Measurement of Current with the National Bureau of Standards Current Balance

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Prior to the adjustment of the electrical units in 1948, the value of a current had been determined in absolute units by means of a current balance and simultaneously measured in NBS amperes by comparison with standard resistors and standard cells. This work was reported in RP1449. Similar measurements made recently with an electrodynamometer indicate a possible change in the values of the standards. The present paper reports a repetition of the work described in RP1449. The purpose of this remeasurement was to determine whether or not the standards had changed. Only minor changes were made in the equipment in order that factors which might have introduced small systematic errors in the results would remain unchanged.

According to the work described in this paper, 1 NBS ampere = 1.000008 absolute amperes. Recent work with the Pellat electrodynamometer gave the result 1 NBS ampere = 1.000013 absolute amperes. The weighted mean of these two values is

1 NBS ampere-1.000010 $\pm$ 0.000005 absolute amperes

The results given above for the current balance differ by 6 ppm from those obtained in 1942. This indicates, in view of the uncertainties of measurement, that any change in the ampere as maintained by standard resistors and standard cells does not exceed a few parts in a million.

1. Introduction

The accuracy to which the electrical units as maintained at the National Bureau of Standards are known is under a continual process of improvement. A history of the development of the electrical units up to the adoption of the absolute units in 1948 [1] [1] has been presented by Silsbee [2]. Since the 1948 revision, two absolute determinations of electric current have been made at the Bureau.

The recent determination of current with a Pellat-type electrodynamometer [3] led to the result that the NBS unit of current was larger than the absolute ampere by 13 ppm (parts per million). The difference was not much more than the estimated uncertainty of the absolute measurement; but, since the values assigned to the NBS primary standard cells depend largely upon an earlier determination of current with the NBS current balance [4], it was thought necessary to repeat the earlier work in order to determine whether an appreciable drift in the electrical standards had taken place. This work was done as soon as possible after the completion of the measurement using the electrodynamometer, to assure as far as possible that both sets of absolute measurements were referred to the same electrical standards.

Photographs of the current balance used in 1942 and again in this determination appear in figures 1, 2, and 3. Briefly, the equipment consists of a helical fixed coil designated H_1 (fig. 6) in which current flows into the coil through a lead in the center of the helix, and out through leads on each end. A smaller helical coil designated P_1 hangs from an arm of a

[1] Figures in brackets indicate the literature references at the end of this paper.

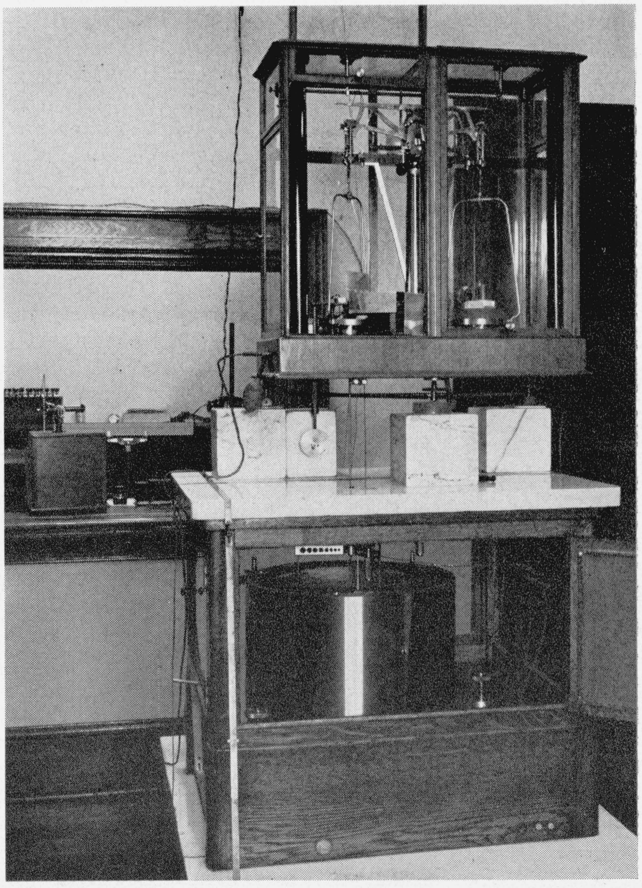

FIGURE 1. *Rear view of current balance showing fixed coil in operating position and operating room in background.*

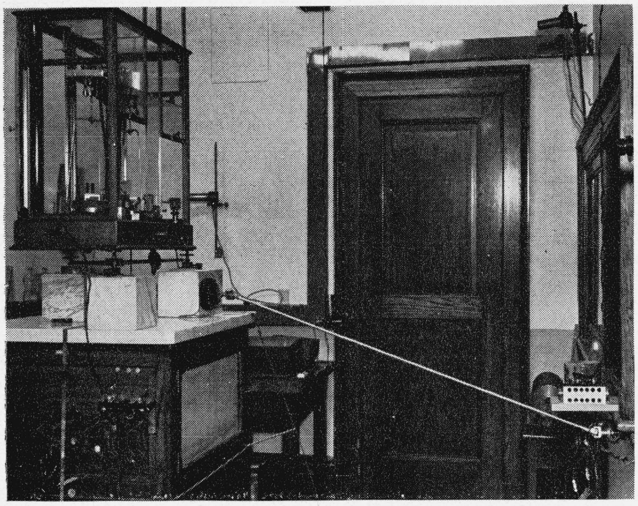

FIGURE 2. *Side view of current balance showing reversing switches and operating rod.*

sensitive balance so as to be concentric and coaxial with the fixed coil. A current flowing in the movable coil produces a force between the two coils, and tends to deflect the beam.

