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Capacitor Calibration by Step-Up Methods

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Step-calibration methods are used in many physical laboratories for the extension of measurements to quantities far removed from the magnitude of greatest accuracy at which absolute determinations are made. The excellent precision of repetitive substitution procedures is exploited by step-up or step-down methods to extend measurements to higher or lower magnitudes without serious degradation of accuracy. The application of step-up techniques to the calibration of variable air capacitors is described in this paper as a practical example of the method.

1. Introduction

One of the important statutory functions of the National Bureau of Standards is the calibration of physical standards of measurement used in science and industry. The chain of measurements connecting this calibration service to the national prototype standards of length, mass, and time is complex, and for electrical measurements involves meticulous experiments to assign numerical values to calibration standards and corrections to standard instruments. These devices, designed for excellent stability and definitude, serve as comparison standards basic to the calibration services rendered by the Bureau. Equally important to accurate scientific work is the proper use of these standards to overcome their inherent limitations. Frequently an appropriate choice of method and the employment of suitable techniques are as important as the judicious selection of equipment. The close association between methods, techniques, and equipment is particularly evident when, in the course of calibration activities, it is necessary to obtain accurate measurements at magnitudes far removed from that at which absolute determinations are made. The extension of range of electrical measurements is sometimes accomplished by the establishment of accurately known ratios. For example, ratios very nearly equal to the squares of integers may be obtained through the successive measurement in series and parallel of resistors having nearly equal values [1].[1] Resistance ratios of approximately 10:1 may be realized by the successive measurement of 11 resistors in arbitrary units and the use of these as the 10:1 ratio arms of a bridge. A unique 10:1 ratio apparatus used with a special resistance bridge is described by Wenner [2].

The building-up to ratios larger than 10:1 is particularly well exemplified by the procedure followed in calibrating the standard volt box at the Bureau [3] in which a group of sections of nominally equal resistance is intercompared. These sections, connected in series, form the first section of a group of larger denomination. The buildup to large ratios is rapid and exact. The standard volt box was designed specially for self-calibration by this method.

The calibration of resistance decade boxes and the resistance decades of bridges by the step-substitution or step-up method illustrates yet another technique of obtaining accurate measurements over wide ranges [4]. A similar process is used by the Bureau for the calibration of the capacitance bridges that are used daily to measure standards of capacitance.

In order to obviate the concern over connection errors and avoid the detailed consideration of connectors, it is customary and convenient to use as standards of low grounded capacitance such devices as variable air capacitors and capacitance decade boxes which may be calibrated accurately for capacitance difference from some arbitrary setting. The calibration of such variable capacitors may be accomplished quite effectively by the step-up method employing fixed standards or standards of capacitance difference.

An excellent description of a step-up method applied to the calibration of decade capacitors for both capacitance and dissipation factor has been described by Ford and Astbury of the British National Physical Laboratory [5].

2. Equipment

Very little special equipment is needed to calibrate a variable capacitor by step-up methods. If the variable air capacitor, X, having a range from 100 to 1,100 pf, is to be calibrated at every 100-pf division mark, it is necessary to have a fixed air capacitor, S, of approximately 100 pf that can be connected in parallel with the variable capacitor under test in a precisely repeatable manner. This can be achieved if the mating connectors introduce no significant uncertainties to the capacitance added to the circuit and if the connectors are designed to couple quickly and easily to either the variable capacitor or the

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[1] Figures in brackets indicate the literature references at the end of this paper.

bridge that will be used, or if a capacitor can be connected or disconnected by a precise switching arrangement. The 100-pf capacitor should be adjusted close to the nominal value, but it need not be calibrated. It must, however, be free from significant drift over the period of a quarter-hour or so during which the test is being run.

A calibrated 1,000-pf air capacitance standard, S', is needed to relate the results of the step-up test to the national reference standard of capacitance.

The bridge used for this step calibration need not have great accuracy but must be stable, for it is used with a sensitive detector for substitution measurements. A small variable capacitor, V, is required, having a least count (smallest readable increment) one-tenth that of X or smaller. It is advantageous to choose the smallest possible precision variable capacitor, V, consistent with other limitations so that the corrections to V are negligibly small relative to the corrections to X. The total range of V must be at least a little larger than the range of errors in the capacitor to be calibrated. The readable accuracy of this capacitor, if expressed in percent of total range, need not be very great.

