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Effect of Atmospheric Horizontal Inhomogeneity Upon Ray Tracing [1]

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The tracing of radio rays is normally carried out under the assumption that the refractive index varies only in the vertical direction. Although this assumption appears to be quite reasonable in the average or climatic sense, it is seldom satisfied under actual conditions and is strongly violated by horizontal airmass changes occurring near frontal and land-sea interfaces. This latter case is investigated by tracing rays through two instances of observed marked horizontal variation of the refractive index. The bending for these ray paths was then compared with values obtained under the normal assumption of horizontal homogeneity.

Although at 1 kilometer and above these horizontal changes appear to have little effect, rays emitted at low elevation angles are sensitive to extreme horizontal variations of the atmosphere near the surface, such as those associated with ducting conditions. However, since it appears that such conditions occur less than 15 percent of the time at most locations, the majority of ray-path calculations may be carried out under the normal assumption of horizontal stratification of the refractive index.

1. Introduction and Background

It is common in ray tracing studies to assume that the refractive index of the atmosphere is spherically stratified with respect to the surface of the earth [1, 2, 3, 4, 5].[2] Thus, the effect of refractive index changes in the horizontal direction is normally not considered, although recently Wong [6] has considered the effect of mathematically smooth horizontal changes in airborne propagation problems.

Neglecting the effect of horizontal gradients seems quite reasonable in the troposphere because of the relatively slow horizontal change of refractive index in contrast to the rapid decrease with height. In fact, examination of climatic data indicates that one must compare sea level stations located 500 km from each other on the earth's surface in order to observe a difference in refractive index values which would be comparable to that obtained by taking any one of these locations and comparing its surface value with the refractive index 1 km above the location. Although the assumption of small horizontal changes of the refractive index appears to be true in the average or climatic sense, there are many special cases such as frontal zones and land-sea breeze effects where one would expect the refractive index to change abruptly within the 80-odd kilometers of horizontal distance traversed by a tangential ray passing through the first kilometer in height.

It is these latter variations that are investigated in the present paper. Two cases of marked horizontal change of refractive index conditions were studied; one which occurred over the Canterbury Plain in New Zealand and the other at Cape Canaveral, Fla. Although these particular sites were chosen for several reasons such as land-to-sea paths and a subtropical location (where marked changes in refraction conditions are common) the major consideration was that detailed aircraft and ground meteorological observations were available for prolonged periods.

These detailed meteorological measurements allow a quantitative evaluation of the error apt to be incurred by assuming that the refractive index is horizontally stratified. The procedure used was to determine the refractive index structure vertically over the transmitter and assume that this same structure described the atmosphere everywhere. Rays were then traced through this horizontally laminated atmosphere. These ray paths were then compared with those obtained by the step-by-step ray tracing through the detailed convolutions of refractive index structure in the two cases under study.

In the sections that follow we will discuss the two cases chosen for study, the methods of calculations used to evaluate refraction effects, and the degree of confidence to which standard prediction methods may be used under conditions of horizontal inhomogeneity.

2. Canterbury

The Canterbury data was compiled by a radio meteorological team working from September 1946, through November 1947, on the South Island of New Zealand under the leadership of R. S. Unwin [7]. This report proved invaluable in this investigation as it was very carefully prepared, giving minute details of the experiment on a day-to-day basis.

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[2] Figures in brackets indicate the literature references at the end of this paper.

Anson aircraft and a trawler were used for meteorological measurements over the sea, and three mobile sounding trucks for observations on land. The trucks and the trawler carried wired-sonde equipment, whereby elements for measuring temperature and humidity up to a height of 150 m to 600 m (depending on wind conditions) were elevated by means of balloons or kites. Standard meteorological instruments provided a continuous record of wind, surface pressure, temperature, and humidity at stations at the coast and 14 km and 38 km inland. The headquarters of the project were at Ashburton Aerodrome, and the observations extended out to sea on a line perpendicular to the coastline of Canterbury Plain. Aircraft were equipped with a wet- and dry-bulb psychrometer, mounted on the portside above the wing. Readings were taken three or four times on each horizontal flight leg of 2 or 3 min duration. Special lag and airspeed corrections were applied, resulting in accuracy of $\pm 0.1°$ C. It was found that, under the variety of conditions in which observations were made, the aircraft flights were more or less parallel to the surface isobars; hence, the sea-level pressure as recorded at the beach site was considered to hold over the whole track covered by the aircraft. The relationship used for calculating the pressure; P, in millibars at a height h in feet was:

$$P(h) = P_0 - h/30$$

where P_0 is the surface pressure. This approximation (determined by averaging the effect of the temperature and humidity distributions on pressure in a column of air) resulted in a maximum error in the refractivity of 0.5 percent at 900 m. Radiosonde ascents at Hokitika on the west coast of South Island and Paraparaumuo and Auckland on North Island were used to supplement the aircraft measurements, particularly in the altitude levels above 1 km.

The observations, diagrams and meteorological records were studied, and a profile of unusually heterogeneous nature was chosen. The synoptic situation for the morning of November 5, 1947 was selected, as it revealed a surface-ducting gradient near the coast with an elevated layer about 100 km off shore. A cross section of the area from Ashburton to a point 200 km off shore was plotted with all available data, and isopleths of modified refractive index, M, were drawn to intervals of 2.5 units.

$$M = (n - 1 + K_e h)\ 10^6, \qquad (1)$$

where $K_e = (15.70)\ (10^{-8})/\text{m}$ and

$$N = (n-1)10^6 = \frac{77.6}{T}\left[P + \frac{4{,}810 e_s\, RH}{T}\right], \qquad (2)$$

where P is the station atmospheric pressure in millibars, RH is the percent of the saturation vapor pressure, e_s, in millibars at the absolute temperature, T, in degrees Kelvin and n is the refractive index [8]. A simplified version of the lower portion of this cross section with the corresponding M curves is accompanied by a sketch of the general location of the experiment in figure 1. Some smoothing was necessary, particularly near the sea surface and in those areas where aircraft slant ascents and descents caused lag errors in altimeter readings and temperature and humidity elements. Isopleths over land were plotted above surface rather than above sea level with an additional adjustment in the scale ratios of height and distance in an attempt to simplify the reading of values from the diagram.

3. Cape Canaveral

The second area studied was the Cape Canaveral to Nassau path for the period of April 24 to May 8, 1957. This material was supplied by the Wave Propagation Branch of Naval Research Laboratories and the University of Florida. The particular case chosen for study was the meteorological profile of May 7, 1957, (2000 e.s.t.) due to its heterogeneous nature, showing a well-defined elevated layer

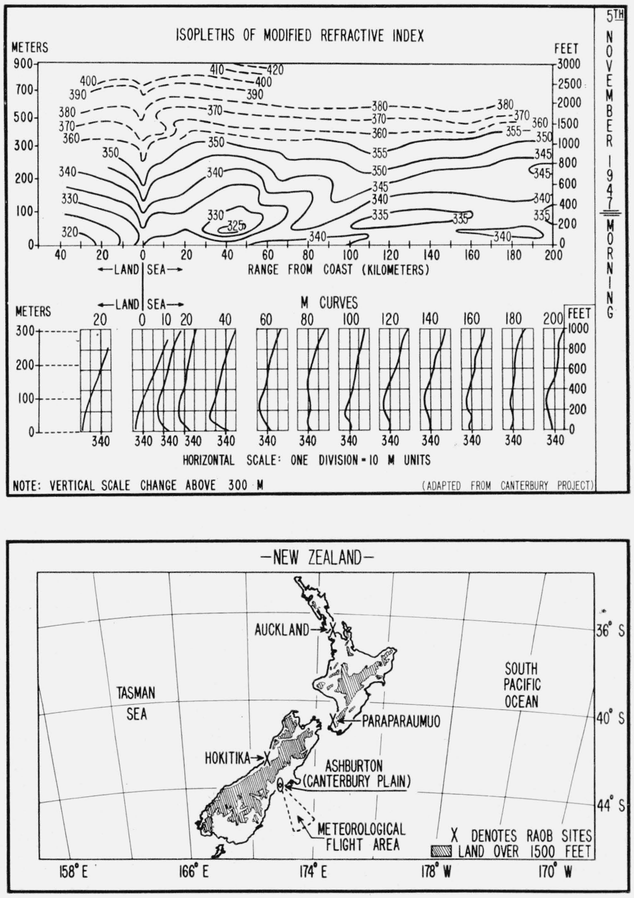

FIGURE 1. *Isopleths and curves of refractive index for Nov. 5, 1947, at Canterbury, with a map locating sources of meteorological data. (Courtesy of the Canterbury project.)*