In practice, the current is held constant and evaluated in NBS units by comparing the potential drop it produces across a known resistance with the emf of a standard cell which is known with reference to the NBS primary standard cells. The balance is adjusted to equilibrium witb this current flowing in both coils. Then, the current in the fixed coil only is reversed, and simultaneously a weight is placed on the balance pan. The weight is adjusted to equal as closely as possible the change in force caused by reversing the current. The small difference between the forces is observed as a change in the rest point of the balance. A switch for reversing the current is mounted on the coil case. A rod extends from this switch to the operating room; a cam and other connecting linkages enable the observer by turning this rod to raise and lower the weight on the balance pan and reverse the current at the same time.

The change in force caused by reversing the current is measured by comparison with the force exerted by gravity on the mass placed on the balance pan. This force is equal to the square of the current times a calculable function of the physical dimensions of the coils. From these equivalent expressions for the force, the current flowing can be determined in the mechanical units of length, mass, and time.

2. Changes in Equipment

Inasmuch as the redetermination of the ampere by means of the current balance was intended primarily as a check on the stability of the NBS standards, the principal features of the equipment were kept intact. The only geometrical change in the arrangement of the coils was a change in α, the angle between the movable and fixed coil leads, which has only a very small effect on the mutual force.

The standard cells were moved from the underground compartment to a "standard celler" [5] where their temperatures were thermostatically controlled near 34° C. This arrangement was used also for the Pellat electrodynamometer, and made it possible to regulate the cell temperatures and hence the cell voltages more precisely than had been possible before.

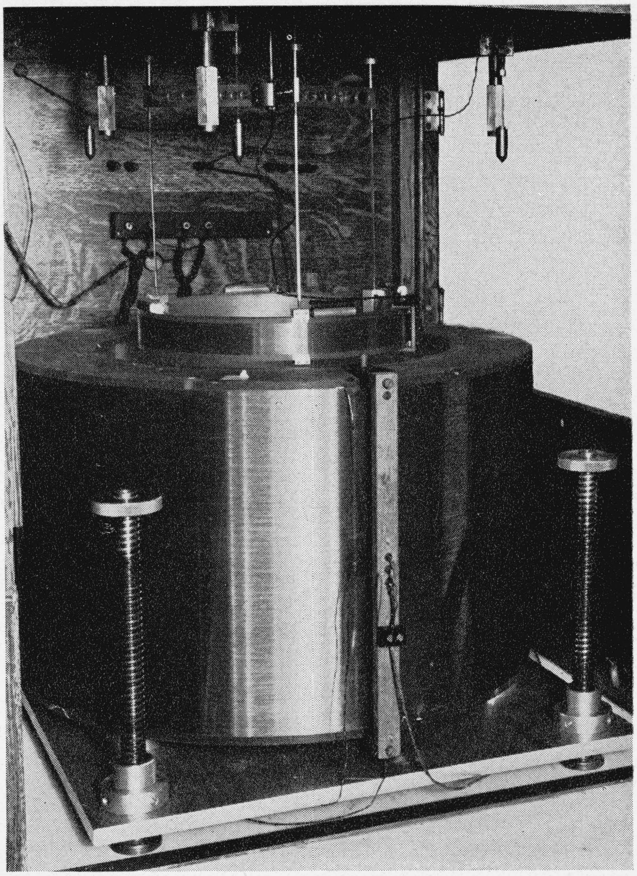

FIGURE 3. *Coil case with fixed coil lowered to show movable coil.*

Changing the temperatures of the cells also changed their voltages, and made it necessary to decrease the size of the platinum weight that had been used with the balance in the earlier work.

The turning points of the balance are observed on the scale in the operating room by a beam of light reflected to the scale from a mirror mounted on the balance beam. A scheme in which the beam of light was reflected twice from a moving prism had been used before, in order to increase the balance sensitivity. We preferred to use a singly reflecting mirror instead of the doubly reflecting prism, because the hair line at the light source could be focused more sharply at the balance scale. The sensitivity of the balance dropped from 1.21 mg/cm to 2.33 mg/cm, but the reliability of the readings was improved.

During the preliminary measurements it was noticed that throwing the reversing switch mounted on the coil case gave the case a push that changed the apparent rest point of the balance as observed on the scale in the operating room. It was decided that the coil case was too shaky to be reliable, so copper straps were bound around it to make it more rigid. These can be seen in the photographs. Also, a sliding joint was put into the switch rod. The performance of the balance was then checked with no current in the coils, and it was found that the position of the reversing switch had no effect on the rest point of the balance.

The turning points of the current balance have always been subject to random fluctuations. These are attributed to fluctuations in the air flow around the movable coil. Much experimentation has been done with ventilation of the coil case in an effort to steady the swings of the balance. The most satisfactory arrangement found was used for the final runs. This consisted of a honeycomb baffle under the movable coil and a fan to draw air from the top of the coil case. The fan was located about 20 feet from the coils and was connected with the coil case by means of a tube.

