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Sample Calculations of Gamma-Ray Penetration Into Shelters: Contributions of Sky Shine and Roof Contamination [1]

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An approximate method is presented for calculating the penetration of gamma radiation in shelters. Sample calculations, for an assumed source energy of 1 million electron volts, are given for the following problems: (1) Dose rate inside houses and underground shelters whose roofs are covered with radioactive fallout, and (2) dose rate in open holes due to reflected radiation (sky shine) from fallout contamination on the surrounding ground. A detailed examination is made of the dependence of the dose rate in a shelter on the shape of the shelter, and on the position of the detector within the shelter. The estimated accuracy of the calculations is ±30 percent.

1. Introduction

The shielding against gamma radiation provided by shelters, houses, and similar structures is of considerable practical interest. The experimental data on this problem are usually obtained under complex circumstances, so that they are difficult to interpret. It is desirable, therefore, to obtain some theoretical understanding of the shielding problem. Calculations with the required accuracy (say 30 to 50%) appear feasible, and are perhaps easier than experimental determinations.

The well-developed semianalytic theory of the penetration and diffusion of gamma radiation [1,3] [2] treats only infinite homogeneous media, and is thus not directly applicable to the problem under consideration which involves more complicated boundary conditions. Yet the theory provides the foundations on which the calculations can be carried out. It would be possible, but rather laborious, to take the boundary conditions into account by means of random sampling (Monte Carlo). Another approach appears more promising—the adaptation of infinite-medium results to realistic conditions through a suitable schematization.

It is the purpose of this paper to explore one such schematization that can be applied to calculations of *shielding against fallout radiation*. In this schematization, the radiation dose inside a shelter is computed as a suitable weighted integral over the angular distribution of the radiation dose in an infinite medium. The present paper contains only pilot calculations. More extensive and systematic calculations are now in progress as part of a National Bureau of Standards program of shelter evaluation, carried out under the auspices of the Federal Civil Defense Administration.

[1] Work supported by the Federal Civil Defense Administration.
[2] Figures in brackets indicate the literature references at the end of this paper.

2. Schematization

The nature of the schematization can be most easily explained by applications to specific problems. Generalization to other situations is then straightforward. Figure 1 illustrates the schematization to be employed for three typical situations that will form the basis for our discussion.

2.1. Underground Shelter With Roof Covered by Fallout

The shelter roof and the surrounding ground are assumed to be covered with radioactive material that constitutes a *plane isotropic source of gamma radiation* of constant strength per unit area, which emits gamma rays uniformly in all directions.

Simplification 1. Different materials of low average atomic number, such as air, concrete, earth, or water, have nearly the same gamma-ray mass attenuation coefficients. Provided that all distances from the source are expressed in units of mass per unit area, it is therefore a good approximation to replace the combination air-concrete (or earth) by a concrete medium that is infinite and homogeneous, except that it contains an air cavity (the inside of the shelter).

Simplification 2. In order to reach a detector somewhere inside the shelter, radiation must penetrate varying amounts of concrete or earth, depending on its path. Radiation coming through the ceiling has the least amount of material to traverse and is therefore least attenuated. There are two types of paths on which photons could reach the detector through the floor. They can detour the shelter to come up from below, but in this case the attenuation is so strong that the contribution of the detoured photons is negligible. There is also the more important possibility for photons to enter the shel-

