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A Method for Measuring Local Electron Density from an Artificial Satellite

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A method is proposed for measuring the electron density at known points in the outer ionosphere, by the use of vlf receiving equipment in an artificial satellite, in conjunction with a vlf transmitter on the ground. The transmitter would radiate continuous waves, which would be propagated through the ionosphere in the "whistler" mode. The basis of the method is a measurement of the local wave admittance of the medium, by comparison of the signals received on an electric dipole and on a loop.

A further proposal is made for an integrated vlf satellite experiment, in which several different types of observation would be made simultaneously.

1. Introduction

Until recently, most of our knowledge of the ionosphere had been obtained by indirect methods, such as vertical-incidence soundings with radio waves. Now, however, the ionosphere can be investigated directly, with equipment borne aloft in rockets or satellites. These two vehicles lend themselves to making complementary studies: a rocket, which goes straight up and then falls back to earth, is better for studies of the variations of the properties of the ionosphere with height at a particular time and place, while a satellite, which may stay in orbit for a long period, is better for synoptic studies of slow variations with time and latitude at particular heights. Rocket and satellite experiments are especially valuable as a source of information about the outer ionosphere, above the peak of the $F2$ layer, which is beyond the reach of conventional vertical soundings.

In studying the ionosphere, the number density of free electrons is a quantity of major interest. Already there are several ways in which the influence of the ionospheric electrons has been observed on transmissions from the artificial satellites; examples are the Faraday rotation of the plane of polarization, and the difference in refraction at disparate frequencies. Such observations, however, all measure integrated effects of the distribution of electrons along the entire path from the satellite to the observer, rather than their density at any one point. But the special advantage of a satellite is that it provides the opportunity for making local measurements at known points in the ionosphere, and so the question arises of how to equip it to measure local electron density. This paper sets out a proposal for one new method. Elsewhere, other methods have been suggested (Hoffmann [1]).[1] Since few have been put to practical test, it is too early to assess their relative merits.

[1] Figures in brackets indicate the literature references at the end of this paper.

The basis of the proposed method is the measurement of the local *wave admittance* of the ionospheric medium. To understand this concept, consider first a linearly-polarized plane wave in free space. The electric ($\underline{E}$) and magnetic ($\underline{H}$) vectors, which comprise the field of the wave, lie at right-angles to each other in the plane of the wavefront (fig. 1). At any fixed point, the variations of the two vectors are always in phase, so that their ratio E/H is a constant. This constant, which has the dimensions of a resistance, is called the *wave impedance of free space* (R_0); its value is about 377 ohms. Its physical significance may be appreciated from the fact that if the wave were to impinge directly upon a uniform sheet of lossy material, that had this resistance per unit square area of surface, then it would be absorbed without reflection. Thus such a sheet would be "matched" to free space.[2]

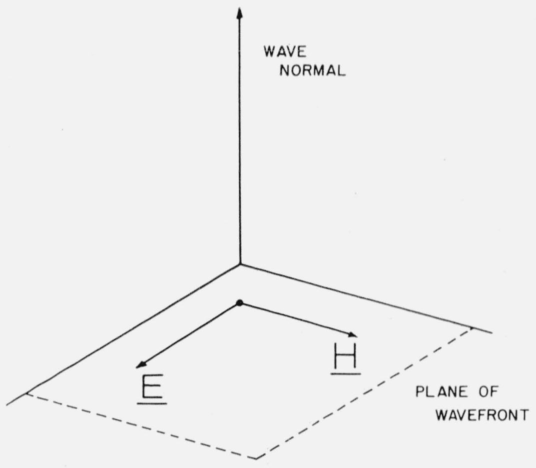


FIGURE 1. *A linearly-polarized plane wave in free space.*

[2] Strictly, for the absorption to be complete the resistive sheet would have to be backed by a perfectly reflecting sheet, at a distance of a quarter-wavelength.

Within a uniform dielectric medium, the ratio of E to H is different from its value in free space. It is given by

$$E/H=R=R_0/\mu \qquad (1)$$

where μ is the refractive index. In words, the wave impedance of the medium is inversely proportional to its refractive index. Henceforth, therefore, it will be more convenient to talk in terms of the wave admittance, which is the reciprocal of the impedance, and so is directly proportional to the refractive index:

$$H/E=A=\mu A_0. \qquad (2)$$

Evidently, if the wave admittance of the medium could be measured in some way, this measurement would yield the value of the local refractive index.

In applying this result to the measurement of local electron density in the ionosphere, the following experimental arrangement is envisaged: There is to be a transmitter on the ground, radiating continuous waves of constant amplitude. The waves travel upward into the ionosphere, where an orbiting satellite is equipped to receive them. The satellite receives the waves on two separate antennas, a loop for the magnetic field and a dipole for the electric field. Information about the strength of the two fields is telemetered back to ground. At the ground station, the ratio of the two field strengths is computed, giving the wave admittance of the ionosphere in the immediate neighborhood of the satellite. By comparing the observed wave admittance with the value to be expected for free space, the local refractive index is obtained. Now the refractive index is related to the electron density by the expressions of the magneto-ionic theory. Hence, given the local refractive index, it is possible to work backwards and arrive finally at the value of the local electron density. These are the essential features of the proposed experiment, though there are many problems of detail.

The first problem to consider is the choice of the frequency of the transmitted waves. In principle, the experiment would work at any frequency. But if measurements are to be made above the maximum of the $F2$ layer, then the choice is restricted to those frequencies that are capable of passing through this layer: either the frequency must exceed the "critical frequency" of the layer, which varies roughly in the range 5 to 15 Mc, or it must be less than the gyrofrequency (about 1.5 Mc), in which case the waves can penetrate the layer in the "whistler" (ordinary) mode. It is preferable to choose low frequencies and the "whistler" mode, on the following two counts:

First, at low frequencies only this mode is propagated, whereas at high frequencies both the ordinary and extraordinary mode would be propagated; their mutual interference would produce a complicated pattern of field in the ionosphere.

Second, at low frequencies the waves are affected very strongly by the ionospheric electrons, so that the refractive index differs greatly from unity; high frequencies are affected much less. To illustrate this point, suppose that at some level above the $F2$ layer the electron density is such as corresponds to a plasma frequency of 1.5 Mc (see sec. 2.1.). For the transmitted frequencies, take 15 kc as a typical low value and 15 Mc as a typical high value. The refractive indices for these two frequencies are then as given in table 1 below:

TABLE 1. *Refractive indices of the ionosphere, for a plasma frequency of 1.5 Mc*

Wave frequency	Refractive index
15 kc	12.5 (whistler mode).
15 Mc	0.99 (ignoring magnetic field).

In this example, the change of the refractive index from unity is more than a thousand times greater at the lower frequency. There can be no doubt, therefore, that low frequencies are preferable. However, since the "whistler" mode requires the presence of a magnetic field to support its propagation, the use of this mode does confine the experiment to the domain of the earth's magnetic field.

Having chosen the frequency and mode of propagation of the transmitted waves, the remaining problems arise from the fact that, at low frequencies, the ionosphere does not behave at all like a simple dielectric, so that the whistler mode has quite a complicated field structure. The outstanding questions are: (a) What is the structure of the wave field in the whistler mode, and how is the wave admittance defined for this mode? (b) How should the satellite be equipped to explore the structure of the field, and measure the wave admittance? (c) What sort of records would be obtained at the ground? (d) How should the records be interpreted? These questions are discussed successively in sections 2 to 5 of this paper. Section 6 describes a proposal for an integrated vlf satellite experiment, in which several different types of observation would be made simultaneously, so as to obtain the fullest information possible from the vlf transmissions; it includes an independent method for obtaining another measure of the local electron density. Finally, section 7 reviews the proposals and examines some possible sources of error.

2. Structure of Waves in the Whistler Mode

2.1. Introduction

In the proposed experiment, a transmitter on the ground radiates continuous waves at a very low fixed frequency. Some of these waves reach the lower boundary of the ionosphere, where their energy is divided between the two magneto-ionic modes. The ordinary mode is reflected, with some absorption. The extraordinary mode also is partly reflected and partly absorbed, but in addition it experiences a mode coupling effect around the ordinary reflection level. The coupling is brought about by collisions, and by the rapid variation of the electron density with height in the ionosphere at this low level. Its effect is to transfer some energy from the extraordinary mode through to that branch of the ordinary that is propagated freely at large electron densities; this branch is the mode of propagation of "whistlers." Judging from measurements on the "whistlers" (Storey [2]), energy is coupled into the ionosphere over an area around the transmitter that is about 1,000 km in radius. Thus, there is a vlf field in the ionosphere above this whole area, due to waves that have penetrated the lower boundary and traveled upwards in the "whistler" mode, together perhaps with some downcoming waves, that would arise from partial reflections at any steep gradients of electron density. In the upper atmosphere, where the medium is probably fairly uniform, there is likely to be just a single upgoing plane wave. Moreover, owing to the strong refraction at the base of the ionosphere, the wave is likely to be traveling more or less radially outwards from the earth (Storey [2]). The structure of the field of this wave must be considered now.