Reversing the current and changing the weight sometimes gives the balance an impulse which, if unchecked, would make the balance amplitude unsatisfactory. Previously the balance had been steadied after reversing the current by injecting short blasts of air under the balance pans. It was found that the turning points of the balance were more regular if the adjustments in balance amplitude were made by changing briefly the current through the coils. Two switches were installed in the operating room, one to increase, and one to decrease the current. All of the runs reported in this paper were obtained without the use of air jets.

3. Mechanical Dimensions

The mechanical dimensions of the coils were remeasured, using for the most part the methods that had been used in 1942. The end standards used to measure the diameters of the coils were re-evaluated by the NBS Gage Section. Summaries of the coil dimensions appear in table 1.

3.1. Diameter

Figures 4 and 5 illustrate the coherence between the earlier and recent measurements of the diameters

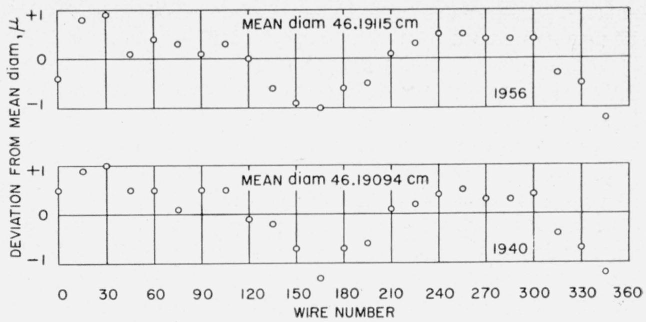

FIGURE 4. *Coherence between 1942 and 1956 measurements of fixed coil wire diameters.*

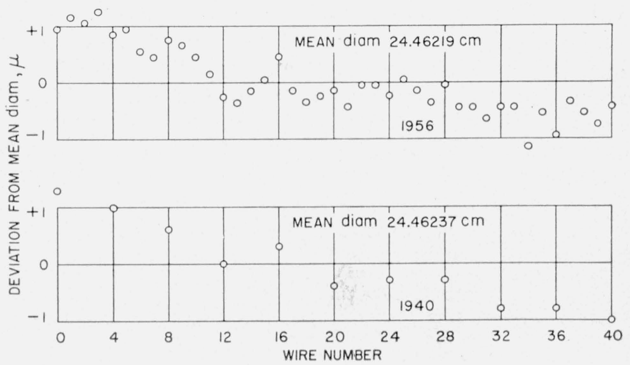

FIGURE 5. *Coherence between 1942 and 1956 measurements of movable coil wire diameters.*

of the two coils at 30° C. It can be seen that apparently the fixed coil became larger and the movable coil smaller. Some changes in dimensions are to be expected, and could be caused by a gradual relaxation of the strains in the wires or forms.

The diameter and electrical resistance of each coil were measured at three temperatures: near 25°, 30°, and 35° C. From these measurements it was possible to estimate, from measurements of the resistances of the wires, the diameters of the coils when they were in the balance case under different ambient conditions.

The newly determined temperature coefficients of expansion agree very well with the values found in

TABLE 1. *Constants of the fixed helix H_1 and the movable helix P_1*

[All values reduced to 30° C]

	H_1		P_1	
	1942	1956	1942	1956
Average outside diameter of coil ____ cm	46. 26090	46. 26111	24. 51360	24. 51342
Diameter of wire ____ cm	0. 06996	0. 06996	0. 05123	0. 05123
Mean diameter of coil ____ cm	46. 19094	46. 19115	24. 46237	24. 46219
Current distribution correction ____ cm	−0. 00003	−0. 00003	−0. 00003	−0. 00003
Weighting correction ____ cm	+. 00002	+. 00002	. 0	. 0
Effective mean diameter ____ cm	46. 19093	46. 19114	24. 46234	24. 46216
Axial length of coil ____ cm	27. 51654	27. 51642	2. 6650	2. 6649
Number of turns	344	--------	41	--------
Mean pitch ____ cm	0. 0799899	0. 0799896	0. 065000	0. 064998
Resistance of winding {Upper, Lower} ____ ohms	11. 8016 11. 7974	11. 7806 11. 7808	2. 8220	2. 8169
Resistance-temperature relationship {Upper, Lower} ____ ohms/°C	0. 0443 . 0443	0. 0436 . 0442	0. 01079	0. 01034
Temperature coefficient of expansion ____ ppm/°C	3. 6	3. 5	3. 9	3. 7
Winding tension ____ kg	10	--------	3	--------

the old measurements. The temperature coefficients of resistance do not agree, but this is because they were assumed, not measured, in the earlier work. At that time the temperature coefficients were taken from tables of copper-wire characteristics. Since the wires are under considerable strain, it is not surprising that the measured temperature coefficients differ from the values assumed in RP1449.

A new measurement of the diameter of the wire on the movable coil was made, and the result agreed with the previous measurement. In view of the excellent agreement it was felt unnecessary to remeasure the fixed coil wire diameter.

