Journal of Research of the National Bureau of Standards Vol. 61, No. 6, December 1958 Research Paper 2920

Evaluation of Lens Distortion by Visual and Photographic Methods [1]

Francis E. Washer, William P. Tayman, and Walter R. Darling

The evaluation of lens distortion by photographic and visual methods is discussed. Measurements made on a single lens using the two methods are reported. The precision of measurement of each method is determined which shows that the observed differences must be attributed to systematic error. Various sources of systematic error are considered. Uncompensated differential plate tipping is identified as the most probable cause of the observed differences. A method of correction is developed. It is concluded that when work is done with extreme care and with due account taken of various insidious sources of error, it is possible to achieve comparable results with either method.

1. Introduction

The measurement of radial distortion in the focal plane of photographic objectives has been the subject of intensive study since the advent of aerial mapping from photographs. This particular aberration is of prime interest as its magnitude determines the accuracy with which the final negative maintains the correct relationships among the array of point images making up the photograph of the corresponding array of points in the area photographed. It was early realized that the reliability of the quantitative information obtained from a photograph increased as the distortion in the taking lens decreased. Consequently, the development of improved lenses was encouraged with the result that succeeding series of new lenses were characterized by ever lower values of distortion.

During this period of change, diverse methods of evaluating the distortion of lenses came into use at various laboratories. The reason for such diversity was primarily the availability of given types of measuring instruments in various laboratories. There are now three principal methods of measuring distortion plus numerous additional methods that are either the inverse of one of the principal methods or a variation thereof. The nodal slide bench is one of the oldest methods; this is a visual method capable of high accuracy and is perhaps the most widely used. The photographic method is more recent and arose out of the desire to make measurements under conditions approximating that of use. The third principal method is the goniometric method which is used to considerable extent in Europe.

Because of the diversity of methods being used in the evaluation of distortion, it seems worthwhile to investigate the results of measurement made in a single laboratory on a number of lenses by a variety of methods to determine whether or not the values so obtained varied appreciably with method. This is being done for several different methods and some preliminary results are reported herein for two methods.

The two methods are as follows:

A. Photographic, precision lens testing camera,

B. Visual, nodal slide optical bench.

[1] This work was performed in connection with the research project sponsored by the U. S. Air Force.

2. Methods of Measurement

Methods A and B have been described at some length in the literature. However, for purposes of clarity, in the following section, a brief description of these methods is given.

2.1. Precision Lens Testing Camera. Method A

The precision lens testing camera [1][2] shown in figure 1 was developed at the Bureau by I. C. Gardner and F. A. Case. It is one of the earliest successful devices developed to measure the performance of lenses by photographic means. It consists of a bank of collimators spaced at 5-deg intervals having resolution test charts as reticles. The lens under test is mounted at the center of convergence of the collimator fan and can be aimed at any one of them by rotation of a carriage which carries the lens holder and camera back whereon the photographic recording plate is mounted. As presently constituted, the lens testing camera has 10 collimators covering a total angle of 45 deg. When used to test wide angle lenses, the camera is aimed at one of the extreme collimators (position I) and the test made. In order to cover a complete diameter, a second test is made with the camera aimed at the collimator at the opposite ex-

[2] Figures in brackets indicate the literature references at the end of this paper.

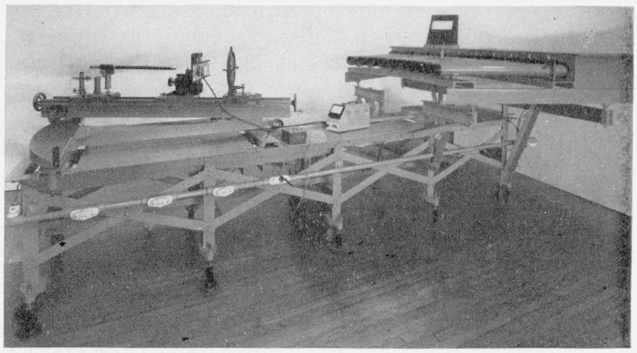

FIGURE 1. *Precision lens testing camera (Method A).*

This photograph shows the camera back with a light metering device in place on the plate holder. The values of illumination in the focal plane are read from the large meter above the collimator. Use of this device enables the operator to adjust for uniform exposure in making the test negatives from which the values of distortion, D_A, are obtained.

treme (position II). It has been found by experience that the results obtained from two negatives made in this manner are quite reliable.