In describing the field structure, the rectangular coordinate system that is shown in figure 2 will be adopted. The coordinate axes are labeled 1, 2, and 3, and in that order form a right-handed set: axis 3 lies along the direction of propagation, that is to say, the wave normal; axis 1 is directed so that the geomagnetic field vector $\underline{H}_0$ lies in the 1–3 plane, and has a positive 1-component: axes 1 and 2 define the plane of the wavefront. The various components of the vectors $\underline{H}$ and $\underline{E}$ in the field of the wave will be indicated thus: $\underline{H}=(H_1, H_2, H_3)$ and $\underline{E}=(E_1, E_2, E_3)$.

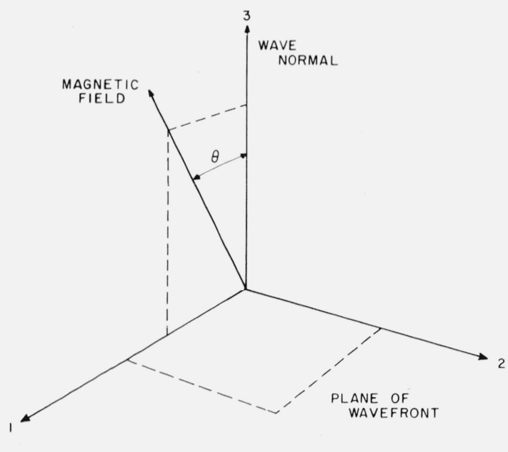


FIGURE 2. *The coordinate system.*

The term "structure" is used here to mean the relative values of the various axial components of the wave field at a point, and how these components vary from point to point and from one instant of time to another. The structure, in this sense, is independent of the amplitude of the wave, because the medium is linear. For determining the electron density, the absolute amplitudes of the field components need not be known, and the only requirement is that they be large enough to overwhelm external atmospheric noise and circuit noise in the receivers. Nevertheless, the absolute amplitudes would be measured in the course of the experiment, and they are of interest for other reasons.

The structure of a plane wave of constant frequency can be described fully by the following quantities:

(a) *Refractive index.* If there is no absorption, all components of the wave vary in time and space as

$$\exp\{j\omega(t-\mu x_3/c)\}$$

where μ is the refractive index, $\omega(=2\pi f)$ is the angular frequency of the wave, t is the time, c is the speed of light in free space, and x_3 is the component of distance along the wave normal.

(b) *Polarization.* The polarization is specified by the ratios of the components of $\underline{H}$ among themselves $(H_1: H_3: H_3)$, and of the components of $\underline{E}$ among themselves $(E_1: E_2: E_3)$.

(c) *Wave admittance.* The wave admittance is the ratio of one component of $\underline{H}$ to one of $\underline{E}$. Here use will be made of the admittance looking in the direction of propagation, which is defined as H_2/E_1.

These quantities are functions of the frequency of the wave, its direction of propagation relative to the earth's magnetic field, and the local properties of the medium. Expressions for their values are provided by the magneto-ionic theory; the expressions quoted here are taken, with some change of notation, from the work of Booker [3]. Rationalized units are used, with the following definitions:

Constants:

e, charge of electron (a negative quantity)
m, mass of electron
ϵ_0, electric permittivity of free space
μ_0, magnetic permittivity of free space.

Variables:

f, frequency of wave
θ, angle between wave normal and earth's magnetic field
N, number density of electrons
H_0, strength of earth's magnetic field.

Characteristic frequencies:

$$\omega_p^2=(2\pi f_p)^2=\left(\frac{e^2}{\epsilon_0 m}\right)N$$

f_p is the electron plasma frequency.

$$\omega_H=2\pi f_H=-\left(\frac{\mu_0 e}{m}\right)H_0$$

f_H is the electron gyrofrequency.

Dimensionless parameters:

$$X=(\omega_p/\omega)^2=(f_p/f)^2$$

$$Y=\omega_H/\omega=f_H/f$$

$$Y_L=Y\cos\theta$$

$$Y_T=Y\sin\theta.$$

With the aid of this notation, the refractive index, polarization, and wave admittance are discussed in sections 2.2, 2.3, and 2.4, respectively. The influence of collisions is ignored, because they are infrequent in the outer ionosphere.

To illustrate the results of the theory, and to examine the adequacy of any approximations, numerical calculations will be made for the following particular set of conditions:

$$f=15.5\text{ kc}\qquad f_p=1.55\text{ Mc}$$

$$\theta=160°\qquad f_H=1.06\text{ Mc}.$$

The parameters of the magneto-ionic theory then have the values

$$X=10^4\qquad Y_L=-64.5$$

$$Y=68.7\qquad Y_T=23.5.$$

Such conditions might apply at a height of about 1000 km above the vlf radio transmitter NSS (geomagnetic latitude 50° N). The wave is assumed to be traveling straight out from the earth in the plane of the magnetic meridian, so that axis 3 of the coordinate system is vertical, while the other two axes are horizontal; axis 1 points towards magnetic north, and axis 2 towards magnetic east. Note that in this instance the angle θ is greater than 90°, since the earth's magnetic field is directed downwards in the northern hemisphere.

2.2. Refractive Index

When collisions are ignored, the general expression for the refractive index becomes

$$\mu^2=1-\frac{2X(1-X)}{2(1-X)-Y_T^2\pm\sqrt{Y_T^4+4Y_L^2(1-X)^2}}\tag{3}$$

in which the positive sign corresponds to the ordinary mode, and the negative to the extraordinary. At wave frequencies that are low compared to both the local plasma frequency and gyrofrequency, the refractive index for the "whistler" (ordinary) mode is usually given quite accurately by the quasi-longitudinal (QL) approximation:

$$\mu^2\cong 1+\frac{X}{|Y_L|-1}.\tag{4}$$

The condition for the approximation to hold is that

$$|(X-1)Y_L|>>\tfrac{1}{2}Y_T^2.\tag{5}$$

This equality is well satisfied in the example chosen, where the error of the QL approximation to the refractive index is less than 0.1 percent.

The further approximation

$$\mu^2\simeq\frac{X}{|Y_L|}=\frac{f_p^2}{ff_H|\cos\theta|}\tag{6}$$

holds if $X>>|Y_L|>>1$. In the example, this approximation gives μ with an error of about 1 percent.

2.3. Polarization

Consider first the polarization in the plane of the wavefront. The ratios of the transverse components of $\underline{H}$ and $\underline{E}$ among themselves are given by the "polarization ratio"

$$R=-H_2/H_1=E_1/E_2\tag{7}$$

$$=-jk\{1\pm\sqrt{1+k^{-2}}\}\tag{8}$$

where

$$k=\frac{g}{1-X}\tag{9}$$

and

$$g=\frac{1}{2}\frac{Y_T^2}{Y_L}=\frac{1}{2}Y\frac{\sin^2\theta}{\cos\theta}\tag{10}$$

and where the positive and negative signs correspond to the ordinary and extraordinary modes respectively.

Now the "whistler" mode is the branch of the ordinary mode on which X is greater than unity, so that the denominator in (9) is negative; hence k is negative or positive according to whether θ is less or greater than 90°. Also, at vlf the quasi-longitudinal condition (5) is satisfied, and this condition implies that $|k|\ll 1$. With these restrictions on the value of k, the polarization ratio for the whistler mode is approximately

$$R\simeq+j\{1+|k|\}\tag{11}$$

where the sign is positive or negative according as θ is less or greater than 90°, that is, according as the direction of propagation makes an acute or obtuse angle with the direction of the earth's magnetic field.

In general, the polarization is elliptical, with the axes of the ellipse parallel to axes 1 and 2 of the coordinate system; this conclusion follows from the fact that R is purely imaginary, so that H_1 and H_2 are in phase quadrature. The vector sum traces out an ellipse with its minor axis parallel to axis 1. The sum of E_1 and E_2 traces out an ellipse of the same shape, but in this case the minor axis is parallel to axis 2.

The axial ratios of these ellipses are very close to unity; in the example, they differ from unity by less than 0.1 percent. So the polarization is almost circular, as it always is under QL conditions.

The sense of rotation around the circle is specified by the sign of the imaginary value of R, a positive sign meaning right-handed rotation. For the whistler mode, the rotation is right-handed when θ is less than 90°, and left-handed when θ is more than 90°. Of course, the senses of rotation for the $\underline{\mathrm{H}}$ and $\underline{\mathrm{E}}$ vectors are the same.

Now consider the longitudinal field components, the components along axis 3. The $\underline{\mathrm{H}}$ vector has no such component; indeed, radio waves in a plasma have the general property that their magnetic field is always wholly transverse to the direction of propagation. Their electric field, however, in general does have a longitudinal component, which is given by

$$E_3/E_2=j\frac{(\mu^2-1)Y_T}{1-X} \tag{12}$$

$$\cong -j\left(\frac{Y_T}{|Y_L|-1}\right)\left(\frac{X}{X-1}\right). \tag{13}$$

The approximation holds under QL conditions. Furthermore, if $X \gg 1$ and $|Y_L| \gg 1$,

$$E_3/E_2 \cong -j\frac{Y_T}{|Y_L|}=-j|\tan\theta|. \tag{14}$$

To the same approximation

$$E_3/E_1 \cong -\tan\theta. \tag{15}$$

These expressions mean that E_3 is in phase with E_1, and of such a magnitude and sign that the sum of these two components is roughly perpendicular to the geomagnetic field; in the numerical example, the angle between the sum and the perpendicular is about 3/4°. Now E_2 also lies in the plane perpendicular to the field, so that the total $\underline{\mathrm{E}}$ vector traces out an ellipse in this plane, with the minor axis parallel to axis 2 of the coordinate system. The explanation of this result is that the electrons in the ionosphere are able to move much more rapidly in the direction of the geomagnetic field than at right angles to it, and at low frequencies they have sufficient time during a period of oscillation to move and neutralize any component of the electric field in this direction. Thus the result is not true in general, but only at low frequencies.

