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System Loss in Radio Wave Propagation

Kenneth A. Norton

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A summary is presented of the ways in which the concept of system loss and the closely related concepts of transmission loss, basic transmission loss, propagation loss, and path antenna gain may be used for precise, yet simple, descriptions of some of the characteristics of radio wave propagation which are important in the design of radio systems. Definitions of various terms associated with the concept of system loss are given which introduce a greater flexibility into its use without any loss in precision. It is shown that the use of these added terms and concepts makes feasible the extension of the use of this method of description to any portion of the radio spectrum. A more general formula for the system loss is given which may be used for antennas with an arbitrarily small separation. Using this formula it is shown that the system loss between small electric or magnetic dipoles separated by a distance $d \ll \lambda$ can be made arbitrarily small even though the individual antennas have large circuit losses. Formulas are developed for the percentage of time that a desired signal is free of interference, and these are used to demonstrate methods for the efficient use of the spectrum. In particular, contrary to general belief, it is shown that efficiency is promoted by the use of high power and high antênnas and, in the case of a broadcast service, sufficiently *small* separations so that there is appreciable mutual interference. An analysis is made of the variance of the path antenna gain in ionospheric scatter propagation. Methods are given for the calculation of the transmission loss for the ground wave and tropospheric scatter modes of propagation through a turbulent model atmosphere with an exponential gradient. Examples of such calculations are given which cover a wide range of frequencies and antenna heights. Finally, examples are given of the expected range of various tropospheric point-to-point scatter systems such as an FM multichannel teletype system, a television relay or an FM broadcast relay.

1. Introduction

Although the idea of a radio circuit transmission loss had been in use by engineers concerned with the design of communication systems for many years prior to that time, it was in a paper by the author [1] in 1953 that its great advantages, particularly in connection with ionospheric or tropospheric scatter systems, were first explicitly pointed out. Since that time this concept has been used extensively in the radio propagation studies at the Central Radio Propagation Laboratory and elsewhere, and a fairly large body of conventions has grown up around this usage. It is the purpose of this paper to describe the precise meaning attached to these conventions by the engineers at CRPL, with the hope that these usages will prove equally useful and will be adopted in other laboratories throughout the world. In addition, its use is illustrated by numerous examples throughout the usable portion of the radio spectrum ranging from 3 to 100,000,000 kc.

2. Definitions of System Loss, Transmission Loss, Basic Transmission Loss, Path Antenna Gain, and Path Antenna Power Gain

The *system loss* of a radio circuit consisting of a transmitting antenna, receiving antenna, and the intervening propagation medium is defined as the dimensionless ratio, p_t/p_a, where p_t is the radiofrequency power input to the terminals of the transmitting antenna and p_a is the resultant radiofrequency signal power available at the terminals of the receiving antenna. Both p_t and p_a are expressed in watts. The system loss is usually expressed in decibels:

$$L_s = 10 \log_{10} (p_t/p_a) = P_t - P_a \qquad (1)^1$$

Note that the system loss, as above defined, excludes any transmitting or receiving antenna transmission line losses since it is considered that such losses are readily measurable. On the other hand, the system loss includes all of the losses in the transmitting and receiving antenna circuits, including not only the transmission loss due to radiation from the transmitting antenna and reradiation from the receiving antenna, but also any ground losses, dielectric losses, antenna loading coil losses, terminating resistor losses in rhombic antennas, etc. The inclusion of all of the antenna circuit losses in the definition of system loss provides a quantity which can always be accurately measured and which is directly applicable to the solution of radio system design problems.

For many applications, however, particularly in the study of radio wave propagation, it is convenient to have a definition of a system loss which excludes all of the antenna circuit losses except those associated with the antenna radiation resistances; thus, the *transmission loss* of a radio circuit consisting of a transmitting antenna, receiving antenna, and the intervening propagation medium is defined as the dimensionless ratio, p'_t/p'_a, where p'_t is the radiofrequency power radiated from the transmitting antenna, and p'_a is the resultant radiofrequency signal power which would be available from the

[1] Throughout this paper, capital letters are used to denote the ratios, expressed in decibels, of the corresponding quantities designated with lower-case type; e.g., $P_t = 10 \log_{10} p_t$ is the input power to the transmitting antenna expressed in decibels above 1 w.

receiving antenna if there were no circuit losses other than those associated with its radiation resistance. The transmission loss is usually expressed in decibels:

$$L \equiv 10 \log_{10}(p_t'/p_a') = L_s - L_{tc} - L_{rc}, \qquad (2)$$

where L_{tc} and L_{rc} are the losses, expressed in decibels, in the transmitting and receiving antenna circuits, respectively, excluding the losses associated with the antenna radiation resistances; i.e.,

$$L_{tc} = 10 \log_{10}(r_t'/r_t) = 10 \log_{10}(1+\Delta_{tc})$$

and

$$L_{rc} = 10 \log_{10}(r_r'/r_r) = 10 \log_{10}(1+\Delta_{rc}),$$

where r' is the actual resistive component of the antenna circuit, r is the radiation resistance, and the subscripts t and r refer to the transmitting and receiving antennas, respectively.

In order to separate the effects of the transmitting and receiving antenna gains and circuit losses from the effects of the propagation, it is convenient to define the *basic transmission loss*, L_b (sometimes called path loss) as the transmission loss expected between fictitious loss-free isotropic transmitting and receiving antennas at the same locations as the actual transmitting and receiving antennas. This serves also to define the *path antenna directive gain*, G_p:

$$G_p \equiv L_b - L, \qquad (3)$$

$$L_b = L + G_p = L_s + G_p - L_{tc} - L_{rc}. \qquad (4)$$

In some cases it may be quite difficult to measure the antenna circuit losses; thus it is convenient to define the *path antenna power gain*, G_{pp}, as

$$G_{pp} \equiv L_b - L_s = G_p - L_{tc} - L_{rc}. \qquad (5)$$

It is seen that the path antenna power gain is the change in the system loss when loss-less isotropic antennas are used at the same locations as the actual antennas; note that G_{pp} will be negative when the antenna circuit losses exceed the path antenna directive gain. Throughout the remainder of this paper the term path antenna gain and symbol G_p are often used when a distinction between the directive gain and the power gain is unnecessary.

In some idealized situations the path antenna power gain, G_{pp}, is simply the sum $(G_{tp}+G_{rp})$ of the free space power gains G_{tp} and G_{rp} of the transmitting and receiving antennas relative to loss-less isotropic antennas. However, in most practical situations G_{pp} is less than $G_{tp}+G_{rp}$ because of the complex nature of the received field. The path antenna power gain may be measured by determining the increase in the system loss when both the transmitting and receiving antennas are replaced *simultaneously* by simple standard antennas such as short electric or magnetic dipoles, and then adding the calculated path antenna power gain corresponding to the use of the standard antennas. In the case of ionospheric or tropospheric scatter propagation, the path antenna power gain is sometimes substantially smaller than the sum of the free space power gains G_{tp} and G_{rp}. In such cases the path antenna power gain cannot be determined accurately as the sum of the effective power gains of the transmitting and receiving antennas (as determined by replacing first one antenna by a standard antenna and then the other antenna by a standard antenna) since such effective power gains depend upon the gain of the antenna used at the other terminal.

In the case of ionospheric or tropospheric propagation, the transmission loss L, the basic transmission loss L_b, and the path antenna gain G_p, are all random variables with respect to time and tend to be normally distributed about their expected values. Furthermore, L and G_p are typically negatively correlated with each other, and thus the variance of L_b is usually substantially less than the sum of the variances of L and G_p (see sec. 9); for this reason it will often be more practical simply to measure the transmission loss with the particular antennas intended for use rather than attempt to calculate the expected transmission loss and its variance with time in terms of the measured or calculated values of the basic transmission loss and the path antenna gain.

Note also that the path antenna gain may actually be negative. For example, the path antenna gain will usually be negative for ground wave or tropospheric wave propagation between a vertically polarized and a horizontally polarized antenna, and the concept of path antenna gain should prove to be useful for expressing the results of such cross polarization measurements.

3. Transmission Loss in Free Space

As an example of the simplicity of transmission loss calculations in some cases, we may consider the transmission loss between two isotropic antennas in free space. At a distance, d, very much greater than the wavelength, λ, the field intensity, expressed in watts per square meter, is simply $p_t'/4\pi d^2$ since the power is radiated uniformly in all directions. Since the effective absorbing area of the receiving antenna is $\lambda^2/4\pi$, the available power at the terminals of the loss-free isotropic receiving antenna is given by

$$p_a' = (\lambda^2/4\pi)\cdot(p_t'/4\pi d^2). \qquad (6)$$

Consequently, the basic transmission loss in free space may be expressed

$$L_{bf} = 10 \log_{10}(4\pi d/\lambda)^2 \qquad \text{(free space; } d >> \lambda). \qquad (7)$$

Since the free space gain of a short electric loss-less dipole is $g_t = g_r = 1.5$, the path antenna gain for two optimally oriented short electric loss-less dipoles in free space is

$$G_p = G_t + G_r = 3.52 \text{ db}. \qquad (8)$$

Consequently, the transmission loss between two optimally oriented short electric loss-less dipoles in free space is

$$L = 10 \log_{10}(4\pi d/\lambda)^2 - 3.52. \qquad (9)$$

4. Influence of the Antenna Environment and Definition of Propagation Loss

In order to illustrate the influence of changes in the impedances of the antennas caused by environmental factors which may, in part, be independent of the antenna circuit losses, we will consider the transmission loss between two short vertical electric loss-less dipoles at heights h_t and h_r, respectively, above a plane perfectly conducting surface and separated by a distance, d, along the surface large with respect to the wavelength. Although we will treat here in detail only the case of antennas over a perfectly conducting plane, the concepts are identical in the case of any other environmental conditions and only the magnitudes of the effects will be different. In free space the field strength e, expressed in volts per meter at a distance d, expressed in meters, in the equatorial plane of a short electric loss-less dipole radiating p'_t watts is given by

$$\frac{e^2}{\eta_0}=\frac{1.5p'_t}{4\pi d^2}, \tag{10}$$

where $\eta_0=4\pi c\cdot 10^{-7}$ = impedance of free space expressed in ohms, and $c=2.997925\cdot 10^8$ m/sec = velocity of light in free space. Note that the factor (1.5) is just the free space gain of the transmitting dipole antenna. The radiation resistance of a short vertical electric dipole of effective length l and at a height h_t above a perfectly conducting plane is given by

$$r=\frac{2\pi\eta_0 l^2}{3\lambda^2}[1+\Delta_t]=r_f[1+\Delta_t], \tag{11}$$

$$\Delta_t=\frac{3}{(2kh_t)^2}\left[\frac{\sin(2kh_t)}{2k\,h_t}-\cos(2kh_t)\right]. \tag{12}$$

In the above $k=2\pi/\lambda=2\pi f/c$, i.e., λ is the wavelength in free space. Note that Δ_t approaches zero at large heights above the surface and r approaches its free space value, r_f. On the other hand, $\Delta_t=1$ for $h_t=0$, and the radiation resistance is then just twice its free space value. The field intensity expressed in watts per square meter for a short vertical electric loss-less dipole over the perfectly conducting surface may be expressed

$$\frac{e^2}{\eta_0}\cong\frac{1.5p'_t[2\cos^2\psi\cos(k\,h_t\sin\psi)]^2}{4\pi d^2[1+\Delta_t]}. \tag{13}$$

In the above expression $\tan\psi=h_r/d$ and the distance d along the surface must be large with respect to both λ and h_t; in this case the distance between the antennas is approximately $d/\cos\psi$. Equations (11), (12), and (13) were derived by Schelkunoff in chapters VI and IX of [2]. Since $\Delta_t=1$ for $h_t=0$, the field intensity is 3 db greater when $\psi=0$ and the dipole is on the surface of a perfectly conducting plane (i.e., $e^2/\eta_0=3p'_t/4\pi d^2$) than when it is in free space; note that in free space (10) must be used rather than (13), since the ground reflection influences the radiation for all values of h_t. In more familiar units, when $h_t=h_r=0$, (13) may be expressed

$$e(\mu v/m)=2.998962\cdot 10^5\sqrt{p'_t(kw)}/d(km). \tag{14}$$

The effective absorbing area of a short vertical electric loss-less dipole receiving antenna at a height h_r above the perfectly conducting plane may be expressed

$$a_e=\frac{1.5\lambda^2\cos^2\psi}{4\pi[1+\Delta_r]}, \tag{15}$$

where Δ_r is defined by (12) with h_t replaced by h_r. Since $p'_t=e^2a_e/\eta_0$ we find by combining (13) and (15) that the transmission loss between two short vertical electric loss-less dipoles at heights h_t and h_r above a plane perfectly conducting surface may be expressed

$$L\cong 10\log_{10}\left\{\frac{[4\pi d/\lambda\cos\psi]^2[1+\Delta_t][1+\Delta_r]}{(1.5)^2[2\cos^2\psi\cos(kh_t\sin\psi)]^2}\right\} \tag{16}$$

or

$$L_s=L=L_{bf}-G_p-6.02+A+L_t+L_r\equiv L_p+L_t+L_r, \tag{17}$$

where $L_{t,r}=10\log_{10}[1+\Delta_{t,r}]$, $G_p\cong 20\log_{10}[(3/2)\cos^2\psi]$ and $A\cong -20\log_{10}[\cos(kh_t\sin\psi)]$. Note that $L_s=L$ in this case since there are no losses other than radiation losses. The factor L_t is shown graphically as a function of (h/λ) in figure 1. It is of interest to note that the transmission loss between the dipoles on the plane perfectly conducting surface, $h_t=h_r=0$, is the same as if the dipoles were separated by the same distance in free space, although the field intensity is 3 db greater for the same power radiated. On the other hand, when the dipoles are several wavelengths above the perfectly reflecting surface ($h_t=h_r>>\lambda$) and are separated by a large distance ($\psi\cong 0$), the transmission loss is 6.02 db less than for dipoles separated by the same distance in free space. Equation (17) illustrates the definition of L_p, the *propagation loss*, i.e., the transmission loss expected if the antennas had gains and circuit resistances the same as if they were in free space. More generally, L_t and L_r are defined as

$$L_{t,r}=10\log_{10}(r'/r_f), \tag{18}$$

where r' is the actual resistance of the antenna in the presence of its environment and r_f is the radiation resistance it would have had if it were in free space. When we note that L_t and L_r may vary substantially from one antenna installation to the next, depending upon the circuit losses, polarization, ground conditions, whether or not a ground screen is used, and upon other environmental factors such as the presence of trees or overhead wires, it becomes clear why it is desirable to separate these components from the system loss and to have a