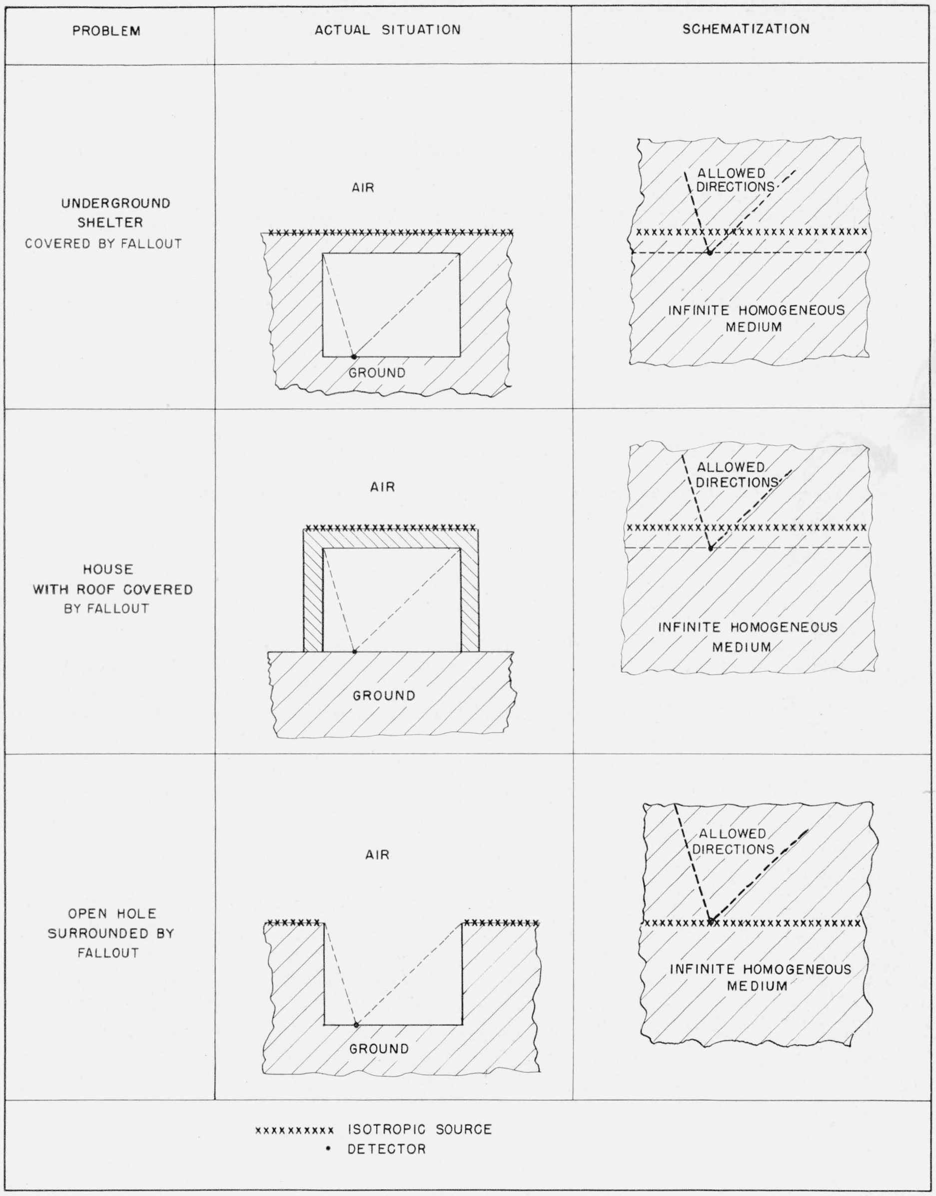

FIGURE 1. *Schematization.*

ter through the ceiling, to penetrate into the floor (or side walls) and then to be backscattered. Some radiation will also penetrate to the detector through the side walls of the shelter. But because of the small size of a mean free path in concrete or earth (a few inches) only a small side-wall region near the ceiling can make a nonnegligible contribution, resulting in an effective ceiling area somewhat larger than the true ceiling area. Both the backscattering correction and the side-wall penetration correction are small. They are briefly discussed in sections 6.1 and 6.2 but are omitted from the initial schematization. It will be a good first approximation to consider only radiation entering through the ceiling. The mean free path in air is of the order of several hundred feet, and is very large compared to the shelter dimensions. The radiation thus travels in a straight line inside the shelter and can reach a detector only if its direction of motion lies within the solid angle subtended by the ceiling with respect to the detector. This solid angle is indicated in figure 1 by the label "allowed directions." The notion of "allowed directions" will be formalized later through the introduction of an "angular response function."

