JOURNAL OF RESEARCH of the National Bureau of Standards—D. Radio Propagation
Vol. 64D, No. 3, May–June 1960

On the Calculation of the Departures of Radio Wave Bending From Normal

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(October 29, 1959; revised December 10, 1959)

The calculation of nonnormal tropospheric bending of radio waves is treated in terms of a reduced-to-sea-level value of the refractive index. This method emphasizes departures of bending from the average bending for the United States and consists of visualizing ray bending as consisting of two parts; an "average" component and a "departure-from-average" component. The "average" component comprises most of the bending and is obtained accurately from refraction tabulations while the component due to departures is easily obtained by graphical means.

1. Introduction and Background

The angular bending, τ, of a radio ray passing from the earth's surface where the refractive index is n_s to any point in the atmosphere of refractive index n is given by

$$\tau = -\int_{ns}^{n} \frac{\cot \theta}{n} \, dn \tag{1}$$

where θ is the local elevation angle of the ray. (The geometry is shown on fig. 1.) In practice, (1) is evaluated by numerical integration, assuming that n is stratified in layers concentric with the earth and decreases linearly between measured or assigned values of n [1, 2].[1] Although at any instant the vertical distribution of n may be quite complex, it has been customary [3] to assume that n decreases on the average linearly with height up to ½ km, thus vastly simplifying many practical problems encountered in radio engineering applications involving antennas at these heights. Even though the assumption of a linear decrease of n is adequate for the majority of applications, the advent of space communications has forced a closer consideration of actual atmospheric n structure [4, 5, 6] with the general conclusion that it is more realistically represented by an exponential decrease with height than by a linear decrease. Millington [7] and Norton [8, 9, 10] have given numerous examples of the use of these more realistic atmospheres in the calculation of transmission loss in both ionospheric and tropospheric propagation, while Wait [11] has given rigorous series formulas for the distance to the horizon in such an atmosphere.

In fact, the International Radio Consultative Committee has recently recommended for international use a basic reference atmosphere to describe the average refractive index structure of the atmosphere by

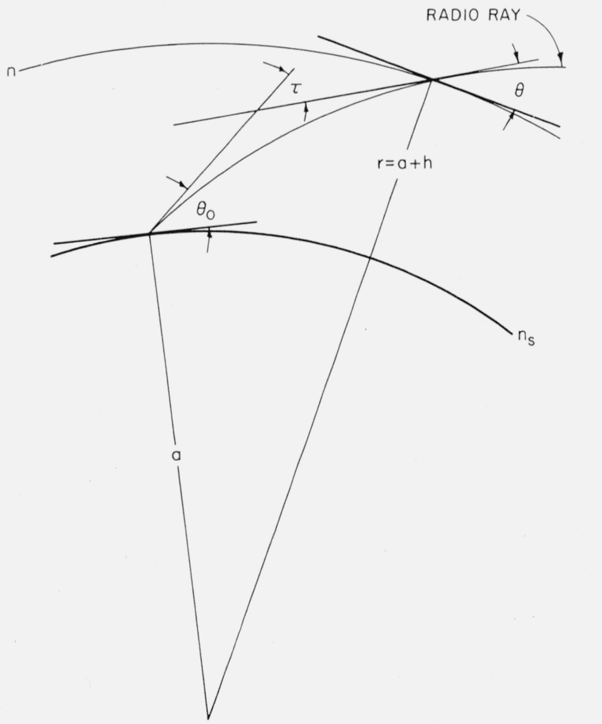

FIGURE 1. *Geometry of atmospheric refraction of radio waves.*

$$\overline{N(h)} = [\overline{n(h)} - 1]\, 10^6 = N_s \exp(-ch). \tag{2}$$

Tables of refraction variables have been prepared [12] for atmospheres of this form for specific values of N_s and c throughout their normal range of variation.

The systematic exponential decrease of the refractive index with height is relatively large compared to variations introduced by fronts and airmass discontinuities. To obtain a clear picture of such variations one should eliminate the systematic height changes from the actual observed value,

[1] Figures in brackets indicate the literature references at the end of this paper.

