Journal of Research of the National Bureau of Standards—B. Mathematics and Mathematical Physics
Vol. 63B, No. 1, July–September 1959

Tables of Transport Integrals: A Supplement

William M. Rogers,[1] William J. Hall, and Robert L. Powell

(April 23, 1959)

Tables of values of the transport integrals, defined by

$$J_n(x) \equiv \int_0^x \frac{e^z z^n dz}{(e^z-1)^2},$$

were previously published in NBS Circular 595 for values of n from 2 through 17. In this paper, values are given for the integrals where n is 18 and 20 and where x ranges from 0.2 to 50.0 in steps of 0.2.

Soon after the publication of the Tables of Transport Integrals [2] an unanticipated need arose for tables of the integrals for n equal to 18 and 20. The integrals with even values of n are useful in the problem of inverting heat capacity results to obtain lattice frequency spectra for solids.[3] The technique is similar to the method of moments developed by Montroll.[4] The integrals with odd values of n are not used in this method, and since there was no other need for the integrals with n above 17, the integrals with $n=19$ were not calculated. The additional values were calculated on the IBM 650 computer using the already existent programs and auxiliary numbers. The computed values of the integrals are presented in the following tables.

The mathematical formulation, analyses of errors, and tables of auxiliary numbers were discussed in the previous publication. For x below 10, interpolation should be performed by the technique of using an interpolating polynomial in a table of divided differences of the logarithms of both the arguments and the integral values. For x between 0 and 4, a fifth-degree polynomial should be used; between 4 and 10, a third-degree polynomial. For x above 10, straightforward Lagrangian interpolation should be performed. For x between 10 and 20, a 4-point formula should be used; between 20 and 35, a 3-point; and above 35, a 2-point.

[1] Present address: Headquarters, Strategic Air Command, Offutt Air Force Base, Nebr.
[2] William M. Rogers and Robert L. Powell, Tables of transport integrals, NBS Circ. 595.
[3] J. A. Morrison, private communication.
[4] E. W. Montroll, J. Chem. Phys. **11**, 481 (1943).

(Tables follow)

TRANSPORT INTEGRALS : $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2} dz$; $n = 18, 20$

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
.2	5.0000	7.68716	-14	2.75110	-15
.4	2.5000	9.98611	- 9	1.42940	- 9
.6	1.6667	9.69389	- 6	3.12152	- 6
.8	1.2500	1.26317	- 3	7.22945	- 4
1.0	1.0000	5.46290	- 2	4.88381	- 2
1.2	.8333	1.17362	0	1.51031	0
1.4	.7143	1.55318	1	2.71939	1
1.6	.6250	1.43986	2	3.29112	2
1.8	.5556	1.01589	3	2.93725	3
2.0	.5000	5.77334	3	2.05955	4
2.2	.4545	2.75197	4	1.18710	5
2.4	.4167	1.13372	5	5.81587	5
2.6	.3846	4.12961	5	2.48433	6
2.8	.3571	1.35385	6	9.43812	6
3.0	.3333	4.05157	6	3.23956	7
3.2	.3125	1.11950	7	1.01753	8
3.4	.2941	2.88300	7	2.95532	8
3.6	.2778	6.97382	7	8.00642	8
3.8	.2632	1.59495	8	2.03807	9
4.0	.2500	3.46812	8	4.90503	9
4.2	.2381	7.20417	8	1.12206	10
4.4	.2273	1.43552	9	2.45095	10
4.6	.2174	2.75377	9	5.13255	10
4.8	.2083	5.10162	9	1.03403	11
5.0	.2000	9.15275	9	2.01034	11
5.2	.1923	1.59414	10	3.78208	11
5.4	.1852	2.70137	10	6.90194	11
5.6	.1786	4.46243	10	1.22443	12
5.8	.1724	7.19868	10	2.11576	12
6.0	.1667	1.13584	11	3.56723	12
6.2	.1613	1.75544	11	5.87786	12
6.4	.1563	2.66088	11	9.47887	12
6.6	.1515	3.96048	11	1.49801	13
6.8	.1471	5.79463	11	2.32279	13
7.0	.1429	8.34235	11	3.53773	13
7.2	.1389	1.18285	12	5.29775	13
7.4	.1351	1.65316	12	7.80755	13
7.6	.1316	2.27919	12	1.13335	14
7.8	.1282	3.10193	12	1.62174	14
8.0	.1250	4.17019	12	2.28921	14

TRANSPORT INTEGRALS: $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2}\, dz$; $n = 18, 20$ (cont'd.)

