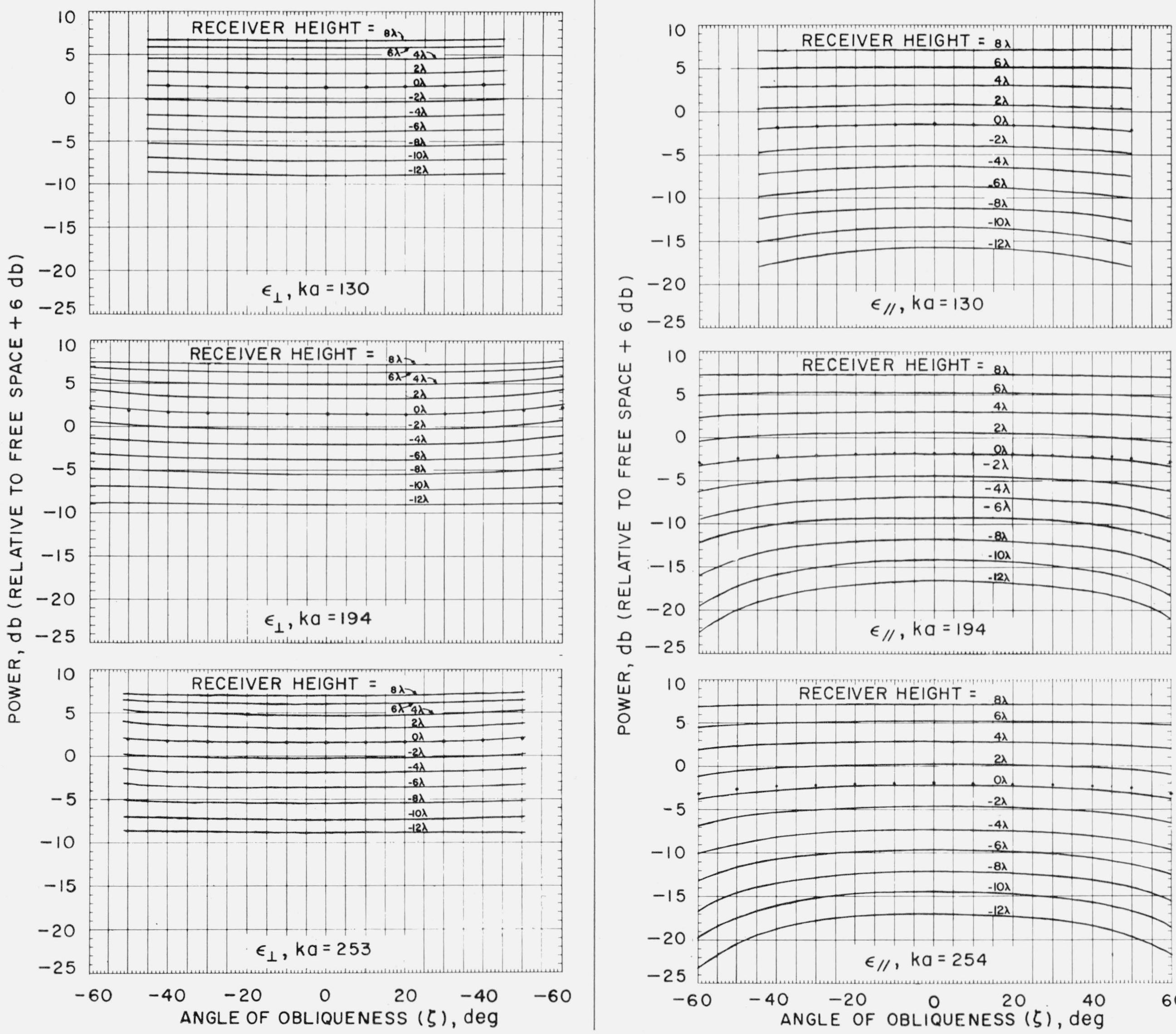

FIGURE 3. *Experimental measurements of power variation with angle of obliqueness for a smooth perfectly conducting obstacle of uniformly varying radius of curvature (15° half cone angle) for both horizontal polarization ($\epsilon \parallel$) and vertical polarization ($\epsilon\perp$).*

The dots represent theoretical values. ($d_1=150\lambda$, $d_2=113\lambda$, $\lambda=1.25$ cm.)

b. Horizontal Polarization

For horizontally polarized waves incident on the obstacles the diffracted power behavior is:

1. Illuminated region—the power is nearly constant with angle of obliqueness, showing only a small increase at large ka_{eff} values and large oblique angles.