$N(h)$, and thus emphasize the departures from normal of the actual N structure. This may be accomplished by

$$A(h, N_s)=N(h)+N_s\,[1-\exp(-ch)] \tag{3}$$

which has been found [13] to be more effective than previous corrections involving the assumption of a linear $N(h)$ distribution [14 through 17].

The purpose of this paper is to consider departures of n structure from normal, as specified by (2), the effect of these departures upon ray refraction, and to present a new method for calculating τ based upon these departures which will differ from other approximate methods already in the literature [1, 2, 5, 12, 18, 19, 20].

2. Calculation of Nonnormal Refraction

In evaluating the bending of a ray one is concerned with the refractive index gradient:

$$dA(h, N_s)=dN(h)+N_s c \exp(-ch)dh \tag{4}$$

or

$$10^6 dn \equiv dN(h)=dA(h, N_s)-N_s c \exp(-ch)dh. \tag{5}$$

The integral for the bending of radio waves,

$$\tau_{1,2}=-\int_{n_1}^{n_2}\frac{\cot\theta}{n}\,dn,$$

now becomes

$$\tau_{1,2}=-\int_{n_1}^{n_2}\frac{10^{-6}\cot\theta\, dA(h, N_s)}{n} -\int_{n_1}^{n_2}\frac{10^{-6}\cot\theta}{n}\,[-N_s c\exp(-ch)]\,dh. \tag{6}$$

The height variation of θ, although dependent upon the refractive index distribution through Snell's law, is at least four times more sensitive to purely geometric changes than to refractive index changes [2]. This then suggests that we may simplify our problem by assuming that θ may always be determined by its value in the $\overline{N(h)}=N_s\exp(-ch)$ atmosphere for the height under consideration. When this assumption is made the second term on the right-hand side of (6) becomes simply the bending in the $\overline{N(h)}=N_s\exp(-ch)$ atmosphere, designated $\tau(h, N_s)$. Equation (6) may now be written

$$\tau_{1,2}=-\int_{n_1}^{n_2}\frac{10^{-6}\cot\theta}{n}\,dA(h, N_s)+\tau(h, N_s). \tag{7}$$

Since $\tau(h, N_s)$ normally comprises most of the bending and is accurately tabulated (for several specific values of N_s), approximate or graphical methods may be used to determine

$$-\int_{n_1}^{n_2}\frac{10^{-6}\cot\theta}{n}\,dA(h, N_s). \tag{8}$$

The perturbation term represented by eq (8) presumably contributes only a small portion of τ which is taken as justification for the further approximations that $n=1$ and that $\cot\theta=1/\theta$, then

$$-\int_{n_1}^{n_2}\frac{10^{-6}\cot\theta}{n}\,dA(h, N_s)\approx-\int_{n_1}^{n_2}\frac{10^{-6}}{\theta}\,dA(h, N_s)$$

$$\approx\frac{-10^{-6}}{\overline{\theta}}\,[A(h, N_s)]_{n_1}^{n_2}=-\frac{2}{\theta_1+\theta_2}\,[A(h, N_s)]_{n_1}^{n_2}\,10^{-6}. \tag{9}$$

The problem now consists of determining the best method of estimating $\tau(h, N_s)$ and the variation of θ with height. One method would be to refer the method to a single exponential N profile typical of the United States:

$$\overline{N(h)}=313\exp(-0.144h) \tag{10}$$

where h is in kilometers. Equation (10) represents average conditions and would have the advantage of referring all N structure to a single "standard" atmosphere. However, climatic and synoptic changes in refractive index profiles can be accounted for by utilizing the range of refraction variables tabulated in reference [12]. These tabulations are in terms of specific values of N_s ranging from $N_s=200$ (extremely dry and high locations) to $N_s=450$ (extremely hot and humid locations). However, refraction effects are more sensitive to initial N *gradient* conditions than to the value of N_s [5], particularly at small elevation angles. Thus, in what follows, the choice of a reference atmosphere will be determined by the initial gradient of the N profile for which we wish to determine the bending. Table 1 lists the exponential profiles for which refraction tabulations are available. The N_s value