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
8.2	.1220	5.54145	12	3.18984	14
8.4	.1190	7.28248	12	4.39041	14
8.6	.1163	9.47008	12	5.97241	14
8.8	.1136	1.21916	13	8.03411	14
9.0	.1111	1.55454	13	1.06928	15
9.2	.1087	1.96411	13	1.40870	15
9.4	.1064	2.45996	13	1.83786	15
9.6	.1042	3.05533	13	2.37590	15
9.8	.1020	3.76441	13	3.04320	15
10.0	.1000	4.60253	13	3.86510	15
10.2	.0980	5.58606	13	4.86890	15
10.4	.0962	6.73205	13	6.08530	15
10.6	.0943	8.05832	13	7.54830	15
10.8	.0926	9.58320	13	9.29500	15
11.0	.0909	1.13254	14	1.13656	16
11.2	.0893	1.33040	14	1.38045	16
11.4	.0877	1.55380	14	1.66582	16
11.6	.0862	1.80461	14	1.99760	16
11.8	.0847	2.08468	14	2.38115	16
12.0	.0833	2.39576	14	2.82181	16
12.2	.0820	2.73954	14	3.32528	16
12.4	.0806	3.11761	14	3.89742	16
12.6	.0794	3.53141	14	4.54416	16
12.8	.0781	3.98224	14	5.27146	16
13.0	.0769	4.47120	14	6.08533	16
13.2	.0758	4.99927	14	6.99174	16
13.4	.0746	5.56715	14	7.99644	16
13.6	.0735	6.17539	14	9.10518	16
13.8	.0725	6.82427	14	1.03233	17
14.0	.0714	7.51388	14	1.16559	17
14.2	.0704	8.24402	14	1.31077	17
14.4	.0694	9.01431	14	1.46831	17
14.6	.0685	9.82409	14	1.63858	17
14.8	.0676	1.06725	15	1.82193	17
15.0	.0667	1.15583	15	2.01862	17
15.2	.0658	1.24803	15	2.22887	17
15.4	.0649	1.34370	15	2.45283	17
15.6	.0641	1.44264	15	2.69057	17
15.8	.0633	1.54468	15	2.94210	17
16.0	.0625	1.64960	15	3.20737	17

TRANSPORT INTEGRALS: $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2}\, dz$; $n = 18, 20$ (cont'd.)

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
16.2	.0617	1.75718	15	3.48623	17
16.4	.0610	1.86717	15	3.77849	17
16.6	.0602	1.97933	15	4.08386	17
16.8	.0595	2.09340	15	4.40200	17
17.0	.0588	2.20911	15	4.73249	17
17.2	.0581	2.32619	15	5.07485	17
17.4	.0575	2.44436	15	5.42855	17
17.6	.0568	2.56336	15	5.79297	17
17.8	.0562	2.68289	15	6.16747	17
18.0	.0556	2.80270	15	6.55134	17
18.2	.0549	2.92250	15	6.94383	17
18.4	.0543	3.04204	15	7.34417	17
18.6	.0538	3.16107	15	7.75153	17
18.8	.0532	3.27933	15	8.16507	17
19.0	.0526	3.39658	15	8.58393	17
19.2	.0521	3.51262	15	9.00721	17
19.4	.0515	3.62720	15	9.43404	17
19.6	.0510	3.74015	15	9.86352	17
19.8	.0505	3.85127	15	1.02948	18
20.0	.0500	3.96039	15	1.07269	18
20.2	.0495	4.06735	15	1.11590	18
20.4	.0490	4.17201	15	1.15902	18
20.6	.0485	4.27423	15	1.20198	18
20.8	.0481	4.37390	15	1.24469	18
21.0	.0476	4.47093	15	1.28707	18
21.2	.0472	4.56522	15	1.32905	18
21.4	.0467	4.65671	15	1.37056	18
21.6	.0463	4.74534	15	1.41152	18
21.8	.0459	4.83107	15	1.45189	18
22.0	.0455	4.91386	15	1.49159	18
22.2	.0450	4.99369	15	1.53058	18
22.4	.0446	5.07056	15	1.56881	18
22.6	.0442	5.14447	15	1.60622	18
22.8	.0439	5.21543	15	1.64278	18
23.0	.0435	5.28346	15	1.67846	18
23.2	.0431	5.34860	15	1.71322	18
23.4	.0427	5.41089	15	1.74703	18
23.6	.0424	5.47037	15	1.77987	18
23.8	.0420	5.52709	15	1.81173	18
24.0	.0417	5.58111	15	1.84259	18

