Journal of Research of the National Bureau of Standards—D. Radio Propagation
Vol. 63D, No. 3, November–December 1959

On the Correlation of Solar Noise Fluctuations in Harmonically Related Bands

L. R. O. Storey

(May 25, 1959)

A method is proposed for the study of the solar corona, by observing a delayed correlation between rapid fluctuations of enhanced solar radio emission in harmonically related frequency bands. The correlation is expected in those types of emission that are produced by nonlinear plasma oscillations. The delay of the fundamental frequency with respect to the harmonic would be brought about by dispersive group retardation in the corona. The method appears to be most suitable for use with type II bursts, though it might also be applied to other types of nonthermal solar emission.

A solar noise burst of type II consists of a narrow band of intense radio emission, that drifts steadily downwards in frequency over a period of several minutes (Wild and Macready [1] and[1] Wild [2]). Often two such bands are observed simultaneously, with center frequencies roughly in the ratio 1:2. This fact suggests that the burst originates in nonlinear plasma oscillations in the solar corona (Wild, Murray, and Rowe [3]; Maxwell, Swarup, and Thompson [4]). The oscillations are thought to be excited by the motion through the corona of a bunch of particles that has been ejected violently from the photosphere. Usually the burst is preceded by a solar flare which, in these cases, may well be the original source of the fast particles. At each point along their course outward through the corona, the impact of the particles excites oscillations at the local plasma frequency, which radiate the noise observed in the lower of the two bands. Because the oscillations are supposedly nonlinear, energy is radiated also at the second harmonic of the plasma frequency, and this corresponds to the upper band. As the bunch of particles continues outward, it encounters progressively decreasing local plasma frequencies, and this circumstance explains the downward progression of the frequency of the burst.

Now it is possible that the noise in these bursts may exhibit fairly rapid nonstatistical fluctuations, of duration 0.1 sec or less, due to irregularities either in the distribution of electrons in the corona or in the strength of the source of excitation for the oscillations. If so, then the same fluctuations would be expected to occur on both the fundamental frequency and on the harmonic, since the two frequencies are generated simultaneously at the same point by the same process. However, the fluctuations would not be exactly simultaneous when observed at the earth, because, as the waves travel outward through the corona, the different frequencies would experience different amounts of group delay; the fundamental would be delayed more than the harmonic. If the differential delay between the fluctuations on the two frequencies could be observed and measured, it might prove to be a source of useful information about the solar corona.

To find out whether this differential delay is likely to be observable, its size has to be estimated. In the appendix, some calculations are made on the assumptions that the electrons in the corona are distributed with spherical symmetry about the center of the sun, and that their number density N varies with distance from the center according to the power law

$$N=N_0R^{-b}, \tag{1}$$

where N_0 is the nominal density at the surface of the photosphere, and the distance R is measured in solar radii. The source of excitation for the burst is assumed to be traveling radially outwards from the sun, along the line directly towards the earth. In calculating the group delays, no account was taken of the possibility that magnetic fields might be present. On these assumptions, the differential delay between a fluctuation on the fundamental frequency f and the same fluctuation on the second harmonic $(2f)$ is

$$\Delta T(f)=Kf^{-2/b}, \tag{2}$$

where the factor of proportionality K depends on the values of N_0 and b. These parameters were estimated by fitting the distribution (1) to the densities given by Van de Hulst [5], which were obtained experimentally by measurement of the brightness and polarization of coronal light at different angular distances from the sun. The densities used were those that apply at sunspot maximum, and the fit was made at the level where the density corresponds to a plasma frequency of about 100 Mc; this process gave the values $N_0=4\times10^8$ cm^{-3}, and $b=9$. Thence $K\approx1$, when ΔT is measured in

[1] Figures in brackets indicate the literature references at the end of this paper.

seconds, and f in megacycles. Equation 2 then yields the following estimates of the delay at various frequencies of the fundamental:

f	ΔT
80 Mc	0.40 sec
100 Mc	.38 sec
120 Mc	.36 sec

These are lower estimates because, above the 100 Mc level, the densities given by (1) are rather less than those of Van de Hulst's distribution; yet, if that distribution is correct, then they are not likely to be excessively low, as most of the differential delay of the waves is produced just above their level of origin.[2]

It appears that a differential delay of about 0.4 sec should be expected, and that, for the assumed distribution, it should vary only slowly with frequency. A delay of this order should be observable readily, provided that the noise itself contains fluctuations of much shorter duration. If observed on just one harmonically related pair of frequencies, the delay would give a weighted measure of the total electron content of the corona above the level of origin of the waves. If the delay could be measured at several such pairs of frequencies, and the same assumptions were made as in the calculations above, then the observed variation with frequency would give the value of the index b in the power law for the distribution of electrons. More generally, it should be possible to infer the distribution of electrons in height with no prior assumptions about its form, by a variant of the "lamination" methods that have been developed for analyzing the group delays observed in ionospheric sounding (Thomas [8]). This information, when combined with the observed frequency/time relation for the burst, would show how the height of the source of excitation in the corona increased with time. At present, the radial movement of the source can be traced only by assuming some model for the distribution.