TABLE 1. *Initial N gradient, ΔN_ϵ, in the C.R.P.L. exponential reference atmosphere* [a]

Range of ΔN_ϵ (N units/km)	$N(h)$
$-\Delta N_\epsilon \leq 27.55$	$200\ \exp(-0.1184h)$
$27.55 < -\Delta N_\epsilon \leq 35.33$	$252.9\ \exp(-.1262h)$
$35.33 < -\Delta N_\epsilon \leq 42.13$	$289\ \exp(-.1357h)$
$42.13 < -\Delta N_\epsilon \leq 49.52$	$313\ \exp(-.1438h)$
$49.52 < -\Delta N_\epsilon \leq 59.68$	$344.5\ \exp(-.1568h)$
$59.68 < -\Delta N_\epsilon \leq 71.10$	$377.2\ \exp(-.1732h)$
$71.10 < -\Delta N_\epsilon \leq 88.65$	$404.9\ \exp(-.1898h)$
$88.65 < -\Delta N_\epsilon$	$450\ \exp(-.2232h)$

[a] Note: height, h, is in kilometers.

and the range of initial N gradients assigned to each atmosphere are also listed. Since, theoretically, there are an infinite number of exponential atmospheres but, in practice, only a few with tabulated refraction variables, it will be necessary to assign each actual N profile encountered to a specific exponential reference atmosphere according to its initial N gradient. The initial N gradient of

$$\overline{N(h)}=N_s\exp(-ch)$$

is given by

$$\left(\frac{dN(h)}{dh}\right)_0 = -cN_s \tag{11}$$

For any actual case, however, one would not be able to define $(dN/dh)_0$ since the radiosonde reports at only certain points above the ground and not continuously. Thus if the first reporting level is at a height of ϵ, the actual initial gradient is determined from

$$-\Delta N_\epsilon = \frac{N_s - N_\epsilon}{\epsilon}. \tag{12}$$

For the practical application that is the objective of this paper we shall approximate $(dN/dh)_0$ by ΔN_ϵ, and the choice of an exponential atmosphere from table 1 will be based upon the observed value of ΔN_ϵ. As a consequence of this choice N_s will usually differ from $N(0)$.

The notation $A(h, 313)$ and $\tau(h, 313)$ indicates, for example, that the initial gradient of the particular profile under study falls into the 313 exponential atmosphere category, and that A, τ, and θ are determined from that particular atmosphere. Note that $A(0, 313)$ is not equal to 313 but $A(h, 313)$ will approach 313 at sufficiently large heights.

Radiosonde data from Santa Maria, California (a good test station because the refractive index profile departs quite radically from any smooth model) were graphically converted into $A(h, 313)$ units (see fig. 2) and the bendings calculated for several values of θ_0. The $A(h, 313)$ profile is compared on figure 3 with the departure of the bendings from the values expected in the 313 exponential atmosphere and emphasizes that τ is an integrated effect of all refractive index departures from the surface to the point under consideration while $A(h, 313)$ is determined solely from data at the height h alone. For instance, a strong surface-based refracting layer will bias all bending values above the layer while $A(h, 313)$ is independent of the values of N at any other height. It is quite

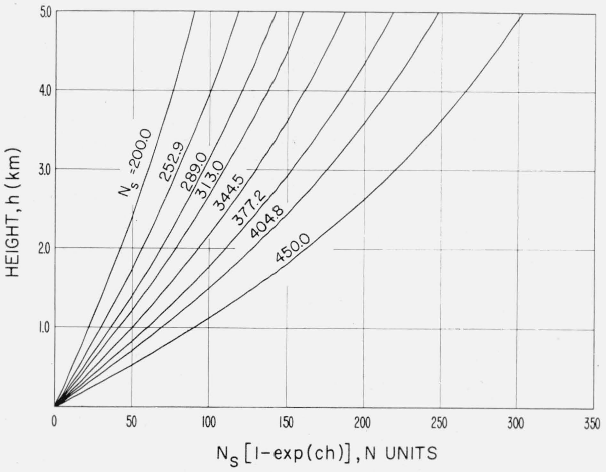

FIGURE 2. N_s *[1-exp(-ch)] versus height for conversion of* N(h) *profiles into* A(h, N_s) *profiles.*