The equipment described is assembled as shown in figure 1. It is most important that the cables used to connect components be shielded and rigid, or if flexible cables are used, it should be ascertained that variations in cable capacitance are negligible. The cables must be fixed in position and must not be disturbed during the entire calibration. This precaution is intended to emphasize the importance of particular care to one of those sources of systematic error that could impair good calibration accuracy. The operator must have a good technical appreciation of the apparatus and quantities measured, gained through study and experience.

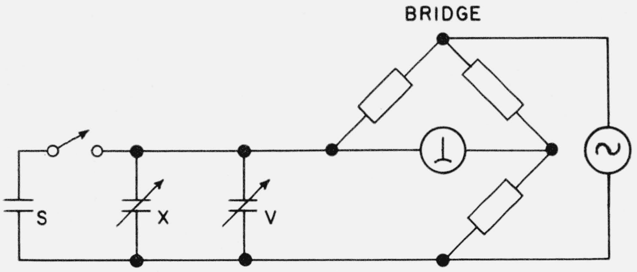

FIGURE 1. *The variable capacitor under text,* X, *is calibrated by a step-up method employing a fixed capacitor,* S, *and a small variable capacitor,* V.

3. Procedure and Computations

The true value of each of the capacitors involved in the calibration may be defined as the nominal value plus a correction; thus, the true capacitance of the uncalibrated 100-pf capacitor is $S=S_n+s$, where $S_n=100$ (exactly) and s is the small correction. Similarly the calibrated 1,000-pf standard has the value $S'=S'_n+s'$, and the variable air capacitor to be calibrated may be represented by $X=X_n+x$, where x is the correction to the reading (or setting) X_n. The calibration will consist of the determination of the relatively small correction, x, to each 100-pf division mark, X_n, and since only capacitance differences are of interest, the observer is free to choose any one of the division marks as a reference point. It is frequently convenient to choose as a reference the first marked point on the dial. In this case the 100-pf mark is considered as a reference and a correction of 0.00 is arbitrarily assigned to it. For this example it will be assumed that the small variable capacitor, V, has corrections that are negligibly small.

The capacitor under test is first carefully set to the 100-pf mark ($X_{n1}=100$ pf) avoiding backlash errors by approaching the mark in the direction of increasing dial readings. The small variable capacitor, V, is set to any convenient mark near the center of its range. The 100-pf capacitor, S, is connected in parallel with X and V. Now the bridge must be balanced using the controls on the bridge itself. If a balance cannot otherwise be obtained, V may be used to attain balance. When the bridge is balanced the reading V_A is recorded. The fixed capacitor, S, is then removed and X set to the 200-pf mark ($X_{n2}=200$ pf), again approaching the mark in the same direction. Without changing any other component the circuit is rebalanced by changing V alone, and the reading V_B recorded as before. In the first balance the external bridge arm consisted of $S+X_1+V_A$, and for the second balance, with the bridge unchanged, the external arm consisted of X_2+V_B. These can be equated to yield

$$S+X_1+V_A=X_2+V_B \tag{1}$$

The cable and connector capacitance, as well as residuals within the bridge, contribute equally to both balances and are therefore deliberately disregarded.

It is convenient to work with small numbers, and eq (1) can be expressed as

$$S_n+s+X_{n1}+x_1+V_A=X_{n2}+x_2+V_B \tag{2}$$

and since

$$S_n=X_{n2}-X_{n1} \text{ and } x_1=0 \tag{3}$$

$$x_2=(V_A-V_B)_2+s \tag{4}$$

where the subscript is appended to the difference (V_A-V_B) to distinguish this set of data from other sets and to appropriately correlate the difference with the setting of X in the second balance of each set.

The quantity x_2 is the desired correction to X_n when $X_n=200$ pf. The difference $(V_A-V_B)_2$ is easily computed from the recorded data.