In this kind of analysis, height would be measured with respect to the level of origin of the highest fundamental frequency that was observed. The height of this particular level with respect to the photosphere would have to be estimated in some other way. For instance, if the radio burst was preceded by a flare, then the time interval might be measured between the occurrence of the flare and the onset of the burst; this information, when combined with the calculated velocity of movement of the source of excitation, would give the height.

Such an analysis would also be complicated by the fact that the source would not, in general, be moving directly towards the earth; nor would it necessarily, even, be moving radially outwards from the sun (Wild, Murray, and Rowe [3]). However, it is worth noting that any motion of the source transverse to the line of sight can be observed directly by other means (Payne-Scott and Little [7]). If such observations were combined with those of the differential delay, it might then be possible to infer the distribution on the assumption of spherical symmetry. This assumption itself may seem naïve, considering that the corona is known to have an irregular structure, but nonetheless it may still be useful as a basis for deriving a first approximation to the distribution.

The first step in implementing these proposals would be to find out whether the noise in the type II bursts does exhibit nonstatistical fluctuations that are sufficiently rapid to be of use for measuring the differential delay. This question cannot be answered from the results of previous observations, since all these were made using receivers with too large bandwidths. In detecting a signal that varies in frequency, the best bandwidth to use is roughly equal to the square root of the rate of change of frequency (Storey and Grierson [10]). Judging from the spectral characteristics of the type II bursts, the best frequency range in which to observe them on the fundamental would be 80 to 120 Mc, and in this range their rate of change of frequency is usually about 1 Mc^2. Hence the RF bandwidth of the receiver should be about 1 kc. With so narrow a bandwidth, a fairly large antenna would be required to provide adequate sensitivity.

If rapid fluctuations were observed, the next step would be to search for the anticipated correlation between the fluctuations in harmonically related bands. The antenna would be connected to a pair of receivers, one tuned to a fixed frequency in band 80 to 120 Mc, and the other to twice this frequency; the high-frequency receiver should have an RF bandwidth double that of the low-frequency receiver (the criterion of rate of change of frequency would require the bandwidth at the harmonic to be $\sqrt{2}$ times that at the fundamental, but this requirement is not at all critical, so it seems better to arrange that the limits of the two bands are related harmonically). The outputs from the detectors of the two receivers would be recorded on magnetic tape, together with timing signals. If a particular section of tape was identified as the record of a type II burst, then this record would be played back at reduced speed, the information on the two data tracks converted to digital form, and the cross-correlogram of the two sets of fluctuations evaluated in a digital computer. In this way a search would be made for a correlation between the fluctuations observed in the two bands, and for a possible systematic time displacement that would give the best correlation. If this search were to prove successful, then the measurements would be extended to cover more than one pair of frequencies. There would be advantage to making these measurements in conjunction with recordings on a panoramic radio spectrograph, which could be used to identify the type II bursts.

[2] However, some recent studies indicate that Van de Hulst's figures may be too low. Newkirk [6], on the basis of further optical measurements, concludes that even on the quiet disk the figures should be multiplied by a factor of 2.2, while over active regions the factor should be 4.5. Maxwell [7], on evidence from the radio bursts themselves, has suggested that the multiplying factor for active regions may be as high as nine. If such factors indeed apply, then the estimates of the differential delay ΔT should be multiplied by these same factors raised to the power of $1/b$ (see eq 11 in the appendix); for the given value of b, the increase in the estimates is less than 25 percent.

Possibly the same techniques could be used to study other types of enhanced solar radiation. Thus the noise bursts of type III also exhibit harmonic structure (Wild, Murray, and Rowe [3]), but the rate of change of frequency is much faster, of the order of 50 Mc^2; hence the receiver would need to have a rather larger RF bandwidth, about 7 kc, for recording these bursts. A further possibility is the study of noise storms. Though these disturbances are not localized in frequency at any one time, as are the bursts, but to the contrary are spread over a wide frequency band (Wild and Macready [1]), nevertheless they also may be generated by the same process involving nonlinear plasma oscillations. The difference between the two kinds of disturbance may lie only in the nature of the excitation; a continuous stream of particles for the storm, as against a discrete bunch for the burst. If so, then there would be the same grounds for expecting a delayed correlation between fluctuations of noise power in harmonically related bands; in this case, however, there is no way of telling in advance what RF bandwidth would be best for observing the correlation. I am indebted to R. S. Lawrence for this suggestion.

In this work, there is one possible source of difficulty that needs to be recognized. It is the presence of irregularities in the corona, which may cause the radiation to be propagated from the source to the earth by more than one path. Multipath propagation could give rise to rapid and severe fading that would not be correlated in harmonically related frequency bands, and any correlated fluctuations might thus be masked. A similar difficulty has been met with already in an attempt to measure the electron density at great heights above the earth by observing the dispersion of radar echoes from the moon (Eshleman, Gallagher, and Barthle [11]). However, it is hard to judge whether this factor will present a serious obstacle to the proposed studies.

In writing this paper, I have been helped considerably by advice from Dr. Constance S. Warwick of this Laboratory, and from Dr. G. Newkirk of the High Altitude Observatory of the University of Colorado.