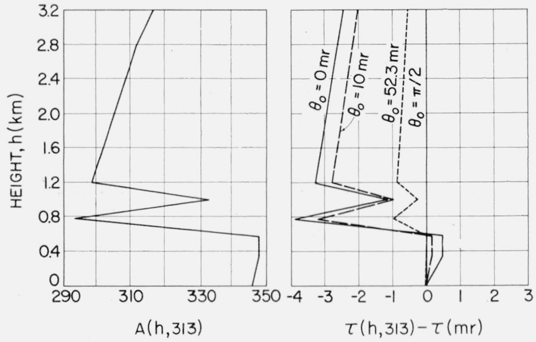

FIGURE 3. *Comparison of* A(h, *313) with the departures of ray bending from the value in the 313 exponential atmosphere for Santa Maria, Calif., Sept. 4, 1952, 1900 local time.*

Note that *negative* gradients of $A(h, 313)$ produce *positive* values of ray bending.

clear that the agreement in height structure between the bending departures and $A(h, 313)$ is very pronounced on figure 3, especially in the layers nearest the ground and for small values of θ_0, indicating that a knowledge of $A(h, 313)$ alone could enable one to obtain a visual impression of the effect of nonnormal atmospheric structure upon the refraction of radio waves.

The above illustration has served to introduce a method of calculating τ in terms of departures of N structure from normal. The following section will be concerned with testing the accuracy of the method.

3. Comparison of Bendings

Significant level data for the radiosonde observations taken at 11 geographically and climatically diverse U.S. Weather Bureau observatories were analyzed to give adequate coverage of climatic conditions found within the country. Individual radiosonde observations taken during the years 1951 to 1952 were examined for each observing site, and profiles were selected to give a measure of the maximum variation of bending due to both geographic and local refractive index profile variations. The bending was then calculated for each of these profiles by both the standard numerical integration of (1), [1], and the departures-from-normal method described above. The differences between the bendings obtained by the two methods are given in table 2 as a percentage of the value obtained by conventional methods. A classification of the data of table 2 was made as regards height. Since most of the total bending to 70 km occurred within the first 3 km [5], the integral correction involving A was only applied to that height, the bending in the 313 atmosphere being used to estimate the average bending above 3 km thus assuming that the effect of N departures above this height is negligible. The mean errors are quite close to zero, indicating the absence of any marked bias in the method of calculation. One also notes from table 2 that the maximum percentage difference occurs at $\theta_0 = 0$ and generally decreases with increasing initial elevation angle.

TABLE 2. *Percentage deviation of ray bending values calculated by the departures-from-normal method from conventionally calculated values*

θ_0(mr)	0		10		52.3(3°)	
h(km)	3	70	3	70	3	70
	A. Bendings calculated relative to multiple exponential atmospheres					
Mean % of error	−0.84	−0.35	+0.49	+0.30	−0.007	−0.80
Std deviation of % errors	3.19	3.65	3.59	3.71	.97	3.95
Max % error	−10.0	+8.6	+9.4	+9.4	+2.5	+8.5
	B. Bendings calculated relative to the 313 exponential atmosphere					
Mean % of error	−0.03	+0.04	−3.04	−1.14	+0.78	−0.87
Std deviation of % errors	7.57	7.15	6.36	5.19	1.14	4.29
Max % error	−14.5	−14.0	−14.6	+10.9	−2.7	+9.0