To determine the value of the distortion, the best row of images is selected on the negative made in position I. The separation of all images on this row from the image formed by light from the 0° collimator is measured. The equivalent focal length (*EFL*) is then determined from the relations

$$f_1 = d_5 \cot \beta_5, \tag{1}$$

$$f_2 = d_{10} \cot \beta_{10}, \tag{2}$$

$$EFL = \frac{f_1 + f_2}{2}. \tag{3}$$

Where d_5 and d_{10} are the measured distances on the negative separating the images formed by the lens for the target in the 0° collimator from the corresponding images formed by the lens for the targets in the collimators located at $\beta = 5°$ and 10° respectively from the 0° collimator. One then determines the distances from the 0° image to the other images using the relation

$$d_\beta = EFL \tan \beta. \tag{4}$$

The difference between the distances found with the aid of eq (4) and the corresponding measured distance for a given value of β is the value of the distortion at that point. It is positive if the measured location of the image is farther from the central image than the computed location. Similar computations are made for the negative obtained for position II and the results averaged and accepted as final.

It has always been maintained that the probable error of values of distortion obtained by this method did not exceed ± 0.020 mm. The sources of error arising from error in angle in plate measurement are discussed in an earlier publication [2] where it was established that errors from this source should not exceed ± 0.013 mm.

In making the negatives, the collimator targets are illuminated by light from a tungsten source after passing through a K–3 filter. Eastman Kodak Spectroscopic plates, emulsion Type V–F, are used to record the image formed by the lens under test. The exposed plates are processed in trays containing Eastman Developer D–19 and 68° F for 3 min with continuous agitation.

2.2. Visual Optical Bench, Direct Nodal Slide. Method B

The visual optical bench has long been the basic tool for evaluating the optical constants of lenses. The one used at the Bureau has been in existence for approximately 30 yr and can still be regarded as a precision instrument [3]. For measuring distortion, one uses a collimator, nodal slide lens holder shown in figure 2, and micrometer microscope. The lens is carefully alined in the holder and the axial image formed by the lens under test of the illuminated

