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Error Analysis of a Standard Microwave Phase Shifter*

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A standard microwave phase shifter has been proposed which utilizes an adjustable short circuit attached to a tunable three-arm waveguide junction. Ideally, the change of phase of the emergent wave from the third port can be made to equal the change of phase of the equivalent load attached to the second port, whether the generator and detector are matched or not. The difference between the change of phase of the emergent wave from port 3 and the change of phase of the equivalent load attached to arm 2 because of imperfect tuning is termed tuning error. This analysis relates the tuning error to amplitude changes which are observed at the detector attached to arm 3 during the tuning procedure. Graphs are presented for determining the parameters needed to estimate limits of tuning error from the observations of amplitude changes during the tuning procedure.

One calculates the change in phase of the reflection coefficient produced by the short circuit from its observed displacement and from the guide wavelength. This procedure results in errors called dimensional errors because of uncertainties in determining the axial motion of the short circuit and because of slight variations from the nominal broad dimension of the waveguide. Limits of these dimensional errors are calculated for WR–90 waveguide in the recommended frequency range of 8.2 to 12.4 kilomegacycles per second, and presented in graphical form.

1. Introduction

A standard microwave phase shifter has been proposed [1] which utilizes an adjustable short circuit attached to a tunable three-arm waveguide junction. This standard is illustrated in figure 1. It can be shown [2] that the change of phase of the emergent

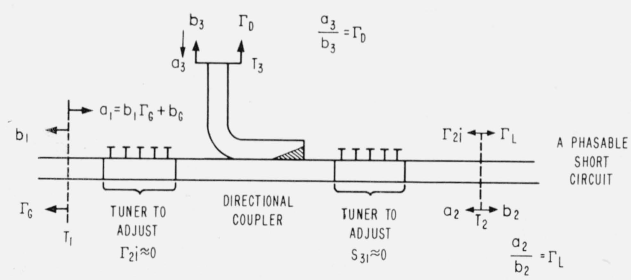

FIGURE 1. *A standard microwave phase shifter.*

wave from the third arm can be ideally made to equal the change of phase of the equivalent load attached to the second arm, whether the generator and detector are matched or not. Practical limitations in tuning the junction result in departures from this ideal behavior which leaves an error to be evaluated. The difference between the change of phase of the emergent wave from arm 3 and the change of phase of the load attached to arm 2 because of imperfect tuning is termed the tuning error. This analysis relates the tuning error to amplitude changes which are observed at the detector attached to arm 3 during the tuning procedure. Graphs are presented for determining parameters needed to estimate the limits of tuning error from observations of amplitude variations during the tuning procedure.

Other sources of error which are considered are those which enter in determining the change of phase of the reflection coefficient of the load attached to arm 2. This load is a short circuit whose axial motion in the waveguide can be measured. The change of phase produced depends on the waveguide wavelength and the length of travel of the short circuit. For the dominant mode of propagation in lossless rectangular waveguide, the only dimension which affects the waveguide wavelength is the broad dimension. The difference between the phase shift of the reflection coefficient of the short-circuit and the calculated phase shift of the load caused by broad dimension nonuniformity and by inaccuracy in determining the motion of the short circuit is termed dimension error. Limits of dimension errors are calculated for WR–90 waveguide in the recommended frequency range of 8.2 to 12.4 kMc/s, and presented in graphical form.

2. Tuning Errors

It has been shown (see footnote 2) that the amplitude of b_3, the emergent wave from arm 3 (connected to the detector), may be expressed in the form

$$\frac{b_3}{b_G}=\frac{\begin{vmatrix}S_{21}S_{22}\\S_{31}S_{32}\end{vmatrix}\Gamma_L+S_{31}}{\begin{vmatrix}(1-S_{11}\Gamma_G) & S_{13}\Gamma_D\\S_{31}\Gamma_G & (1-S_{33}\Gamma_D)\end{vmatrix}(1-\Gamma_{2i}\Gamma_L)},\tag{1}$$

*Contribution from the Radio Standards Laboratory, National Bureau of Standards, Boulder, Colo.

[1] Milton Magid, Precision microwave phase shift measurements, IRE Trans. on Instr. **I**-7, 321–331 (Dec. 1958).