Since the mean deviations are less than 1.0 percent the present system of calculation appears to be adequate for normal applications. One would probably not wish to use it for estimating τ for $\theta_0>52.3$ milliradians (mr) since at such high angles the total bending is as accurately estimated solely from ground level values by [18]:

$$\tau=N_s\times 10^{-6}\cot\theta_0. \tag{13}$$

One might also wish to use the departures-from-normal method referenced to a single average N profile typical of the entire United States. This approach was investigated by use of such an average, the 313 exponential atmosphere, and the results are tabulated in table 2. It is quite evident that the use of multiple reference profiles, classified according to ΔN_ϵ, produces significantly smaller standard deviations and more consistent mean errors than does the use of a single average profile. This is, of course, what one would expect since the ΔN_ϵ method adjusts the profile to the initial layer where a significant percentage of the total bending occurs. Interestingly enough, the maximum errors for $\theta_0=0$ arise from profiles such as shown on figure 2 where a very strong elevated layer exists well above the surface layer. Under this condition, the classification of profiles according to initial gradient does not account for the major anomaly of the profile. The maximum errors for $\theta_0=10$ mr are due to a quite different cause; the presence of ducting profiles in the test profiles. Under ducting conditions rays may be traced only for initial elevation angles greater than the angle of penetration [21] which, for radiosonde observed ducts, is less than 6 mr [22]. For this reason the data for $\theta_0=10$ mr includes many profiles that represent near ducting conditions and consequently maximum departures from normal refraction. In fact, the maximum errors for $\theta_0=10$ mr are associated with profiles having very thin surface ducts with normal or subnormal profiles above the duct, a condition that produces overestimation of the bending above the duct.

It should be observed that the $A(h, N_s)$ units as used in this paper are always referenced to zero *height* above ground level. We are able to ignore the *altitude* of the ground level by using multiple reference atmospheres which have, in effect, a built-in dependence of N_s upon ground level altitude. This effect may be accounted for with a single exponential atmosphere by adding the altitude of the ground level, b, to the height, h. Thus the equation for bending becomes

$$\tau=-\int_{n_1}^{n_2}\frac{10^{-6}\cot\theta}{n}\,dA(h,313)+\exp(-cb)\tau(h,313). \tag{14}$$

The exponential correction factor to $\tau(h, 313)$ is a constant for any particular station and is important only when b is large and consequently N_s appreciably less than 313.0.

One should bear in mind, however, that the above errors are about one-half that of other simple methods of estimating the bending of radio rays for $\theta_0<52.3$ mr such as regression techniques [18] or model atmospheres based upon the surface value of initial gradient conditions [5]. It would seem then that an additional advantage of the present system lies in the facility with which the user would come to appreciate the relative importance of various profile departures.

The data required for use of the present method are contained in reference [12]. Values of N_s [1-exp(-ch)] are given in table 3.

TABLE 3. *Values of* N_s *[1-exp(-ch)]*

Ht. (km)	200.0	252.9	289.0	313.0	344.5	377.2	404.9	450.0
0.01	0.2	0.3	0.4	0.4	0.5	0.6	0.8	1.0
.02	.5	.6	.8	.9	1.1	1.3	1.6	2.0
.05	1.2	1.6	2.0	2.2	2.7	3.2	3.9	5.0
.10	2.4	3.1	3.9	4.5	5.3	6.5	7.6	9.9
.20	4.7	6.3	7.7	8.9	10.6	12.8	15.1	19.7
.50	11.5	15.4	19.0	21.7	26.0	31.3	36.6	47.5
1.00	22.3	30.0	36.7	41.9	50.0	60.0	69.9	90.0
2.00	42.2	56.4	68.7	78.3	92.7	110.4	127.6	162.1
5.00	89.4	118.4	142.4	160.5	187.2	218.6	247.8	302.6

4. Conclusions

The present study has shown that deviations from normal ray bending are mirrored by departures of refractive index from values expected in average atmospheres of exponential form. Further, this departures-from-average method emphasizes that n-profile anomalies cause the greatest change in the bending when they occur near the ground, these changes being larger the smaller the initial elevation angle. A graphical method based upon these departures may be used to calculate the bending of radio rays.

The authors extend their gratitude to K. A. Norton and G. D. Thayer for their many critical and stimulating reviews of this work.

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BOULDER, COLO. (Paper 64D3–56)