Without changing X, S is reconnected and the bridge rebalanced using the bridge controls and V, if necessary, to attain exact balance. The reading, V_A, is then recorded. Capacitor S is then removed and X set to the 300-pf mark ($X_n=300$). The bridge is rebalanced using V alone and the reading,

V_B, recorded. When the first balance is equated to the second balance

$$S+X_2+V_A=X_3+V_B \tag{5}$$

or

$$S_n+s+X_{n2}+x_2+V_A=X_{n3}+x_3+V_B \tag{6}$$

and since

$$S_n=X_{n3}-X_{n2} \tag{7}$$

$$x_3=x_2+(V_A-V_B)_3+s. \tag{8}$$

Substituting eq (4) in eq (8)

$$x_3=(V_A-V_B)_2+(V_A-V_B)_3+2s. \tag{9}$$

Continuing this process step-by-step, in general for the mth step

$$x_m=\sum_2^m (V_A-V_B)+(m-1)s \tag{10}$$

and finally

$$x_{11}=\sum_2^{11} (V_A-V_B)+10s \tag{11}$$

A tabulation of the differences and the cumulative sum of the differences is shown in table 1, which shows the data and computations for a typical calibration. It remains to determine the value of s so that the corrections x_2 through x_{11} can be evaluated.

TABLE 1. *Observations and calculations*

All values in picofarads

S	X	V	V_A-V_B	$\Sigma(V_A-V_B)$	ns	x
100	100	5.00	0.31	0.31	−0.14	0.17
0	200	4.69				
100	200	4.84	−.02	.29	−.29	.00
0	300	4.86				
100	300	4.99	.06	.35	−.43	−.08
0	400	4.93				
100	400	4.92	.41	.76	−.58	.18
0	500	4.51				
100	500	4.65	.01	.77	−.72	.05
0	600	4.64				
100	600	4.81	.05	.82	−.86	−.04
0	700	4.76				
100	700	4.87	.27	1.09	−1.01	.08
0	800	4.60				
100	800	4.73	.31	1.40	−1.15	.25
0	900	4.42				
100	900	4.57	.04	1.44	−1.30	.14
0	1000	4.53				
100	1000	4.67	.55	1.99	−1.44	.55
0	1100	4.12				
1000	100	5.04	0.92	----------	----------	----------
0	1100	4.12				

$s'=-0.37$ pf $\quad 10s=-1.44$ pf

The 1,000-pf standard capacitor, S', accurately calibrated for insertion capacitance, is now connected in parallel with X and V. With X set at 100 pf, the bridge is balanced with the bridge controls and V, if necessary, and the reading, V_A, recorded. S' is removed, X is set to the 1,100-pf mark, the bridge rebalanced using V alone, and the reading, V_B, recorded. Then

$$S'+X_1+V_A=X_{11}+V_B \tag{12}$$

or

$$S'_n+s'+X_{n1}+V_A=X_{n11}+x_{11}+V_B \tag{13}$$

and since

$$S'_n=X_{n11}-X_{n1} \tag{14}$$

$$x_{11}=(V_A-V_B)_T+s' \tag{15}$$

where the subscript T denotes the V difference obtained when the capacitor, S', is used.

In this way x_{11} is determined accurately in terms of a small difference reading of the variable capacitor, V, and the known correction, s', to the standard capacitor, S'. The correction, s, to the fixed capacitor, S, can now be computed from eq (11)

$$10s=x_{11}-\sum_2^{11} (V_A-V_B) \tag{16}$$

or

$$10s=s'+(V_A-V_B)_T-\sum_2^{11} (V_A-V_B) \tag{17}$$

The quantity $10s$ is then added algebraically to the sum of the V differences corresponding to the test of the 1,100-pf mark, the result being the correction to this reading. Similarly $9s$ is added to the sum of the V differences corresponding to the 1,000-pf mark, and so on, until only s is added to the V differences corresponding to the 200-pf mark. These small corrections are listed in table 1 under the heading ns.

The observations can be made rapidly and the computations are simple, since only small differences appear. A second complete calibration, preferably by another observer, enables one to appraise the precision of the measurements including the stability and resetability of the capacitor under test, and serves to reveal measurement and arithmetic errors that might otherwise remain undetected.