2. Halo region—the diffracted power decreases with increasing angle of obliqueness and increasing ka_{eff} values.

3. Shadow region—the power decreases rapidly with angle of obliqueness and increasing effective radius.

The diffracted power behind a knife edge obstruction ($ka_{\mathrm{eff}}=0$) is also shown in figure 2. It should be noted that the diffracted power in this case is independent of both the angle of obliqueness of the incident energy and of polarization.

Comparison of the data for obstacles of constant radius and varying radius of curvature shows that the diffracted power variation with angle of obliqueness is indeed similar provided their effective radii of curvature (a_{eff}) are the same. In all cases the horizontally polarized field results in the diffracted power in the halo and shadow regions being considerably less than for a knife edge obstruction. The vertically polarized energy, on the other hand, is greater than the values for a knife edge obstacle in the halo region but can be less than the knife edge values deep in the shadow region. These results will be discussed further in section 3.3. Theoretical values of the diffracted power for grazing incidence ($\psi_{sc}=0$) are shown in each case and are found to agree well with experiment up to angles of obliqueness exceeding 50°.

3.3. Effect of Oblique Incidence on Power Variation With Scattering Angle

The variation of the diffracted power with scattering angle is shown for normal incidence ($\zeta=0°$) and an angle of obliqueness of 60° for obstacles of constant radius in figure 4 and for obstacles of varying radius of curvature in figure 5.

In the illuminated region, the horizontally polarized energy is considerably greater than the vertically polarized energy. The power for a knife edge obstruction is intermediate to the values for the two polarizations. The effect of oblique incidence is small for either polarization in the illuminated region.

For small scattering angles, the diffracted power for vertical polarization is greater at oblique angles, while the reverse is true for horizontal polarization. Comparison with theory shows satisfactory agreement.

At large scattering angles, the vertically polarized energy decreases with angle of obliqueness and becomes *less* than for normal incidence. The horizontally polarized energy, however, continues to decrease more rapidly with angle of obliqueness. This behavior for large ka values has been predicted for normal incidence in earlier publications [2, 8] but not verified experimentally. The theory for diffraction by conical obstacles [6] appears to hold up to scattering angles of 5 deg or more, then deviates from experimental measurements as the geometric approximations used in numerical evaluation of the theory become significant.

The power distribution behind a uniform cylindrical obstacle has been found experimentally to be the same as the power distribution behind an equivalent cylinder of uniformly varying radius of curvature.

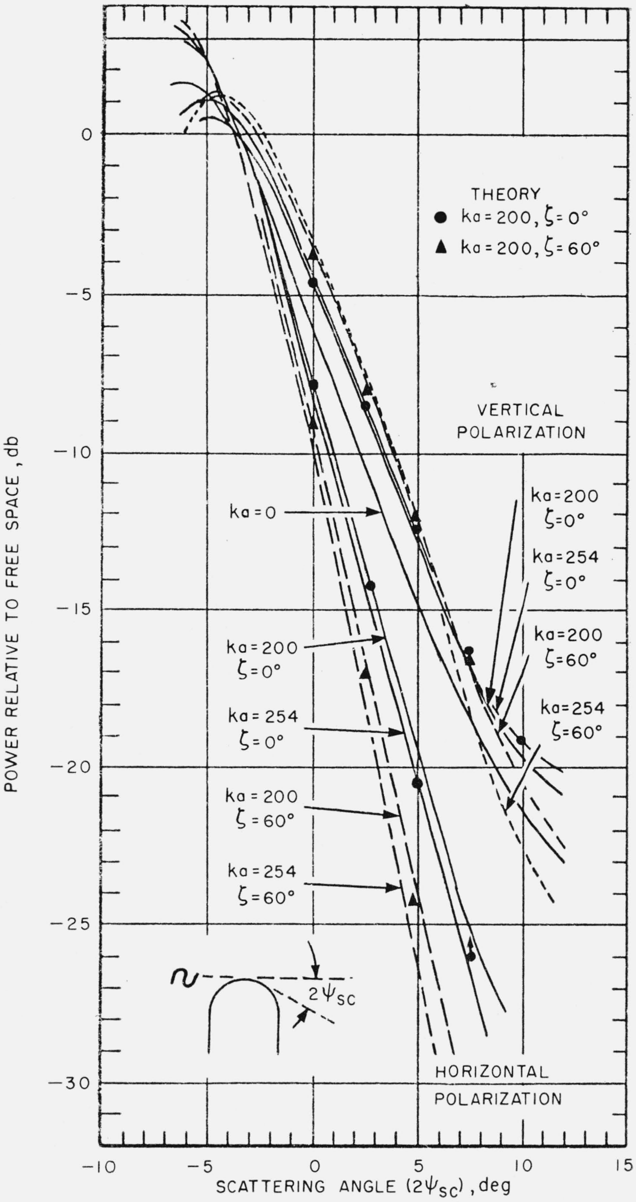

FIGURE 4. *Variation of diffracted power with scattering angle for angles of incidence of 0 and 60°.*