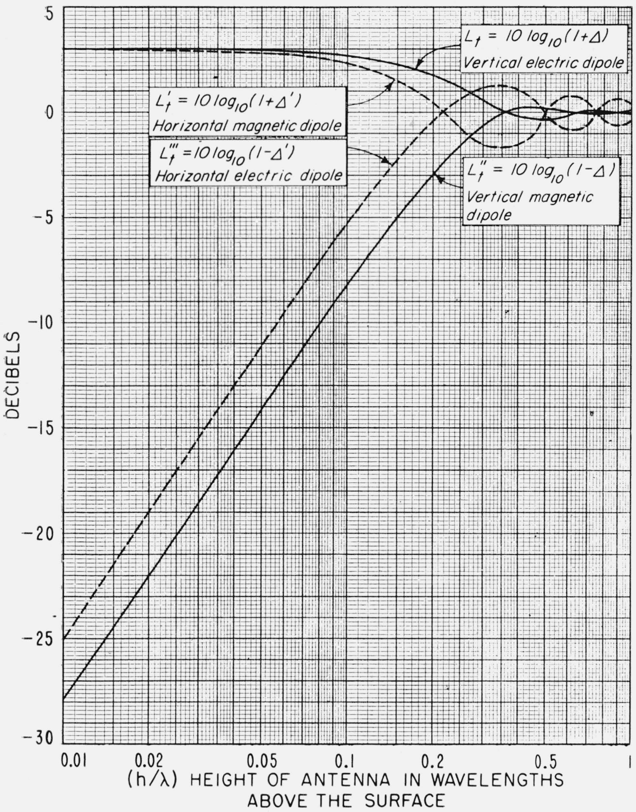

FIGURE 1. *Transmission loss arising from a change in the radiation resistance of short dipole antennas near a perfectly-conducting surface.*

propagation loss, L_p, independent of these antenna environmental conditions.

$$L_p \equiv L_s - L_t - L_r \tag{19}$$

The above is not the only logically consistent method of describing the gains and losses of antennas in the presence of an environment. Thus Schelkunoff [2] considered the perfectly reflecting surface to be an integral part of the transmitting and receiving antennas, and set $G_{ts}=10 \log_{10}\{3/[1+\Delta_t]\}$ and $G_{rs}=10 \log_{10}\{3/[1+\Delta_r]\}$ where the subscript s refers to Schelkunoff's usage. In other words, he referred his maximum gains to those expected for isotropic antennas with the earth removed; this leads to a path antenna gain, referred to that expected between isotropic antennas with the earth removed, given by $G_{ps}=G_p+6.02-L_t-L_r-A$, and his tranmission loss is then simply $L=L_{bf}-G_{ps}$. However, this method of approach is *not recommended* since (a) it is impracticable to remove the earth in order to measure G_{ps} and (b) it would lead to antenna gains 3 db larger than their free space values even when they are many wavelengths above a perfectly reflecting plane surface, and this is inconsistent with the present usage of the concept of antenna gain in the higher frequency ranges.

Although it appears to be desirable to separate the effects of earth reflections and sometimes other environmental effects from the gains of the antennas by means of the propagation loss concept described earlier, there will be other situations in which such a separation of environmental effects is undesirable. For example, the power gain of an antenna mounted on an aircraft, satellite, or space vehicle should be considered as the gain which would be determined by comparison of the fields obtained in various directions as the vehicle is rotated in free space with those obtained from a standard antenna with the vehicle removed; i.e., in these cases the vehicle is to be considered an integral part of the antenna.

Suppose now that we use small loop antennas of area S, with their axes normal to the plane of propagation, parallel to the perfectly conducting surface and at heights h_t and h_r, respectively. In this case the electric vector lies in the horizontal plane and the radiation resistance is [2]

$$r'_m = \frac{8\pi^3\eta_0 S^2}{3\lambda^4}[1+\Delta'_r] = r_{mf}[1+\Delta'_r]. \tag{20}$$

The subscript m refers to the magnetic dipole, and the prime to the effect of the surface.

$$\Delta'_r = (3/2)\left[\left(1-\frac{1}{(2kh_r)^2}\right)\frac{\sin(2kh_r)}{2kh_r}+\frac{\cos(2kh_r)}{(2kh_r)^2}\right], \tag{21}$$

$$a_e = \frac{\lambda^2(3/2)}{4\pi[1+\Delta'_r]}, \tag{22}$$

$$G'_p = 20 \log_{10}[3/2] = 3.52, \tag{23}$$

$$A' = -20 \log_{10}[\cos(kh_t \sin\psi)], \tag{24}$$

$$L'_{t,r} = 10 \log_{10}[1+\Delta'_{t,r}]. \tag{25}$$

The factor $L'_{t,r}$ is also shown graphically on figure 1. Note that Δ'_r approaches zero at large heights and $\Delta'_r=1$ for $h_r=0$.

Consider next the tranmission loss between two small loop antennas at heights h_t and h_r, respectively, above a perfectly conducting surface with their axes normal to this surface. In this case the electric vector lies in the horizontal plane and the radiation resistance is [2]

$$r''_m = \frac{8\pi^3\eta_0 S^2}{3\lambda^4}[1-\Delta] = r_{mf}[1-\Delta], \tag{26}$$

$$G''_p = 20 \log_{10}[(3/2)\cos^2\psi], \tag{27}$$

$$A'' = -20 \log_{10}[\sin(kh_a \sin\psi)], \tag{28}$$

$$L''_{t,r} = 10 \log_{10}[1-\Delta_{t,r}]. \tag{29}$$

The factor $L''_{t,r}$ is also shown as a function of (h/λ) on figure 1.

Finally, consider the transmission loss between two short horizontal electric dipoles of effective length l, normal to the plane of propagation and at heights h_t and h_r, respectively, above a perfectly conducting plane surface. In this case [2],

$$r''' = \frac{2\pi\eta_0 l^2}{3\lambda^2}[1-\Delta'] = r_f[1-\Delta'], \tag{30}$$

$$G_p''' = 20 \log_{10}(3/2) = 3.52, \tag{31}$$

$$A''' = -20 \log_{10}[\sin(kh_a \sin\psi)], \tag{32}$$

$$L_{t,r}''' = 10 \log_{10}[1-\Delta_{t,r}']. \tag{33}$$

The factor $L_{t,r}'''$ is also shown graphically on figure 1. Note that L''' and L'' both approach $(-\infty)$ as h approaches zero, but the radiation resistance simultaneously approaches zero, and it would be difficult in practice to keep the radiated power constant as the antennas are brought nearer and nearer to the surface. When h_t and h_r are both much less than a wavelength, $A''+L_t''+L_r''$ for horizontal loops becomes independent of these heights and equal to $20 \log_{10}(kd/10)$; similarly $A'''+L_t'''+L_r'''$ for horizontal electric dipoles approaches $20 \log_{10}(kd/5)$ for h_t and h_r much less than a wavelength.

All of the above specific results refer to the case of a perfectly conducting plane surface and to distances $d \gg \lambda$. For a finitely conducting ground, the attenuation A in the above expressions will be substantially modified. For example, for vertical electric dipoles over a flat earth of finite conductivity and with $h_t = h_r = 0$,

$$A = -20 \log_{10}\left|1 - i\sqrt{\pi p}\exp(-p)\,\mathrm{erfc}\,(i\sqrt{p})\right| \tag{34}$$

Here p denotes Sommerfeld's numerical distance as defined for a time factor $\exp(+i\omega t)$. In reference [3] a comprehensive discussion is given of the radiation fields of electric and magnetic dipoles over a finitely conducting plane earth; in this reference a time factor $\exp(-i\omega t)$ was used, but in the present paper $\exp(+i\omega t)$ is used in order to conform with current engineering practice. Furthermore, Δ and Δ' will be modified when the antennas are located over a finite ground [4, 5, 6, 7], but this difference will largely be cancelled if a large ground screen is used under the antennas. Although (26) indicates that r_m'' approaches zero as the vertical magnetic dipoles approach the perfectly conducting surface, Wait [4] has shown that r_m'' actually becomes very large when such loops are brought near a finitely conducting ground.

5. Relation Between Propagation Loss, L_P, and Field Strength, E

The results of many previous studies of radio wave propagation in the low and medium frequency range have been expressed in terms of the field strength E, expressed in decibels above 1 μv/m for an unattenuated field I, at a unit distance, i.e., to the radiation field expected at a unit distance on a perfectly conducting plane surface. It is convenient to express I in decibels above 1 v, e.g., 1 v/m at 1 m. We will relate such values to the propagation loss, L_p.

In free space the inverse distance field I_f may be obtained from (10); when we note that $P_t' = P_t - L_t$,

$$I_f = 10 \log_{10}(p_t' c \cdot 10^{-7} g_t) = P_t - L_t + G_t + 14.77. \tag{35}$$

Over a perfectly conducting ground plane the inverse distance field I, may be obtained from (13):

$$I = P_t - L_t + G_t + 20.79. \tag{36}$$

The relation between the available power, p_a, from the receiving antenna and the field strength, e, may be expressed in decibels as follows:

$$E = 10 \log_{10}(4\pi\eta_0 p_a [r'/r_f] \cdot 10^{12}/\lambda^2 g_r)$$
$$= P_a + 20 \log_{10} f_{\mathrm{Mc}} - G_r + L_r + 107.22. \tag{37}$$

If we solve (36) for P_t and (37) for P_a and combine the results, we obtain the following expression:

$$L_s = P_t - P_a = I - E_I - G_p + 20 \log_{10} f_{\mathrm{Mc}} + 86.43 + L_t + L_r \tag{38}$$

In the literature E is sometimes referred to an inverse distance field of 300 v, i.e., 3×10^5 μv/m at 1 km (i.e., by (14) to 1 kw radiated from a short vertical electric dipole on a perfectly conducting plane surface) and in this case $I = 49.54$ db so that

$$L_p = L_s - L_t - L_r = 135.97 - G_p + 20 \log_{10} f_{\mathrm{Mc}} - E_{I=49.54} \tag{39}$$

In the higher frequency ranges, however, E is usually referred to an inverse distance field in free space of 222 v (i.e., by (35) to 1 kw radiated from a half wave dipole in free space) and in this case $I_f = 46.92$ db; if we now solve (36) for $P_t - L_t$, solve (37) for $P_a + L_r$, we obtain the following relation:

$$L_p = P_t - L_t - P_a - L_r = 139.37 - G_p$$
$$+ 20 \log_{10} f_{\mathrm{Mc}} - E_{I_f=46.92} \tag{39a}$$

One of the important advantages of expressing propagation results in terms of the propagation loss L_p rather than in terms of E is the fact that L_p is dimensionless and so does not require a specification of the reference inverse distance field or of the radiated power.

6. System Loss, Transmission Loss, Propagation Loss, and Relative Phase of Currents in Two Antennas Expressed in Terms of Their Self and Mutual Impedances

All of the preceding analyses are limited to the case where the antennas are sufficiently far apart ($d \gg \lambda$) so that the magnitude of their mutual impedance is small compared to the self resistances of each of the antennas. In this section a completely general expression will be obtained for the system loss. It is convenient to use the T-network of figure 2 to represent the impedances of the antennas. The accessible terminals of the transmitting antenna *a* and of the receiving antenna *b* are denoted by AA and BB, respectively. The central member of the T is the mutual impedance z_m between the two antennas while z_a and z_b are the self-impedances of the two antennas; thus the impedance at the terminals AA, with BB open circuited, is z_a while the impedance at the terminals BB, with AA open circuited, is z_b.

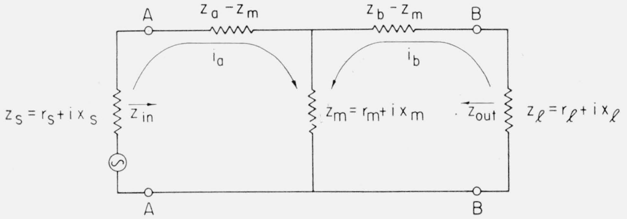

FIGURE 2. T-*network for system loss analysis.*

Let z_l indicate the receiving antenna load impedance at BB. From figure 2 it is easy to show that the impedance z_{in} at AA is given by

$$z_{in} = z_a - \frac{z_m^2}{z_b + z_l}. \tag{40}$$

Since there are no source voltages in the mesh in which the current i_b is flowing, we may write

$$i_b(z_b + z_l) + i_a z_m = 0. \tag{41}$$

The above relation provides a means for determining the relative magnitude and phase of the current i_b in antenna *b* relative to the magnitude and phase of the current i_a in antenna *a*. The fact that i_a and i_b have the same direction of flow through z_m may be established by considering the limiting case as the spacing between the two antennas approaches zero. The ratio of the power $|i_a^2|$ Re z_{in} from the source to the power $|i_b^2|$ Re z_l delivered to the load may now be expressed

$$\frac{|i_a^2| \text{ Re } z_{in}}{|i_b^2| \text{ Re } z_l} = \frac{\text{Re } z_{in}}{\text{Re } z_l}\left|\frac{z_b + z_l}{z_m}\right|^2. \tag{42}$$

Note that the maximum power will be delivered to the load, i.e., the above ratio will be minimized for a constant power input to the transmitting antenna, when the load impedance z_l is adjusted so that it is equal to the complex conjugate z_{out}^* of the output impedance $z_{out} = z_b - z_m^2/(z_a + z_s)$ where z_s is the impedance of the source. In this case i_b^2 Re z_l is equal to the available power from the receiving antenna and the system loss is equal to

$$L_s = 10 \log_{10}\left[\frac{r_a'|z_b + z_{out}^*|^2}{\text{Re } z_{out}|z_m^2|} - \frac{|z_b + z_{out}^*|^2}{|z_m^2|} - \frac{|z_b + z_{out}^*| \cos(2\phi - \theta)}{\text{Re } z_{out}}\right], \tag{43}$$

where r_a' and r_b' denote the real parts of the antenna self impedances z_a and z_b, $z_m \equiv |z_m| \exp(i\phi)$ and $(z_b + z_{out}^*) = 2r_b' - [z_m^2/(z_a + z_s)]^* \equiv |z_b + z_{out}^*| \exp(i\theta)$. Equation (43) is the most general expression for the system loss; this same equation may be used for calculating the transmission loss L simply by replacing the actual antenna resistances r_a' and r_b' by the radiation resistances r_a and r_b; furthermore, (43) gives the propagation loss L_p if we replace r_a' and r_b' by the radiation resistances r_{fa} and r_{fb} which the antennas would have if they were located in free space.