at about 1,500 m. Fourteen refractometer soundings from aircraft measurements taken at various locations along the 487-km path (fig. 2) and six refractive index profiles (deduced from radiosonde ascents from Cape Canaveral, Grand Bahama Island, and Eleuthera Island) were read in order to plot a cross section of the atmosphere which would represent as closely as possible the actual refractive conditions at that time. Unfortunately, the data near the surface (up to 300 m) were quite sparse compared to those recorded in the Canterbury Project, and calibration and lag errors had not been noted as carefully in this preliminary report; therefore, some interpolation and considerable smoothing of refractive index values were necessary when drawing isopleths.

4. Ray Bending

The classic expression for the angular change, τ, or the bending of a ray passing from a point where the refractive index is n_1 to a second point where the refractive index is n_2 is given by [5]

$$\tau_{1,2} = -\int_{n_1}^{n_2} \frac{dn}{n} \cot \theta, \tag{3}$$

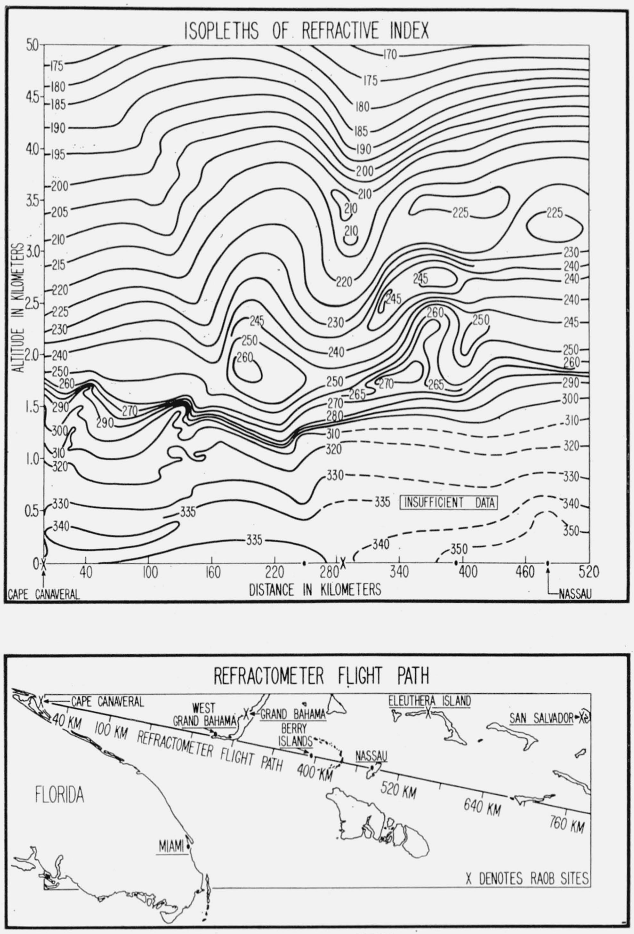

FIGURE 2. *Isopleths of refractive index and map of refractometer flight for May 7, 1957, Cape Canaveral to Nassau.*

where θ is the local elevation angle. Equation (3) was evaluated by use of

$$\Delta\tau_{1,2} = \frac{(N_1 - N_2)}{\bar{n}_{1,2}} \times 10^{-6} \cot \bar{\theta}, \tag{4}$$

where

$$\bar{\theta} = \frac{\theta_1 + \theta_2}{2}.$$

The value of θ at each point was determined from Snell's law:

$$n_1\, r_1 \cos \theta_1 = n_2\, r_2 \cos \theta_2 = \text{constant}, \tag{5}$$

where r is the radial distance from the center of the earth and is given by $a+h$, where a represents the radius of the earth and h the altitude of the point under consideration. For simplicity one may rewrite (5) as

$$(1+N_1\times10^{-6})(a+h_1) \cos \theta_1 = (1+N_2\times10^{-6})(a+h_2) \cos \theta_2. \tag{6}$$