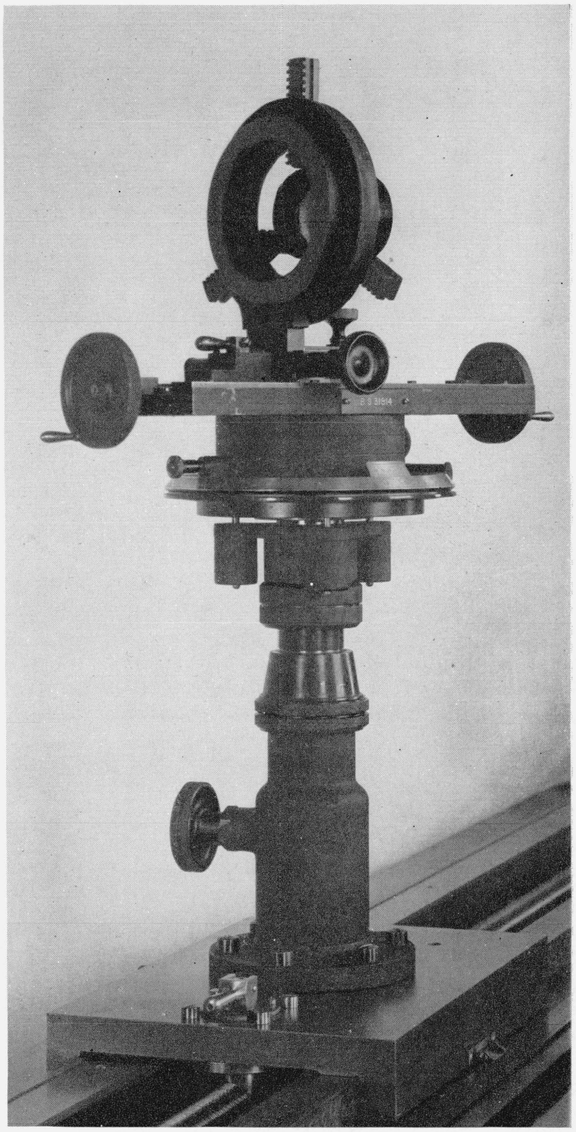

FIGURE 2. *Visual nodal slide (Method B).*
This photograph shows the nodal slide in which the lens under test is mounted.

reticle of the target is brought into coincidence with the object plane of the viewing microscope. The reticle is illuminated by filtered light from a tungsten source. The effective wavelength is approximately 575 mμ. By a series of successive adjustments, a condition is found for which a small rotation of the lens about a vertical axis does not produce a displacement of the axial image viewed. The rear nodal point of the lens is then considered to coincide with the center of vertical rotation of the nodal slide.

Assuming the equivalent focal length, f, to be known, the nodal slide is rotated by amount β about the vertical axis using the calibrated circle of the nodal slide to position it exactly. The entire saddle carrying the nodal slide and lens is then moved away from the microscope toward the collimator by an amount, $f(\sec \beta - 1)$. The viewing microscope is shifted laterally to the new position of the image and its dial read and recorded as reading R. The nodal slide is then rotated to position $-\beta$ and a second setting of the microscope, L, is made. The distortion, D_β, is then obtained from the relation

$$D_\beta = \frac{(R-L)}{2} \sec \beta. \tag{5}$$

3. Results of Measurement

At the time this study was initiated, it was planned to make measurements of distortion on a series of seven or more lenses by different methods and to compare the results. Measurements have been made on seven lenses by two different methods and comparative tables of results prepared. All values of distortion are referred to the calibrated focal length [4] for ease of comparison. In general the results are comparable and all values fall within the range of ± 0.02 mm which has been established here as the accepted maximum value of the probable error for distortion measurements. However, the agreement fell somewhat short of that expected in that systematic differences were observed. Because of these systematic differences, it was deemed wise to concentrate on the analysis of the results of measurement made on a single lens by the two methods, in the belief that more information on the causes of lack of agreement could be gained thereby. This has proved to be the case, and in the following pages some of the steps in this investigation and analysis are reported.

3.1. Values of the Distortion by Methods A and B for Wide Angle Lens No. 3

The values of the distortion referred to the calibrated focal length obtained photographically with the precision lens testing camera (D_A) and visually with the nodal slide optical bench (D_B) are shown in table 1. These values are based on a single run by each method. For method A, a single run provides two negatives, one each for positions I and II; each negative is measured 5 times and the results are averaged. For method B, a single run consists of two independent sets of measurements; the accepted values of distortion are based on the average of two sets. It is clear from table 1, that

TABLE 1. *Measured values of the distortion versus angular separation β from the axis for wide angle lens No. 3*

The values obtained with the precision lens testing camera are designated D_A while those obtained on the nodal slide optical bench are designated D_B. All values are referred to the calibrated focal length and are given in microns.

β	D_A	D_B	D_B-D_A
deg			
0	0	0	0
5	−11	−6	5
10	−18	−8	10
15	−17	1	18
20	−2	19	21
25	32	50	18
30	65	89	24
35	97	104	7
40	76	80	4
45	−97	−104	−7

the values of D_A and D_B do not depart from their average by more than 12 μ at most and accordingly either set of values can be regarded as accurate within the usual tolerance of ± 20 μ. However, the differences in the values obtained by the two methods D_B-D_A which are also shown in table 1 do not appear to be random but rather appear to indicate the presence of systematic error. This is shown more strikingly in figure 3 where D_B-D_A is plotted as a function of β.