Note now that $z_{out} \simeq z_b$ provided $|z_m^2| \ll |z_b| \cdot |z_a + z_s|$ and this will be the case (a) at large distances or (b) even at short distances if the source impedance z_s is sufficiently large; in this case $z_l = z_{out}^* \simeq z_b^*$. Note that this same relation $z_l = z_b^*$ would apply in case the load impedance were matched to the receiving antenna impedance with the terminals of the transmitting antenna open or with the transmitting antenna at a sufficiently large distance so that it has a negligible effect on z_{out}. In this special case (41) and (43) become

$$i_b(z_l = z_b^*) = -i_a \frac{|z_m| \exp(i\phi)}{2r_b'}, \tag{44}$$

$$L_s(z_l = z_b^*) = 10 \log_{10}\left\{\frac{4r_a' r_b'}{|z_m^2|} - 2\cos(2\phi)\right\}. \tag{45}$$

It will be of interest to illustrate the use of (45) by means of two simple examples. We will consider first the system loss between two short electric dipoles, each perpendicular to the line joining their centers, and separated by a distance *d* in free space. It may be shown [2] that the mutual impedance z_m between the dipoles of effective lengths l_a and l_b, which are each much shorter than a wavelength, is given by

$$z_m = \frac{i\eta_0 l_a l_b}{2\lambda d}\left[1 - \frac{1}{k^2 d^2} - \frac{i}{kd}\right] \exp(-ikd) \equiv |z_m| \exp(i\phi). \tag{46}$$

Here $kd = 2\pi\, d/\lambda$. The antenna self resistances are given by

$$r_a' = 2\pi\eta_0 l_a^2(1 + \Delta_{tc})/3\lambda^2, \tag{47}$$

$$r_b' = 2\pi\eta_0 l_b^2(1 + \Delta_{rc})/3\lambda^2. \tag{48}$$

The factors $(1 + \Delta_{tc})$ and $(1 + \Delta_{rc})$ in (47) and (48) allow for the circuit losses.

$$|z_m|^2 = \frac{\eta_0^2 l_a^2 l_b^2}{4\lambda^2 d^2}\left\{\left(1 - \frac{1}{k^2 d^2}\right)^2 + \frac{1}{k^2 d^2}\right\}, \tag{49}$$

$$\phi=\frac{\pi}{2}-kd-\tan^{-1}\left\{\frac{1}{kd}\Big/\left(1-\frac{1}{k^2d^2}\right)\right\}, \tag{50}$$

$$L_s\ (z_l=z_b^*)=10\ \log_{10}\left[\frac{(2kd)^2\ (1+\Delta_{tc})\ (1+\Delta_{rc})}{(3/2)^2\left\{\left(1-\frac{1}{k^2d^2}\right)^2+\frac{1}{k^2d^2}\right\}}-2\ \cos\ (2\phi)\right]. \tag{51}$$

When the distance $d\gg\lambda$, the second term in (51) is negligible compared with the first term and (51) approaches $(9)+L_{tc}+L_{rc}$ as expected. At the other extreme when $d\ll\lambda$, the first term in (51) is negligible compared to the second term, ϕ approaches $\pi/2$ and $L_s(z_l=z_b^*)=10\ \log_{10}\ 2=3.01$ db. Thus, in this case, half of the power from the source is available in the load resistance. The above formulation of the transmission loss problem in terms of the antenna self and mutual impedances is due to Wait [8] and (51) is similar to the solution Wait obtained in appendix II of his paper for two vertical dipoles on a flat perfectly-conducting ground plane.

The transmission loss may be obtained from (51) if we set $\Delta_{tc}=\Delta_{rc}=0$ and we find that $L(z_l=z_b^*)$ between short dipoles is then dependent only on kd. Figure 3 gives the transmission loss versus (d/λ) for this special case, and the system loss for the case $(1+\Delta_{tc})=(1+\Delta_{rc})=2$. By the same method used for deriving (51) it may be shown that it applies also to small magnetic dipoles, i.e., to two small loop antennas with the planes of each loop lying in the same plane.

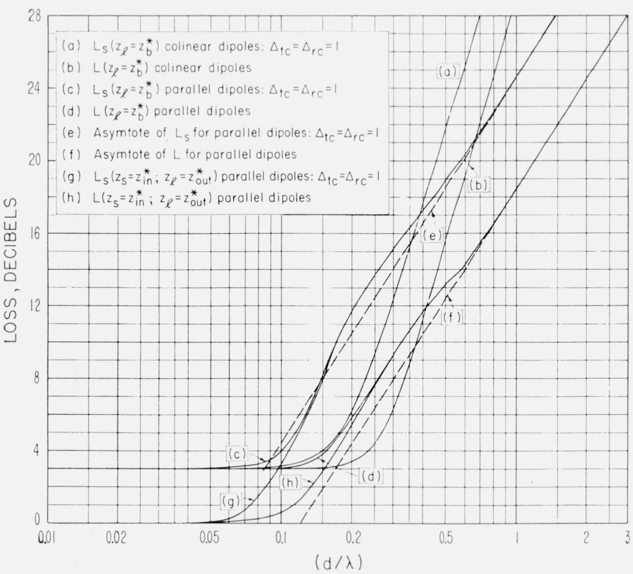

FIGURE 3. *System loss and transmission loss between small electric or magnetic dipoles in free space.*

Consider next two short electric colinear dipoles separated by a distance d in free space. In this case,

$$|z_m|^2=\frac{\eta_o^2 l_a^2 l_b^2}{\lambda^2 d^2 k^2 d^2}\left\{1+\frac{1}{k^2d^2}\right\}, \tag{52}$$

$$\phi=\frac{\pi}{2}-kd+\tan^{-1}(kd), \tag{53}$$

$$L_s(z_l=z_b^*)=10\ \log_{10}\left[\frac{4k^4d^4\ (1+\Delta_{tc})\ (1+\Delta_{rc})}{9\left\{1+\frac{1}{k^2d^2}\right\}}-2\ \cos\ (2\phi)\right]. \tag{54}$$

Figure 3 also shows the transmission loss, as given by (54) with $\Delta_{tc}=\Delta_{rc}=0$, as well as the system loss for the case $(1+\Delta_{tc})=(1+\Delta_{tr})=2$, between short electric colinear dipoles; (54) also applies to small colinear magnetic dipoles, i.e., small loop antennas with their axes colinear.

Equation (45) and consequently (51) and (54) are not very realistic expressions for the system loss when the distance (d/λ) is small since they are strictly applicable; i.e., the power delivered to the load is the available power, only if z_s increases without limit as (d/λ) decreases. One other example will be used to illustrate the problem. Thus we will assume that the impedance of the source is matched to the transmitting antenna input impedance: $z_s=z_{\text{in}}^*$ while the impedance of the load is *simultaneously* matched to the receiving antenna output impedance: $z_l=z_{out}^*$. In this case we may eliminate z_s^* from the two equations: $z_l=z_b^*-(z_m^2)^*/(z_a^*+z_s^*)$ and $z_s^*=z_a-(z_m^2)/(z_l+z_b)$, and in this way find the following expression for $z_l\equiv r_l+iz_l$:

$$z_l=r_b'\left\{1-\frac{|z_m^2|\ \cos\ (2\phi)}{r_a'r_b'}-\frac{|z_m^4|\ \sin^2\ (2\phi)}{4r_a'^2r_b'^2}\right\}^{1/2}-ix_b+ir_b'\left\{\frac{|z_m^2|\ \sin\ (2\phi)}{2r_a'r_b'}\right\}. \tag{55}$$

We may determine $z_s\equiv r_s+ix_s$ from (55) simply by interchanging r_a' and r_b' and x_a with x_b. The system loss for this case may now be determined by introducing the above expressions in (42):

$$L_s(z_s=z_{\text{in}}^*;z_l=z_{\text{out}}^*)=10\ \log_{10}\left[\frac{2r_a'(r_b'+r_l)}{|z_m^2|}-\cos\ (2\phi)\right]. \tag{56}$$

The above is the general expression for the system loss when the source is matched to the input impedance z_{in} and the load is simultaneously matched to the output impedance z_{out}. This is, of course, the most favorable condition for the transfer of power from the source to the load. Equations (55) and (56) are applicable to any kind of antennas and we see that the system loss depends only on the self resistances r'_a and r'_b of the two antennas and their mutual impedance $z_m \equiv \| z_m \| \exp(i\phi)$.

When $d \gg \lambda$, r_l approaches r'_b and (56) approaches (45) since the second terms in (45) and in (56) are negligible compared to the first terms; thus we conclude that the system loss does not depend upon the condition of matching of the source to the input impedance when d is sufficiently large so that $|z_m^2| \ll r'_a r'_b$.

The behavior of (55) and (56) when $d \ll \lambda$ will depend markedly upon the particular characteristics of the antennas involved. At such short spacings one would normally be interested only in antennas with dimensions small with respect to their spacing and thus small with respect to the wavelength; thus, when $d \ll \lambda$, we are led to consider the special case of short electric or small magnetic dipoles. When (47), (48), (49), and (50) are used in conjunction with (55) and (56) we find that the system loss, $L_s(z_s = z^*_{\text{in}};\ z_l = z^*_{\text{out}})$, approaches zero when $d \ll \lambda$, i.e., when $(kd)^6 \ll (9/16)/\{(1+\Delta_{tc})(1+\Delta_{rc})\}$, even if r'_a and r'_b are very large with respect to their radiation resistance components; see figure 3 for a graph of the transmission loss and for the system loss with $(1+\Delta_{tc}) = (1+\Delta_{rc}) = 2$ for this type of impedance matching. A system with near zero loss is very remarkable, but may be understood when we note that (r_l/r'_b) and (r_s/r'_a) both increase in this case without limit as d approaches zero; thus the power from the source $i_a^2 r_s$ is much larger than the power $i_a^2 r'_a$ dissipated in the transmitting antenna while simultaneously the power $i_b^2 r_l$ delivered to the load is very much larger than the power $i_b^2 r'_b$ dissipated in the receiving antenna. Although the system loss is nearly equal to zero, only about half (actually a fraction $[r'_b/(r'_a+r'_b)]$) of the power is available in the load while the remaining fraction $[r'_a/(r'_a+r'_b)]$ is dissipated in the impedance of the source.

It might appear that a very efficient short range communication system could be developed based on the above analysis by using sufficiently low frequencies that $\lambda \gg d$; note that the signal-to-noise ratio should be quite favorable since the small antennas used would pick up very little noise power, and this small noise power would be in competition essentially with the large power directly available from the source. Note also that this communication system would create very little interference since the power radiated $i_a^2 r_a + i_b^2 r_b$ is much smaller than the power $i_b^2 r_l^2$ delivered to the load. However, the distance out to which the system loss is less than 3db for such a system is quite small as may be seen from the following equation which applies when $r'_a \gg r_a$ and $r'_b \gg r_b$:

$$d(L_s = 3\text{ db}) = \frac{\lambda}{2\pi}\left\{\frac{(9/16)r_a r_b}{r'_a r'_b}\right\}^{1/6}. \tag{57}$$

If we assume that the antennas are grounded vertical radiators and substitute (11) with $\Delta_t = 1$ for r_a and r_b in (57), we obtain

$$d(L_s = 3\text{ db}) = 1.68\lambda^{1/3}\, l_a^{1/3}\, l_b^{1/3}/(r'_a r'_b)^{1/6} \tag{58}$$

If we assume $\lambda = 100{,}000$ m (f=3 kc), $l_a = l_b = 100$ m, i.e., with actual heights without top loading equal to 200 m, and $r'_a = r'_b = 10$ ohms, then $d(L_s = 3\text{ db}) =$ 781 m; since very good ground systems would be required to reduce r'_a and r'_b to only 10 ohms, it appears that such a system has little promise. This statement should not be interpreted to mean that low frequencies are not useful for short range communication systems, but merely that such systems with less than 3-db system loss will have a very limited range of operation. The propagation losses encountered at low and medium frequencies are discussed in a recent paper by the author [32].

Finally it is of interest to examine briefly the behavior of the system loss at short spacings ($d \ll \lambda$) when the antennas have dimensions which are comparable to the wavelength and thus large with respect to their spacing. For this analysis it will be more convenient to express (55) in the following form:

$$z_l = r'_b\left\{\left(1 - \frac{r_m^2}{r'_a r'_b}\right)\left(1 + \frac{x_m^2}{r'_a r'_b}\right)\right\}^{1/2} - ix_b + i\,\frac{r_m x_m}{r'_a}. \tag{59}$$

If it is assumed that the transmitting and receiving antennas are each dipoles (either electric or magnetic) with idential dimensions (comparable to the wavelength) and identical orientations, than as d approaches zero, r_m approaches the antenna self resistances $r'_a = r'_b$. Since the antennas are large, they may have small reactances and large resistances (e.g., half-wave dipoles in free space); in this case the load resistance r_l and the source resistance r_s will both be very small and, since ϕ is now also very small, the system loss will by (56) still be approximately equal to zero. In this case again the power delivered to the load is much larger than the power radiated. This unusual situation may be understood when it is noted that $i_b \cong -i_a$ so that the net power radiated from two such closely spaced antennas is approximately proportional to $(i_a + i_b)^2$ and thus extremely small. It is, of course, well known that it is difficult to radiate much power from a directional antenna consisting of a pair of closely spaced elements with the currents in the two elements out of phase.