Appendix: Calculation of the Differential Delay

In calculating the differential delay, the following symbols will be used to refer to points, first, at the surface of the photosphere, and second, at some level in the corona:

	Photosphere	*Corona*
Distance from center of sun	r_0	r
Local electron density	N_0	N
Local plasma frequency	F_0	f_p

The electron density in the corona will be assumed to depend on r only, according to the power law

$$N=N_0R^{-b} \tag{1}$$

where

$$R=r/r_0 \tag{3}$$

The plasma frequency f_p is proportional to the square root of N, so its distribution is

$$f_p=f_0R^{-b/2}. \tag{4}$$

By hypothesis, the oscillations that produce the noise emissions take place at the local plasma frequency; emissions of frequency f are produced at the level, $r(f)$ say, where $f_p=f$. The difference in the group delay experienced by the fundamental of frequency f and the harmonic of frequency $2f$, in traveling from this level out through the corona to the earth, is

$$\Delta T(f)=1/c\int_{r(f)}^{\infty}\{\mu'(f)-\mu'(2f)\}dr \tag{5}$$

where c is the speed of light, and μ' is the group refractive index. If magnetic fields are absent, as will be assumed, then μ' depends only on the relative values of the wave frequency and local plasma frequency, according to the law

$$\mu'(f)=(1-X)^{-\frac{1}{2}} \tag{6}$$

where

$$X=(f_p/f)^2 \tag{7}$$

Along the path of the waves, the dimensionless quantity X varies from a value of unity at the source to zero at infinity. It is convenient to adopt X as the variable of integration:

$$\Delta T(f)=1/c\int_1^0\{\mu'(f)-\mu'(2f)\}\left(\frac{dr}{dX}\right)dX$$

$$=1/c\int_0^1\{\mu'(f)-\mu'(2f)\}\left(-\frac{dr}{dX}\right)dX\cdot \tag{8}$$

The components of the integrand are obtained as follows: first, from (6),

$$\mu'(f)-\mu'(2f)=(1-X)^{-\frac{1}{2}}-(1-X/4)^{-\frac{1}{2}}. \tag{9}$$

Also, from (7), (4), and (3),

$$-\frac{dr}{dX}=r_0/b\ (f_0/f)^{2/b}X^{-(1+1/b)}. \tag{10}$$

Hence the differential delay is

$$\Delta T(f)=Kf^{-2/b} \tag{2}$$

where

$$K=\frac{r_0f_0^{2/b}}{bc}I \tag{11}$$

and

$$I=\int_0^1\{(1-X)^{-\frac{1}{2}}-(1-X/4)^{-\frac{1}{2}}\}X^{-(1+1/b)}dX. \tag{12}$$

Now the factor K must be evaluated. The constants are $c=3\times 10^5$ kmps, and $r_0=7\times 10^5$ km, while, from the data of Van de Hulst [5], $b\simeq 9$ and $f_0\simeq 180$ Mc. The integral I can be evaluated numerically, though with some difficulty, since one or other of the factors in the integrand becomes infinite at each limit. The result is that, when f is measured in megacycles, $K=1$. (1).

References

[1] J. P. Wild and L. L. Macready, Observations of the spectrum of high-intensity solar radiation at metre wavelengths, Part I—The apparatus and spectral types of solar burst observed, Australian J. Sci. Research [A] **3**, 387 (1950).

[2] J. P. Wild, Observations of the spectrum of high-intensity solar radiation at metre wavelengths, Part 2—Outbursts (type 2), Australian J. Sci. Research [A] **3,** 399 (1950).

[3] J. P. Wild, J. D. Murray, and W. C. Rowe, Harmonics in the spectra of solar radio disturbances, Australian J. Phys. **7,** 439 (1954).

[4] A. Maxwell, G. Swarup, and A. R. Thompson, The radio spectrum of solar activity, Proc. Inst. Rad. Engrs. **46,** 142 (1958).

[5] H. C. Van de Hulst, *in* The Sun (ed. G. H. Kuiper) p. 262, table 10 (Chicago, 1953).

[6] G. Newkirk, A model of the electron corona with reference to radio observations, High Altitude Observatory, U. of Colorado, Solar Research Memo No. 109.

[7] A. Maxwell (private communication).

[8] J. O. Thomas, The distribution of electrons in the ionosphere, Proc. Inst. Rad. Engrs. **47,** 162 (1959).

[9] R. Payne-Scott and A. G. Little, The position and measurement on the solar disc of sources of radiation at a frequency of 97 Mc. Part 3—Outbursts, Australian J. Sci. Research [A] **5,** 32 (1952).

[10] L. R. O. Storey and J. K. Grierson, Time-symmetric filters, Part 2—Response to gliding tones, Electronic Eng. **30,** 648 (1958).

[11] V. R. Eshleman, P. B. Gallagher, and R. C. Barthle, On the use of HF moon echoes to determine cislunar ion densities, presented at URSI Spring Meeting (May 5, 1959).

BOULDER, COLO. (Paper 63D3–25)