2.4. Wave Admittance

The admittance looking in the direction of propagation is defined as

$$A=H_2/E_1=-H_1/E_2. \tag{16}$$

It is given by

$$A=\mu A_0 \tag{17}$$

where μ is the refractive index, given by (3), and A_0 is the admittance of free space

$$A_0=(\epsilon_0/\mu_0)^{\frac{1}{2}}=(377 \text{ ohms})^{-1}. \tag{18}$$

These expressions show that the field components H_2 and E_1 are in phase. From this and previous results it follows that, in the plane of the wavefront, the rotating magnetic vector leads the projection of the electric vector by 90° when there is propagation along the earth's magnetic field ($\theta<90°$), and lags it by 90° when the propagation is against the field ($\theta>90°$).

2.5. Summary of Approximations

To summarize, the structure of waves in the whistler mode can be represented quite well by the following approximate expressions:

For the refractive index,

$$\mu^2=1+\frac{X}{|Y_L|-1}. \tag{19}$$

For the polarization,

$$H_1=\pm jH_2 \tag{20}$$

$$E_1=\pm jE_2 \tag{21}$$

$$E_3=-E_1\tan\theta \tag{22}$$

where the sign in (20) and (21) is positive or negative according as θ is less or greater than 90°. The assumptions now are that the $\underline{\mathrm{H}}$ vector traces out an exact circle in the plane of the wavefront, and the $\underline{\mathrm{E}}$ vector an ellipse in the plane exactly perpendicular to the earth's magnetic field, such that the projection of the ellipse onto the plane of the wavefront is a circle also. This state of polarization is illustrated in figure 3; here θ has been taken as acute, for the sake of clarity, whereas this angle is obtuse in the worked example.

Finally, for the wave admittance,

$$A=H_2/E_1=\mu A_0 \tag{23}$$

with μ given by (19).

These approximations are used throughout the rest of this paper.

3. Details of the Satellite

3.1. General Description

The satellite would be equipped with two separate antennas for receiving the vlf waves. One would be a loop, responding to the magnetic field of the

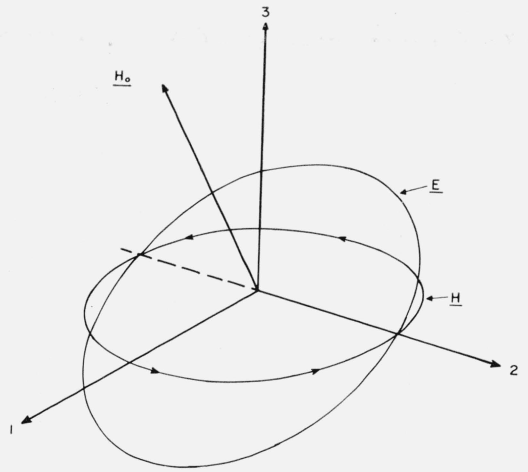


FIGURE 3. *The polarization of waves in the whistler mode.*

wave, while the other would be a dipole, for the electric field. The two antennas would be mounted coaxially, that is to say, the plane of the loop would be perpendicular to the axis of the dipole. With this arrangement, the satellite would receive components of the magnetic and electric vectors resolved in the same direction, that of the common axis of its two antennas. The mechanical design and layout of the satellite are shown in figure 4.

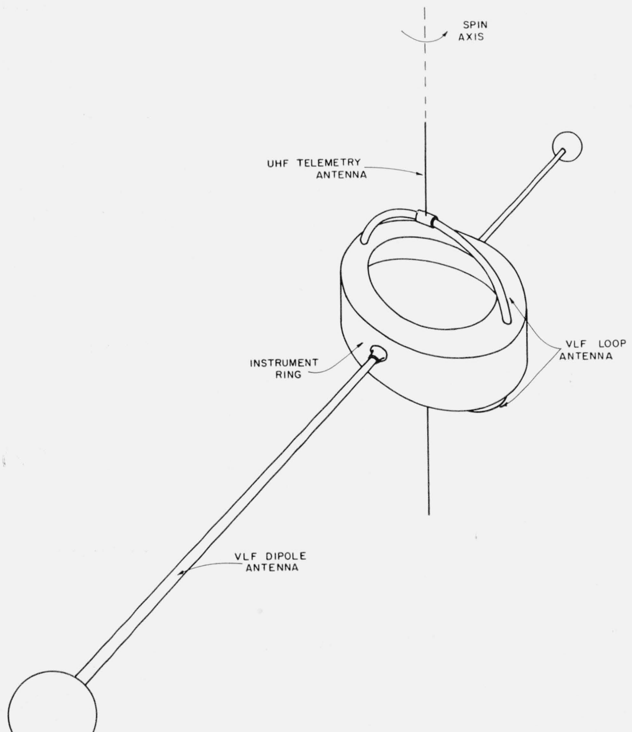


FIGURE 4. *The mechanical design of the satellite (the outer cover is not shown).*

In order to explore the structure of the wave field, the satellite would be set spinning, so that the direction of the antenna axis, and hence the amplitude and phase of the received signals, would change continually. Information is required on the variations of the amplitudes of the signals and of the sign of their correlation. These variations would be easiest to interpret if the spin axis of the satellite was at right angles to the axis of the antennas; then the motion of the antenna axis would be a simple rotation in a plane, and all variations would be periodic. The theory for interpreting these variations, as developed in sections 4 and 5, assumes that this relation between the spin axis and the antenna axis is established exactly.

Details of various aspects of the design of the satellite now follow.

3.2. VLF Antennas

The design of the loop antenna is quite straightforward. At vlf, the most efficient arrangement is an air-cored loop, connected to the receiver through a matching transformer. The basis for optimum design, subject to limitations of weight and space, is provided by the work of Helliwell [4].

The design of the electric dipole is more difficult, since this antenna is in direct contact with the ionospheric plasma, and its properties are affected thereby. The aims of design are to ensure that the effective height of the antenna is the same in the medium as in free space, and that its impedance does not vary excessively as the satellite spins. Consideration of the probable effects of the medium suggest that the best arrangement is one where each arm of the dipole consists of a conducting sphere supported by a long insulating rod; connection is made to the sphere by a thin wire running down the center of the rod. The merits of this arrangement are that the sensitive parts of the antenna are well localized, and are removed as far as possible from the disturbing influence of the body of the satellite. Furthermore, the effective height of this antenna should not be altered by the formation of a "positive-ion sheath" around the satellite (Seddon [5]), though its impedance doubtless would be altered. To minimize the effects of changes of antenna impedance, the input impedance of the receiver should be as high as possible.

3.3. Electronic Equipment

A block diagram of the electronic equipment is given in figure 5. It comprises vlf receivers, detectors, a telemetry system, and a generator for calibration signals.

Two separate vlf receivers are needed, one for each antenna. They should be designed for low noise, stability of phase shift, and wide dynamic range. Their pass-bands should be fairly narrow, to reduce atmospheric noise, but not so narrow that the phases of their outputs fail to reproduce the changes in phase of the received signals that accom-

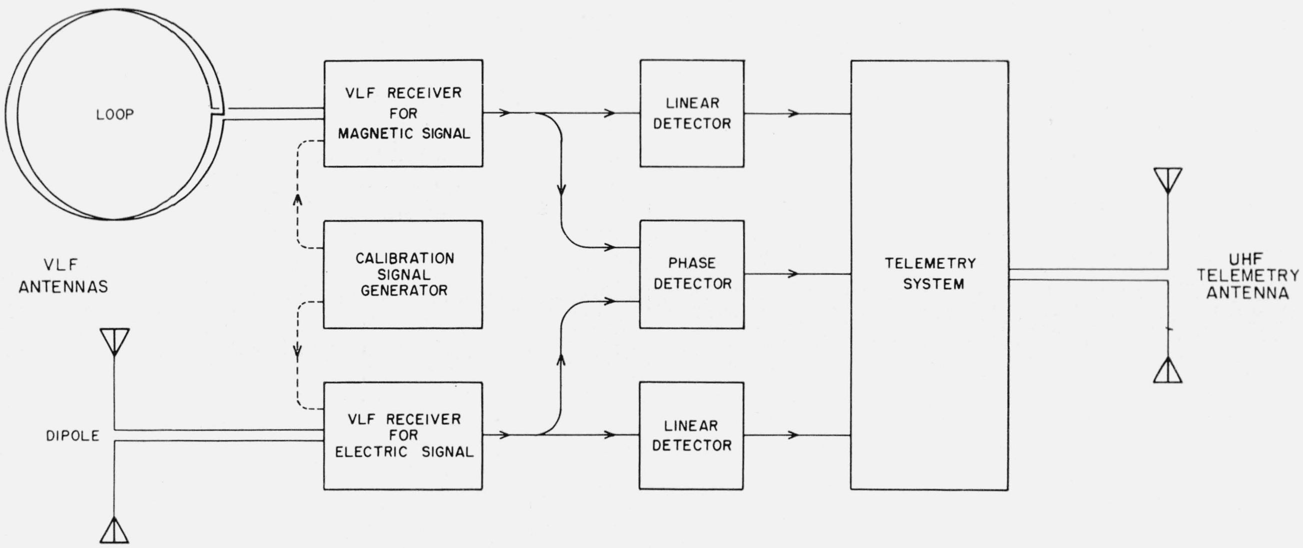


FIGURE 5. *The electronic equipment of the satellite.*

pany the spinning of the satellite. They should incorporate negative feedback to improve both the stability and the dynamic range.