7. Variations of the System Loss with Time

The instantaneous signal power available on long circuits involving transmission through the ionosphere or troposphere will vary with time (i.e., fade) due, in part, to phase interference between the components arriving along various transmission paths. Over short periods of time, during which transmission conditions may be regarded as constant, the instantaneous available power may, in some cases, be regarded as distributed in a Rayleigh distribution, to a good approximation. This distribution may be expressed

$$Q(p_a > y) = \exp(-y/\overline{p}_a), \tag{60}$$

where p_a is the instantaneous available power, $\overline{p}_a$ is the average power and Q is the probability that the instantaneous power exceeds some given value y. A discussion of the physical conditions under which the Rayleigh distribution may be expected to apply, together with an extension to the case in which a large nonfading component of power is present at the receiver terminals, is given in a recent paper by Norton, Vogler, Mansfield, and Short [9].

It has been found experimentally that $\overline{p}_a$ varies relatively slowly with time, and it is customary to determine the hourly median values p_h which, by (60), are related to $\overline{p}_a$ by $p_h = (\log_e 2)\overline{p}_a$. It has been determined experimentally that the hourly median values, p_h, for ionospheric or tropospheric propagation at a given time of day and a given season of the year are log-normally distributed [10, 11, 12]. It follows from this that the hourly median system loss is normally distributed with a standard deviation which may be expressed in decibels. For example, if the distribution of the 30 daily values of the hourly median transmission loss for 8 to 9 p.m. in June is studied, it is found that these values will appear to be a sample from a normal distribution of such values. Thus a reasonably satisfactory description of the expected distribution of the hourly median (or average) transmission loss for a specified time of day and season of the year can be given in terms of the two parameters, the median (of the hourly medians from day to day) and the standard deviation (of these hourly medians), both expressed in decibels.

8. Percentage of Time That a Desired Signal is Free of Interference

The effective range of a radio system is determined when the desired signal becomes so weak that it is obscured by the presence of noise or of other types of interference for too large a percentage of time. Consider first the effect of noise. This is most readily determined by means of a generalization [13] of Friis' definition [14] of the noise figure of a radio receiver. Consider the network of figure 4. It is convenient to compare the desired signal power with the noise power in network (a), i.e., in the loss-free receiving antenna. Let p_n denote the average external noise power and set

$$p_n \equiv f_a k_B t b \qquad \text{(watts)} \tag{61}$$

where k_B is Boltzmann's constant and is equal to 1.38×10^{-23}, t is the absolute temperature in degrees Kelvin, and b is the effective bandwidth in cycles per second as defined by Friis [14]. Thus (61) effectively defines the noise figure f_a of network (a). Network (c) has a noise figure $f_c = r'/r$ which is simply the loss factor of the receiving antenna circuit. Similarly the transmission line loss is equivalent to a noise figure f_t which is the ratio of the power in-and-out of the line. Finally we may

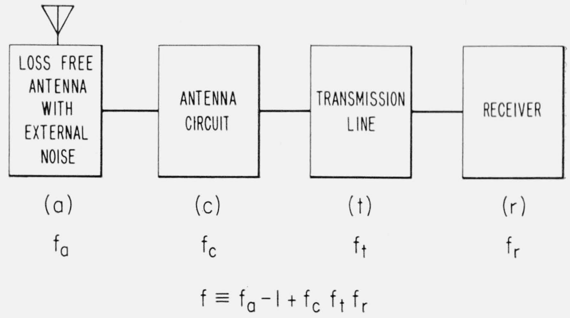

FIGURE 4. *Network defining the effective receiver noise figure including the effects of external noise.*

denote the noise figure of the receiver itself by f_r. Using Friis' method for combining the noise figures of several networks in tendem, we obtain for the effective noise figure at the input to the loss-free receiving antenna

$$f = f_a - 1 + f_c f_t f_r. \tag{62}$$

Finally, if we write r_n for the minimum signal-to-noise power ratio which will provide a given grade of reception, e.g., teletype reception with an 0.01 percent binary error rate, then the minimum signal power available at the terminals of the receiving antenna which will provide this grade of reception may be expressed

$$p_m = r_n f k_B t b \qquad \text{(watts)}. \tag{63}$$

Expressed in decibels (63) becomes

$$P_m = R_n + F + B - 204, \tag{64}$$

where $R_n = 10 \log_{10} r_n$, $F = 10 \log_{10} f$, $B = 10 \log_{10} b$, and $10 \log_{10}(k_B t) = -204$ if we use a reference temperature $t = 288.48°$ K.

The power P'_{ta} radiated from the transmitting antenna of a desired station at a which is required for a specified grade of reception by a receiver at b with a given transmission loss L_{ab} may now be expressed

$$P'_{ta} = L_{ab} + P_{mb} = L_{ab} + R_n + F_b + B - 204 \tag{65}$$

If the signal has a well-defined short term fading characteristic, e.g., representable by the distribution corresponding to that of a constant component plus a Rayleigh-distributed component [9], it is customary to generalize R_n to be the minimum median-signal to median-noise ratio required for a specified grade of reception of such a fading signal. In this case (65) gives the required radiated power if L_{ab} and F_b in (65) are considered to be hourly median values or, more generally, median values over a sufficiently short period of time that the short term variations of the signal and the noise each have the characteristics specified in determining R_n. In order to allow for the long-term fading characteristic, we recall that the hourly median values of the transmission loss at a specific time of day, season of the year, and period in the sunspot cycle, tend to be normally distributed about their long-term median value L_{mab} with a standard deviation σ_{Lab}. Now let $\lambda(x)$ denote a standard normal deviate and x the probability that a randomly chosen normally distributed variable with unit standard deviation will not exceed $\lambda(x)$:

$$x = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\lambda(x)} \exp(-y^2/2) dy. \quad (66)$$

Note that $\lambda(x)$ varies from $-\infty$ to $+\infty$ as x varies from 0 to 1 and that $\lambda(0.5)=0$. Now we may express the value of the hourly median transmission loss not exceeded with probability x by

$$L_{ab}(x) = L_{mab} + \sigma_{Lab}\,\lambda(x). \quad (67)$$

Similarly F_b tends to be normally distributed about its long-term median value F_{mb} with a standard deviation σ_{Fb}:

$$F_b(x) = F_{mb} + \sigma_{Fb}\lambda(x). \quad (68)$$

Equations (67) and (68) are not valid in the limit as x approaches zero since L_{ab} and F_b are inherently positive, but this restriction is unimportant in practical applications. The random variables L_{ab} and F_b may be correlated (usually negatively) with correlation coefficient ρ_{abn} since diurnal, seasonal, and secular variations in propagation conditions sometimes cause L_{ab} to decrease at the same time that they cause F_b to increase. Now the above results may be combined and the following relation determined by standard statistical methods:

$$(\sigma_{Lab}^2 + \sigma_{Fb}^2 + 2\rho_{abn}\sigma_{Lab}\sigma_{Fb})^{1/2}\lambda(0.01 q_{abn}) = P'_{ta}(q_{abn}) - R_n - L_{mab} - F_{mb} - B + 204. \quad (69)$$

In the above, q_{abn} is the percentage of hours during which the specified (or better) grade of reception of the signals from station a is possible at station b when noise is the only source of interference. Note that q_{abn} varies from 0 to 100 percent as the right hand side of (69) varies from $-\infty$ to $+\infty$. When $P'_{ta}(q_{abn})$ is fixed, (69) may be solved for q_{abn}; alternatively, by fixing q_{abn}, it is possible to solve (69) for the transmitter power $P'_{ta}(q_{abn})$ required to provide the specified (or better) grade of service for q_{abn} percent of the hours at a given time of day, season of the year, and period of the sunspot cycle. For example, if service is required for $q_{abn}=99$ percent of the hours, we find from tables of the standard normal deviate [*15*] that $\lambda(0.99)=2.326$ and thus $2.326(\sigma^2_{Lab}+\sigma^2_{Fb}+2\rho_{abn}\sigma_{Lab}\sigma_{Fb})^{1/2}$ db more power is required than would be required for $q_{abn}=50$ percent of the hours since $\lambda(0.5)=0$. If L_{mab}, σ_{Lab}, F_{mb}, σ_{Fb}, ρ_{abn}, and R_n are all known as a function of time, then $P'_{ta}(q_{abn})$ can be determined as a function of time.

For a proper determination of F_{mb} and σ_{Fb} it is necessary to know the magnitude and variance of the external noise f_a at the location b. Predictions of f_a over a wide range of frequencies and geographical locations were originally published by Crichlow et al. [16]. Revised predictions of f_a are available in a recent report [17] of the Consulting Committee on International Radio (C.C.I.R.). Using the results in a recent paper by Cottony and Johler [18], values of f_a may also readily be determined for cosmic radio noise simply by dividing their equivalent blackbody temperatures by 288.48. It should be noted that f_a will usually be somewhat dependent upon the receiving antenna directivity and the published values of f_a quoted above correspond to reception on a short vertical electric antenna in references [16] and [17] and to reception on a horizontal half-wave electric dipole in [18].

It is, of course, possible to determine the instantaneous distribution of the transmission loss by combining the short term and long term fading distributions, and a solution of this problem for the case of signals with a Rayleigh distribution over short periods of time and with a log-normal distribution of hourly medians has been given by Garner McCrossen [19]. However, it is usually more convenient to follow the procedure described above and absorb the effects of the short term fading distribution into the required signal-to-noise ratio.

To the degree that efficient use is made of the radio spectrum, interference from other radio stations rather than noise will often limit the effective range of the desired station. It is convenient in formulating this problem to consider initially the interference caused at receiving location b as a result of the operation of another system including a transmitter at c and a receiver at d. This special case will then be extended to determine the interference between additional pairs of stations and ultimately to a mobile or other broadcast type of system. If we write r_u for the minimum median-signal to median-interference power ratio at the terminals of the receiving antenna which will provide a given grade of reception of a desired signal with appropriate allowance for the modulation characteristics and the short term fading characteristics of both the desired and undesired carriers, then we find that the criterion for the absence of interference at the receiving location b from the undesired transmitter at c may be expressed

$$L_{cb} - L_{ab} + P'_{ta} - P'_{tc} \geq R_u. \quad (70)$$

In the above $R_u = 10 \log_{10} r_u$, L_{ab}, and L_{cb} denote the hourly median values of the transmission loss at the time in question for the paths *ab* and *cb*, respectively, while P'_{ta} and P'_{tc} are the powers radiated from the transmitting antennas at *a* and *c*, respectively. Now let L_{mcb} denote the long term median value of L_{cb}, σ_{Lcb} its standard deviation while ρ_{abcb} denotes the correlation coefficient between the variations of L_{ab} and L_{cb}. Now we may determine from the following equation the percentage of the hours q_{abc} at a given time of day, season of the year and phase of the sunspot cycle, during which reception of a given grade *or better* is possible at station b of signals from station a on the assumption that the effects of all other sources of interference may be neglected except that from the station at c:

$$(\sigma_{ab}^2+\sigma_{cb}^2-2\rho_{abcb}\sigma_{ab}\sigma_{cb})^{1/2}\lambda(0.01q_{abc})$$
$$=L_{mcb}-L_{mab}+P'_{ta}-P'_{tc}-R_u. \quad (71)$$

For the efficient use of the radio spectrum it is most important to notice that the percentage of hours q_{abc} free of interference from the undesired station at c depends on the *ratio* $P'_{ta}-P'_{tc}$ of the desired and undesired station powers, whereas the percentage of hours q_{abn} free of interference from noise depends only on P'_{ta}.

An expression is also required for the percentage of hours q_{cda} of satisfactory reception of station c at station d on the assumption that the effects of all other sources of interference may be neglected except that from the station a:

$$(\sigma_{cd}^2+\sigma_{ad}^2-2\rho_{cdad}\sigma_{cd}\sigma_{ad})^{1/2}\lambda(0.01q_{cda})$$
$$=L_{mad}-L_{mcd}+P'_{tc}-P'_{ta}-R_u. \quad (72)$$

Now suppose that q_{abc} and q_{cda} are both sufficiently large (say greater than 99%) so that satisfactory service would be available in both systems except for the effects of noise, i.e., suppose that q_{abn} is less than 99 percent and that q_{cdn} is possibly, but not necessarily, also less than 99 percent. In this case we should, if this is feasible, increase P'_{ta} until $q_{abn}>99$ percent. However, if this is done, it may cause interference to the reception of station c by station d since q_{cda} will be decreased by an increase in P'_{ta} *unless* P'_{tc} is increased in the same proportion so that $P'_{tc}-P'_{ta}$ remains constant. The above argument is easily extended to any number of station pairs, and thus we have shown that a net improvement in the use of the spectrum for a given geographical configuration of radio stations can *always* be obtained by increasing the power of all of the stations by the same number of decibels until the power of each of the stations is adequate to reduce the level of the noise well below the level of the interference caused by the other stations. In practice there will be an economic limit above which such horizontal increases in station power are no longer feasible. When this limit is reached, then still further improvement of the use of the spectrum can *usually* be obtained by allocating additional stations or, in the case of a broadcast or mobile service, by reducing the spacing between the existing stations until the interference between them is comparable to the interference from noise.

The above analysis is, of course, only qualitative; thus, for example, some types of service might have to be available for much more than 99 percent of time. Nevertheless the principles of efficient allocation described above appear to be universally applicable and it is concluded that a satisfactory allocation of facilities to any portion of the spectrum can be considered to exist only when it is impossible to find a geographical location at which radio noise, rather than radio signals, may be observed. It also follows from the above discussion that an efficient allocation of broadcast stations necessarily implies the existence of large areas in which there will be mutual interference between the stations since otherwise it would be possible to observe radio noise, rather than radio signals, in these interference regions.