Then, when θ is small, one may expand (6), neglect second order terms and obtain the convenient expression:

$$\theta_2 = \left\{ \theta_1^2 + \frac{2(h_2-h_1)}{a} - 2(N_1-N_2)\times10^{-6} \right\}^{1/2}, \tag{7}$$

where all values of θ are in milliradians.

After obtaining τ by use of (6) or (7), one may determine the distance, d, along the earth's surface that the ray has traveled from:

$$d_{1,2} = a[\tau_{1,2} + (\theta_2 - \theta_1)]. \tag{8}$$

Thus by successive application of the above formulas, one may trace the progress of the radio wave as it traverses its curved path through the atmosphere. Normally the use of these equations is quite straightforward. When considering horizontal changes in n, however, one must satisfy these equations by iterative methods. In the present application, since n had to be determined by graphical methods, it was felt to be sufficient to assume a constant distance increment of 250 to 500 m, solve for appropriate height increment from

$$\Delta h = \Delta d \; tan\, \theta_1 \left[1 + \frac{h}{a}\right], \tag{9}$$

graphically determine N for the point $d_1+\Delta d$, $h_1+\Delta h$ and then determine θ_2 and $\tau_{1,2}$.

This latter type of ray tracing was done for various rays of initial elevation angles between 261.8 milliradians (mr) (15°) and 10 mr (~0.5°). The calculations were not carried to smaller elevation angles since this type of ray tracing is not valid within surface ducts for initial elevation angles below the angle of penetration [9, 10].

5. Comparisons

Although both of the calculated ray paths consisted of an oversea itinerary with coastal transmission sites, they are quite different in other aspects. Canterbury Plain is located southeast of the 10,000-ft chain of the Southern New Zealand Alps at a latitude of 44° S (the equatorward edge of the westerly belt of winds in November). Cape Canaveral is located on a sea level peninsula at 28° N (the poleward edge of the northeast trade circulation in May). While the Canterbury profile showed superrefractive tendencies, the Canaveral profile illustrated subrefraction at the surface counterbalanced by an elevated trade wind inversion layer, indicating that the total bending values of Canterbury would be higher than normal, while the Canaveral example would have values near or lower than normal.

These differences are illustrated by figures 3 and 4 where the bending, τ, in milliradians is plotted versus altitude in kilometers. The effect of horizontal changes is most pronounced for rays with initial elevation angles of 10 mr. On these figures the term "vertical" ray is used to designate the ray path through the horizontally homogeneous n structure determined from the refractive index vertically over the station. The term "horizontal" ray designates the ray path through the complex actual n structure. It is quite evident that a consistent difference in bending of about 1 mr exists between the "vertical" and "horizontal" rays at Canterbury above 1 km for $\theta_0=10$ mr. This would be expected since the vertical M profile (fig. 1) at the beach (our hypothetical transmitter site) is nearly normal in gradient while as little as 10 km to sea a duct exists, thus indicating a near maximum difference between the "horizontal" and the "vertical" rays at any initial elevation angle small enough to be affected by the duct. This is in contrast, however, with the case of Cape Canaveral where, except for the region of the elevated duct centered at about 1,500 m, the "vertical" and "horizontal" rays are in quite close agreement. These two examples illustrate that horizontal variations must be near the surface to be most effective. The importance of the altitude of the variation is due to the fact that refraction effects are very heavily weighted toward the initial layers [10].