TRANSPORT INTEGRALS: $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2}\, dz$; $n = 18, 20$ (cont'd.)

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
24.2	.0413	5.63250	15	1.87243	18
24.4	.0410	5.68132	15	1.90126	18
24.6	.0407	5.72765	15	1.92907	18
24.8	.0403	5.77155	15	1.95585	18
25.0	.0400	5.81312	15	1.98162	18
25.2	.0397	5.85242	15	2.00638	18
25.4	.0394	5.88954	15	2.03014	18
25.6	.0391	5.92455	15	2.05290	18
25.8	.0388	5.95755	15	2.07470	18
26.0	.0385	5.98862	15	2.09553	18
26.2	.0382	6.01783	15	2.11543	18
26.4	.0379	6.04526	15	2.13441	18
26.6	.0376	6.07101	15	2.15249	18
26.8	.0373	6.09514	15	2.16969	18
27.0	.0370	6.11774	15	2.18604	18
27.2	.0368	6.13889	15	2.20157	18
27.4	.0365	6.15865	15	2.21629	18
27.6	.0362	6.17710	15	2.23025	18
27.8	.0360	6.19431	15	2.24345	18
28.0	.0357	6.21034	15	2.25593	18
28.2	.0355	6.22528	15	2.26772	18
28.4	.0352	6.23917	15	2.27885	18
28.6	.0350	6.25208	15	2.28933	18
28.8	.0347	6.26407	15	2.29921	18
29.0	.0345	6.27519	15	2.30849	18
29.2	.0342	6.28550	15	2.31722	18
29.4	.0340	6.29505	15	2.32542	18
29.6	.0338	6.30388	15	2.33311	18
29.8	.0336	6.31205	15	2.34031	18
30.0	.0333	6.31960	15	2.34706	18
30.2	.0331	6.32657	15	2.35337	18
30.4	.0329	6.33299	15	2.35927	18
30.6	.0327	6.33892	15	2.36478	18
30.8	.0325	6.34437	15	2.36992	18
31.0	.0323	6.34939	15	2.37472	18
31.2	.0321	6.35401	15	2.37918	18
31.4	.0318	6.35825	15	2.38333	18
31.6	.0316	6.36214	15	2.38720	18
31.8	.0314	6.36572	15	2.39079	18
32.0	.0313	6.36899	15	2.39412	18

TRANSPORT INTEGRALS: $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2}\, dz$; n = 18, 20 (cont'd.)

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
32.2	.0311	6.37199	15	2.39722	18
32.4	.0309	6.37474	15	2.40008	18
32.6	.0307	6.37726	15	2.40274	18
32.8	.0305	6.37956	15	2.40520	18
33.0	.0303	6.38166	15	2.40747	18
33.2	.0301	6.38358	15	2.40957	18
33.4	.0299	6.38533	15	2.41151	18
33.6	.0298	6.38692	15	2.41330	18
33.8	.0296	6.38838	15	2.41496	18
34.0	.0294	6.38970	15	2.41648	18
34.2	.0292	6.39091	15	2.41788	18
34.4	.0291	6.39200	15	2.41917	18
34.6	.0289	6.39300	15	2.42036	18
34.8	.0287	6.39391	15	2.42145	18
35.0	.0286	6.39473	15	2.42245	18
35.2	.0284	6.39548	15	2.42337	18
35.4	.0282	6.39615	15	2.42421	18
35.6	.0281	6.39677	15	2.42499	18
35.8	.0279	6.39732	15	2.42570	18
36.0	.0278	6.39783	15	2.42634	18
36.2	.0276	6.39828	15	2.42694	18
36.4	.0275	6.39869	15	2.42748	18
36.6	.0273	6.39907	15	2.42798	18
36.8	.0272	6.39940	15	2.42843	18
37.0	.0270	6.39971	15	2.42884	18
37.2	.0269	6.39998	15	2.42922	18
37.4	.0267	6.40023	15	2.42956	18
37.6	.0266	6.40045	15	2.42988	18
37.8	.0265	6.40065	15	2.43016	18
38.0	.0263	6.40083	15	2.43042	18
38.2	.0262	6.40099	15	2.43066	18
38.4	.0260	6.40114	15	2.43087	18
38.6	.0259	6.40127	15	2.43107	18
38.8	.0258	6.40139	15	2.43124	18
39.0	.0256	6.40150	15	2.43140	18
39.2	.0255	6.40159	15	2.43155	18
39.4	.0254	6.40168	15	2.43168	18
39.6	.0253	6.40175	15	2.43180	18
39.8	.0251	6.40182	15	2.43191	18
40.0	.0250	6.40188	15	2.43201	18