In the procedure described above the fixed increment of calibration was 100 pf. It is well to point out that other increments can be accommodated as well. A 50-pf capacitor, if used as a fixed step, would permit calibration at 50-pf intervals. Although the procedure has been described using a fixed capacitor as a step, a continuously variable capacitor or decade capacitor would also be satisfactory if it were used in such a manner as to provide a repeatable difference of capacitance. Care would be necessary to avoid setting errors caused, for example, by backlash in the control mechanism, or by careless setting to the index.

A variable capacitor calibrated for capacitance difference by this method can be used as a standard for extending the method to capacitance calibrations of still lower magnitudes.

4. Dual Calibration

Reviewing the calibration described above, it is noticed that for each set of two balances, one balance is obtained with the bridge controls and V, if necessary. The fact that the change in the bridge reading is always an amount approximately equal to S (or S') leads to the consideration of calibrating two variable capacitors having the same range with practically no extra work.

If X and U, the variable capacitors to be calibrated, are connected as shown in figure 2, the procedure is similar to that described above except that the bridge need not be changed after the initial setting. The settings of X and U are listed in table 2. Care must be taken to apply the proper sign to the differences and to cumulatively add the differences for the calibration of U beginning at the bottom of the table rather than the top.

Lower range capacitors can be calibrated similarly, but extreme attention must be paid to good mechanical rigidity in all parts of the circuit.

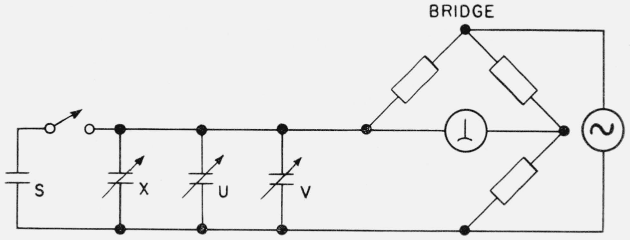

FIGURE 2. *Two variable capacitors,* X *and* U, *may be calibrated simultaneously by a step-up method.*

5. Discussion

If attention is confined to the calibration of two-terminal variable air capacitors having a capacitance range from several picofarads to about 1,000 pf, the procedure outlined in this paper demonstrates the ability of step-calibration methods to provide accurate calibrations of capacitors at levels at which good accuracy is otherwise difficult to obtain.

TABLE 2. *Observations and calculations (dual calibration)*

All values in picofarads

S	X	U	V	V_A-V_B	$\Sigma(V_A-V_B)$	ns	x [a]	V_B-V_A	$\Sigma(V_B-V_A)$	ns	u [a]
0	100	1100	5.00								
								0.34	2.05	−1.38	0.67
100	100	1000	5.34								
				0.31	0.31	−0.14	0.17				
0	200	1000	5.03								
								.04	1.71	−1.24	.47
100	200	900	5.07								
				−.03	.28	−.28	.00				
0	300	900	5.10								
								.07	1.67	−1.10	.57
100	300	800	5.17								
				.06	.34	−.41	−.07				
0	400	800	5.11								
								.24	1.60	−0.97	.63
100	400	700	5.35								
				.37	.71	−.55	.16				
0	500	700	4.98								
								.36	1.36	−.83	.53
100	500	600	5.34								
				.02	.73	−.69	.04				
0	600	600	5.32								
								.31	1.00	−.69	.31
100	600	500	5.63								
				.05	.78	−.83	−.05				
0	700	500	5.58								
								.26	0.69	−.55	.14
100	700	400	5.84								
				.26	1.04	−.97	.07				
0	800	400	5.58								
								.29	.43	−.41	.02
100	800	300	5.87								
				.31	1.35	−1.10	.25				
0	900	300	5.56								
								.01	.14	−.28	−.14
100	900	200	5.57								
				.03	1.38	−1.24	.14				
0	1000	200	5.54								
								.13	.13	−.14	−.01
100	1000	100	5.67								
				.54	1.92	−1.38	.54				
0	1100	100	5.13								
1000	100	1100	5.37								
				.91							
0	1100	1100	4.46								
								1.04			
1000	1100	100	5.50								
$s'=-0.37$ pf				$10s=-1.38$ pf				$10s=-1.38$ pf			

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[a] The columns x and u are the corrections to the dial readings of the variable capacitors X and U. The good precision of the method is noticeable by comparing the test of X in table 2 with table 1, which represents a test of the same capacitor about an hour earlier.