The current distribution corrections contained in table 1 correct for the variation of current density over the cross section of each wire; no corresponding corrections were made in the work reported in RP1449 because the net effect based on Snow's assumption of the "natural" distribution [6] was small. Recently Wells [7] has measured the resistance-strain relation in copper wire, making it now possible to give further expression to the variation of current density over the cross section of the wires. Using Snow's formula for the helix equivalent to a helical wire, we have

$$\bar{r}_1=r_1+\frac{\rho_1^2}{8}\left(\frac{1}{r_1}+2A_1\right),$$

where $\bar{r}_1$=the effective coil radius,

r_1=the mean coil radius,

ρ_1=the wire radius,

$$A_1=\frac{\left[\frac{du(r_1')}{dr_1'}\right]_{r_1'=r_1}}{u(r_1)},$$

and $u_1(r_1')$ is the volume density of current in the wire as a function of the distance r_1' from the x axis. A relationship similar to the above holds for the movable coil, whose coefficients will be denoted in what follows by the subscript "2." Snow has shown that the radius corrections do not depend upon the second derivative of $u(r)$. This means that a first order expansion of $u(r)$ will lead to a radius correction which is correct to second order.

The current density at a given point in the fixed coil is given by $u_1(r_1')=G/\sigma r_1'=(G/\sigma r_1)[1-(y_1/r_1)]$ to first order in y_1/r_1, where σ is the resistivity of the copper, $y_1=r_1'-r_1$, and G is a constant. The analysis to follow is carried out for the fixed coil only, wire 1, but it is to be understood that the equations are valid for the movable coil, wire 2, also.

The resistivity of the copper has been measured in terms of the strain by Wells [7]. He finds that

$$\frac{\frac{\Delta\sigma}{\sigma_0}}{\frac{\Delta l}{l_0}}=\beta+\gamma\left(\frac{\Delta l}{l_0}\right)^2,$$

where $\beta=1.13$ and $\gamma=-2.5\times10^4$;

or

$$\sigma=\sigma_0\left[1+\beta\frac{\Delta l}{l_0}+\gamma\left(\frac{\Delta l}{l_0}\right)^3\right].$$

$\frac{\Delta l}{l_0}$ at a point in wire 1 is related to the initial strain in the wire, K_1, and the position of the point, y_1, by $\Delta l/l_0=(y_1/r_1)+K_1$, where $K_1=2.5\times10^{-3}$) (and $K_2=1.4\times10^{-3}$). We then have to first order in y_1/r_1

$$\sigma=\sigma_0(1+\beta K_1+\gamma K_1^3)\left[1+\frac{y_1}{r_1}\frac{(\beta+3\gamma K_1^2)}{(1+\beta K_1+\gamma K_1^3)}\right],$$

and

$$\frac{1}{\sigma}=\frac{1}{\sigma_0(1+\beta K_1+\gamma K_1^3)}\left[1+\frac{y_1}{r_1}\frac{(\beta+3\gamma K_1^3)}{(1+\beta K_1+\gamma K_1^2)}\right]$$

leading to the result

$$u_1(r_1')=\frac{G}{r_1\sigma_0(1+\beta K_1+\gamma K_1^3)}\left[1-\frac{y_1}{r_1}\frac{(1+\beta K_1+\gamma K_1^3+\beta+3\gamma K_1^2)}{(1+\beta K_1+\gamma K_1^3)}\right].$$

We then have

$$A_1=-\frac{1+\beta+3\gamma K_1^2+\beta K_1+\gamma K_1^3}{1+\beta K_1+\gamma K_1^3},$$

and similarly for A_2. For the fixed coil, we have $A_1=-0.07$, $\Delta r_1=-1.7\times10^{-5}$ cm; and for the movable coil, $A_2=-0.13$, $\Delta r_2=-1.5\times10^{-5}$ cm. Δr_1 and Δr_2 are doubled and applied in table 1 as diameter corrections.

An attempt has been made to determine higher order corrections to the effective diameter based on Wells' resistivity determinations, but for the coils used here such corrections are negligible.

3.2. Pitch

The pitch of H_1 was measured as described in RP1449, and found to be insignificantly different from the earlier value. The pitch of P_1 was last measured in 1934, and was not remeasured for the 1942 work. A new determination was felt to be in order for the completeness of this determination, even though the force constant is not strongly dependent upon the pitch of the movable coil.

For the measurement of the movable-coil pitch a meter bar was set up vertically, parallel to the coil axis. A telescope was clamped to a vertical bar which was free to pivot in such a way as to swing the telescope from the meter bar to horizontal graduations ruled on the wires of the coil. The telescope was equipped with a filar micrometer eyepiece. A reading was made of the distance between a graduation ruled on a wire and a graduation on the meter bar. Then, the telescope was raised to measure the position of another wire. The distance between the two wires is given by the distance between the two meter bar graduations plus the difference between the readings of the filar micrometer eyepiece.

The accuracy of the measurement depended upon how well the coil and meter bar remained fixed with respect to each other, and upon the repeatability of the pivot of the vertical bar. The measurement is not as good as that used to measure the fixed-coil pitch, which used two telescopes, both of which were mounted on the vertical bar. It is felt, though, that the method is better than that used in 1934, which used a single telescope mounted on a carriage with a calibrated screw movement. The two-telescope method is better than either method used, but the movable coil was too short to be viewed by both telescopes at the same time. The result of the pitch measurements is that the changes found were too small to make any change in the balance constant as large as 1 ppm.

4. Calculation of the Force Constant

The force between the two helices is computed from the formula given by Snow [6]. With the notation of RP1449,

r_1=mean radius of fixed helix.

r_2=mean radius of movable helix.

l_1=axial length of fixed helix (pitch×number of turns).

l_2=axial length of movable helix (pitch×number of turns).