Summary of the Calculation. The calculation consists of the following steps: (1) Replacement of the air-shelter complex by an infinite concrete medium; (2) computation of the amount and angular distribution of the radiation in such a medium, at a distance from an infinite plane isotropic source equal to the thickness of the shelter roof; (3) computation of the angular response function for a specific shelter-detector configuration; (4) computation of the total amount of radiation received by the detector, by an integral over the angular distribution weighted by the angular response function; and (5) corrections to account for additional radiation reaching the detector through the side walls, and by the reflection from interior surfaces of the shelter.

2.2. House With Roof Covered by Fallout

As indicated in figure 1, the schematization for this problem is essentially the same as for the previous problem. A number of points require comment, however.

(1) The contamination now is confined to the roof so that the plane-isotropic source is *finite*. Yet the situation is practically equivalent to that of an underground shelter underneath an infinite contaminated plane. Note that in the underground shelter, fallout from regions other than that directly above the roof does not contribute appreciably to the dose, because of the large amount of intervening earth.

The equivalence is, of course, only approximate. To illustrate the order of magnitude of the error involved, one may consider the following point. In the schematization, approximately 10 to 15 percent of the energy emitted by the source is reflected back toward the roof. In the actual situation, with air above the roof, this reflected radiation would spread laterally, and most of it would come down on the ground surrounding the roof. The reflected radiation assumed incorrectly to be incident on the roof is low in energy and diffuse in direction, so that its penetrating power is much less than that of the source radiation. Hence, the resulting dose-overestimate will only be a few percent.

(2) There is a somewhat better chance than in the case of an underground shelter, that radiation originating on the roof will reach the detector through the side walls. But this radiation consists mainly of photons which after repeated air scattering are incident on the side walls with low energies and therefore are not very penetrating.

(3) If the ground surrounding the house is also covered with fallout, the penetration of radiation through the side walls becomes very important. The numerical details of this case are not worked out in the present paper, but the same schematization could be applied as in the case of a roof contamination, through the introduction of appropriate allowed directions and angular response functions defined with respect to the side walls, or perhaps the windows.

2.3. Open Hole Surrounded by Fallout

Again, the schematization is essentially the same as in 2.1. The detector is now in a position where it cannot "see" the source, i. e., only *scattered* radiation can reach it. This scattered radiation will consist mainly of photons that have gone up into the air and are reflected into the hole by scattering. This radiation component is customarily called *sky shine*. The radiation distributions that must now be used in steps (2) and (4) of the schematized calculation are those pertaining to the *scattered* flux at the source plane.

3. Mathematical Formulation

An isotropic source of gramma radiation is assumed to be located in the plane $z=0$. Let $I_D(z, \theta) \sin \theta d\theta$ denote the dose delivered to a detector in an infinite homogeneous medium at a distance z from the source plane, by photons incident on the detector at angles between θ and $\theta+d\theta$ (with respect to the z-axis). Dose means the quantity technically called "absorbed air dose", i. e., energy dissipated in air, measured in rads (100 ergs/g) [2]. It will be convenient to express the dose distribution in the form

$$I_D(z,\theta)=f(z)g(z,\theta), \tag{1}$$

where $g(z,\theta)$ is an angular distribution normalized to unity:

$$\int_0^\pi g(z,\theta) \sin \theta d\theta=1. \tag{2}$$

The factor $f(z)$ will represent the attenuation of radiation by the roof of thickness z in the problems of the house or underground shelter. For the open hole, one must replace $f(z)$ and $g(z,\theta)$ by quantities $f_s(z)$ and $g_s(z,\theta)$ pertaining to *scattered* radiation.