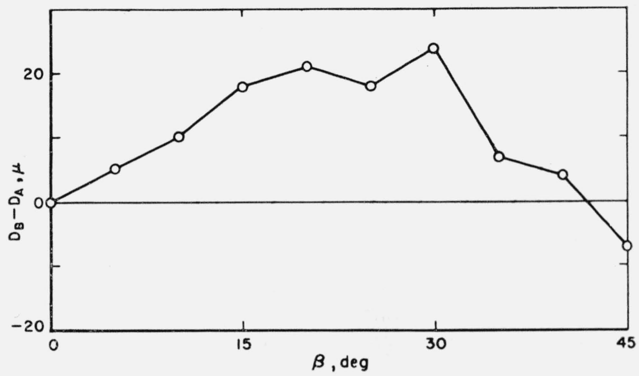

FIGURE 3. *Variation of distortion difference, D_B-D_A,* with *angular separation from the axis β.*

3.2. Precision of Measurement for Visual Nodal Slide

One's first thought when confronted with discrepancies in the results of measurement obtained by two different methods is to make additional measurements using the method most likely to be suspected. The visual nodal slide optical bench in use at the time of these measurements was somewhat antiquated so it seemed proper to check its performance first. Five additional independent sets of measurements were made. A pinhole reticle was used initially, but a transparent crossline was substituted for it during two sets of measurements. Some time elapsed between the first two sets of measurements and the 3d, 4th, 5th, and 6th calibration. During this time, the lens cells were removed from the test barrel and replaced. A similar operation occurred between calibrations 6 and 7. Additional care was taken in checking alinement in the course of calibration 3 and 7. Each set of data was carefully processed and the values of distortion adjusted to a calibrated focal length with maximum plus at 35° and maximum minus at 45°. The results are shown in table 2. Cursory examination of these values indicates quite good agreement, so an average for all seven including the first two calibrations was made. The table also shows the probable error of a single determination, and the value of PE_s. The values of PE_s are so low that it seems there is no cause to question the precision of method B. While there may be a systematic error present that impairs the accuracy, none comes to mind at the present writing.

Table 2. *Values of the distortion, D_B, versus β for seven sets of measurement using method B*

These values of D_B were obtained for wide angle lens No. 3 with the visual nodal slide bench. All values are referred to the calibrated focal length and are given in microns. The values of PE_s are also shown.

β	Values of D_B obtained from calibration							Average value of D_B	PE_s
	1	2	3	4	5	6	7		
deg									
0	0	0	0	0	0	0	0	0	±0
5	−6	−6	−7	−7	−7	−8	−8	−7	1
10	−7	−8	−9	−8	−6	−6	−11	−8	1
15	1	1	1	4	3	3	−7	1	2
20	19	19	21	24	23	27	13	21	3
25	48	52	52	53	60	53	45	52	3
30	88	89	85	88	85	87	84	87	1
35	103	104	105	107	104	105	111	106	2
40	83	78	86	82	80	69	77	79	4
45	−103	−104	−105	−107	−104	−105	−111	−106	2

3.3. Precision of Measurement for Lens Testing Camera

a. First Trial

Comparison of the average values of distortion based on the seven separate determinations shown in table 2 with the values of D_B for the first two determinations shown in table 1 indicates that the difference between D_B and D_A can not be explained by lack of precision in method B. The average value of D_B for the seven calibrations differ at most by 2 μ from the average of the first two calibrations. Accordingly further tests by method A were made. Five new negatives were made in position I on the precision lens testing camera and five new negatives were made in position II. Test negatives 1 to 4 were made under identical conditions in position I so also were test negatives 5 to 8 in position II. The lens was removed, replaced, and realined for test before making negatives 9 and 10. Each negative was measured and the distortion evaluated separately and the results are shown in table 3. Part (a) of table 3 shows the results for position I, and part (b) shows the results for position II. In part (c), the results shown in parts (a) and (b) are combined by pairs and results of this combination are shown. The values of the precision index, PE_s, are somewhat higher in parts (a) and (b) than those shown in part (c) where the results are averaged in the usual manner. The differences in the average value of D_A for positions I and II are not regarded as serious as it is presumed to arise from a small amount of plate tipping of a nature that becomes negligible on averaging. The final values of the average, D_A, shown in part (c) of table 3, differ by a few microns from the values of D_A shown in table 1, but have not changed in a manner that appreciably reduces the values of D_B-D_A. The values of the precision index PE_s shown in part (c) compare favorably with the similar values shown in table 2 for method B. However, the magnitudes of PE_s for both methods are too small to justify the existence of differences in the values of the distortion as great as are found.