[2] R. W. Beatty and D. M. Kerns, Recently developed microwave impedance standards and methods of measurement, IRE Trans. on Instr. **I**-7, 319–321 (Dec. 1958).

where the S's are the scattering coefficients of the junction. The voltage reflection coefficients of the generator, detector, and the load are Γ_G, Γ_D, and Γ_L, respectively. Γ_{2i} is the reflection coefficient of the equivalent generator connected directly to the load, and b_G is the component of the input wave supplied by the generator. The phase of b_3 with respect to an arbitrary reference, b_G, may be defined as θ_{3G}.

Adjustments [3] of the junction are made to render as nearly as possible $S_{31}=0$ and $\Gamma_{2i}=0$. Under these ideal conditions, eq (1) reduces to

$$\frac{b_3}{b_G}=\frac{S_{21}S_{32}\Gamma_L}{(1-S_{11}\Gamma_G)(1-S_{33}\Gamma_D)}=C\Gamma_L, \tag{2}$$

where C is a constant. The change of phase of the emergent wave, ${}^f\theta_{3G}-{}^i\theta_{3G}$, when the load is changed from initial to final settings, ${}^i\Gamma_L$ to ${}^f\Gamma_L$, may be determined from the ratio of the final to initial values of b_3 as obtained from eq (2)

$$\frac{{}^fb_3}{{}^ib_3}=\frac{{}^f\Gamma_L}{{}^i\Gamma_L}, \tag{3}$$

This may be written to show the changes of phase explicitly as

$$\frac{|{}^fb_3|}{|{}^ib_3|}\,e^{j({}^f\theta_{3G}-{}^i\theta_{3G})}=\frac{|{}^f\Gamma_L|}{|{}^i\Gamma_L|}\,e^{j({}^f\psi_L-{}^i\psi_L)}, \tag{4}$$

where ${}^f\psi_L$ and ${}^i\psi_L$ are the phases of the reflection coefficient of the equivalent load at final and initial settings, respectively. From which it is apparent that the change of phase of the emergent wave is equal to the change of phase of the load attached to arm 2, ${}^f\psi_L-{}^i\psi_L={}^{fi}\psi_L$.

If the tuning errors are small, then departures from this ideal response because of $S_{31}\neq 0$ and $\Gamma_{2i}\neq 0$ can be considered separately and the contributions added. The following analysis uses this first order approximation.

Case I. $S_{31}=0$, but $\Gamma_{2i}\neq 0$. The ratio ${}^fb_3/{}^ib_3$, for these conditions may be derived from eq (1) and written as

$$\frac{{}^fb_3}{{}^ib_3}=\frac{{}^f\Gamma_L}{{}^i\Gamma_L}\,\frac{1-\Gamma_{2i}{}^i\Gamma_L}{1-\Gamma_{2i}{}^f\Gamma_L}, \tag{5}$$

from which it is apparent that the change of phase of the emergent wave, ${}^f\theta_{3G}-{}^i\theta_{3G}$ differs from the change in phase of Γ_L by $\epsilon_{T,\mathrm{I}}$, where

$$\epsilon_{T,\mathrm{I}}=\text{argument of }\frac{1-\Gamma_{2i}{}^i\Gamma_L}{1-\Gamma_{2i}{}^f\Gamma_L}$$

$$=\text{argument of }(1-\Gamma_{2i}{}^i\Gamma_L)-\text{argument of }(1-\Gamma_{2i}{}^f\Gamma_L)+2n\pi, \tag{6}$$

[3] G. F. Engen and R. W. Beatty, Microwave reflectometer techniques, IRE Trans. on Microwave Theory Tech. **MTT-7**, 351-355 (July 1959).

where n is an integer. In order to evaluate $\epsilon_{T,\mathrm{I}}$ from eq (6), one would need to know Γ_{2i}, ${}^f\Gamma_L$ and ${}^i\Gamma_L$. It is more convenient to calculate a limit of error assuming that one knows the magnitudes of of these quantities and the change in phase of Γ_L, which one controls during the measurement. The phases are then assumed to have the values which would give maximum $\epsilon_{T,\mathrm{I}}$. Referring to figure 2, in which the phase of Γ_{2i} and the initial phase of Γ_L are chosen to give the maximum tuning error ($\lim\epsilon_{T,\mathrm{I}}$) for a given ${}^{fi}\psi_L$, one obtains

$$\frac{\sin\left(\dfrac{\lim\epsilon_{T,\mathrm{I}}}{2}\right)}{\sin\left(\dfrac{{}^{fi}\psi_L}{2}+n\pi\right)}=\frac{|\Gamma_{2i}||\Gamma_L|}{1-|\Gamma_{2i}||\Gamma_L|}\leq|\Gamma_{2i}||\Gamma_L|. \tag{7}$$