The amplitude of the signal that emerges from each receiver is measured by a detector, the output from which is telemetered to ground. Also, the two signals are compared in a phase detector, and the polarity of the output relayed to ground on a third telemetry channel. The ground station records these three data, which are proportional to the amplitudes of the received signals and the sign of their correlation. The factors of proportionality would have to be determined by calibration of the whole system prior to flight. Also, to confirm that the system is working properly during flight, it should be calibrated automatically from time to time by injecting standard signals at the inputs to the receivers.

3.4. Orbital and Spin Motion

The orbit of the satellite should be eccentric, so that measurements are obtained over a range of height; an apogee height of 1 to 2000 km and a perigee height of about 300 km would be suitable. At this height, the perigee would lie below the maximum of the $F2$ layer from time to time, and then the measurement of electron density could be checked by comparison with ionospheric soundings. There are no special requirements for the inclination, except that it must be large enough to allow the orbit to pass over the transmitter, and that the value 63.4°, at which the line of apsides does not precess, should be avoided.

The necessary rate of spin is determined by the requirement that the satellite, in moving along its orbit, should not experience much change in local electron density during one rotation. If the distribution of electrons in the outer ionosphere is assumed to contain no irregularities smaller in scale than 1 km, and the speed of the satellite is taken to be roughly 5 kmps, then the rate of spin should not be less than 5 rps. If the smallest irregularities actually are larger than assumed here, then proportionately lower rates would be acceptable.

The absolute direction of the spin axis in space should be chosen so that the satellite is oriented favorably for making measurements at the times when it passes over the transmitter. At such times, the spin axis should be more or less horizontal, and should lie roughly in the plane of the orbit. It has to be horizontal so that the spinning motion will bring the antenna axis through the vertical, where the electric dipole is coupled strongly to the field component E_3; for it is on this coupling that the measurement of the angle θ depends (see secs. 4 and 5). The spin axis also has to lie close to the plane of the orbit, to ensure that the elements of the electric dipole do not pass behind the body of the satellite, where they might encounter a wake of disturbed ionization. However, it is doubtful whether such a wake would be formed in the outer ionosphere, where the mean free path of the particles is much larger than the dimensions of the satellite, so this requirement may not be crucial.

3.5. Mechanical Design

Bracewell and Garriott [6] have considered the influence of mechanical design on the free rotation of a satellite. They point out that the characteristics of the free rotation are determined by the relative values of the principal moments of inertia, and that if one moment is much larger than the other two, then the satellite will tend to spin about the axis associated with that moment. Therefore, the mass of the satellite should be arranged so that its moment of inertia is greatest about the desired axis of spin. If a satellite of this type was set spinning initially in a direction slightly incorrect, then its subsequent motion would include a wobble at twice the spin frequency. This wobble could be damped out by such means as suitably disposed tubes of viscous fluid, whereupon the satellite would be left spinning smoothly about the correct axis.

In the present case, the main aim is to ensure that the spin axis is transverse to the axis of the vlf antenna system. If this were the only aim, and the particular location of the spin axis in the transverse plane could be arbitrary, then it would follow that the satellite should be long and thin, extended along the axis of the vlf antenna system. However, it is better to fix the direction of the spin axis in the frame of the satellite completely, so as to provide a stable direction for the antenna of the telemetry system; to this end the mass should be distributed as in a disk, to make the moment of inertia greatest about the chosen direction.

Once the spinning motion has been established about the desired axis in the frame of the satellite, the absolute direction of this axis in space should remain fixed by gyroscopic action. However, if any force were to exert a systematic couple on the satellite as it moved along its orbit, then the spin axis would precess. Possibly the forces due to atmospheric drag, or to electric currents induced in the body of the satellite as it spins in the earth's magnetic field, could exert such couples. The couple due to atmospheric drag could be eliminated by designing the satellite with reflection symmetry about a plane perpendicular to the spin axis, while induced currents could be prevented by making the body of the satellite in several parts that were insulated from one another.

A mechanical design that embodies these principles is illustrated in figure 4; in this drawing, the protective outer shell of the satellite is supposed to have been removed.

4. Received Signals and Their Variations

4.1. General Behaviour

This section deals with the signals received on the two vlf antennas, and how they vary as the satellite spins. Specifically, expressions are derived for the amplitudes of the two signals and for the sign of their correlation, since this is the information that is relayed from the satellite to the ground.

Now the loop antenna and the electric dipole antenna are set on a common axis (sec. 3), so the signals that they respond to are, respectively, the components of $\underline{H}$ and of $\underline{E}$ along this axis; call these components H_a and E_a. Take the coordinate system of figure 2, and let the instantaneous direction of the antenna axis be specified by its direction cosines a_1, a_2, and a_3 relative to the coordinate axes. Then the received signals are

$$\textit{magnetic signal: } H_a = a_1 H_1 + a_2 H_2 \qquad (24)$$

$$\textit{electric signal: } E_a = a_1 E_1 + a_2 E_2 + a_3 E_3. \qquad (25)$$

It is convenient to express H_a in terms of H_2 alone, and E_a in terms of E_1, while recalling that the field components H_2 and E_1 are in phase, and that their ratio is the wave admittance (sec. 2.4.). In eliminating the other components, the structure of the field will be assumed to be that given by the approximations of section 2.5. After making the appropriate substitutions, the following expressions are obtained for the received signals:

$$\textit{magnetic signal: } H_a = \{a_2 \pm j a_1\} H_2 \qquad (26)$$

$$\textit{electric signal: } E_a = \{a_1 - a_3 \tan\theta \mp j a_2\} E_1. \qquad (27)$$

The sign depends on the direction of propagation with respect to the geomagnetic field; the upper sign applies when there is a component of propagation along the field.

As the satellite moves along its orbit, and spins about its axis, the received signals vary in amplitude and phase. The variations due to the orbital motion are slow and systematic, while those due to the spinning motion are relatively rapid and are, for the most part, periodic. In considering the variations over a single rotation period of the satellite, only the effects of spin need to be considered.

One general feature of these variations can be seen immediately; it is that they all must be periodic at twice the spin frequency, since a reversal of the direction of the antenna does not alter the amplitudes of the received signals, nor their relative phase (except possibly by 360°).

To find the exact form of the variations, expressions must be obtained for the direction cosines of the antenna axis as functions of time.

4.2. Direction of the Antenna Axis

The geometry of the situation is represented in figure 6. Figure 6a is a perspective drawing that gives the notation for the angles that specify the directions of the various vectors. In figure 6b the same information is given in the form of a stereographic projection onto the plane of the wavefront. Details of this projection can be found in any standard work on crystallography, such as that of Tutton [7] or Bunn [8]. It maps the surface of a sphere into the interior of a circle on a plane, and has the property that arcs of great circles on the sphere are represented by arc of circles on the plane; those representing great circular arcs on the upper hemisphere are drawn as solid lines, while the projections of arcs on the lower hemisphere are drawn as broken lines. Points in the upper hemisphere are represented by solid dots, and in the lower hemisphere by small open circles. In the present application, the orientations of vectors and planes in space are represented by the projections of their intersections with an imaginary sphere that is concentric with the satellite.

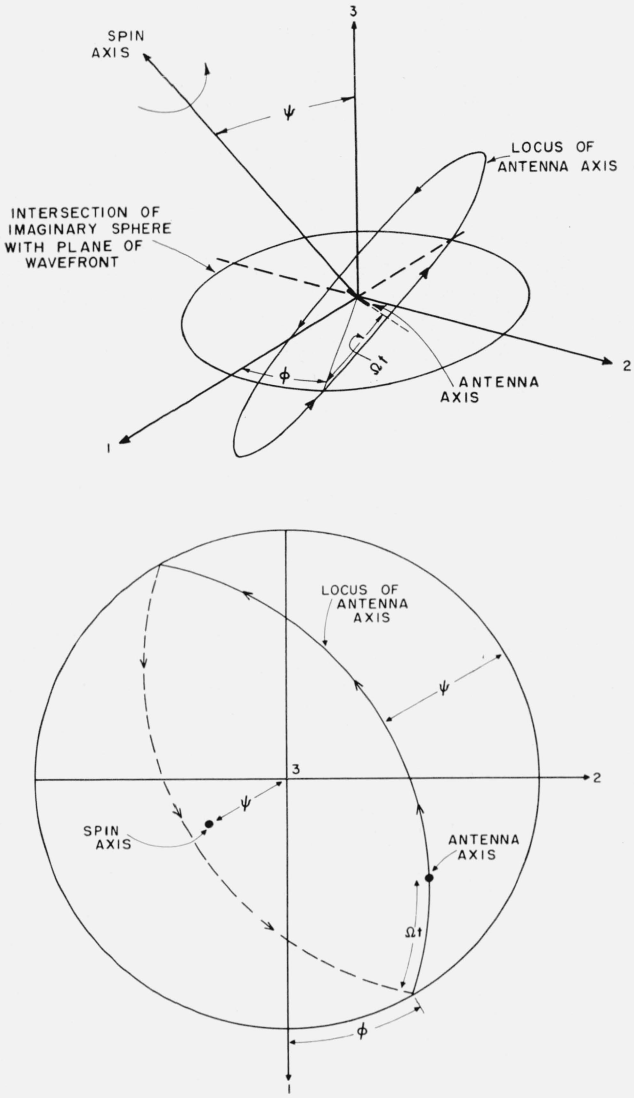


FIGURE 6. *The geometry of the spinning motion.*

(a) In perspective, (b) on a stereographic projection.