Table 3. *Values of the distortion, D_A, versus β for the first trial using method A*

These values of distortion were obtained for Wide Angle Lens No. 3 with the precision lens testing camera. All values are referred to the calibrated focal length and are given in microns. In part (a) values are given for position I for five negatives, together with the average value of D_A and the value of PE_s. In part (b), similar results are given for position II, and in part (c) average values for paired negatives for position I and II are given.

β	(a) Values of D_A obtained in position I from negatives					Average value of D_A	PE_s
	1	2	3	4	10		
deg							
0	0	0	0	0	0	0	±0
5	−13	−13	−13	−11	−9	−12	1
10	−21	−18	−22	−18	−15	−19	2
15	−16	−12	−15	−9	−5	−11	3
20	2	1	1	3	7	3	2
25	20	24	28	26	32	26	3
30	53	60	63	58	64	60	3
35	87	91	96	86	83	89	4
40	81	75	84	77	83	80	3
45	−87	−91	−96	−86	−83	−89	4

β	(b) Values of D_A obtained in position II for negatives					Average value of D_A	PE_s
	5	6	7	8	9		
deg							
0	0	0	0	0	0	0	±0
5	−10	−12	−16	−12	−12	−12	1
10	−15	−22	−22	−19	−18	−19	2
15	−10	−23	−20	−16	−18	−17	3
20	6	−9	−6	−4	2	−2	5
25	34	22	22	22	33	26	5
30	72	50	63	56	75	63	8
35	108	102	100	101	117	106	5
40	88	86	82	84	95	87	3
45	−107	−103	−101	−100	−117	−106	5

β	(c) Values of D_A obtained in position I and II for negative pairs					Average value of D_A	PE_s
	1 and 5	2 and 6	3 and 7	4 and 8	10 and 9		
deg							
0	0	0	0	0	0	0	±0
5	−12	−12	−14	−12	−10	−12	1
10	−18	−20	−22	−18	−16	−19	2
15	−13	−18	−18	−12	−12	−15	3
20	4	−4	−4	0	4	0	3
25	27	23	25	24	32	26	2
30	62	55	63	57	68	61	4
35	98	96	98	94	100	97	2
40	84	80	83	80	89	83	2
45	−97	−97	−98	−93	−100	−97	2

b. Second Trial

While the first values of the distortion obtained in section 3.3a compare favorably with the values shown in table 1 for method A, it is disquieting to see the large discrepancies that exist between values obtained in positions I and II. While it is probable that these discrepancies arise from actual differences in performance along the opposing radii, nonetheless it seemed worthwhile to determine whether or not any maladjustment of the lens testing camera could produce this effect. Careful analysis showed that when the camera was properly alined so as to point properly at the center of the collimator reticle for position I that it did not so point when swung into position II. Instead there was an error of 36 sec in the pointing for position II. Computation indicated that this defect in alinement in position II could produce small errors in the distortion although the indicated magnitude was too small to change the values of D_A by more than a few microns. This alinement error was corrected, the angles were recalibrated, 10 additional negatives were made, and the measurements described in section 3.3a were repeated. The new results are shown in table 4.

TABLE 4. *Values of the distortion, D_A, versus β for the second trial using method A*

These values of the distortion were obtained for Wide Angle Lens No. 3 with precision lens testing camera following its recalibration. All values are referred to the calibrated focal length and are given in microns. In part (a) values are given for position I for five negatives together with the average value of D_A and the value of PE_s. In part (b) similar results are given for position II, and in part (c), averaged values for paired negatives for positions I and II are given.