TRANSPORT INTEGRALS: $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2}\, dz$; n = 18, 20 (cont'd.)

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
40.2	.0249	6.40194	15	2.43210	18
40.4	.0248	6.40199	15	2.43218	18
40.6	.0246	6.40203	15	2.43225	18
40.8	.0245	6.40207	15	2.43232	18
41.0	.0244	6.40211	15	2.43238	18
41.2	.0243	6.40214	15	2.43243	18
41.4	.0242	6.40217	15	2.43248	18
41.6	.0240	6.40219	15	2.43252	18
41.8	.0239	6.40222	15	2.43256	18
42.0	.0238	6.40224	15	2.43260	18
42.2	.0237	6.40225	15	2.43263	18
42.4	.0236	6.40227	15	2.43266	18
42.6	.0235	6.40228	15	2.43268	18
42.8	.0234	6.40230	15	2.43270	18
43.0	.0233	6.40231	15	2.43273	18
43.2	.0231	6.40232	15	2.43274	18
43.4	.0230	6.40233	15	2.43276	18
43.6	.0229	6.40234	15	2.43278	18
43.8	.0228	6.40234	15	2.43279	18
44.0	.0227	6.40235	15	2.43280	18
44.2	.0226	6.40235	15	2.43281	18
44.4	.0225	6.40236	15	2.43282	18
44.6	.0224	6.40236	15	2.43283	18
44.8	.0223	6.40237	15	2.43284	18
45.0	.0222	6.40237	15	2.43285	18
45.2	.0221	6.40237	15	2.43285	18
45.4	.0220	6.40238	15	2.43286	18
45.6	.0219	6.40238	15	2.43286	18
45.8	.0218	6.40238	15	2.43287	18
46.0	.0217	6.40238	15	2.43287	18
46.2	.0216	6.40239	15	2.43288	18
46.4	.0216	6.40239	15	2.43288	18
46.6	.0215	6.40239	15	2.43288	18
46.8	.0214	6.40239	15	2.43288	18
47.0	.0213	6.40239	15	2.43289	18
47.2	.0212	6.40239	15	2.43289	18
47.4	.0211	6.40239	15	2.43289	18
47.6	.0210	6.40239	15	2.43289	18
47.8	.0209	6.40239	15	2.43289	18
48.0	.0208	6.40239	15	2.43289	18

TRANSPORT INTEGRALS: $J_n(x) \equiv \int_0^x \frac{e^z z^n}{(e^z-1)^2}\, dz$; $n = 18, 20$ (cont'd.)

x	1/x	$J_{18}(x)$		$J_{20}(x)$	
		COEF.	EXP.	COEF.	EXP.
48.2	.0207	6.40239	15	2.43290	18
48.4	.0207	6.40239	15	2.43290	18
48.6	.0206	6.40240	15	2.43290	18
48.8	.0205	6.40240	15	2.43290	18
49.0	.0204	6.40240	15	2.43290	18
49.2	.0203	6.40240	15	2.43290	18
49.4	.0202	6.40240	15	2.43290	18
49.6	.0202	6.40240	15	2.43290	18
49.8	.0201	6.40240	15	2.43290	18
50.0	.0200	6.40240	15	2.43290	18

BOULDER, COLO.

(Paper 63B1–5)