N_1=number of turns on fixed coil.

N_2=number of turns on movable coil.

α=angle between movable coil and fixed coil leads.

$$X_1=\frac{l_2}{2}.$$

$$X_2=\frac{l_1-l_2}{2}.$$

$$X_3=\frac{l_1+l_2}{2}.$$

The force in dynes between the movable coil and the upper half of the fixed coil with unit cgs current flowing in each of them is given by

$$f=2\omega'(X_1)+\omega'(X_2)-\omega'(X_3),$$

where

$$\omega'(X)=\omega'_\theta(X)+\omega'_a(X;\alpha)+\overline{\omega}'_X(X,\alpha).$$

Also,

$$\omega'_\theta(X)=\frac{2\pi N_1N_2}{l_1l_2}\left\{X\sqrt{X^2+(r_1+r_2)^2}\,[\mathrm{K}-\mathrm{E}]+\frac{X(r_1-r_2)^2}{\sqrt{X^2+(r_1+r_2)^2}}\left[\mathrm{K}-\Pi\right]\right\};$$

$$\omega'_a(X,\alpha)=\frac{\pi X}{6\sqrt{X^2+(r_1+r_2)^2}}\left[\frac{2-k^2}{1-k^2}\mathrm{E}-2\mathrm{K}\right]$$

$$+\frac{\pi X}{4\sqrt{r_1r_2}}\left(\cos\frac{\alpha}{2}\right)\sin^{-1}\left(k\cos\frac{\alpha}{2}\right)-\frac{k(2-k^2)}{2(1-k^2)}\sqrt{1-k^2\cos^2\frac{\alpha}{2}}$$

$$+\left(\sin\frac{\alpha}{2}\right)\log_e\left[\frac{k\sin\frac{\alpha}{2}+\sqrt{1-k^2\cos^2\frac{\alpha}{2}}}{\sqrt{1-k^2}}\right];$$

$$\overline{w}'_X(X,\alpha)=\frac{X}{\sqrt{X^2+(r_1+r_2)^2}}\log_e\frac{r_1}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\alpha}};$$

$$k^2=\frac{4r_1r_2}{X^2+(r_1+r_2)^2};\text{ and } k_0^2=\frac{4r_1r_2}{(r_1+r_2)^2}.$$

K, E, and Π are the complete elliptic integrals of the first, second, and third kind, respectively, to the modulus k and parameter k_0.

As in RP1449, the force in dynes between the helices with one ampere in the wires, F_{HH}, taking account of both halves of the fixed helix and of reversal of the current, is $F_{HH}=4f/100$.

The formula for the calculation of the force constant assumes that the diameter of each coil is uniform throughout its length. Clearly some turns affect the force constant more strongly than others. Use of the mean diameter in the calculations attaches undue importance to certain turns of wire, such as those near the center of the fixed coil, which have little effect upon the force constant. A plot was made of the calculated force, $f(x)$, between the movable coil and a turn of the fixed coil, as a function of the distance x between the center of the moving coil and the turn. It was decided to weight the radius of the turn at position x with the factor $f(x)$.

Let $r(x)$=the radius of a wire as a function of its axial position, $\bar{r}$=the average radius of the coil, and r_{eff}=the weighted mean radius. Then, summing over all the turns,

$$r_{eff}=\frac{\sum_n f(x)r(x)}{\sum_n f(x)};$$

and,

$$\Delta r=r_{eff}-\bar{r}=\frac{\sum_n f(x)r(x)-\sum_n f(x)\bar{r}}{\sum_n f(x)}=\frac{\sum_n f(x)[r(x)-\bar{r}]}{\sum_n f(x)}$$

$r(x)-\bar{r}$ for the fixed coil is plotted in figure 4. Δr is found by simple summation to be +0.1 micron, and the effect is entered in table 1 as a diameter correction. A similar correction for the movable coil would be much smaller, and was not considered worth calculating.

Table 2 summarizes the calculations of the force constant F_{HH}. Comparison with table 7 of RP1449 shows the difference in F_{HH} to be close to that calculated with the variation coefficients $\partial F/\partial r_1$, $\partial F/\partial r_2$, etc.

TABLE 2.—*Summary of computations on force due to unit currents*

The following values of the independent variables were used in the computation:

$r_1=23.09556.\quad l_1=27.51642.\quad N_1=344.\quad \alpha=\frac{\pi}{4}.$

$r_2=12.23109.\quad l_2=\ 2.6649.\quad N_2=\ 41.$

Quantities used in computing functions	Values of terms for—		
	$X_1=1.33245$	$X_2=12.42576$	$X_3=15.09066$
X^2	1.775 423	154.399 512	227.782 019
$X^2+(r_1+r_2)^2$	1 249.747 623	1 402.371 71	1 475.700 22
k^2	0.904 130 939	0.805 731 806	0.765 694 466
k	.950 858 001	.897 625 649	.875 039 694
$1-k^2$	.095 869 061	.194 268 194	.234 305 534
$\sqrt{1-k^2}$	.309 627 294	.440 758 658	.484 051 168
$X\sqrt{X^2+(r_1+r_2)^2}$	47.104 465 6	465.323 014	579.705 346
K	2.597 959 64	2.270 429 96	2.185 621 83
K−E	1.496 584 83	1.095 820 99	0.984 555 860
$X(r_1-r_2)^2/\sqrt{X^2+(r_1+r_2)^2}$	4.448 943 10	39.165 933 3	46.368 842 4
K−Π	−8.986 743 63	−6.951 426 49	−6.469 109 60
$X\sqrt{X^2+(r_1+r_2)^2}(\mathrm{K}-\Pi)$	70.495 828 7	509.910 727	570.752 295
$\frac{X(r_1-r_2)^2}{\sqrt{X^2+(r_1+r_2)^2}}(\mathrm{K}-\Pi)$	−39.981 511 1	−272.259 106	−299.965 123
Sum of the two preceding terms	30.514 317 6	237.651 621	270.787 172
$\omega'_\theta(X)$	36 876.779	287 203.747	327 248.307
$\omega'_a(X, \alpha)$	0.069 6	0.254 2	0.233 5
$\overline{\omega}'_x(X, \alpha)$	.119 1	.104 8	.124 1

Principal term: $[2\omega'_\theta(X_1)+\omega'_\theta(X_2)-\omega'_\theta(X_3)]=33{,}708.998$ dynes.

Azimuthal correction term: $[2\omega'_a(X_1,\alpha)+\omega'_a(X_2,\alpha)-\omega'_a(X_3,\alpha)]=+0.1600.$

Axial correction term: $[2\overline{\omega}'_x(X_1,\alpha)+\overline{\omega}'_x(X_2,\alpha)-\overline{\omega}'_x(X_3,\alpha)]=+0.0045.$

$f=33{,}709.163$ dynes.

$F_{HH}=1348.3665$ dynes.

The computation of the coefficients in the variation formula for these two helices gave the equation:

$$\frac{\delta F_{HH}}{F_{HH}}=2.73\,\frac{\delta r_2}{r_2}-2.81\,\frac{\delta r_1}{r_1}+0.091\,\frac{\delta l_1}{l_1}-0.015\,\frac{\delta l_2}{l_2}.$$

The following adjustments were made for differences between the values of dimensions used in the above computation and the dimensions given in table 1:

Fixed coil diameter $\Delta r_1=+0.1$ micron; $\Delta F=-0.0016$ dynes.

Movable coil diameter $\Delta r_2=-0.1$ micron; $\Delta F=-0.0030$ dynes.

Total force adjustment $=-0.0046$ dynes.

Hence for H_1 and P_1 at the dimensions of the coil corresponding to 30° C,

$F_{HH}=1348.3619$ dynes.

5. Experimental Determination of the Force Between the Coils

The experimental determination of the force between the coils was made as described in RP1449. The wiring diagram is shown in figure 6 for more convenient reference.

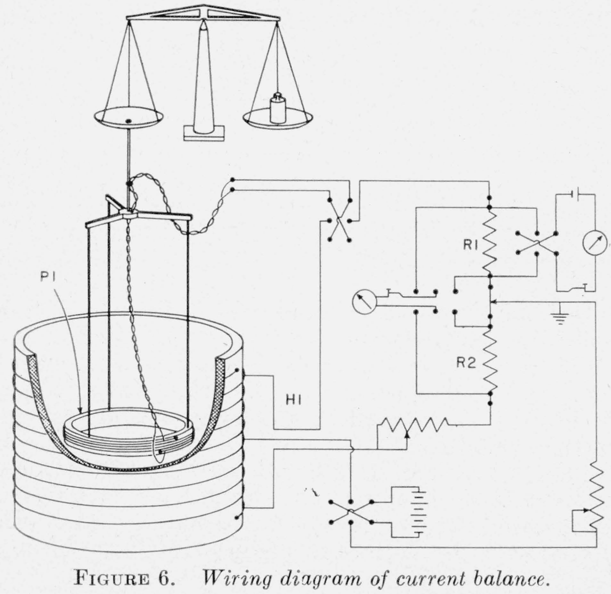

FIGURE 6. *Wiring diagram of current balance.*

Readings were made of nine turning points of the balance with the current in one direction, then the current in the fixed coil was reversed and the measurement of turning points repeated. A set of ten measurements involving nine reversals of current was averaged and entered in table 3 as one determination.

The force between the fixed-coil leads and the movable coil was measured by removing the fixed coil from the circuit without changing the lead-wire configuration. The movable-coil lead effect was measured in a similar way. These forces must be subtracted from the total force between the coils, and appear in table 4.

It was found that the mechanical dimensions of the coil supports were not as stable as had been hoped. Even with the straps around the case as described earlier, the vertical position of the movable coil with respect to the fixed coil changed about 0.2 mm in one month. The change was ascribed to dimensional changes in the wooden case due to a change in humidity, and the observed balancing mass was corrected for this change under the assumption that the shift was proportional to time. The numbers given in table 3 are corrected for the effect, which was never more than 4 ppm in the current.

The temperatures listed in table 3 are the temperatures of the wires, computed from their resistances and measured temperature coefficients. Because of temperature gradients in the coil forms, the mean

TABLE 3. *Results of measurememts of force*

Observed difference in rest points equals average difference in scale reading of rest points of the balance, δ, corresponding to "on" and "off" positions of weight, multiplied by the sensitivity of the balance (2.33 mg/cm). Mass of weight (a platinum cylinder): 1.425 569 g.