The direction of the spin axis of the satellite is taken as that direction about which the rotation is right-handed. It is assumed to be perpendicular to the antenna axis, as specified in section 3. The angle that it makes with axis 3 of the coordinate system is called Ψ, and the angular velocity of the spinning motion is called Ω.

The time t is measured from an instant when the antenna axis lies in the plane of the wavefront (the 1–2 plane). The position of the antenna axis in this plane is then given by the angle ϕ, which is measured from axis 1 to that end of the antenna axis that is going upwards, toward axis 3, at this particular instant. It is reckoned positive if it is measured toward axis 2, and negative otherwise; thus, in figure 6, ϕ is positive.

In these terms, the three direction cosines are given by the expressions

$$a_1=\cos\phi\cos\Omega t-\sin\phi\cos\Psi\sin\Omega t \tag{28}$$

$$a_2=\sin\phi\cos\Omega t+\cos\phi\cos\Psi\sin\Omega t \tag{29}$$

$$a_3=\sin\Psi\sin\Omega t. \tag{30}$$

The angles Ψ, ϕ, and θ define completely the relative directions of the spin axis, the wave normal, and the geomagnetic field. They vary progressively as the satellite moves along its orbit, but these variations are slow compared to the spinning motion itself.

4.3. Amplitude of the Magnetic Signal

The real and imaginary parts of the magnetic signal, relative to H_2 as the standard of phase, may be found by substituting the expressions for the direction cosines into (26). Their values, normalized with respect to H_2, are

$$\mathrm{R}\{H_a/H_2\}=a_2=\sin\phi\cos\Omega t+\cos\phi\cos\Psi\sin\Omega t \tag{31}$$

$$\mathrm{I}\{H_a/H_2\}=\pm a_1=\pm(\cos\phi\cos\Omega t-\sin\phi\cos\Psi\sin\Omega t) \tag{32}$$

Thus the square of the normalized peak amplitude of H_a is

$$|H_a/H_2|^2=a_1^2+a_2^2 \tag{33}$$

$$=1-\sin^2\Psi\sin^2\Omega t \tag{34}$$

Note that this expression does not involve either ϕ or θ. It is made up of a constant part, and a part that varies sinusoidally at twice the spin frequency. The normalized peak amplitude, given by the square root of (34), takes its maximum value of unity when the antenna lies in the plane of the wavefront ($\Omega t=0, \pi$), and its minimum value of $|\cos\Psi|$ at points halfway in between ($\Omega t=\pi/2, 3\pi/2$).

4.4. Amplitude of the Electric Signal

Similarly, the real and imaginary parts of the electric signal E_a, normalized with respect to E_1, are

$$\mathrm{R}\{E_a/E_1\}=a_1-a_3\tan\theta$$
$$=\cos\phi\cos\Omega t-(\sin\phi\cos\Psi+\tan\theta\sin\Psi)\sin\Omega t \tag{35}$$

$$\mathrm{I}\{E_a/E_1\}=\mp a_2=\mp(\sin\phi\cos\Omega t+\cos\phi\cos\Psi\sin\Omega t). \tag{36}$$

Hence the square of the normalized peak amplitude is

$$|E_a/E_1|^2=(a_1-a_3\tan\theta)^2+a_2^2 \tag{37}$$

$$=1-(1-\tan^2\theta)\sin^2\Psi\sin^2\Omega t-2\tan\theta\sin\Psi\sin\Omega t(\cos\phi\cos\Omega t-\sin\phi\cos\Psi\sin\Omega t. \tag{38}$$

This is more complicated than the corresponding expression for the magnetic signal, because the electric field has a longitudinal component. However, it also represents the sum of a constant term, and a sinusoidal term of angular frequency 2Ω. The normalized amplitude of E_a also is equal to unity when $\Omega t=0$ or π, while its value at the points halfway in between is

$$|E_a/E_1|^2=\cos^2\Psi+\tan\theta\sin\phi\sin2\Psi+\tan^2\theta\sin^2\Psi. \quad (39)$$

In general, however, these are not its maximum and minimum values. The times of stationary amplitude are separated by one quarter of a rotation, and are given by

$$\Omega t=\tfrac{1}{2}\cot^{-1}\{\sec\phi\,(\sin\phi\cos\Psi-\cot2\theta\sin\Psi)\}. \quad (40)$$

The question of which time corresponds to the maximum may be decided by inspecting the geometry of the situation. In most situations, maximum amplitude occurs near to the time when the antenna axis passes through the plane perpendicular to the geomagnetic field, because this is the plane of the electric vector.

4.5. Sign of the Correlation

Here the term "correlation" is being used rather loosely, to mean the average of the instantaneous product of the signals H_a and E_a, taken over one complete cycle of the radiofrequency. If the two signals were in phase, its value would be $\frac{1}{2}|H_a|\,|E_a|$. In general, it is

$$\overline{H_aE_a}=\tfrac{1}{2}[\mathrm{R}\{H_a\}\mathrm{R}\{E_a\}+\mathrm{I}\{H_a\}\mathrm{I}\{E_a\}] \quad (41)$$

$$=\tfrac{1}{2}|H_a|\,|E_a|\cos(\alpha_H-\alpha_E) \quad (42)$$

where α_H and α_E are the phases of the magnetic and electric signals, respectively, measured in relation to the phase of H_2 or E_1. From (42), it appears that the correlation is positive when $(\alpha_H-\alpha_E)<90°$, negative when $(\alpha_H-\alpha_E)>90°$, and zero when the two signals are exactly in phase quadrature.

The correlation could be determined by applying the voltages that represent H_a and E_a to the inputs of an electronic multiplier, and then smoothing the output with a time constant that is long compared to the period of the radiofrequency, but short compared to the period of the satellite spin. The proposed satellite would probably use some simpler type of phase detector, rather than a true multiplier. The output from such a detector would not necessarily be proportional in magnitude to the correlation, but it would have the same sign. For the present purpose, only the sign is of interest, so it is satisfactory to discuss the correlation.

The correlation of the received signals is given as a function of the direction cosines by the expression

$$\frac{\overline{H_aE_a}}{\frac{1}{2}|H_2|\quad|E_1|}=-a_2a_3\tan\theta \quad (43)$$

which contains the correlation of H_2 and E_1 as a normalizing factor. From this expression it appears that the sign of the correlation depends jointly on two circumstances: first, whether θ is greater or less than 90°, and second, which of four quadrants of the sphere contains the direction of the antenna axis. These quadrants are bounded by the 1–2 plane and the 1–3 plane (fig. 7a). When the antenna axis lies in either of these planes, passing from one quadrant to another, the correlation is zero. The rules for the signs are given in figure 7b.

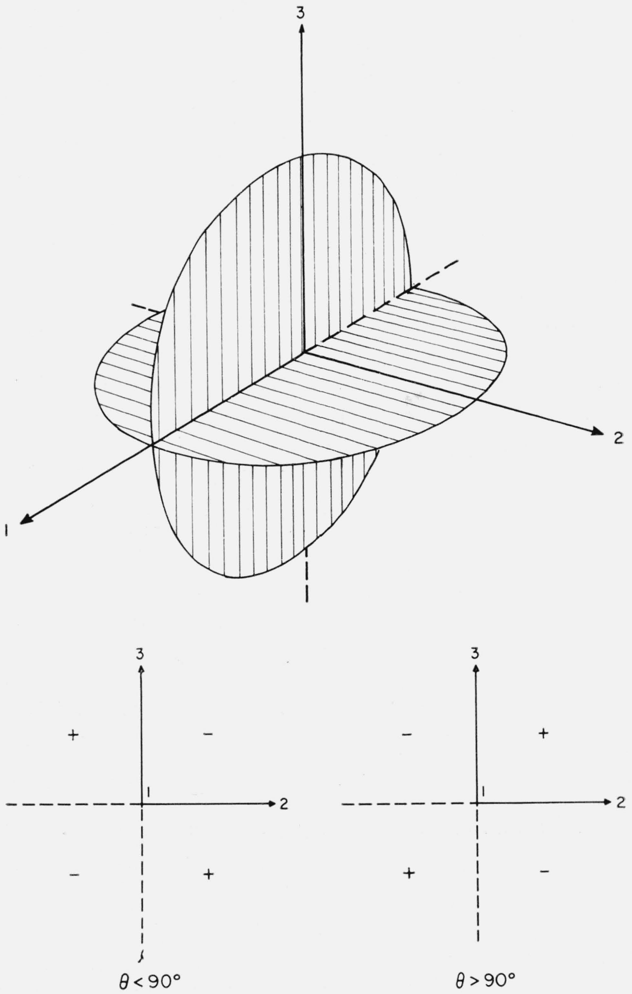


FIGURE 7. *How the sign of the correlation is governed by the direction of the antenna axis.*