Standards of grounded capacitance, often called two-terminal capacitors, are characterized by having one of the capacitor electrodes connected to the case, in contrast with standards of direct capacitance (three-terminal capacitors) having both capacitive electrodes insulated from the case. The direct capacitance, C_D, between the two active electrodes, as shown in figure 3, is definite to the extent that the separate terminals and associated leads are shielded from each other. Adequate shielding that does not interfere with the direct capacitance is relatively easily obtained, and excellent accuracy in direct capacitance measurements is possible to a fraction of a micropicofarad (10^{-18} f).

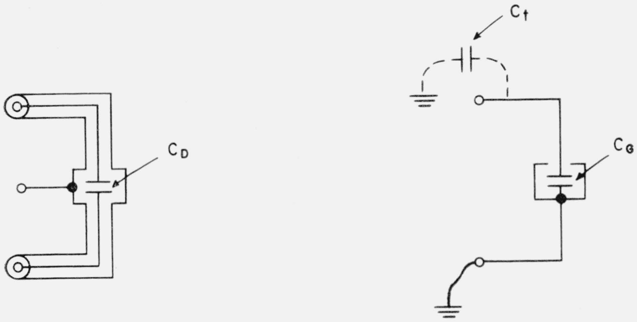

FIGURE 3. *The direct (three-terminal) capacitance,* C_D, *is made definite by complete shielding. The grounded (two-terminal) capacitance,* C_G, *is indefinite because of variations in the stray capacity.*

The grounded capacitance, C_G, shown in figure 3, is more difficult to define in a precise manner, because the capacitance between the ungrounded terminal and all grounded objects, C_t, is indistinguishable from C_G unless a separate "zero-balance" of the measuring apparatus is made with the leads attached but the capacitor disconnected. Even this procedure will not insure good precision unless care is used to connect the capacitor to the measuring apparatus in identically the same way every time. The best precision in practical measurements of grounded capacitance is possible only if the method of connection to the measurement circuit is well defined. An adequately shielded rigid adapter or connector is necessary as an auxiliary part of the capacitor and must be used with it for every measurement. If the capacitor is to be used as a standard for accurate capacitance measurements, the same connector must be used with the capacitor when it is calibrated, and the assembly becomes a standard of, capacitance added to a circuit, or in other words, capacitance difference. Thus, good precision of repeated measurements is simply obtained in any laboratory if the successive measurements are accomplished using rigid wiring and the same connectors every time. The best accuracy in terms of the national reference standards can be obtained only for magnitudes sufficiently large that negligible errors result from differences between the connectors used in the several laboratories involved. Accurate calibration of small fixed capacitors can only be accomplished if mating connectors are submitted.

The accuracy of measurements on fixed standards of grounded capacitance having electrodes terminated in binding posts or unshielded plugs is limited by the variation in the geometrical design of the instrument panels, cables, and connectors to which the standard can be attached. Differences as large as several tenths of a picofarad are possible with present commercially available standards and apparatus with which they may be used. It is understandable that differences of several tenths of a percent are to be expected if a fixed 100-pf standard of this type is measured in several laboratories or on different equipment, while if measurements are performed on 1,000-pf standards the uncertainties at the connectors would be only several hundredths of a percent of the quantity measured. In the step-up procedure described in this paper it is evident that the precision of repeated measurements, the freedom from the effects of residuals in test apparatus obtainable by substitution methods, and the accuracy of measurements at magnitudes closer to optimum, are combined in a manner favorable to the accurate calibration of the capacitance differences of variable capacitors. The method is quite applicable at any frequency although at higher frequencies residuals in components can be troublesome and may require special attention. For example, it may be necessary to apply corrections for errors introduced by residual inductance in the cables connecting the apparatus.

6. Conclusion

Step-calibration methods can be employed for the calibration of variable capacitors. The few necessary items of equipment are generally available in any electrical measurements laboratory. Reference to the national electrical standards is made through the use of a single fixed capacitance standard that can be transported to other standardizing laboratories more easily, and calibrated less expensively than variable capacitors.

7. References

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