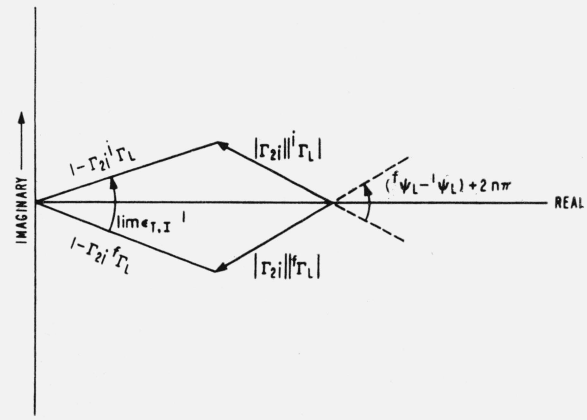

Figure 2. *A representation of* $(1-\Gamma_{2i}{}^i\Gamma_L)/(1-\Gamma_{2i}{}^f\Gamma_L)$, *to show maximum* $\epsilon_{T,\mathrm{I}}$ *for a given* ${}^f\Psi_L-{}^i\Psi_L$.

Since $|\Gamma_L|\approx 1$, and since the errors are assumed small, eq(7) yields

$$\lim\epsilon_{T,\mathrm{I}}\approx 2|\Gamma_{2i}|\left|\sin\frac{{}^{fi}\psi_L}{2}\right|. \tag{8}$$

It is noted that the tuning error from this source cannot exceed $2|\Gamma_{2i}|$ radians for any phase measurement.

One can determine $|\Gamma_{2i}|$ as follows. In the tuning procedure (see footnote 3) for setting $\Gamma_{2i}\approx 0$, the reflection coefficient of the phasable load which is attached to arm 2, $\Gamma_{T,\mathrm{I}}$, is nearly of unit magnitude and $S_{31}\approx 0$; therefore, the ratio of the maximum to minimum response of the detector as the phase of $\Gamma_{T,\mathrm{I}}$ is varied may be obtained from eq(1) as approximately,

$$\frac{|b_3|_{\max}}{|b_3|_{\min}}\approx\frac{1+|\Gamma_{2i}|}{1-|\Gamma_{2i}|}\approx 1+2|\Gamma_{2i}| \tag{9}$$

or,

$$20\log_{10}\frac{|b_3|_{\max}}{|b_3|_{\min}}\approx 20\log_{10}(1+2|\Gamma_{2i}|)\approx 17.4|\Gamma_{2i}|. \tag{10}$$

CASE II. $\Gamma_{2i}=0$, but $S_{31}\neq 0$. For this case, the ratio of the final emergent wave to the initial emergent wave may be derived from eq(1) as

$$\frac{{}^f b_3}{{}^i b_3}=\frac{{}^f\Gamma_L}{{}^i\Gamma_L}\cdot\frac{1+\dfrac{S_{31}}{(S_{32}S_{21}-S_{31}S_{22})\,{}^f\Gamma_L}}{1+\dfrac{S_{31}}{(S_{32}S_{21}-S_{31}S_{22})\,{}^i\Gamma_L}} \tag{11}$$

from which it is apparent that the change of phase of the emergent wave differs from the change of phase of the load by $\epsilon_{T,\mathrm{II}}$ where,

$$\epsilon_{T,\mathrm{II}}=\text{argument of}\,\frac{1+\dfrac{S_{31}}{(S_{32}S_{21}-S_{31}S_{22})\,{}^f\Gamma_L}}{1+\dfrac{S_{31}}{(S_{32}S_{21}-S_{31}S_{22})\,{}^i\Gamma_L}}, \tag{12}$$

which for $|S_{31}S_{22}|\ll|S_{32}S_{21}|$ may be written as

$$\epsilon_{T,\mathrm{II}}\approx\text{argument of}\left(1+\frac{S_{31}}{S_{32}S_{21}\,{}^f\Gamma_L}\right)-\text{argument of}\left(1+\frac{S_{31}}{S_{32}S_{21}\,{}^i\Gamma_L}\right)+2n\pi. \tag{13}$$