Equations (69), (71), and (72) are strictly applicable only when a single source of interference, identified by the last subscript on q, is present at the receiving location which is identified by the second subscript on q. A formal method of extending these results to the case where several sources of interference are present with comparable magnitudes at the receiving location was suggested in [1] and the use of this method for studying broadcast station allocations was described in a paper by Norton, Staras, and Blum [20]. However, because of the complexity of the above formal solution, it is seldom used in practice. Instead, it is usually more practical to consider that a receiving location is satisfactorily free of interference provided the values of q for all of these sources of interference are each independently larger than some predetermined value, say 99 percent.

It is of interest to see how the above interference analysis would be applied to the allocation of broadcast stations. In the first place it seems clear that the maximum economically feasible transmitter power and antenna height should be used so as to overcome the effects of noise for each station over the widest possible area. Having fixed the transmitter power and antenna height the stations should then be located near enough together so as to minimize the areas between the stations where noise may be observed free of radio signals. In fact, studies have been made [21] which indicate that there are optimum separations between broadcast stations with a fixed power; when the stations are located on a regular triangular lattice these optimum separations are quite large but, in the practical case of irregularly located stations, their optimum spacing will be much smaller and will be smallest in the case of isolated pairs of stations.

Variations of Path Antenna Gain With Time in Ionospheric Scatter Propagation

During recent years extensive measurements have been made at the Central Radio Propagation Laboratory on several long transmission paths in the United States, Canada, and Alaska of the system loss for the ionospheric scatter mode of propagation. Some of these measurements were reported in detail in a paper by Bailey, Bateman, and Kirby [22]. In this section a further analysis will be given of some of the data obtained by Bailey, Bateman, and Kirby on the Fargo-Churchill path.

The measurements were made in the following way. At the Fargo end of the path, the transmitter power was switched every half hour alternately to a high gain rhombic antenna and to a half-wave dipole antenna, both at the same height above the ground.

At the Churchill end of the path, continuous recordings were made of the signals from Fargo as received simultaneously on a high gain rhombic antenna and on a half-wave dipole antenna, both at the same height above the ground. For odd half hours, measurements were thus available of the half-hourly median system losses L_{rr} and L_{rh}, and for even half hours measurements were available of the median system losses L_{hr} and L_{hh}, where the first subscript denotes the transmitting antenna while the second subscript denotes the receiving antenna. The half-hourly median path antenna power gain corresponding to the use of a half-wave dipole antenna on the transmitting end of the path and a rhombic antenna on the receiving end of the path may now be determined from:

$$G_{pphr} = L_{hr} - L_{hh} + 4.3. \quad (73)$$

In the above the term 4.3 is the assumed path antenna gain between the two loss-less half-wave dipoles. On the assumption that the system loss does not vary appreciably over a period of 1 hr, it is also possible to estimate the half-hourly median path antenna power gain corresponding to the use of rhombic antennas on both ends of the path from

$$G_{pprr} \simeq L_{rr} - L_{hh} + 4.3, \quad (74)$$

where L_{rr} and L_{hh} denote the values of these median system losses in successive half-hour periods of time. Only these latter values are analyzed here and in what follows we have set $G_{pprr} \equiv G_{pp}$, $L_{rr} \equiv L_s$ and $L_{hh} - 4.3 \equiv L_b$. The observed cumulative distributions of G_{pp}, L_s and L_b as thus defined are given on figure 5 for a 3-day period in February 1952. Note that each of these three observed quantities appears to be a sample from a normal distribution, and on this assumption we find the following sample statistics: $\overline{G}_{pp} = 25.71$ db, $\sigma_{Gpp} = 5.85$ db, $\overline{L}_s = 176.75$ db, $\sigma_{Ls} = 6.98$ db, $L_b = 202.75$ db, and $\sigma_{Lb} = 2.47$ db. The values of L_s and G_{pp} are highly correlated, the correlation coefficient determined from this sample being $\rho = -0.94$. On the assumption that L_s and G_{pp} are normally distributed, the random variable $L_b \equiv L_s + G_{pp}$ will also be normally distributed with expected mean $L_{be} = L_s + G_{pp} = 202.46$ db and expected variance $\sigma^2_{Lbe} = \sigma^2_{Ls} + \sigma^2_{Gpp} + 2\,\rho\,\sigma_{Ls}\,\sigma_{Gpp} = (2.48)^2$. Note that these values are in good agreement with the values of the mean and variance of L_b determined directly from the sample.

It is seen from the above analysis that the variance of the system loss for ionospheric scatter propagation increases with an increase in the path antenna gain. There is some evidence that this may not be the case for tropospheric scatter propagation. For example, Bullington, Inkster, and Durkee [33] measured the variance of 505–Mc and 4090–Mc tropospheric forward scatter over the same path using 28-ft parabolic reflectors in both cases, and thus with much higher gain on the higher frequency, and found an actually smaller variance for the 4090–Mc transmission losses.

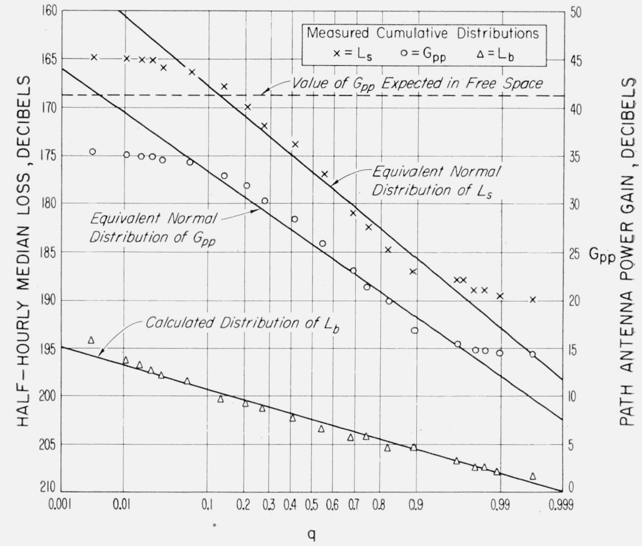

FIGURE 5. *Measurements of* L_s, L_b, *and* G_{pp} *for the Fargo-Churchill ionospheric forward scatter path on Feb. 15, 16, 17, 1952;* d=824 *statute miles,* f=49.7 *Mc.*

10. Transmission Loss for Tropospheric Forward Scatter and Ground Wave Propagation

As was pointed out in an earlier paper [1] the concept of transmission loss is especially appropriate for describing forward scatter propagation, and this idea was developed in some detail in a recent paper by Norton, Rice, and Vogler [12]. Recently the author showed [23, 24] that the $(r/l_0)K_1(r/l_0)$ correlation function could be used for describing the physical nature of atmospheric turbulence and that its use, together with an assumed exponential dependence of the gradient of refractive index with height, provides a theory of tropospheric forward scattering in good agreement with the available data over a wide range of distances, frequencies, and antenna heights.

Recently, Bean and Thayer [25] developed several model tropospheric atmospheres which depend only upon the single parameter, N_s the value of the refractivity at the surface of the earth, i.e., $(n_s-1)\cdot 10^{-6}$ where n_s is the refractive index at the surface. In the present paper use will be made of the particular Bean and Thayer model in which N decreases linearly with height for the first kilometer above the surface, and exponentially with height above 1 km, having scale heights dependent on N_s for the height range from 1 to 9 km but having a fixed scale height above 9 km. Figure 6 compares the trace of radio rays in these model atmospheres for the typical values of $N_s = 250$, 301, 350, and 400 and for several initial angles θ_0 of departure.

Using results of this kind, it is possible to use the methods of geometrical optics to calculate the groundwave fields expected in such atmospheres at points well within the horizon and to use the method described below for points beyond the horizon; it is then easy to join these curves and obtain a solution

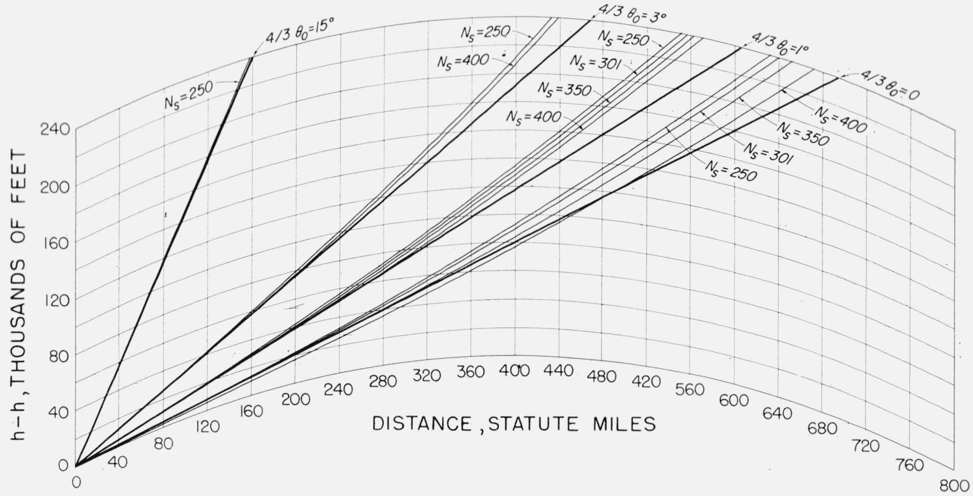

FIGURE 6. *Comparison of rays in the 1958 CRPL reference refractivity atmospheres with those for an earth with no atmosphere and an effective radius* a_e=5,280 *statute miles* ($a_e/a=4/3$).

for the groundwave fields expected for such atmospheres at any distance and for any antenna height. Furthermore, using the methods outlined in [23] and [24] and to be described more fully in a paper [26] to be published in this Journal in the near future, the tropospheric forward scattered fields may be calculated. Figures 7, 8, and 9 are examples of the median basic transmission loss for the groundwave and for the tropospheric scatter modes of propagation over a smooth spherical earth as calculated in this way. Allowance was made at the higher frequencies for atmospheric absorption by using the results of Bean and Abbott [27]. On figure 8 the first two oscillations of the field caused by interference between a direct and ground-reflected wave are shown for d=10 and 20 miles, but for the other distances only one oscillation is shown. The six points of field maxima are shown for all of the distances as circled points. Note that the total number of maxima to be expected (as a function of range for a given height or as a function of height at a given range) for a particular antenna height is equal to the number of half-wavelengths contained in this height; in the present example 30 ft contains 6 half-wavelengths at 100 Mc.

It is well known that approximate allowance may be made for the effects of radio refraction in the troposphere by calculating the fields for an earth with no atmosphere but with an effective radius a_e determined by

$$a_e=a\Big/\left[1+\frac{a}{n_s}\left(\frac{dn}{dh}\right)_s\right], \tag{75}$$

where a is the actual radius, n_s is the value of the refractive index at the surface and $(dn/dh)_s$ is the gradient (usually negative) of the refractive index at the surface. We will see that it is a simple matter to calculate the transmission loss L at or beyond the horizon for a horizontally homogeneous model atmosphere in terms of the transmission loss L_e expected for an earth with no atmosphere but with an effective radius a_e. Figure 10 illustrates the geometry of this more general method of making allowance for the effects of atmospheric refraction on the transmission loss between antennas at a and at b. If the index of refraction is specified as a function of height above the surface in such a way that ray tracing methods [25] may be used for establishing the trajectory of the ray, and such a restriction will rule out very few atmospheric models that are likely to be proposed, then it is possible to calculate for such atmospheres Δh_1 and Δd_1 as a function of the height h_1 and to calculate Δh_2 and Δd_2 as a function of the height h_2. Figures 11 and 12 show Δd and Δh versus h for the above-described Bean and Thayer model atmospheres for N_s=250, 301, 350, and 400, i.e., for a_e=4878.50, 5280, 5896.66, and 6996.67 statute miles, respectively. It may be noted that these particular model atmospheres have discontinuities in their gradients at 1 km above the surface and at 9 km above sea level, but these discontinuities cause no difficulties since we have used ray tracing methods based on geometrical optics. Note that the use of geometrical optics is preferable in this particular application; thus the use of a rigorous wave treatment of the problem would probably have lead to false conclusions since the actual atmosphere cannot have such abrupt discontinuities in its gradient.