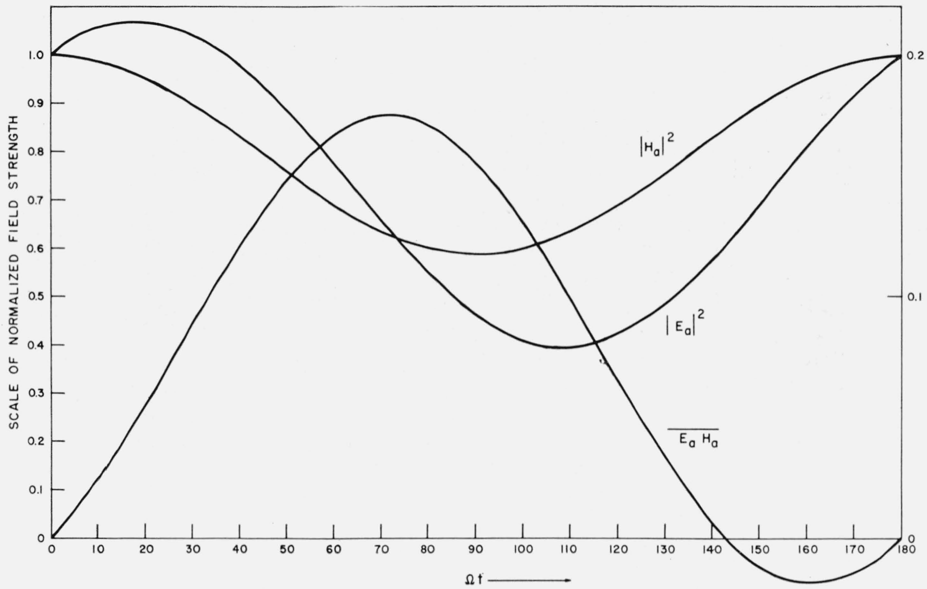
(a) The four quadrants of the sphere, within each of which the sign is constant (b) the rules for the sign.

These results can be explained as follows: The departures of the relative phase of H_a and E_a from the quadrature condition are due entirely to the longitudinal component E_3 of the electric field. If this component was absent, as indeed is the case when $\theta=0$, then the total $\underline{\mathrm{E}}$ vector would be wholly transverse and would execute the same motion as the $\underline{\mathrm{H}}$ vector, with a phase lag or lead of 90°; the received signals H_a and E_a, would then be in phase quadrature for all directions of the antenna axis. When E_3 is finite ($\theta \neq 0$), the signals are exactly in quadrature in just two special situations: the first is the situation where the antenna axis lies in the 1–2 plane ($\Omega t = 0, \pi$), so that the electric dipole is not coupled to E_3: the second is the situation where the antenna axis lies in the 1–3 plane, so that the dipole is not coupled to E_2. Then, since E_3 is in phase with E_1 (see sec. 2.5), the electric signal has the same phase as it would if E_3 were absent. The times of passage of the antenna axis through the 1–3 plane are given by

$$\Omega t = \tan^{-1}\{-\sec\Psi \tan\phi\}. \qquad (44)$$

4.6. An Example of the Variations

The example of figure 8 shows how the properties of the received signals are expected to vary during one half-revolution of the satellite, for a particular orientation of the spin axis. Figure 8a is a graph of the normalized values of the signal strengths $|H_a|^2$ and $|E_a|^2$, and of the correlation $\overline{H_a E_a}$, as observed at the satellite; note that these three quantities all vary sinusoidally. Figure 8b shows how the corresponding records of $|H_a|$, $|E_a|$, and the sign of $\overline{H_a E_a}$ would appear at the ground station.



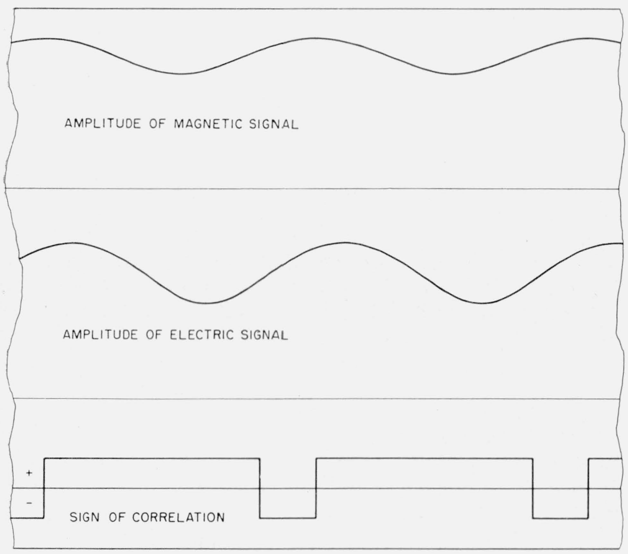


FIGURE 8. *The variations of the properties of the received signals, for the particular case where* $\Psi = 40°$, $\phi = 30°$, *and* $\theta = 160°$.

(a) The variations of $|H_a|^2$, $|E_a|^2$, and $\overline{E_a H_a}$, (b) the corresponding records.

5. Analysis of the Records

5.1. Wave Admittance, Refractive Index, and Electron Density

Records such as that of figure 8b contain almost all the information that is needed for calculating the electron density of the ionosphere in the neighbourhood of the satellite. The calculation, though, is somewhat indirect.

The quantity that can be obtained most directly from the records is the local wave admittance, A. This, it will be recalled, is the ratio of the field components H_2 and E_1. However, since the polarization in the plane of the wavefront is circular, both for $\underline{H}$ and for $\underline{E}$ (sec. 2.3), the wave admittance is given also by the ratio of the electric signal E_a, taken at any instant when the antenna axis passes through this plane. Such instants can be recognized from the fact that the magnetic signal is then at a maximum.

Given the wave admittance, the local refractive index μ can be calculated immediately from (23). Thus

$$\mu = A/A_0. \qquad (45)$$

The relationship between μ and the local electron density N is contained in (19). After rearrangement to express N as a function of the other variables, this equation reads

$$N=\left(\frac{\epsilon_0 m}{e^2}\right)\omega\left\{H_0\left|\frac{\mu_0 e}{m}\cos\theta\right|-\omega\right\}(\mu^2-1). \tag{46}$$

A convenient approximation to this expression may be obtained rearranging (6) similarly; it is

$$N=\frac{\omega}{c^2|e|}H_0|\cos\theta|\mu^2 \tag{47}$$

in which c is the speed of light. Clearly, to calculate the electron density from a given value of μ, it is necessary to know also the local magnetic field strength H_0, and the angle θ between the direction of propagation and the field.

The magnetic field strength cannot be determined from the records. If the position of the satellite is known, the field strength at this point can be estimated roughly from the known distribution of field on the earth's surface, by extrapolating upwards. It would be better, however, to measure the field directly by means of a magnetometer borne in the satellite; this possibility is discussed in section 6.

The angle θ can be obtained from the details of the variations of the recorded data. The argument, which is rather circuitous, depends on the following results from the previous section: (a) The variations of the amplitude of the magnetic signal are governed by Ψ only (34), (b) the times of change of the sign of the correlation are governed by Ψ and ϕ, but are independent of θ (44), (c) the variations of the amplitude of the electric signal depend on all three angles, including θ (38).

Evidently, by analysing these three variations in the given sequence, it is possible first to calculate the angle Ψ, then the angle ϕ, and finally to obtain the angle θ. These calulations are discussed in the rest of this section. The discussion is illustrated by analysing the record of figure 8b.

5.2. The Angle Ψ

Ψ is the angle between the spin axis and the wave normal (sec. 4.2), and its range of possible values is 0 to 180°. The value of Ψ may be obtained from the observed variations of the amplitude of the magnetic signal; in fact, from (34), it is given by the ratio of the extreme values of the amplitude. That is,

$$|\cos\Psi|=\frac{|H_a|\ \text{min}}{|H_a|\ \text{max}}. \tag{48}$$

Hence Ψ is determined, though with ambiguity. For a given value of $|\cos\Psi|$, there are two possible values of Ψ; their sum is 180°.

Thus, in the example of figure 8, the ratio of the minimum and maximum values of $|H_a|$ is 0.766; on this evidence, Ψ is 40° or 140°. The possible locations of the direction of the spin axis are shown on the stereographic projection of figure 9.