Let the shelter-detector configuration be such that only a fraction $\psi(\theta)$ of the photons with direction θ can reach the detector. This function is called the angular response function and is further discussed in section 5. The dose $D(z)$ received by the partially shielded detector in the shelter is expressible as the product of three factors:

$$D(z)=Cf(z)\ G(z), \tag{3}$$

where

$$G(z)=\int_0^\pi g(z,\theta)\, \psi(\theta) \sin \theta d\theta, \tag{4}$$

and C is a correction factor taking into account the contribution of radiation penetrating through the side walls or reflected from the interior surfaces. The determination of the geometrical protection factor G is the main objective of this paper.

4. Calculation of the Radiation Field

The standard solution for an infinite homogeneous medium is provided by the moment method [1]. Tabulations based on this method are available from which one can extract information about the dose $f(z)$ from a plane isotropic source [3].[3] The calculation of the angular distribution $g(z,\theta)$ has been described in [4].

The numerical computations of this paper all pertain to a source emitting 1-Mev photons. This source energy is a little higher than the average energy of fallout radiation, but close to the energy

[3] As part of the work sponsored by FCDA, further tabulations, with applications to shelter evaluation, are now being prepared at NBS.

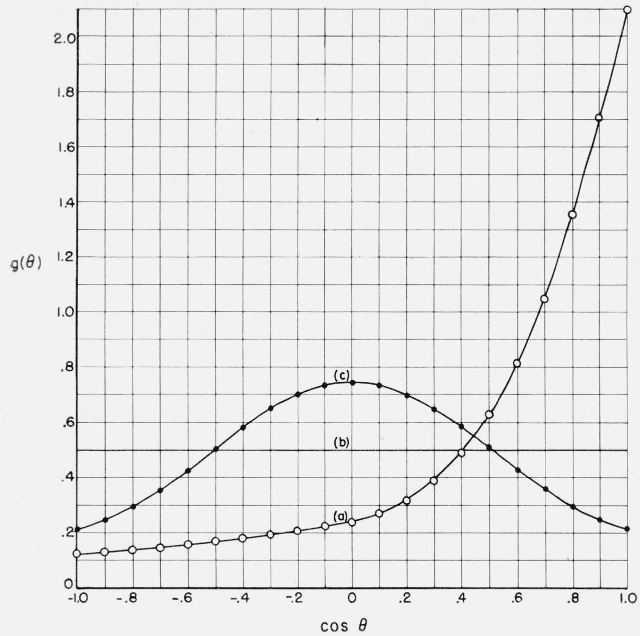

FIGURE 2. *Angular distribution of absorbed air dose from a 1-Mev plane isotropic source in an infinite medium.*

The calculation is for a water medium, but is in very good approximation also applicable to concrete.
(a) $g(z,\theta)$, at a distance $\mu_0 z=4$ from the source plane.
(c) $g_s(z,\theta)$, at the source plane $\mu_0 z=0$ (This distribution represents only the contribution of *scattered* photons.)
The curve (b) represents an isotropic distribution.

of Co^{60}-radiation, which is most suitable for controlled experimentation. The choice was motivated by the ready availability of results for $g(z,\theta)$. In any case, calculations with a 1-Mev source will be sufficient for the illustrative purposes of this paper.

Let $f(z)$ be the dose in an infinite concrete medium at a distance z from a plane isotropic source. The results of a calculation by the moment method can be summarized by the approximate formula

$$f(z)=f_0(z)+f_s(z) \tag{5}$$

with

$$f_0(z)=kE_1(\mu_0 z), \tag{5a}$$

representing the contribution of unscattered radiation, and with

$$f_s(z)=1.04ke^{-0.915\mu_0 z}, \tag{5b}$$

representing the contribution of scattered radiation. The range of validity of (5b) is $0\leq\mu_0 z<8$, E_1 is the exponential integral [5], and $\mu_0=0.0635\ \mathrm{cm}^2/g$ is the mass attenuation coefficient for gamma radiation in concrete at the source energy (1 Mev). The constant k determines the source normalization. If the source plane is covered with 1 curie of gamma emitter per square foot, and the dose rate is expressed in rads per hour, the normalization constant has the value