Since $|\Gamma_L|\approx 1$, one may write $1/\Gamma_L\approx e^{-j\psi_L}$. From a derivation similar to that used for $\epsilon_{T,\mathrm{I}}$, it can be shown that, for small errors,

$$\lim\epsilon_{T,\mathrm{II}}\approx 2\left|\frac{S_{31}}{S_{32}S_{21}}\right|\left|\sin\left(\frac{{}^{fi}\psi_L}{2}\right)\right|. \tag{14}$$

Since $|S_{21}|$ is of the order of unity, $\lim\epsilon_{T,\mathrm{II}}$ is, to the same approximation, proportional to the inverse of the directivity ratio. One can determine this ratio as follows. The observed amplitude variation of the side arm output when tuning adjustments are made to set $S_{31}\approx 0$ can be shown to be

$$20\log_{10}\frac{|b_3|_{\max}}{|b_3|_{\min}}\approx 20\log_{10}\left(1+2\left|\frac{s_{31}}{S_{32}S_{21}}\right|\left|\frac{1}{\Gamma_{T,\mathrm{II}}}\right|\right)=17.4\left|\frac{S_{31}}{S_{32}S_{21}\Gamma_{T,\mathrm{II}}}\right| \tag{15}$$

where $\Gamma_{T,\mathrm{II}}$ is the reflection coefficient of the phasable load which is attached to arm 2 when tuning for the condition $S_{31}=0$. The magnitude $|\Gamma_{T,\mathrm{II}}|$ is small for this adjustment and an estimate of its value must be made in order to evaluate the error.

3. Dimension Errors

Ideally, the change of phase of the standard phase shifter at a single frequency is

$${}^{fi}\psi_L=\frac{4\pi(l_f-l_i)}{\lambda_g}\ \text{radians}, \tag{16}$$

where l_f-l_i is the distance between the final and initial positions of the short circuit within a waveguide in which the waveguide wavelength is λ_g.

The error in the change of phase of Γ_L due to the uncertainty in measurement of the axial motion of the sliding short circuit is termed the motional error, ϵ_l. A small motional error is readily evaluated by considering the partial derivative of ${}^{fi}\psi_L$ with respect to l. In terms of this partial derivative,

$$\epsilon_l=\frac{\partial}{\partial l}({}^{fi}\psi_L)\Delta l=\frac{4\pi}{\lambda_g}(\Delta l_f-\Delta l_i)\ \text{radians}, \tag{17}$$

where Δl's are the errors in setting the positions of the load. If the uncertainty in setting the initial and final positions of the load is $|\Delta l|$, then the limit of motional error, $\lim\epsilon_l$ is

$$\lim\epsilon_l=\frac{8\pi|\Delta l|}{\lambda_g}\ \text{radians}=1440\,\frac{|\Delta l|}{\lambda_g}\ \text{deg}. \tag{18}$$

In general, the waveguide wavelength will not be uniform over a particular path between l_f and l_i because of variations in the dimensions. A limit of this error may be established by calculating the difference between the change of phase of Γ_L in a uniform waveguide with the maximum (or minimum) dimension and the change of phase in a uniform waveguide with the nominal dimension. Let this difference be termed the limit of tolerance error. If the tolerance of the waveguide dimension (maximum variation from the nominal value) is given by Δa, then a small limit of tolerance error, $\lim\epsilon_a$, can be obtained from

$$\lim\epsilon_a=\frac{\partial}{\partial\lambda_g}({}^{fi}\psi_L)\frac{\partial\lambda_g}{\partial a}\Delta a\ \text{radians} \tag{19}$$

which for the dominant mode in a lossless rectangular waveguide of broad dimension "a" becomes

$$\lim\epsilon_a=4\pi(l_f-l_i)\lambda_g\frac{\Delta a}{4a^3}. \tag{20}$$

This error limit is proportional to the total change in phase of Γ_L, and therefore is presented as a fractional error, $\epsilon_a/{}^{fi}\psi_L$, as

$$\lim\frac{\epsilon_a}{{}^{fi}\psi_L}=\frac{\lambda_g^2}{4a^3}\Delta a. \tag{21}$$