β	(a) Values of D_A obtained in position I for negatives					Average value of D_A	PE_s
	11	12	13	14	20		
deg							
0	0	0	0	0	0	0	±0
5	−10	−12	−12	−16	−15	−13	2
10	−18	−23	−21	−19	−19	−20	2
15	−12	−20	−18	−15	−16	−16	2
20	1	−9	−7	0	−2	−3	3
25	27	16	19	25	22	22	3
30	64	57	57	68	62	62	3
35	95	91	94	94	98	94	2
40	86	82	86	83	87	85	2
45	−95	−91	−94	−94	−98	−94	2

β	(b) Values of D_A obtained in position II for negatives					Average value of D_A	PE_s
	15	16	17	18	19		
deg							
0	0	0	0	0	0	0	±0
5	−19	−19	−19	−19	−17	−19	0
10	−28	−26	−25	−26	−26	−26	1
15	−27	−23	−23	−21	−23	−23	1
20	−11	−8	−9	−5	−9	−8	2
25	17	18	21	25	21	20	2
30	58	56	55	62	54	57	2
35	99	102	100	104	96	100	2
40	77	80	84	85	78	81	3
45	−99	−102	−100	−104	−96	−100	2

β	(c) Values of D_A obtained in positions I and II for negative pairs					Average values of D_A	PE_s
	11 and 15	12 and 16	13 and 17	14 and 18	20 and 19		
deg							
0	0	0	0	0	0	0	±0
5	−14	−16	−16	−18	−16	−16	1
10	−23	−24	−23	−22	−22	−23	1
15	−20	−22	−20	−18	−20	−20	1
20	−5	−8	−8	−2	−6	−6	2
25	22	17	20	25	22	21	2
30	61	56	56	65	58	59	3
35	97	96	97	99	97	97	1
40	82	81	85	84	82	83	1
45	−97	−96	−97	−99	−97	−97	1

When the final results obtained from negatives 11 to 20 are compared with those obtained from negatives 1 to 10, it is clear that while there is slightly better agreement at the large angles than before, the agreement is somewhat worsened at the small angles. Finally, the average values for the second trial, shown in part (c) of table 4, are substantially the same as those obtained from the first trial, shown in table 3.

C. Summation

The final accepted values obtained by methods A and B are brought together for comparison in table 5. The values of D_A and D_B differ slightly from the values shown in table 1 but are believed to be more reliable as each value is the average of many more determinations. It is noteworthy that the magnitude of ΔD has increased slightly. This table also shows the probable error of the mean for the values of D_A, D_B, and ΔD. Consideration of these various values indicates that the systematic error still exists and has not been reduced by the multiplication of measurements.

TABLE 5. *Comparison of the average values of distortion, D_A, derived from 20 negatives using method A and the average values of distortion, D_B, derived from sevenc alibrations using method B.*

The difference is shown as $\Delta D = D_B - D_A$. Values of the probable error of the mean for each set of determinations are also shown. All values are expressed in microns.

β	D_A	D_B	$\Delta D = D_B - D_A$	PE_m for D_A	PE_m for D_B	PE_m for ΔD
0	0	0	0	±0.0	±0.0	±0.0
5	−14	−7	7	.6	.4	.7
10	−21	−8	13	.6	.4	.7
15	−17	1	18	.9	.8	1.2
20	−3	21	24	1.0	1.1	1.5
25	24	52	28	0.8	1.1	1.4
30	60	87	27	1.0	0.4	1.1
35	97	106	9	0.3	.8	0.9
40	83	79	−4	.6	1.5	1.6
45	−97	−106	−9	.3	0.8	0.9

3.4. Effect of Plate Curvature

In view of the known effects of plate curvature on the values of distortion, [2] it seemed worthwhile to examine the emulsion surfaces of the negatives used in method A. This was done and a small amount of plate curvature was found. However, in no instance was curvature present in sufficient amount to produce more than one-fourth of the measured differences in distortion values. The average departure from flatness for the 20 negatives could not produce differences in distortion in excess of one-tenth of that found. It may therefore be stated that plate curvature is not a prime cause of the differences in values of distortion found by the two methods.