Also shown on figures 3 and 4 are the values of the bending which would be predicted from the Central Radio Propagation Laboratory corrected exponential reference atmosphere [11]. The values shown are obtained from the value of N at the transmitter site as corrected by the vertical gradient over the first 100 m. It is noted that, for $\theta_0=10$ mr at Canterbury, the value of bending predicted by the model is in essential agreement with the "vertical ray" bending but underestimates the "horizontal ray" (which has the largest variation of n with horizontal distance) by about 1.25 mr. For Cape Canaveral at $\theta_0=10$ mr, the model atmosphere over-

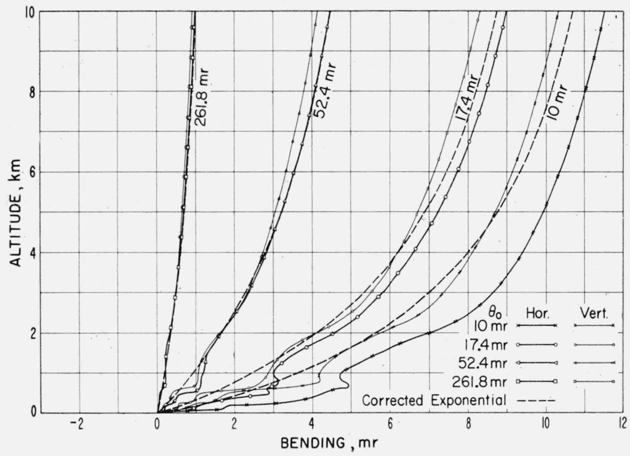

FIGURE 3. *Canterbury, 0 to 10 km, altitude versus ray bending.*

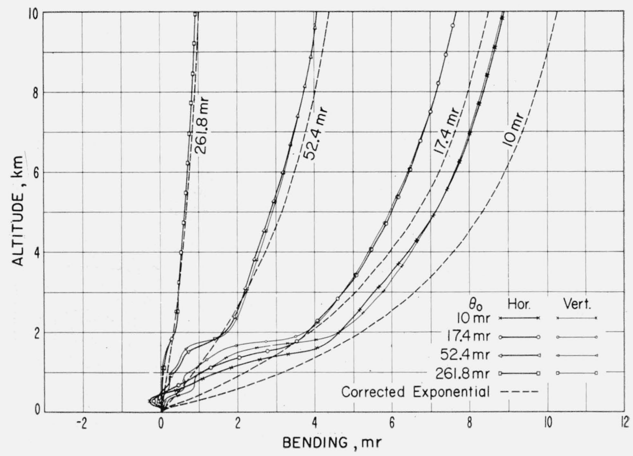

FIGURE 4. *Cape Canaveral, 0 to 10 km, altitude versus ray bending.*

estimates the bending by about 1.25 mr for altitudes in excess of 2 km. It should be emphasized that, although the model exponential atmosphere appears to represent the average of the two specific cases studied, the departure from this average arises from quite different causes in each case. The differences in the Canterbury case arise from the marked effect of horizontal variation of n as is indicated by the agreement of the vertical ray bending with the model atmosphere. The disagreement in the Canaveral case is due to the presence of a very shallow surface layer of nearly normal gradient topped by a strong subrefractive layer; therefore, it represents a shortcoming of the model rather than an effect of horizontal changes of n.

The preceding analysis of bending throws the refractive differences in each case into sharp relief. The effect of refraction, of course, is to vary the ray path. Figures 5 and 6 show the ray paths corresponding to the bendings of figures 3 and 4. Note that for Canterbury at $\theta_0=10$ mr the effect of the horizontal variation of n is to produce a difference in estimation of about 1 km in height or 20 km in ground distance at 300 km from what would be obtained from considering the vertical n profile as a representation of the entire path. The effect of the subrefractive layer at Cape Canaveral is not so large, but it does cause an overestimation of the ground distance by about 5 km and an underestimation of the height by less than one quarter of a kilometer at a ground distance of 300 km by assuming that the vertical profile may be used throughout the entire ray path.