4. Graphical Presentation of Results

It was assumed that the errors in the change of phase were small and therefore the individual contributions to the error could be summed. Two graphs, figures 3 and 4, present values of $|\Gamma_{2i}|$ and $|S_{31}/S_{32}S_{21}|$, respectively, which are used to estimate the limits of error from the two tuning errors given by eqs (8) and (14) respectively. Two more graphs, figures 5 and 6, present limits of dimensional errors. The graphs of $|\Gamma_{2i}|$ and $|S_{31}/S_{32}S_{21}|$ are applicable for any frequency range or waveguide size, while the graphs of limits of dimensional error are only applicable to WR–90 waveguide over the operating range of frequencies noted on the graphs. The equations used to construct these graphs, however, may be used for any size waveguide.

Figure 3 is a graph of the value of $|\Gamma_{2i}|$ plotted against the ratio of the maximum to the minimum response of the detector attached to arm 3, in decibels, as the tuning load (a short circuit) is moved along the waveguide. This value of $|\Gamma_{2i}|$ is to be used in eq (8) to estimate lim $\epsilon_{T,I}$.

Figure 4 is a graph of the value of $|S_{31}/S_{32}S_{21}|$ plotted against the ratio of the maximum to the minimum response of the detector attached to arm 3, in decibels, as the tuning load (having small reflection) is moved along the waveguide. This value of $|S_{31}/S_{32}S_{21}|$ is to be used in eq (14) to estimate lim $\epsilon_{T,II}$. In this portion of the tuning procedure, the magnitude of reflection coefficient of the tuning load usually lies within the range 0.001 to 0.1. Therefore, several curves are plotted for different $|\Gamma_{T,II}|$. It is only necessary to determine an upper limit to the magnitude of $\Gamma_{T,II}$ to estimate limits of error from this source.

Figure 5 is a graph of the limit of motional error plotted against the maximum uncertainty of motion imparted by the drive mechanism to the short circuit. Several curves are plotted for various frequencies throughout the recommended frequency range of WR–90 waveguide.

Figure 6 is a graph of the limit of tolerance error per degree of change of phase, in degrees error per degree of change of phase, applicable for any value of the change of phase of Γ_L. Several curves are plotted for different frequencies throughout the recommended frequency range of WR–90 waveguide.

As an example of the use of the graphs, assume that a standard phase shifter was made and used as follows. The load attached to arm 2 is made with a short-circuit adjustable with a micrometer of 0.0005-in. maximum uncertainty placed in a WR–90 waveguide of standard tolerance (± 0.003 in.). The tuning procedure for Γ_{2i} was carried out to 0.01-db variation in the maximum to minimum response.

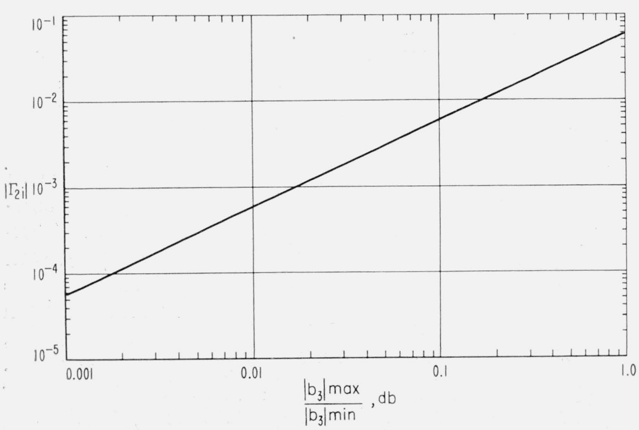

FIGURE 3. *Graph for the determination of* $|\Gamma_{2i}|$.

The tuning for S_{31} was carried out to 1.0-db variation in the maximum to minimum response with a tuning load of maximum VSWR of 1.01. The operating frequency is 9,000 Mc/s. The change of phase is 60°.