3.5. Effect of Plate Tipping

The plate holder in the precision lens testing camera is so constructed that the emulsion surfaces are coplanar for positions I and II. For the coplanar or parallel plane condition, small departures of the plane from true normality to the optical axis of the system under test would not produce variations in the average value of distortion even though the measured distortion on either side of center would be different. However, if a slight warpage of the holder has occurred, the plate in position II would not be coplanar with that in position I and a small amount of asymmetric distortion would persist and adversely affect the final average. Accordingly a procedure was developed for checking this possibility based on the plate tipping analysis described in an earlier paper [5]. The final product of this analysis is shown in table 6. When the values of ΔD in table 5 are divided by the appropriate multiplier M, then the magnitude of $f\epsilon_t$ can be obtained for each value of ΔD, where ϵ_t is the angle between the plane of the plate in position I and that in position II. This was done and the results are shown in table 7.

TABLE 6. *Values of tan $^2\beta$ and the multiplier, M, for a value of $f\epsilon_t = 1.00$ micron*

The multiplier, M, is so calculated that when it is multiplied by a given value of $f\epsilon_t$, it yields the distortion correction referred to the calibrated focal length.

Angular separation, β, from axis	$\tan^2\beta$	M
deg		
0	0.0000	0.0000
5	.0077	−.0690
10	.0311	−.1234
15	.0718	−.1630
20	.1325	−.1865
25	.2174	−.1913
30	.3334	−.1727
35	.4903	−.1234
40	.7041	−.0314
45	1.0000	−.1235

TABLE 7. *Computation of $\overline{f\epsilon_t}$ to determine effective amount of uncompensated plate tipping that may be present in method A*

Values of $f\epsilon_t$ are obtained for each value of ΔD given in table 5; the average $\overline{f\epsilon_t}$ for β ranging from 10° to 30° is −129 microns; this value is used to determine the probable contribution to distortion $M\overline{f\epsilon_t}$. The last column shows the magnitude of difference that remains. All values are given in microns.

β	ΔD	$\overline{f\epsilon_t} = D/M$	$M\overline{f\epsilon_t}$	$\Delta D + M\overline{f\epsilon_t}$
deg				
0	0	--------	--------	--------
5	7	−101	−9	−2
10	13	−105	−16	−3
15	18	−110	−21	−3
20	24	−129	−24	0
25	28	−146	−25	3
30	27	−156	−22	5
35	9	−73	−16	−7
40	−4	127	4	0
45	−9	−73	16	7

The values of $f\epsilon_t$ that are shown in table 7 indicate that some effect of uncompensated plate tipping may be present in the values of distortion obtained by method A. All values of $f\epsilon_t$ are of the same sign and are of the same order of magnitude except that computed for $\beta = 40°$. The value of ΔD for this point is so small (4μ) that little weight can be given to the values of $f\epsilon_t$, derived from it. While there is appreciable variation in the remaining values of $f\epsilon_t$, it must be remembered that the warping that introduces this effect probably bends the plaet slightly as well as tipping it. An average $f\epsilon_t$, for the range 10° to 30° was determined and found to be $f\epsilon_t = -129$ microns which indicates that the effective angle between the plates in positions I and II is approximately 3.0 min of arc. The quantity $M\overline{f\epsilon_t}$ was then evaluated for each value of β. The degree of compensation achieved by assumption of this amount of uncompensated plate tipping is shown graphically in figure 4. The solid curve shows the variation of the quantity $-M\overline{f\epsilon_t}$ with β while the circles show the corresponding values of ΔD. The quantity, $D + M\overline{f\epsilon_t}$, would equal zero if $f\epsilon_t$ was invariant for all values of β. However, the departure from zero is sufficiently small that it is evident that uncompensated differential plate tipping makes an important contribution to the systematic error whose cause is being sought.