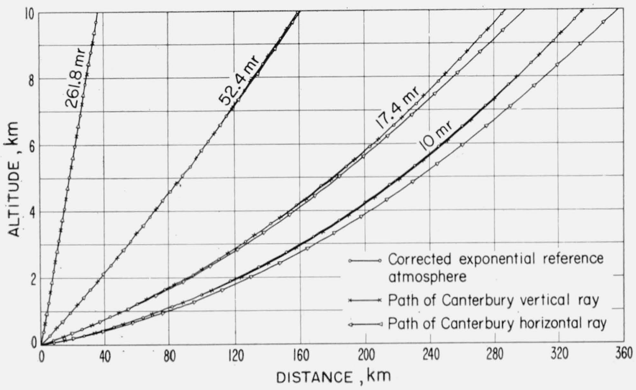

FIGURE 5. *Canterbury, 0 to 10 km, altitude of ray versus distance.*

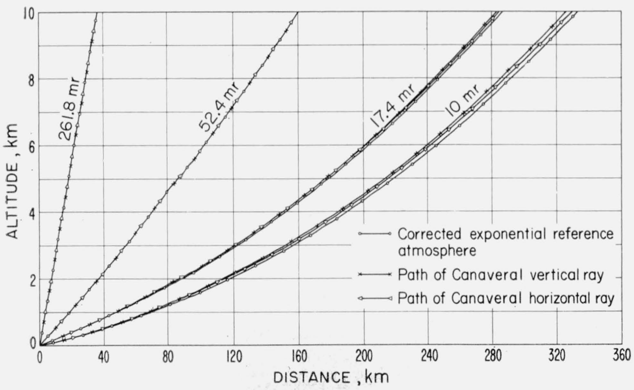

FIGURE 6. *Cape Canaveral, 0 to 10 km, altitude of ray versus distance.*

6. Extension to Other Regions

It should be pointed out that the ducting case at Canterbury represents an extreme refraction condition and is not necessarily typical of conditions observed in other regions or, indeed, at Canterbury. The Canterbury project was purposely restricted to a study of ducting conditions with the result that less than 20 percent of the total observations for the fifteen months are reported. Therefore, because one of the more extreme cases is represented by the November 5th example, one might conclude that much less than 20 percent of the observations would show the same degree of horizontal n change as the profile studied.

If one further hypothesizes that the greatest horizontal n change would be associated with ducting conditions, then the percentage incidence of ducts as evaluated from radiosonde observations, listed for various stations in table 1, would indicate that the effects of horizontal changes of n sufficient to cause variations in the ray path as large as those of the present study would be observed less than 15 percent of the time, regardless of geographic location.

TABLE 1. *Percentage occurrence of surface ducts during the years 1952 to 1956*

Station	% incidence			
	February	*May*	*August*	*November*
Fairbanks, Alaska	9.4	0.4	0.4	6.2
Columbia, Mo	.7	2.5	8.4	1.3
Washington, D.C	.7	4.8	4.3	1.4
Canton Island	10.0	9.2	12.4	11.5
Miami, Fla	0.7	3.5	8.5	2.7

The probable importance of subrefractive layers upon the prediction of refraction effects has emerged as a secondary result of the present study. Although subrefraction is normally neglected, it is potentially a very important refractive factor for distances of, say, less than 40 km. Even though the percentage occurrence of subrefractive layers can be as large as 6 percent (see table 2), this effect is frequently offset by the concurrent occurrence of an adjacent superrefractive layer, as is illustrated by the Cape Canaveral example.

TABLE 2. *Percentage occurrence of surface subrefractive layers during the years 1952 to 1956*

Station	% incidence			
	February	*May*	*August*	*November*
Fairbanks, Alaska	0.0	0.0	1.2	0.4
Columbia, Mo	.0	1.6	0.6	4.0
Washington, D.C	.9	2.2	5.8	2.7
Canton Island	.0	0.0	0.0	0.3
Miami, Fla	.7	.3	.9	.7

7. Conclusions

The conclusions of the present study could be considerably modified by the analysis of many more examples, although it is evident that horizontal variation of n near the earth's surface produces the most marked deviations from the ray paths obtained by assuming horizontal stratification of n. The effect of horizontal changes occurring more than a kilometer above the surface appear from our present examples, to have little effect. Further, the effects of horizontal changes appear to be most pronounced in the presence of surface ducts and at small elevation angles. The tentative conclusion is reached that the effect of horizontal n change is normally small since ducting will occur less than 15 percent of the time.

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