If n were assumed to have a constant gradient $(dn/dh)=-n_s\left(\frac{1}{a}-\frac{1}{a_e}\right)$ at all heights (as determined by solving (75) for $(dn/dh)_s$ and then assuming that this surface gradient is the same at all heights), then it is easy to show for such an atmosphere that the distance to the radio horizon from a height h is given by

$$d=a_e \text{ arc cos } [a_e/(a_e+h)]\simeq\sqrt{2a_e h} \tag{76}$$

The approximation in the above is valid when h is

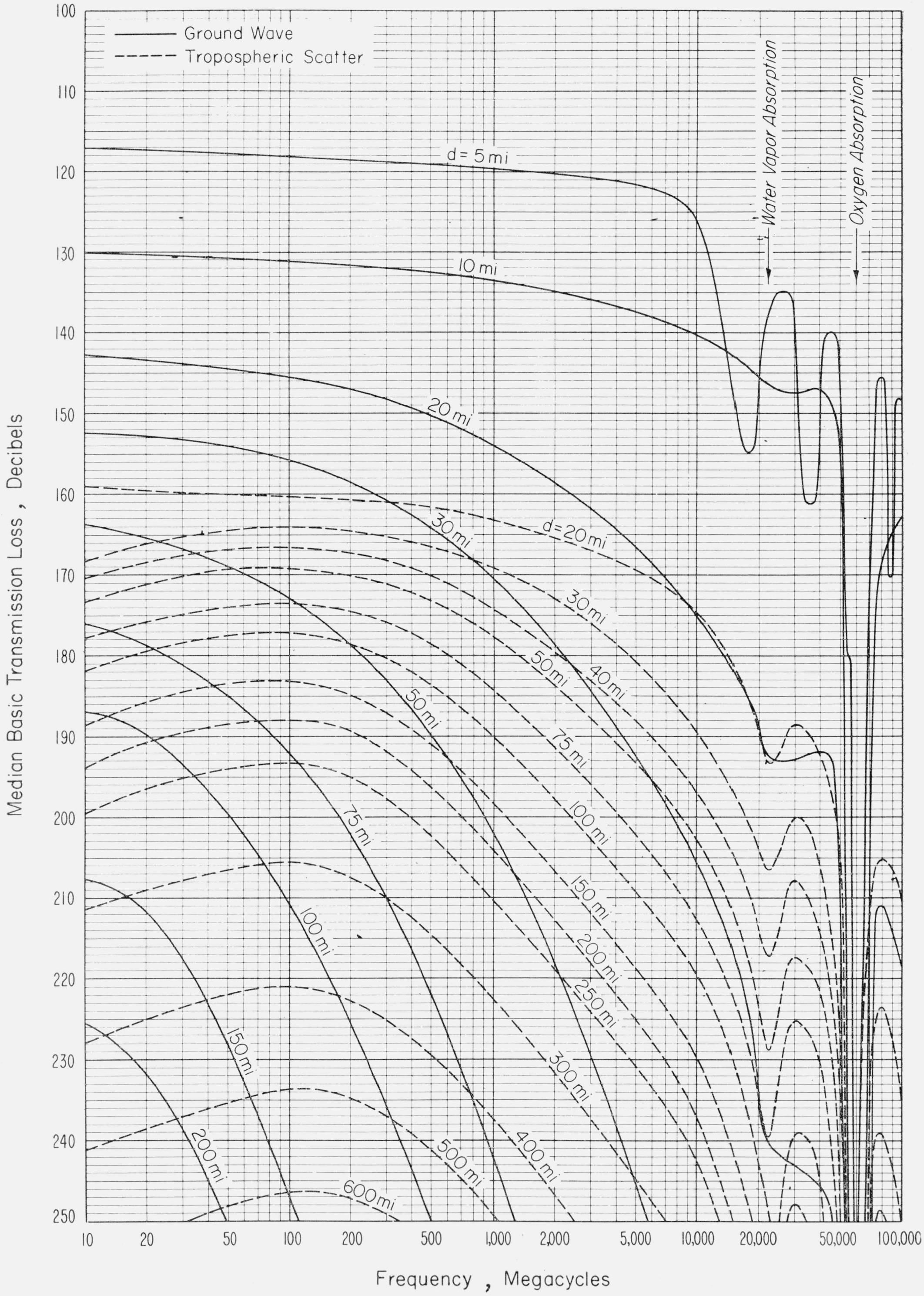

FIGURE 7. *Median basic transmission loss for the groundwave and tropospheric scatter modes of propagation over a smooth spherical earth; over land $\sigma=0.005$ mhos/m, $\epsilon=15$; horizontal polarization; transmitting and receiving antennas both 30 ft above the surface.*

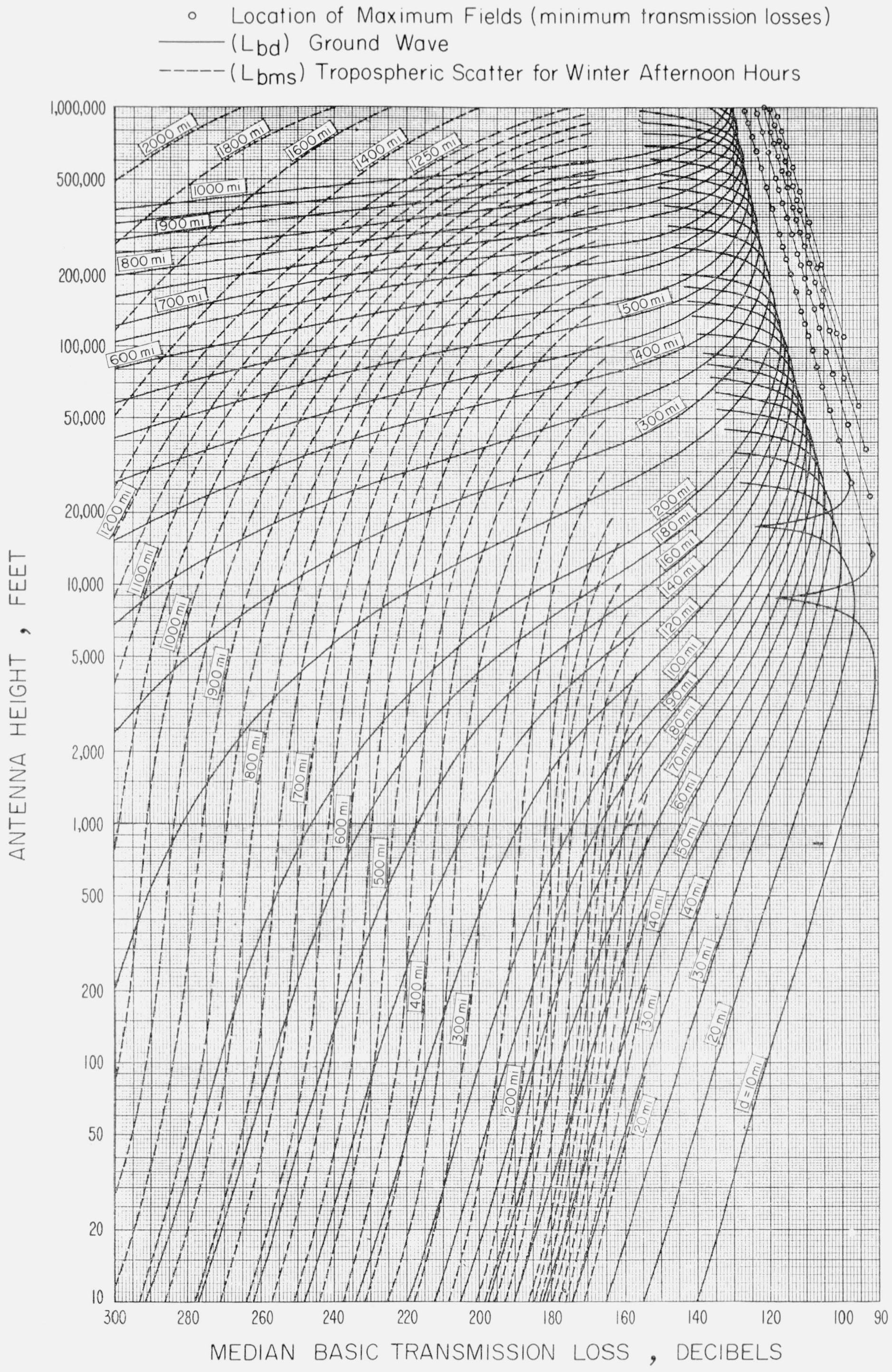

FIGURE 8. *Median basic transmission loss for the groundwave and tropospheric scatter modes of propagation at 100 Mc over a smooth earth in the 1958 CRPL reference refractivity atmosphere with $N_s=301$; horizontal polarization over land; one antenna at 30 ft and the other antenna at the heights indicated.*

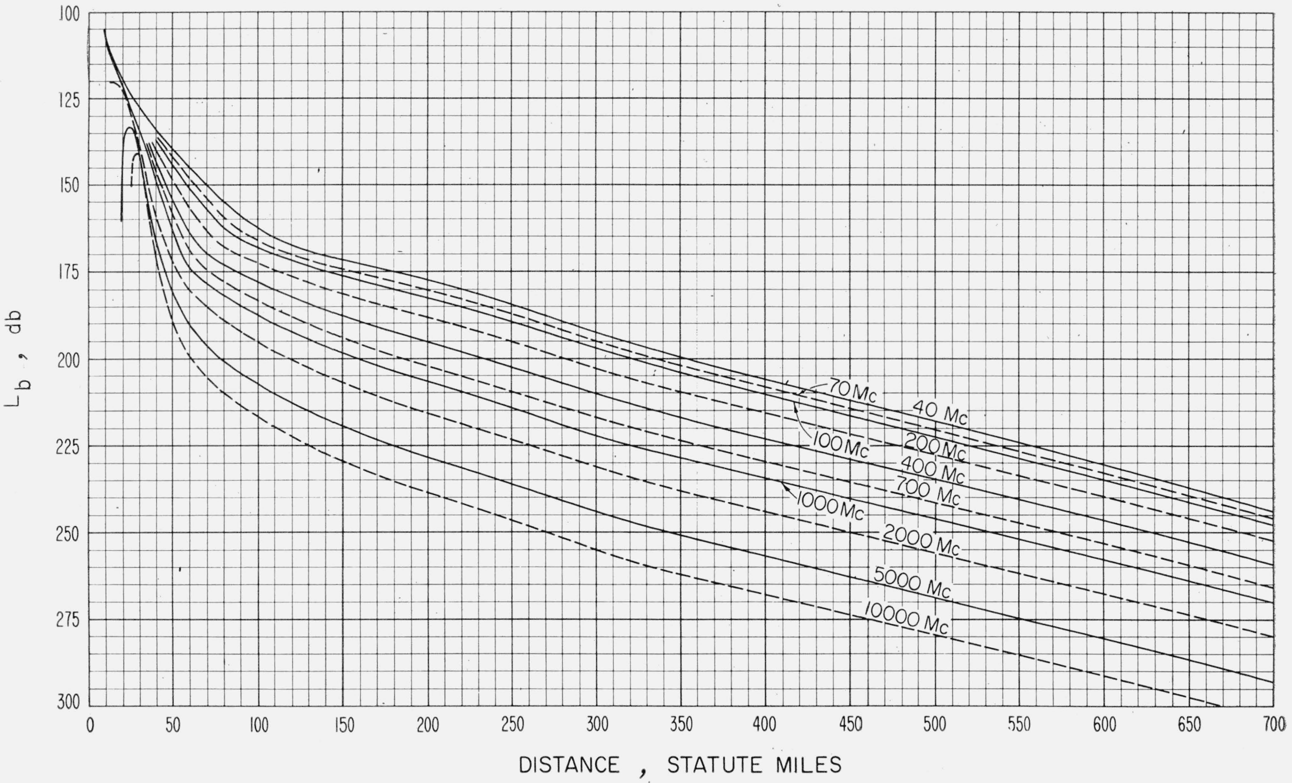

FIGURE 9. *Median basic transmission loss for the groundwave and tropospheric scatter modes of propagation over a smooth earth in the 1958 CRPL reference refractivity atmosphere with* $N_s=301$*; horizontal polarization over land; one antenna at 500 ft and the other antenna at 30 ft above the surface.*

sufficiently small. Since our model atmosphere has just such a linear gradient for the first kilometer above the surface, we see why Δh and Δd on figures 11 and 12 are equal to zero for heights h less than 1 km. At larger heights we see by figure 6 or by figures 11 and 12 how the distance to the radio horizon gradually departs from (76) in the model atmospheres which are much better approximations to the actual atmosphere than the assumption of a linear gradient at all heights.

Since the radio waves travel near the surface over that part of the path (of length d_s on fig. 10) between the radio horizons, it is intuitively clear that their propagation will be affected only by the refractive index gradient at the surface, and thus this portion of the propagation will be independent of the other characteristics of the atmosphere. Thus it is clear that only those portions of the propagation path from the antennas to their radio horizons are affected by that part of the atmosphere much above the surface, and these can be treated by the methods of geometrical optics. In this way the following relations are determined for evaluating the transmission loss $L(d,h_1,h_2)$ between the points a and b in the model atmosphere over an earth of radius a in terms of the transmission loss L_e expected for an earth with no atmosphere but with an effective radius a_e:

$$L(d, h_1, h_2)=L_e(d, h_1', h_2'), \qquad (77)$$

where

$$h_1'=h_1-\Delta h_1 \text{ and } h_2'=h_2-\Delta h_2$$

or

$$L(d, h_1, h_2)=L_e(d', h_1, h_2)-10\log_{10}(d'/d) \qquad (78)$$

where

$$d'=d+\Delta d_1+\Delta d_2$$

Equation (77) was used for determining the median basic transmission losses for the groundwave as shown on figure 8; essentially these same ray tracing methods were also used for determining the scatter losses shown on figures 7, 8, and 9, but in this case they were used also for tracing the rays up to the scattering region as well as for tracing the rays from an antenna to its radio horizon.

J. R. Wait, in a paper to be published shortly, has provided a rigorous basis for our intuitive development of (77) for sufficiently high frequencies and for model atmospheres in which the variation of the refractive index with height is monotonic.

Millington independently derived the above-described method of allowing for the effects of refraction in a horizontally homogeneous atmosphere, and suggested a formula for adjusting groundwave field strength similar to (78) but omitting the inverse distance factor $10 \log_{10}(d'/d)$ which is usually negligible [28].