Obstacles of constant radius of curvature. ($d_1=150\lambda, d_2=113\lambda, \lambda=1.25$ cm.)

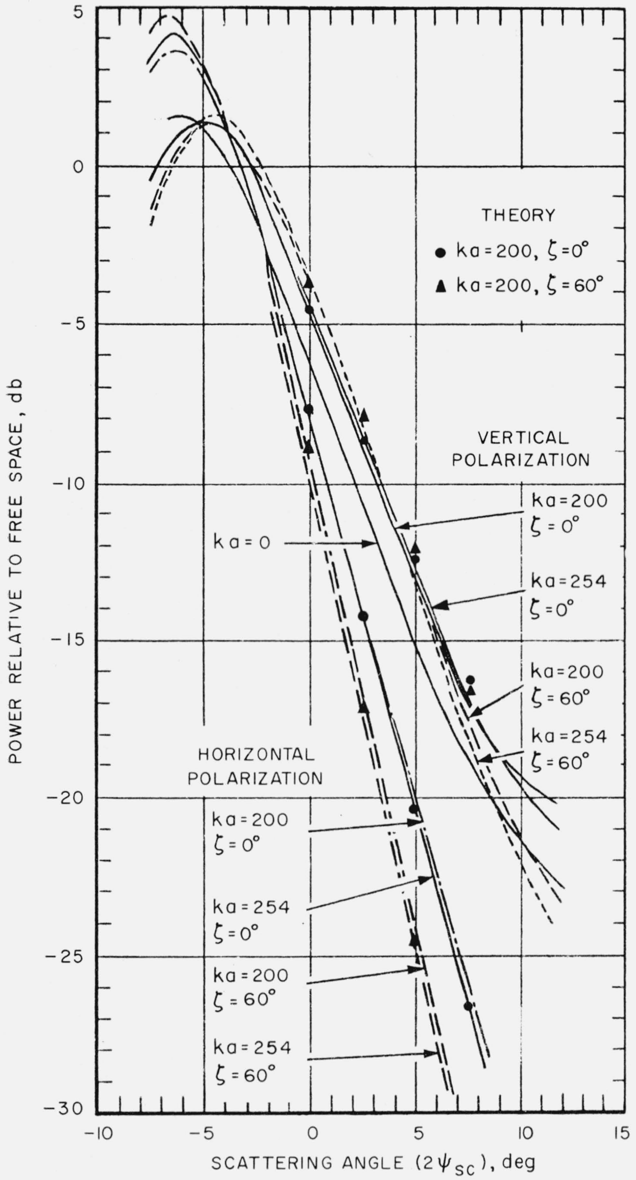

FIGURE 5. *Variation of diffracted power with scattering angle for angles of incidence of 0 and 60°.*

Obstacles of uniformaly varying radius of curvature −15° half-cone angle. ($d_1=150\lambda$, $d_2=113\lambda$, $\lambda=1.25$ cm.)

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4. Conclusions

It is shown that, for propagation of electromagnetic waves at oblique incidence over smooth, perfectly conducting cylindrical obstacles, the effect of oblique incidence can be accounted for by assigning an effective radius of curvature to the obstacle and then considering propagation to occur at normal incidence.

The effect of oblique incidence is to sharply dedecrease the scattered power for horizontally polarized energy, while for vertically polarized energy, the diffracted power is enhanced for small scattering angles and decreased at large scattering angles—a behavior similar to that predicted [2, 8] for normal incidence. In all cases the power in the halo and shadow region is less for horizontal polarization than for vertical polarization.

There is no difference in behavior of the power whether it is diffracted by an obstruction of uniform radius of curvature or by an obstruction which has a varying radius of curvature (half-cone) providing the ka_{eff} values are the same. For small grazing angles the theory for diffraction by smooth conical obstacles [6] agrees well with experimental measurements.

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