From figure 3, $|\Gamma_{2i}|$ for a 0.01-db variation is 0.00058. From eq (8), the limit of tuning error

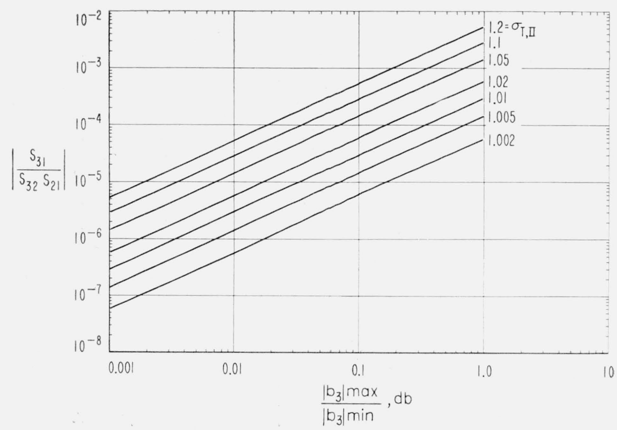

FIGURE 4. *Graph for the determination of* $|S_{31}/S_{32}S_{21}|$.

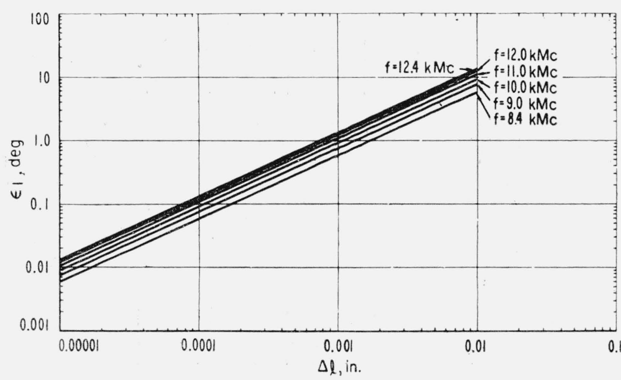

FIGURE 5. *Limit of motional error.*

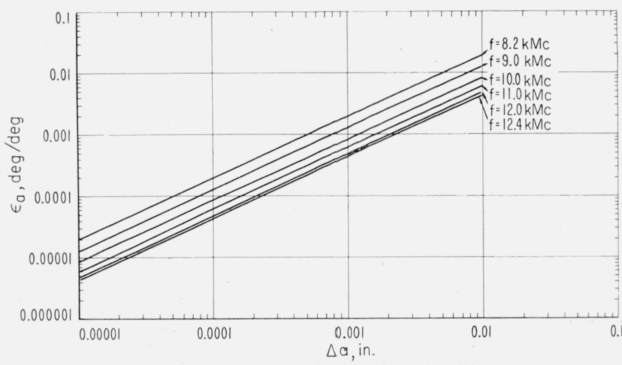

FIGURE 6. *Limit of tolerance error.*

lim $\epsilon_{T,I}$ is therefore 0.00058 radians or 0.033 deg. From figure 4, $|S_{31}/S_{32}S_{21}|$ for a 1.0-db variation with a $|\Gamma_{T,II}|$ of 0.005 (VSWR=1.01) is 0.00029. From eq (14), the limit of tuning error, lim $\epsilon_{T,II}$, is 0.00029 radians or 0.018 deg. The total limit of tuning error is then 0.051 deg. From figure 5, for a tolerance of the micrometer of 0.0005 in., the limit of motional error at 9,000 Mc/s is 0.38 deg. From figure 6, for a tolerance of 0.003 in. in the dimension of the waveguide at 9,000 Mc/s, the limit of waveguide dimension error per degree of change of phase is 0.038 deg/deg. For 60°, this is a limit of waveguide dimension error of 2.28°. The total limit of dimensional error is then 2.66 deg. The total limit of error from these sources is then 2.71 deg.

The above example is considered to be typical of readily constructed phase shifters since the tolerances were typical (WR–90) commercial tolerances and the tuning variations used can be attained in reasonably stable systems. However, precision waveguide sections, and tuning loads of very small reflection coefficients permit constructing standard phase shifters of extremely high accuracy. For example, the motional error may easily be reduced to 0.038 deg while precision waveguides have been constructed which have 0.00013°/deg limit of dimensional error per degree of change of phase. With such improvement in the dimensional errors, the total limit of error in the phase measurement described in the above example would be only 0.097 deg.

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