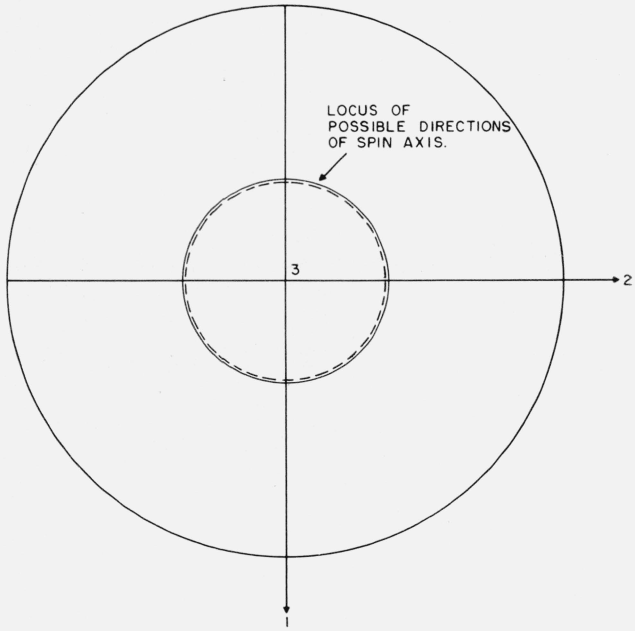


FIGURE 9. *The possible directions of the spin axis, for a given value of* $|\cos\Psi|$.

5.3. The Angle ϕ

ϕ is measured from axis 1 to the upgoing end of the antenna axis, at an instant when the latter passes through the 1–2 plane (sec. 4.2); its possible values range from −180° to +180°. The value of ϕ may be obtained from the times of change of the sign of the correlation. Recall, from section 4.5, that the correlation changes sign at two instants in each half-revolution of the satellite: the first, which can be identified readily since it is also the instant when $|H_a|$ is a maximum, occurs when the antenna axis passes through the 1–2 plane ($\Omega t=0$): the second occurs when it passes through the 1–3 plane. If the observed time delay between the two instants is expressed, in terms of the period of rotation of the satellite, as a phase angle Ωt, then ϕ can be calculated by using (44). After rearrangement, this equation gives

$$\tan\phi=-\cos\Psi\tan\Omega t. \tag{49}$$

Altogether, there are now four possibilities, since for each value of ψ there are two values of ϕ that satisfy (49), and ψ itself has two possible values. The problem now is how to resolve this ambiguity and select the correct combination of ψ and ϕ.

5.4. Resolution of the Ambiguity

If their true values are denoted by the symbols Ψ_t and ϕ_t, the four combinations of Ψ and ϕ that would be possible in the light of the present evidence are the following:

(a)	ψ_t	ϕ_t
(b)	ψ_t	$\phi_t-180°$
(c)	$180°-\psi_t$	$180°-\phi_t$
(d)	$180°-\psi_t$	$-\phi_t$.

Figure 10 illustrates these four possibilities in the analysis of the record of figure 8b; figure 10a shows the possible directions of the spin axis, and figure 10b the corresponding loci of the antenna axis.

What allows this ambiguity to be resolved is the fact that the angle θ is not completely unknown at the outset of the experiment. Thus, in the example, θ is certainly greater than 90°, because the transmitter is located in the northern hemisphere where the lines of magnetic force point downwards, while the waves are traveling upwards. Granted this knowledge, the alternatives (b) and (d) can be eliminated by considering the direction in which the correlation changes sign at $\Omega t=0$; in the example, the sign changes from negative to positive at this instant. Now, since θ is greater than 90°, the sign is governed by the rule on the right-hand side of figure 7b. Applying this rule, it appears that at the instant $\Omega t=0$ the antenna axis is entering the upper right-hand quadrant of the sphere, and hence its locus must be either (a) or (c).

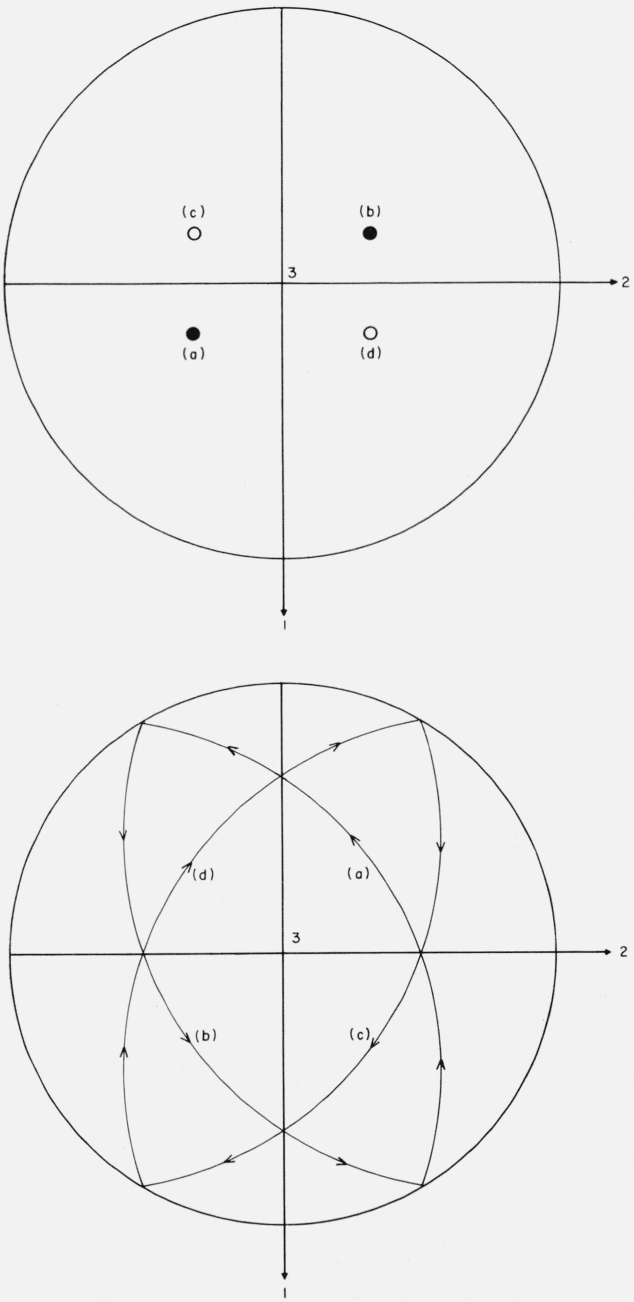


FIGURE 10. *The fourfold ambiguity in the determination of the direction of the spin axis.*

(a) The possible directions of the spin axis, (b) the corresponding loci of the direction of the antenna axis.

The angle θ can be estimated more closely by assuming that the wave normal is directed to the vertical by refraction (see sec. 2.1), and by making use of the known value of the magnetic dip; at a guess, such an estimate would lie within 10° of the true value. If so, then the choice between the two remaining alternatives can be made by considering the variations of the amplitude of the electric signal. The expected variations of $|E_a|$ are calculated for the two possible loci, using the estimated value of θ, and they are compared with the observed variation. The two calculated variations are mutually opposite, so that usually one should fit the record much better than the other, in spite of the inaccuracy of the estimate of θ. Figure 11 shows the comparison for the given example, taking 150° as the estimate of Y. This estimate is in error by 10°, but nevertheless it is clear that (a) is the correct locus. Thus, the ambiguity is resolved.

In practice, the direction of the spin axis in space should be fairly well known from the circumstances of the launching, so that the ambiguity could be resolved immediately, and the chain of reasoning described above would be superfluous.

5.5. The Angle θ

θ is the inclination of the wave normal to the direction of the earth's field, and its possible values lies between 0° and 180°. Now that Ψ and ϕ are known, θ may be found by adjusting its value until the observed and calculated variations of the amplitude of the electric signal agree most closely.

When θ is known, the local electron density N can be calculated from the equations given in section 5.1.

Since θ is estimated from the details of the modulation of the received signals by the spinning motion of the satellite, obviously the depth of this modulation is one of the main factors that control the accuracy of the estimate. Now the depth of the modulation is governed chiefly by the inclination of the spin axis to the wave normal, that is to say, by the value of Ψ. The most favorable condition is that $\Psi \simeq 90°$, which gives 100 percent modulation of the magnetic signal; in section 3.4, the absolute direction of the spin axis in space was chosen so as to fulfill this condition when the satellite passes over the transmitter. On the other hand, the least favorable condition is that $\Psi \simeq 0°$, where neither the magnetic nor the electric signal is modulated. If this latter condition were to arise, then θ could not

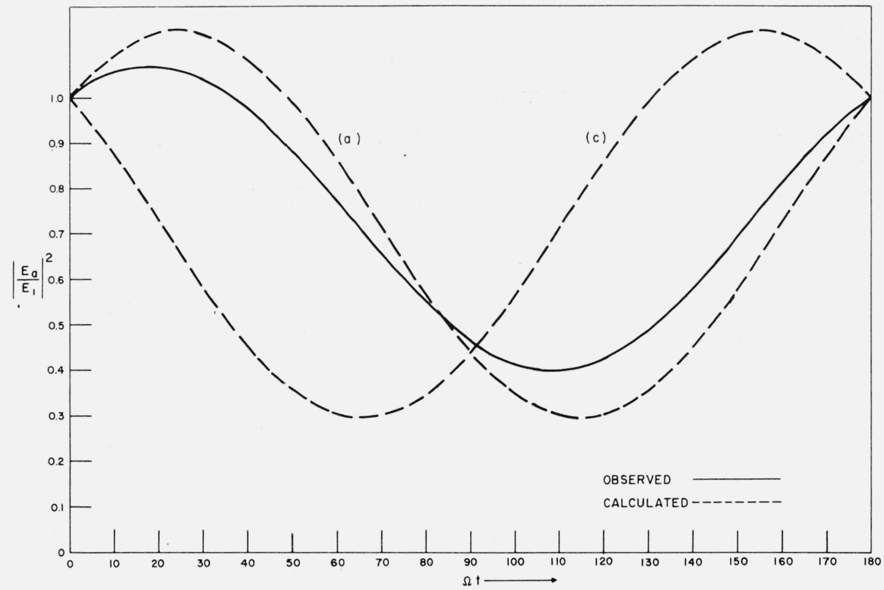


FIGURE 11. *Comparison of the "observed" and calculated variations of the strength of the electric signal; in the calculations, it was assumed that $\theta = 150°$.*

be determined from the records. A theoretical estimate of θ, obtained as described in section 5.4 above, would then have to be used in calculating the electron density N. Note therefore, from (37), that the inferred value of N is roughly proportional to $|\cos\theta|$, so that it should be insensitive to errors in θ around the assumed value of 160°.