The groundwave curves in the C.C.I.R. Atlas [29] have been prepared using the approximately-linear

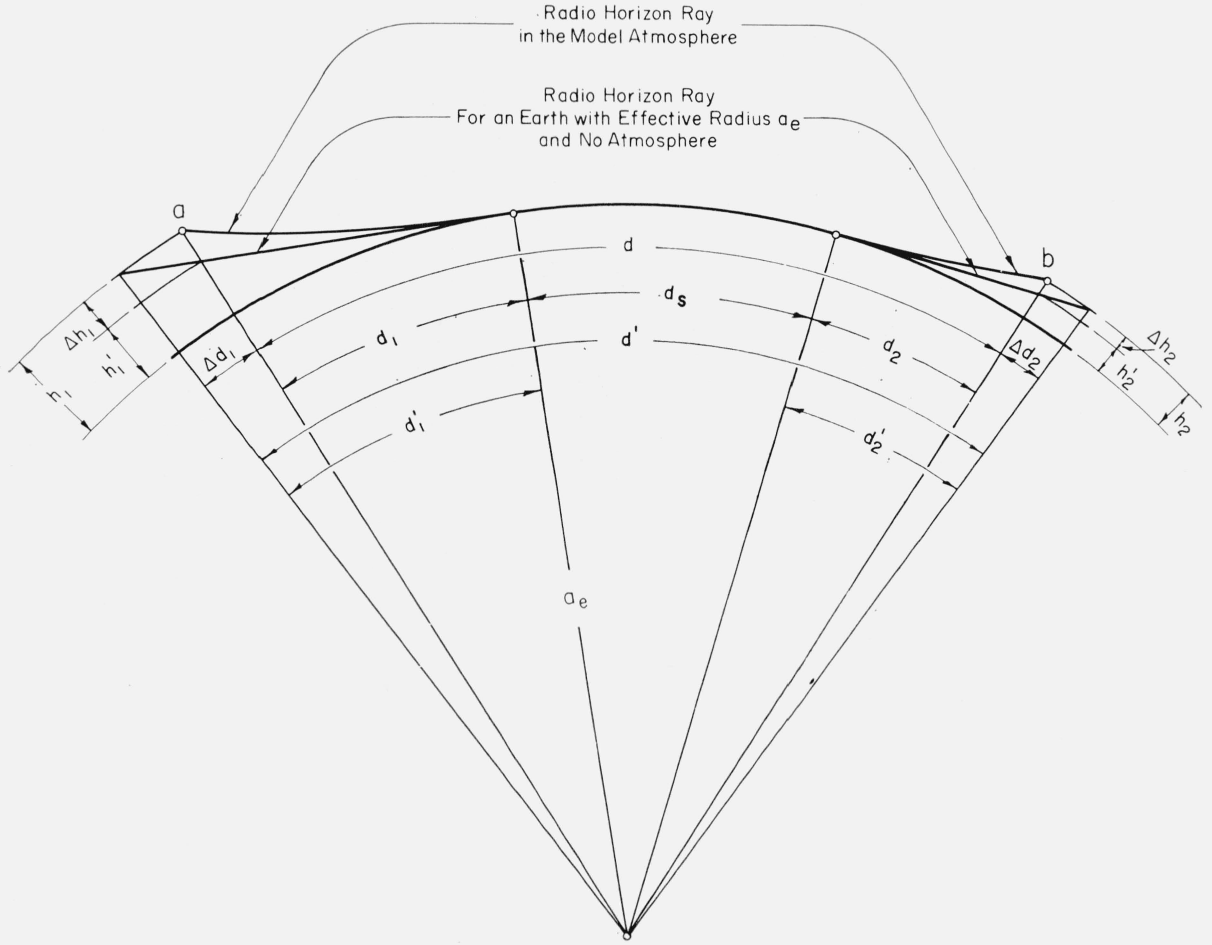

FIGURE 10. *Geometry illustrating the corrections* Δd *and* Δh *in the ray deviations between a model atmosphere and a sphere with no atmosphere but with an effective radius equal to* a_e.

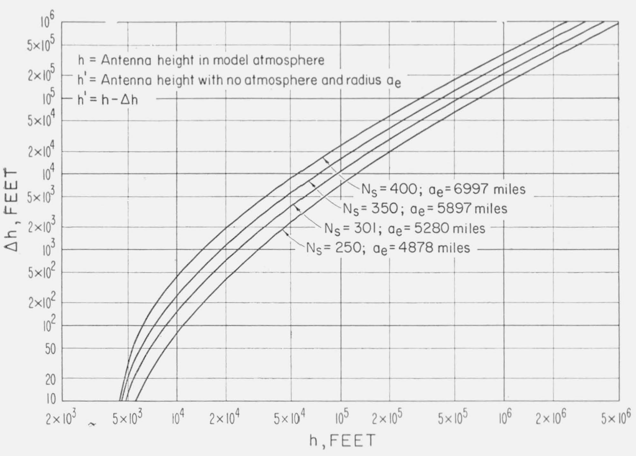

FIGURE 11. *Correction* Δh *to the heights of the radio horizon rays in the 1958 CRPL refractivity atmospheres relative to the heights they would have for spheres with no atmosphere but with effective radii* a_e.

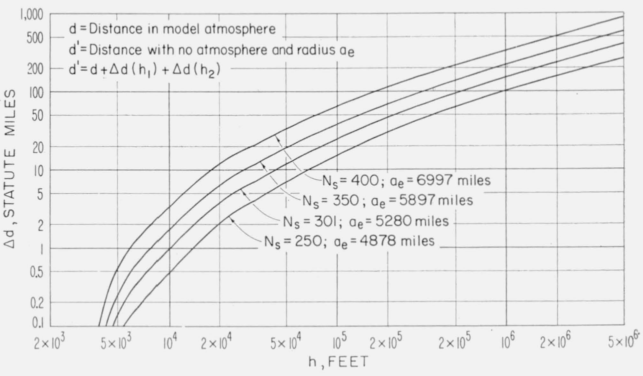

FIGURE 12. *Corrections* Δd *to the distances to the radio horizon rays in the 1958 CRPL refractivity atmospheres relative to the distances they would have for spheres with no atmosphere but with effective radii* a_e.

Eckersley profile $n^2=1-\eta+\epsilon a^2/(a+h)^2$ for which the effective radius is given by $a_e=an_s^2/(1-\eta)$ and for which the distance to the radio horizon is given by $(a\, a_e)^{1/2}$ arc cos $[a/(a+h)]$. This horizon distance differs only slightly from that given by (76) for the atmosphere with a linear gradient, but it is necessary, for extremely high antennas, to use values of Δd and Δh obtained for an appropriate reference atmosphere if accurate results are to be expected. The groundwave curves in the present C.C.I.R. Atlas do not extend to sufficient heights to require much adjustment for nonlinearity of the atmosphere, but the curves in a recent atlas prepared for a linear atmosphere at the Radio Research Laboratories in Japan [30] extend to sufficient heights so that a small adjustment will be required. It is perhaps desirable to emphasize that these systematic corrections for atmospheric bending are not likely to be important in *most* practical applications, but it is aesthetically more satisfying and occasionally might be important to have a precise solution to this interesting problem.

Figures 7 and 8 give the values of median basic transmission loss separately for the diffracted groundwave and for tropospheric scatter. If we assume that the short term variations in the scatter fields are Rayleigh distributed, and that the groundwaves are relatively steady over short periods of time, then we may determine the expected combined median basic transmission loss L_{bm} in terms of the diffracted wave basic transmission loss L_{bd} and the median basic scatter transmission loss L_{bms} as follows:

$$L_{bm}=L_{bd}-R(0.5), \qquad (79)$$

where $K=L_{bd}-L_{bms}+1.592$ is the ratio in decibels of the average scattered power to the diffracted wave power, and $R(0.5)$ is given graphically and in tables in [9]. When K is less than -16.5 db, L_{bm} differs from L_{bd} by less than 0.1 db, and when K is greater than 19.5 db, L_{bm} differs from L_{bms} by less than 0.1 db.

Finally, to determine the expected values $L_b(p)$ not exceeded by p percent of the hourly medians during a year, we may simply subtract $V(p,\theta)$ as given on figure 13 from L_{bm} as calculated from the values given on figures 7 or 8 or as given directly on figure 9. The parameter θ is the angular distance [12] and, over a smooth spherical earth, is given by

$$\theta=\frac{d_s}{a_e}=\frac{d-d_1-d_2}{a_e}, \qquad (80)$$

where d_1 and d_2 are the distances to the horizons in the model atmosphere. The angular distance is a particularly convenient parameter for making appropriate allowance for the effects of irregularities in the terrain as is explained more fully in [12] and [26] and the curves on figure 13 may be used for a rough earth as well.

11. Range of Tropospheric Forward Scatter Systems

As an example of the method of using transmission loss in systems design, the problem of estimating the effective maximum range of a radio relay system using tropospheric scatter is considered. As an illustration of typical ranges to be expected, we will assume that the terrain is smooth, and will base our predictions on the above-described Bean and Thayer [25] model atmosphere with $N_s=301$. We will assume that either two 28-ft or two 60-ft parabolic antennas are used at both ends of the path, with their centers 30 ft above the ground and connected in a quadruple diversity system. With these assumptions, we may use the methods described in the preceding section to determine the transmission loss, $L(99)$, which we would expect one percent of the actual hourly median transmission losses to exceed throughout a period of 1 year; the use of these one percent losses implies that the specified service will be available for 99 percent of the hours. Tables 1 and 2 give for the 28-ft and 60-ft antennas the free space gains G_t+G_r, and the path antenna gains as a function of frequency and distance, while tables 3 and 4 give $L(99)$ as a function of frequency and distance.

The transmitter power P_t required to provide a specified type and grade of service for 99 percent of the hours may be obtained from the equation

$$P_t=P'_{ta}\,(99\%)+L_t=L_t+L(99)+R_n+F+B-204, \qquad (79)$$

where L_t is the loss in the transmitting antenna circuit and this is set equal to 1 db in the following calculations. P'_{ta} (99%) may be determined from (69) and this is equal to the right-hand side of (79) since we will assume that the effective receiver noise figure has the constant value $F=5\log_{10}f_{Mc}-5$ so that $\sigma_{Fb}=0$ and $L(99)=L_{mab}+\sigma_{Lab}\,\lambda(0.99)$; it is assumed that the receiver incorporates gain adequate to ensure that the first circuit noise is detectable. $B\equiv 10\log_{10}(b_o+b_m)$ is the effective receiver bandwidth factor with b_o and b_m expressed in cycles per second; b_o allows for the drift between the transmitter and receiver oscillators, while b_m allows for the band occupied by the modulation.

For the calculations in this paper, the transmitter and receiver oscillators were each assumed to have a stability of one part in 10^8 and to vary independently so that $b_o=\sqrt{2}\,f_{Mc}\cdot 10^{-2}$. Table 5 gives the values of b_m assumed for the various types of service considered. Table 5 also gives the values of R_n for the various kinds of service on the assumption that quadruble diversity is used. The value of R_n for the FM multichannel system is expected to provide a service with less than an 0.01 percent teletype character error rate. The FM multichannel system consists of 36 voice channels, each of which can accommodate sixteen 60 words per minute teletype circuits. The values of R_n given in table 5 were

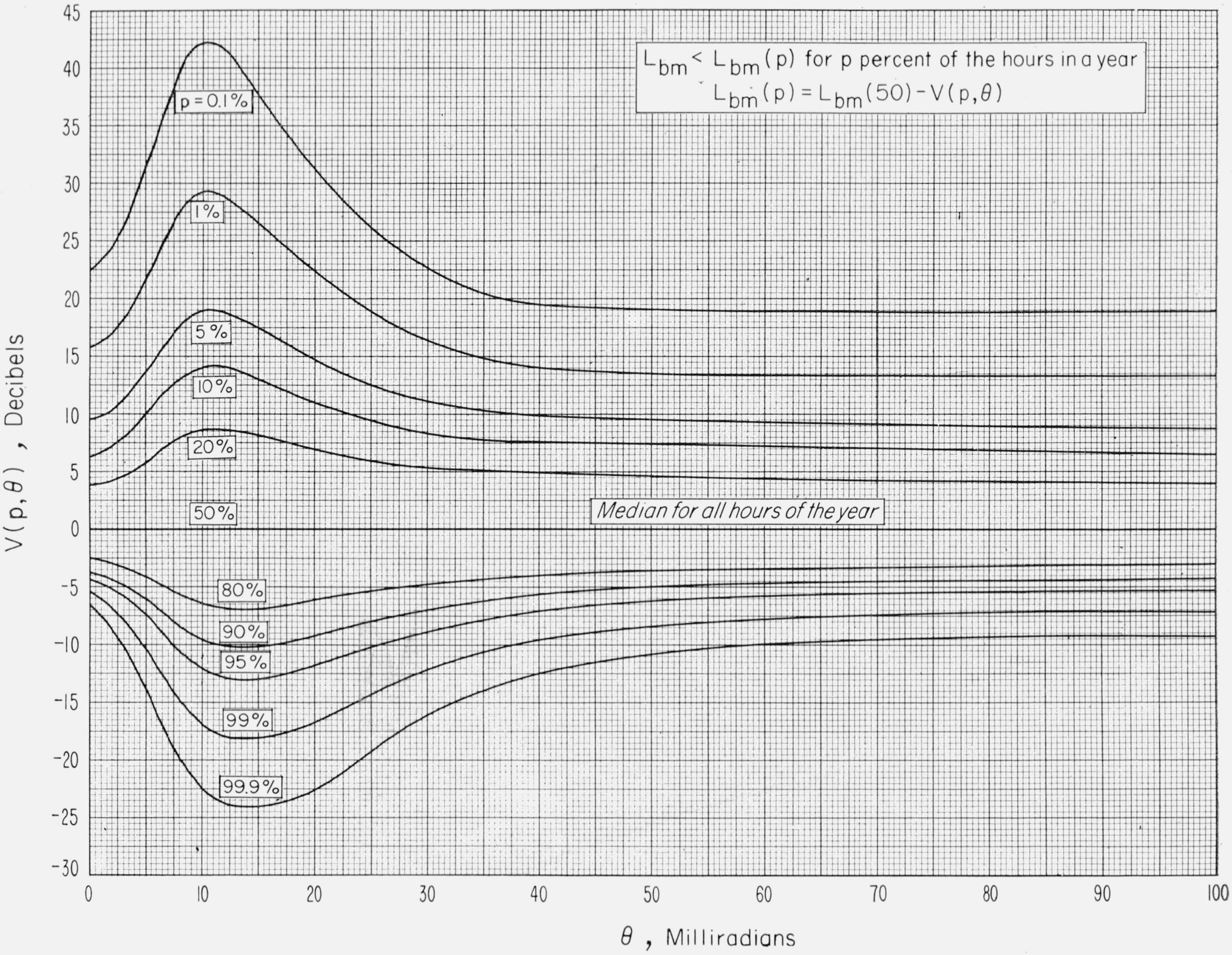

FIGURE 13. *Variance of transmission loss in tropospheric propagation.*

TABLE 1. *Path antenna gain for 28-ft parabolic antennas 30 ft above a smooth spherical earth with a CRPL model radio refractivity atmosphere corresponding to $N_s=301$*