Date (May 1956)	Temperature °C			Observed difference in rest points	Difference in rest points corrected to 30° C
	P_1	H_1			
		Upper	Lower		
				mg	*mg*
18	24.79	29.84	28.60	−0.041	−0.049
21	29.45	29.45	28.26	−.001	−.009
21	29.55	29.56	28.37	−.002	−.010
22	29.54	29.56	28.37	+.004	−.004
22	29.59	29.64	28.42	−.029	−.037
23	29.88	29.93	28.71	−.049	−.057
23	29.92	29.95	28.73	+.014	+.006
23	29.96	30.00	28.78	−.063	−.071
23	29.99	30.05	28.80	−.063	−.071
24	29.95	30.05	28.82	−.046	−.053
24	29.95	30.05	28.82	−.019	−.026
24	29.96	30.06	28.82	−.002	−.009
24	29.97	30.07	28.82	−.007	−.014
Average δ times sensitivity of the balance					−0.031

TABLE 4. *Calculation of final results*

[Value of acceleration of gravity 980.081 cm/sec ²]

	1942	1956
Standard mass (a platinum cylinder) grams	1.427 655	1.425 569
Buoyance correction grams	−0.000 077	−0.000 079
Lead corrections {Moving coil grams	−.000 068	−.000 044
Lead corrections {Fixed coil grams	−.000 018	−.000 022
δ times sensivity of balance grams	+.000 222	−.000 031
Correction for temperature gradients in coil forms grams	−.000 014	−.000 014
Net compensating mass $=M$ grams	1.427 700	1.425 379
Measured force (M times gravity) = F_M dynes	1399.262	1396.987
Calculated force for unit current at 30° C $=F_C$ dynes	1348.394	1348.361 9
Equivalent current, $I_{abs}=\sqrt{\frac{F_M}{F_C}}$ absolute amperes	1.018 688	1.017 871 4
Emf. of standard cell volts	1.018 702	1.017 870 3
Resistance of standard resistor, R_1 ohms	1.000 015	1.000 007 6
Resistance of standard resistor, R_2 ohms	1.000 019	1.000 005 6
Equivalent current NBS amperes	1.018 686	1.017 863 0
$\frac{I_{abs}}{I_{NBS}}$	1.000 002	1.000 008

temperatures of the coils are slightly different from the wire temperatures. A measure of this effect was made and applied to the work reported in RP1449. The 1942 temperature gradient measurements were corrected by the better resistance measurements made recently; and it was found that under equilibrium conditions with one ampere through the coils, the mean fixed coil form temperature was 1.1° C below the fixed coil wire temperature, and the mean movable coil form temperature was 0.1° C below the movable coil wire temperature. Application of the computed force-diameter variation coefficient from table 2 and the temperature coefficients of expansion from table 1 leads to −0.014 mg as the required correction. This will be found applied in table 4.

For the comparison of the present work with the work of 1942 in table 4, both of these determinations have been referred to the same electrical standards and to the same value of the acceleration of gravity. This makes it possible to interpret the results directly as an apparent change in the electrical standards. The measured values of the currents are expressed in "NBS amperes," which is taken in this paper to mean the current with reference to the present NBS standards of resistance and electromotive force, which went into effect in 1948 [1]. The value of the acceleration of gravity is based on the Dryden reduction [8] and a gravity survey made at the National Bureau of Standards by the Geological Survey. To make the comparison complete, the new diameter weighting, current distributions, and temperature gradient corrections are applied in this paper to both the 1942 and 1956 work. It may be pointed out here that these last three corrrections tend to cancel, and do not change the 1942 result by more than 1 ppm.

6. Permeability of the Forms

It was assumed in the earlier work on the current balance that the permeability of the coil forms had a negligible effect on the force constant. Inasmuch as the susceptibility of each form was only -1×10^{-6}, the correction would certainly be small; but an order of magnitude calculation was felt desirable.

In the following computation, the permeabilities of the movable coil and of the fixed coil are treated separately. Unit current (1 amp) is assumed flowing in each of the coils. It is necessary with the method used to compute the magnetic fields of the solenoids at various points. This can be done in all cases by means of formulas given in a paper by Snow [9].

The field of the movable coil serves to induce magnetic poles on the ends of the movable coil form, whose magnitude can be computed through the relation $m=(\overline{H}\chi V)/l$, where $\overline{H}=3.5$ oersteds is the mean field intensity in the form, $\chi=-1\times10^{-6}$ is the susceptibility, V is the volume of the coil form, and l is its length. m is from this approximately -3.0×10^{-4} pole. The axial component of the fixed coil field intensity, H_X, at the end of the movable coil form is computed to be $H_X=0.65$ oersted. The force on each end of the coil form is then $F'=-0.65\times3.0\times10^{-4}$ dyne. Since the total force between the coils is $F=1348/2=674$ dynes without reversal of the current, the permeability of the movable coil has an effect of $-[(2\times2.0\times10^{-4})/674]=-0.6$ ppm in the force, considering both ends of the form.

Because of the complicated field distribution inside the fixed coil form due to current in the fixed coil, a rather elaborate calculation was made of the fixed coil permeability effect. A rough estimate indicated that the form, although diamagnetic, would cause an increase in the radial component of field at the movable coil, which is not what one would at first expect.