6. Integrated VLF Experiment

In the experiment as proposed so far, the satellite measures the least number of quantities that suffice to determine the electron density. Much more could be learned about the ionosphere if, at the same time, measurements were made of certain other quantities; namely, (a) The aspect of the satellite, (b) the local magnetic field, (c) a component of the vlf wave field ($\underline{H}$ or $\underline{E}$) parallel to the spin axis. These measurements will be discussed in turn.

(*a*) *Aspect.* The instantaneous aspect of the satellite could be measured by photoelectric devices which sense the direction of the sun, the moon, or the horizon of the earth. Such measurements would determine the absolute direction of the spin axis in space.

This information, when combined with the record of the amplitude of the magnetic signal H_a, would determine the absolute direction of the wave-normal.

(*b*) *Magnetic field.* Three components of the local magnetic field could be measured by a pair of magnetometers. One, a fluxgate, would measure the component of field along the spin axis. The other, an induction magnetometer, would be set up in the plane transverse to the spin axis; it would make use of the spin of the satellite to induce an alternating voltage that would represent the two components of the magnetic field in the transverse plane. A magnetometer of this type has been used successfully in the lunar probe "Pioneer I" (Sonnett [9]).

From the measurements of the three components, the total local field strength (H_0) can be calculated. These same measurements, when combined with the measurement of aspect, also determine the absolute direction of the field.

Having determined the absolute directions both of the field and of the wave-normal, the angle (θ) between them can be obtained directly. This value is likely to be more accurate than that calculated by the method of section 5, where much use was made of the sign of the correlation between H_a and E_a; this measurement is apt to be sensitive to errors caused by small phase-shifts in the receivers, and by noise also.

It will be recalled that the values of the H_0 and θ are needed for calculating the local electron density from the measured wave admittance (sec. 5.1).

(c) *Component of vlf field along the spin axis.* Finally, following a proposal by Helliwell [10], the satellite could be equipped with a third vlf receiver, connected to an antenna that was alined with the spin axis. The signal received on such an antenna would not be affected by the spinning motion. Its phase (relative to a stable local standard carried in the satellite), and also its amplitude, would be telemetered back to the ground.

The rate of change of phase, when corrected for the change in path length for the telemetry transmission, yields the Doppler frequency shift of the vlf signals as observed at the satellite. This frequency shift is given by

$$\Delta f = (V \cos \delta)/\lambda \qquad (50)$$

where V is the speed of the satellite along its orbit, δ is the angle between the orbit and the wave-normal, and λ is the wavelength in the medium. Now the orbit is presumed known, and the local direction of the wave normal is measured, so the values of V

and δ are both data. Hence, if Δf is appreciable, λ can be calculated from its observed value. Now

$$\lambda = \lambda_0/\mu \tag{51}$$

where λ_0 is the wavelength in free space. Given λ and λ_0, μ can be calculated. Thus the Doppler shift would provide another measure of the local refractive index, and hence of the electron density; this measurement would check the value calculated from the wave admittance (sec. 5.1).

If the vlf transmissions were modulated in some way, the modulation would be received at the satellite with an appreciable delay, because the group velocity of the whistler mode is rather low. The delay could be measured most easily on this third vlf channel, where the received signals are not modulated in addition by the spin of the satellite. From the observed delay, it is possible to determine the integral of the plasma frequency with respect to height up to the level of the satellite (Helliwell [10]).

The proposal is made that there should be an integrated vlf satellite experiment, incorporating all the features described above. The following information about the ionosphere would be obtained at each point on the orbit: (a) Local strength of earth's magnetic field, (b) local direction of field, (c) local strength of vlf waves, (d) local direction of propagation, (e) local electron density, (f) height integral of plasma frequency.

The changes in these quantities, as the satellite moves along its orbit, would be expected to show both systematic and random components. The systematic components would be related to the bulk properties of the ionosphere, while the random variations would give information on ionospheric irregularities.

7. Discussion

The main proposal made in this paper is for a satellite that would determine local electron density in the ionosphere, by exploring the structure of a vlf wave field set up by a transmitter on the ground. The following sources of difficulty are anticipated:

(a) *Noise.* One limit to the accuracy of the experiment will be set by noise. Judging by previous experience at vlf, there should be no difficulty in making the receivers sufficiently quiet that the limit is set by the external atmospheric noise picked up on the antennas, which is unavoidable. However, the actual level of atmospheric noise within the ionosphere is hard to predict. At the ground, vlf noise is observed to come both from the lower atmosphere, where it is produced by lightning strokes, and also from the ionosphere, where its mode of origin is uncertain (Watts [11]). Within the ionosphere, presumably, the former source of noise is less effective and the latter more so. It is hoped that the vlf satellite experiment already planned by Helliwell [10] will provide data on the noise level in the ionosphere.

(b) *Multiple waves.* The theory has assumed that the vlf field in the ionosphere is that of a single plane wave (sec. 2.1). Thus the experiment would not be expected to work near the conjugate point, where the downcoming direct waves would be mixed with upgoing waves reflected from the base of the ionosphere. Difficulty might be experienced even near the transmitter, if the downcoming wave returned from the opposite hemisphere was at all comparable in strength to the upgoing primary wave. This difficulty could be overcome either by working at a higher frequency, which would be more heavily absorbed, or by modulating the transmissions in such a way that the direct signal and the echo never overlap one another; for 15.5 kc signals from the station NSS, the time of travel over the whistler route and back would be about 1 sec (Helliwell and Gehrels [12]), so that a suitable form of modulation would be a ½ sec pulse repeated every 2 sec. A potentially more serious problem is presented by ionospheric irregularities, which would distort the upgoing wavefronts. If the curvature of the wavefronts was appreciable in terms of a wavelength, then the plane-wave theory would not apply. The significance of the measurement of wave admittance in these circumstances has not yet been investigated.

(c) *Effect of the medium on the electric dipole antenna.* There is doubt as to how well the electric dipole antenna will perform when immersed in the anisotropic plasma of the ionosphere. The design of antenna that was described in section 3.2. is intended to minimize the adverse effects of the medium. If such effects persist nevertheless, their presence could be detected from the fact that they will almost certainly cause the variation of $|E_a|^2$ with time to depart from the pure sinusoid predicted by the theory (sec. 4.4).

(d) *Disturbance of the medium by the satellite.* There is a chance that the satellite, by virtue of its rapid orbital motion, may disturb the medium seriously and thus alter the distribution of magnetic and electric fields in its immediate neighbourhood. The purely hydrodynamic disturbance is likely to be small, and confined to the region behind the satellite (sec. 3.2). However, Kraus and Watson [13] have pointed out that the satellite, in its passage through the ionosphere, acquires an electrostatic charge, and that the field of this charge should produce a hydromagnetic disturbance ahead of the satellite. It is uncertain how serious this effect will be. In this connection, it is worth noting that instruments have been developed to measure the state of charge of a satellite (Krassovsky [14]), and doubtless one could be arranged to control some discharging device so as to maintain a state of electrical neutrality, and thus eliminate the effect.

(e) *Incorrect spinning motion.* Finally, there is the possibility of failure to establish the desired spinning motion: the satellite might be launched either with no spin, or spinning too slowly, or again the spin might be fast enough but accompanied by a spurious wobble that would take some time to suppress (see secs. 3.4 and 3.5). If the spin was non-existent or very slow, then the experiment would be spoiled completely. A wobble would not affect the measurement of wave admittance, since the spin would still bring the antenna axis through the plane of the wavefront (see sec. 5.1), but it would spoil the measurement of θ. In such a case the electron density could still be calculated by using a theoretical estimate of θ (sec. 5.4), and this procedure would probably not entail much error (sec. 5.5), unless θ was varying widely and at random along the orbit due to the effects of ionospheric irregularities; this last condition would be obvious from the records of signal amplitude. The presence of a component of wobble in the spin motion could be detected most easily if the satellite carried some device for measuring aspect.

In view of these several difficulties that may beset the determination of local electron density by the wave admittance method, it is clear that this experiment should not be performed in isolation, but only as part of an integrated vlf satellite experiment such as that proposed in section 6, where some of the measurements would provide cross-checks on others.

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