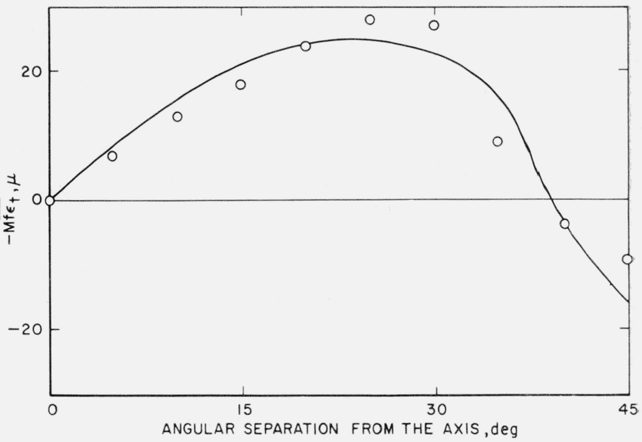

FIGURE 4. *Variation of the quantity $-M\overline{f\epsilon_t}$ with angular separation from the axis β.*

Compensation of the differences in distortion values ($\Delta D = D_B - D_A$) by an assumed differential plate tipping ϵ of approximately 3 min of arc. The solid line is a plot of the function, $-M\overline{f\epsilon_t}$; the circles are corresponding values of ΔD

Following this work careful measurements on the plate holder were made to determine if this warpage actually existed. While measurable values of warpage were found that were somewhat smaller than might be inferred from the results shown in table 7, they appeared large enough to warrant correction. Accordingly a new heavier plate holder has been constructed and has replaced the one in use during this experiment. Recent measurements show that the values of $\Delta D = D_B - D_A$ now obtained are less than one-fourth the values of ΔD obtained prior to the change.

In table 8, the correction $M\overline{f\epsilon_t}$ is applied directly to the values of distortion D_A. The corrected value $D_A' = D_A - M\overline{f\epsilon_t}$ is given in the table and agrees closely with the corresponding values of D_B. The last column shows the differences still remaining between values of D_A' and D_B. It is clear that the remaining discrepancies are so small in comparison with the values of D_A' and D_B that they can be neglected.

TABLE 8. *Comparison of D_A' (the value of D_A after correction for plate tipping) and D_B*

The values of D_A and D_B are taken from table 5; values of $\overline{Mf\epsilon_t}$ are taken from table 7. The last column $\Delta D'$ shows that the discrepancies between the two methods have been reduced to tolerable values. All values are given in microns.

Angular separation from axis	D_A	$\overline{Mf\epsilon_t}$	$D_A'=D_A-\overline{Mf\epsilon_t}$	D_B	$\Delta D'=D_B-D_A'$
deg					
0	0	0	0	0	0
5	−14	−9	−5	−7	−2
10	−21	−16	−5	−8	−3
15	−17	−21	4	1	−3
20	−3	−24	21	21	0
25	24	−25	49	52	3
30	60	−22	82	87	5
35	97	−16	113	106	−7
40	83	4	79	79	0
45	−97	16	−113	−106	7

4. Conclusion

It is evident from the foregoing study that highly accurate values of distortion can be obtained by either the photographic method using the precision lens testing camera or the visual method using the nodal slide bench. For each method, values can be obtained that are precise to within ±4 microns. When proper care is taken, the values obtained by either of the methods do not depart from the common average by amounts exceeding ±5 microns.

It is however clear that to obtain and maintain such high accurarcy it is necessary to be constantly on the alert for various insidious sources of error such as plate curvature, differential plate tipping, and incorrect prime calibration of angles used in the determinations.

5. References

[1] I. C. Gardner and F. A. Case, Precision camera for testing lenses, J. Research NBS **18**, 449 (1937) RP984.

[2] F. E. Washer, Sources of error in various methods of airplane camera calibration, Photogrammetric Eng. XXII, p. 727 (1956).

[3] A. H. Bennett, The distortion of some typical photographic objestives, J. Opt. Soc. Am. and Rev. Sci. Instr. **14,** 245 (1927).

[4] F. E. Washer and F. A. Case, Calibration of precision airplane mapping cameras, Photogrammetric Eng. XVI, 502 (1950); J. Research NBS **45,** 1 (1950) RP2108.

[5] F. E. Washer, Effect of camera tipping on the location of the principal point, J. Research NBS **57,** 31 (1956) RP2691.

WASHINGTON, April 12, 1958.