The magnetic charge distributed over the surface of the form was calculated using the normal component of field given by Snow's formulas, and the form susceptibility. The distribution was broken up into a series of rings of charge one centimeter wide extending around the form, and the total charge per ring was determined. This charge was then assumed concentrated on a circle located at the center of the ring. A formula for the potential of a circle of charge has been given by Smythe [10], in terms of Legendre polynomials, but this did not converge satisfactorily for our purposes. A solution was found in terms of elliptic integrals, which leads to an easier numerical calculation.

It can be shown that the potential of a circle of charge at a point a distance r from the axis of the circle and a distance d from the plane of the circle is given by $V=[2QK(k)]/(\pi\mathscr{R})$, $k^2=(4rR)/\mathscr{R}^2$, $\mathscr{R}^2=(r+R)^2+d^2$, Q is the total charge on the circle, R is the radius of the circle, and K is the complete elliptic integral of the first kind. From this the radial component of field at the point is given by

$$H_r=-\frac{2Q}{\pi\mathscr{R}^3}\left\{\left[2R-\frac{4rR(r+R)}{\mathscr{R}^2}\right]\frac{B}{1-k^2}-K(r+R)\right\}$$

where $B=K[1-(1/k^2)]+(E/k^2)$ and E is the complete elliptic integral of the second kind.

This expression allows one to sum the contributions of the separate circles of charge to the field at the movable coil. One finds the total contribution to be $H_r^{(1)}=1.25\times10^{-6}$ oersted.

The radial field at the movable coil due to the fixed coil itself can be calculated from the force F between the two coils, using the relation $F=I\int ds\times B^{(o)}$, where $B^{(o)}$ is the radial component of magnetic induction at the movable coil due to the fixed coil and I is the current in the movable coil, with the path of integration going around the movable coil. We then have, since the current is not reversed, $F=1348/2=674=(0.1\times41)\times(2\times12.2)B^{(o)}$, or $B^{(o)}=2.2$ gauss. The effect of the permeability of the fixed coil is thus $(1.25\times10^{-6})/2.2=+0.6$ ppm in the force.

A detailed calculation shows that because of the way in which the radial field of a circle of charge drops off at points away from the plane of the circle, those charges near the center of the form have the greatest effect on the field. Since the charges on the outside of the form are concentrated at the center of the coil and the charges on the inside are spread out, the charges on the outside have a slightly larger influence on the radial field at the movable coil. For this reason the fixed coil susceptibility causes an increase in the force.

The effects of the fixed and movable coil forms are in the opposite direction, and are seen to cancel. The calculations were made to a degree of precision which could cause an error of only a fraction of a part per million in the current.

7. Uncertainties

Table 5 contains estimates of known uncertainties in the current. The numbers given are probable errors for those measurements which can be treated statistically, and "50-percent-error estimates" for those cases in which no statistical information is available. Both of these measures of precision will be referred to as 50-percent errors. The estimated total uncertainty in the measurement can be determined by the usual procedure of taking the square root of the sum of the squares of the individual 50-percent errors.

TABLE 5. *Sources of error causing an uncertainty, e, in ratio I_{abs}/I_{NBS} as large as 1 ppm*

	e (50% error)
	ppm
Measurement of the force	2
Lead corrections	2
Radius of fixed coil	2
Radius of movable coil	3
Current distribution over wires	1
Calibration of length standards	1
Calibration of standard mass	1
Calibration of electrical standards	1
Adjustment of coils	1
Temperature of coils	1
Permeability of forms	<1
Acceleration of gravity	3
50-percent error in final result, $\sqrt{\Sigma e^2}$	6

Huntoon and McNish [11] have estimated in an as yet unpublished paper that the probable error of the mean of three gravity determinations, those of Kühnen and Furtwängler (revised), Heyl and Cook, and of Clark is about 2 ppm.

They also estimate that systematic errors could be as great as 15 ppm. We have estimated the total 50-percent error in these gravity determinations to be 6 ppm, which is equivalent to 3 ppm in the current.

Several laboratories have recently completed or are now working on new determinations of the acceleration of gravity. If the presently accepted value for the acceleration of gravity is revised as a result of such work, this paper should be revised accordingly.

8. Comparison with the Pellat Balance

According to the work described in this paper, the ratio of the absolute ampere to the ampere as presently maintained at the Bureau is, as given in table 4,

1 NBS ampere = 1.000008 ±0.000006 absolute amperes.

According to the work done on the Pellat electrodynamometer,

1 NBS ampere = 1.000013 ±0.000008 absolute amperes.

Since the acceleration of gravity is a common factor in these two determinations, it must be taken out before averaging, and reentered after averaging. Doing this and using the appropriate weighting factors, we have

1 NBS ampere = 1.000010 ±0.000005 absolute amperes.

The observed difference between the absolute and NBS units of current could be ascribed to a change in the electrical standards, to inaccuracies in the measurements which were used to define the present standards, or to the uncertainty in the present measurements. The present standards were defined by rounding off the average of several ampere determinations made in various countries to the nearest 10 ppm. The rounding off process combined with the uncertainties of the individual measurements could have caused an error large enough to explain our difference within the estimated 50-percent error. The 6-ppm difference between the results of the present work and the work using the same coils reported in RP1449 is also small enough to be interpreted as a combination of random errors.

It cannot be stated with certainty whether or not the standards have drifted. Our results indicate only that they have not drifted more than a few parts per million and that the NBS unit of current is greater than the absolute ampere by (10 ± 5) ppm.

WASHINGTON, May 13, 1957.

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