Mc	G_t+G_r	G_p						
		d=100 mi	150	200	300	500	700	1,000
	db	*db*	*db*	*db*	*db*	*db*	*db*	*db*
100	33. 02	33. 02	33. 02	33. 02	32. 92	32. 82	32. 72	32. 67
150	40. 07	40. 07	39. 97	39. 97	39. 87	39. 57	39. 47	39. 37
200	45. 06	45. 06	44. 96	44. 86	44. 66	44. 36	44. 16	44. 06
300	52. 11	52. 03	51. 91	51. 71	51. 31	50. 81	50. 51	50. 41
500	60. 98	60. 75	60. 38	60. 08	59. 38	58. 48	58. 18	58. 18
700	66. 83	66. 35	65. 83	65. 23	64. 33	63. 23	62. 83	62. 93
1, 000	73. 02	72. 12	71. 22	70. 42	69. 22	67. 92	67. 32	67. 22
1, 500	80. 07	78. 32	76. 57	75. 87	74. 27	72. 57	71. 67	71. 57
2, 000	85. 06	82. 48	80. 66	79. 26	77. 16	75. 46	74. 36	73. 86
3, 000	92. 11	87. 71	84. 21	83. 41	80. 91	78. 51	77. 61	77. 31
5, 000	100. 98	93. 28	90. 18	87. 68	84. 68	82. 18	81. 28	80. 98
7, 000	106. 83	96. 53	92. 83	90. 03	86. 83	84. 12	83. 33	82. 93
10, 000	113. 02	99. 32	95. 22	92. 52	89. 02	86. 02	85. 22	84. 92

TABLE 2. *Path antenna gain for 60-ft parabolic antennas 30 ft above a smooth spherical earth with a CRPL model radio refractivity atmosphere corresponding to $N_s=301$*

f_{Mc}	G_t+G_r	G_p						
		d=100 mi	150	200	300	500	700	1,000
	db	*db*	*db*	*db*	*db*	*db*	*db*	*db*
100	46. 26	46. 16	46. 16	46. 06	45. 86	45. 46	45. 26	45. 16
150	53. 31	53. 21	53. 11	52. 91	52. 51	52. 11	51. 56	51. 48
200	58. 30	58. 10	57. 90	57. 60	57. 00	56. 30	55. 90	55. 86
300	65. 35	64. 95	64. 45	63. 95	62. 85	62. 05	61. 70	61. 90
500	74. 22	73. 22	72. 67	71. 02	70. 22	68. 72	68. 12	68. 12
700	80. 07	78. 37	77. 07	75. 87	74. 17	72. 57	71. 67	71. 57
1, 000	86. 26	83. 46	81. 46	79. 96	77. 86	76. 26	75. 01	74. 66
1, 500	93. 31	88. 51	85. 91	84. 01	81. 51	79. 11	78. 21	77. 81
2, 000	98. 30	91. 70	88. 80	86. 60	83. 70	81. 10	80. 20	79. 80
3, 000	105. 35	95. 75	92. 25	89. 65	86. 35	83. 55	82. 85	82. 45
5, 000	114. 22	100. 02	95. 62	92. 92	89. 52	86. 52	85. 62	85. 42
7, 000	120. 07	102. 37	97. 87	94. 97	91. 17	88. 22	87. 57	87. 07
10, 000	126. 26	104. 56	99. 96	96. 66	93. 06	90. 06	89. 26	88. 76

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TABLE 3. *Transmission loss L(99) (corresponding to fields exceeded 99 percent of the time) expected between two 28-ft parabolic antennas at a height of 30 ft above a smooth spherical earth with a CRPL model radio refractivity atmosphere corresponding to N_s=301*

	L(99) in decibels						
	d mi						
f_{Mc}	100	150	200	300	500	700	1,000
100	163	167	170	182	208	233	277
150	157	162	165	177	201	226	270
200	153	158	161	173	197	222	266
300	148	153	157	169	193	217	260
500	143	149	153	166	190	214	256
700	140	147	151	165	189	213	254
1,000	138	145	150	164	188	213	252
1,500	136	145	149	165	189	213	254
2,000	136	144	150	166	190	214	256
3,000	136	148	152	168	193	218	258
5,000	138	149	156	173	199	223	264
7,000	140	153	161	180	207	231	272
10,000	148	163	174	196	224	248	289

TABLE 4. *Transmission loss L(99) (corresponding to fields exceeded 99 percent of the time) expected between two 60-ft parabolic antennas at a height of 30 ft above a smooth spherical earth with a CRPL model radio refractivity atmosphere corresponding to N_s=301*

	L(99) in decibels						
f_{Mc}	d=100 mi	150	200	300	500	700	1,000
100	150	154	157	169	195	221	264
150	144	149	152	164	188	214	257
200	140	145	148	161	185	210	254
300	135	141	144	158	182	206	249
500	131	137	142	155	180	204	246
700	127	136	140	155	179	204	245
1,000	127	135	140	156	180	205	244
1,500	126	136	141	157	182	207	248
2,000	126	136	142	159	184	209	250
3,000	128	139	145	163	188	212	253
5,000	131	143	151	169	195	219	260
7,000	134	148	156	176	203	227	268
10,000	142	158	170	192	219	244	285

determined by methods given in a recent report by Watt et al. [31]. The value of R_n for the FM multichannel system corresponds to typical fading encountered at 1,000 Mc, and this value of R_n may change by a few decibels with frequency as the fading changes, but such changes have so far not been evaluated quantitatively; furthermore, R_n will also change as the fading changes from hour to hour.

Table 6 gives as a function of frequency the maximum permissible hourly median transmission loss for a transmitter power of 10 kw: $L_M=204+P_t-L_t-R_n-F-B$ corresponding to the kinds of service described above. By combining the information in tables 3, 4, and 6, we can estimate the maximum range for a quadruple diversity system with 10-kw transmitters. These ranges are shown on figure 14 as a function of the radiofrequency.

TABLE 5

Type of service	b_m	R_n^a	Signal bandwidth	Post detection signal-to-noise ratio
	cps	*db*	*cps*	*db*
Transmission loss measurement.	0	0 [b]	0	
FM multichannel system.	3,750,000	9.5	36 voice channels each capable of use for sixteen 60 words per min teletype circuits.	0.01% teletype character error rate.
FM music	150,000	26.5	15,000	50.[c]
U.S. standard television.	3,750,000	32.7	3,750,000	30.[c]

[a] Ratio between the median intermediate frequency Rayleigh distributed signal and the rms Rayleigh distributed noise.
[b] Diversity reception not involved in this case.
[c] This ratio will be exceeded with a quadruple diversity system for 99 percent of each hour for which the corresponding value of R_n is maintained in each receiver.

TABLE 6. *Maximum permissible transmission loss L_M for 10 kw transmitters using the parameters of table 5*

f_{Mc}	Transmission loss measurement	FM multichannel	FM music	U.S. standard television
100	236.50	162.76	159.74	139.56
150	233.85	161.88	158.86	138.68
200	231.98	161.26	158.23	138.06
300	229.34	160.37	157.35	137.17
500	226.01	159.27	156.25	136.07
700	223.82	158.54	155.51	135.34
1,000	221.50	157.76	154.74	134.56
1,500	218.85	156.88	153.86	133.68
2,000	216.98	156.25	153.23	133.05
3,000	214.34	155.37	152.35	132.17
5,000	211.01	154.27	151.25	131.07
7,000	208.82	153.53	150.51	130.33
10,000	206.50	152.76	149.74	129.56

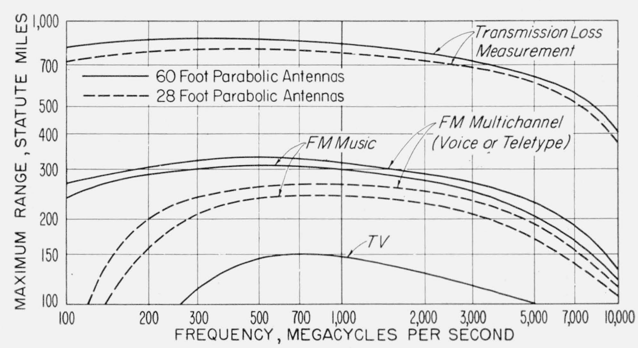

FIGURE 14. *Maximum distance at which satisfactory service of the types indicated may be provided for 99 percent of the hours using 10 kw. transmitters and quadruple diversity; smooth spherical earth; N_s=301; h_t=h_r=30 ft; atmospheric absorption and rain attenuation typical of the Washington, D.C., area.*

12. References

[1] Kenneth A. Norton, Transmission loss in radio propagation, Proc. IRE **41,** 146 (1953).

[2] S. A. Schelkunoff, Electromagnetic waves (D. Van Nostrand Co. 1943). See also S. A. Schelkunoff and Harald T. Friis, Antennas theory and practice (John Wiley & Sons, Inc., New York, N.Y., 1952).

[3] K. A. Norton, The propagation of radio waves over the surface of the earth and in the upper atmosphere, Pt. II, Proc. IRE **25,** 1203 (1937).

[4] James R. Wait, Radiation resistance of a small circular loop in the presence of a conducting ground, J. Appl. Phys. **24,** 646 (1953).

[5] James R. Wait and Walter J. Surtees, Impedance of a top-loaded antenna of arbitrary length over a circular grounded screen, J. Appl. Phys. **25,** 553 (1954).

[6] James R. Wait and W. A. Pope, Input resistance of L. F. unipole aerials, Wireless Engineer **32,** 131 (1955).

[7] James R. Wait, Effect of the ground screen on the field radiated from a monopole, IRE Trans. on Antennas and Propagation AP–**4,** 179 (1956).

[8] James R. Wait, Transmission of power in radio propagation, Electronic and Radio Engineer **36,** 146 (1959).

[9] K. A. Norton, L. E. Vogler, W. V. Mansfield, and P. J. Short, The probability distribution of the amplitude of a constant vector plus a Rayleigh-distributed vector, Proc. IRE **43,** 1354 (1955).

[10] Documents of the VIII Plenary Assembly of the C.C.I.R. (Warsaw, 1956), Published by the International Telecommunication Union, Geneva, 1957.

[11] P. L. Rice and F. T. Daniel, Radio transmission loss versus distance and antenna height at 100 Mc, Trans. IRE PGAP AP–**3,** 59 (1955).

[12] K. A. Norton, P. L. Rice, and L. E. Vogler, The use of angular distance in estimating transmission loss and fading range for propagation through a turbulent atmosphere over irregular terrain, Proc. IRE **43,** 1488 (1955).

[13] K. A. Norton and A. C. Omberg, The maximum range of a radar set, Proc. IRE **35,** 4 (1947).

[14] H. T. Friis, Noise figures of radio receivers, Proc. IRE **32,** 419 (1944).

[15] The Kelley Statistical Tables (Harvard University Press, Cambridge, Mass., 1948).

[16] W. Q. Crichlow, D. F. Smith, R. N. Morton and W. R. Corliss, Worldwide radio noise levels expected in the frequency band from 10 kilocycles to 100 megacycles, NBS Cir. 557 (Aug. 25, 1955).

[17] C.C.I.R. Report No. 65, Report on Revision of Atmospheric Radio Noise Data (Warsaw, 1956 available as a separate document from the International Telecommunications Union, Geneva, or in Vol. I of the Documents of the VIIIth Plenary Assembly of the C.C.I.R. Warsaw, 1956).

[18] H. V. Cottony and J. R. Johler, Cosmic radio noise intensities in the VHF band, Proc. IRE **40,** 1053 (1952).

[19] Garner McCrossen, Long-term cumulative distribution of instantaneous power (to be published).

[20] K. A. Norton, H. Staras and M. Blum, A statistical approach to the problem of multiple radio interference to FM and television service, Trans. IRE Professional Group on Antennas and Propagation PGAP–**1,** 43 (1952).

[21] Report of the Ad Hoc Committee for the Evaluation of the Radio Propagation Factors Concerning the Television and Frequency-Modulation Broadcasting Services in the Frequency Range Between 50 and 250 Mc, Federal Communications Commission Mimeo Nos. 36728; May 26, 1949; 36830; May 31, 1949; and 54382; July 7, 1950.

[22] D. K. Bailey, R. Bateman, and R. C. Kirby, Radio transmission at VHF by scattering and other processes in the lower ionosphere, Proc. IRE **43,** 1181 (1955).

[23] K. A. Norton, Point-to-point radio relaying via the scatter mode of tropospheric propagation, Transactions on Communications Systems **CS–4,** No. 1, 39 (March 1956).

[24] K. A. Norton, Recent experimental evidence favoring the $\rho K_1(\rho)$ correlation function for describing the turbulence of refractivity in the troposphere and stratosphere (to be published in the Proc. of the Joint Commission on Radio Meteorology held under the auspices of the International Scientific Radio Union at New York University, August 1957).

[25] B. R. Bean and G. D. Thayer, On models of the atmospheric radio refractive index, Proc. IRE **47,** 740 (1959).

[26] P. L. Rice, A. G. Longley, and K. A. Norton, Prediction of the cumulative distribution with time of ground wave and tropospheric wave transmission loss (to be submitted to this Journal).

[27] B. R. Bean and R. L. Abbott, Oxygen and water vapor absorption of radio waves in the atmosphere, Geophysica Pura E Applicata **37,** 127 (1957).

[28] G. Millington, Propagation at great heights in the atmosphere, Marconi Review **XXI,** 143 (4th Quarter 1958).

[29] Atlas of Ground-wave Propagation Curves for Frequencies between 30 Mc/s and 300 Mc/s (C.C.I.R. Resolution No. 11, published by the International Telecommunications Union, Geneva, 1955).

[30] Atlas of Radio Wave Propagation Curves for Frequencies between 30 and 10,000 Mc/s (prepared by the Radio Research Laboratories, Ministry of Postal Services, Tokyo, Japan, January 1958).

[31] A. D. Watt, E. F. Florman, and R. W. Plush, Carrier to noise requirements for teletype communications via non-fading or fading UHF carrier, J. Research NBS (in press).

[32] K. A. Norton, Low and medium frequency radio propagation, Proc. of the International Congress on the Propagation of Radio Waves at Liege, Belgium, October 1958 (to be published by the Academic Press).

[33] K. Bullington, W. J. Inkster, and A. L. Durkee, Results of propagation tests at 505 Mc and 4090 Mc on beyond-horizon paths, Proc. IRE **43,** 1306 (1955).

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