$$k=32.2. \tag{5c}$$

If the fallout is spread over a rough surface, the schematization of the source as a perfect geometric plane is not altogether suitable. A possible alternative schematization consists in assuming that the radioactive material is mixed homogeneously with a thin top layer of the ground of thickness t. In this case, the dose function $f(z)$ must be replaced by

$$f^*(z,t)=\frac{1}{t}\int_z^{z+t} f(z')\,dz'. \tag{6}$$

Three angular distributions are shown in figure 2: (a) $g(z,\theta)$ at a distance $\mu_0 z=4$ (0.897 lb of concrete per square inch); (b) a completely isotropic distribution; and (c) the distribution $g_s(z,\theta)$ for *scattered* radiation only, at the source plane $\mu_0 z=0$. The distribution (a) peaks at $\theta=0°$ and is for use in the problems treated in sections 2.1 and 2.2. The distribution (c) peaks at $\theta=90°$ and is for use in the problem in section 2.3. Both of these distributions pertain to an infinite plane-isotropic source and were calculated according to the method of reference [4]. The distribution (b) is flat (when plotted against cos θ) and has been included as an intermediate case. Together, the three cases span a wide range of possible distributions.

5. Calculation of the Response Function

To each angle θ there corresponds a family of unit vectors ($\sin\theta\cos\varphi$, $\sin\theta\sin\varphi$, $\cos\theta$), indicating possible photon directions. If these unit vectors are required by the schematization to lie within a specified solid angle, then not all values of the azimuth φ are permitted, but only azimuths in a region $A_\theta\leq 2\pi$. The response function is defined as the fraction $\psi(\theta)=A_\theta/2\pi$.

5.1. Cylindrical Geometry

As a simple first example, we consider a shelter in the form of an upright cylinder of height H and radius R, with the detector located at the center of the bottom of the shelter (see fig. 3). The response

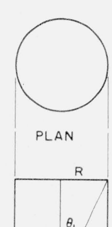

FIGURE 3. *Calculation of the response function for a cylindrical shelter.*

function is in this case particularly simple:

$$\left.\begin{array}{ll} 0 \leq \theta \leq \theta_1 = \cos^{-1} \dfrac{H}{\sqrt{H^2+R^2}}, & A_\theta = 2\pi,\ \psi(\theta)=1 \\ \theta_1 < \theta \leq \pi & , A_\theta = 0, \quad \psi(\theta)=0. \end{array}\right\} \qquad (7)$$

5.2. Parallelepiped Geometry

We consider a box-shaped shelter of length L, width W, and height H. It can be assumed without loss of generality that $L \geq W$. The response function depends on solid angles, and is thus a function of the shape but not of the absolute size of the shelter. Therefore, we eliminate one parameter by dividing all dimensions of the shelter by L, so that the scaled length is unity, the scaled width is

$$W/L = \epsilon \leq 1, \qquad (8)$$

and the scaled height is

$$H/L = h. \qquad (9)$$

The construction of the response function is illustrated in figure 4 for a detector in a bottom corner of the shelter.

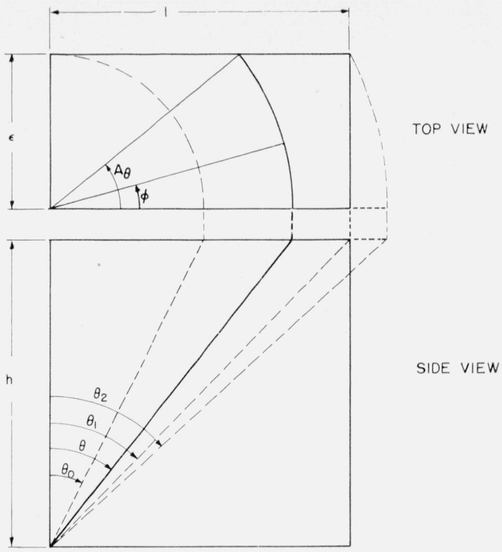

FIGURE 4. *Calculation of the response function for a box-shaped shelter.*

Angular range	Response function, $\psi(\theta, \epsilon, h)$
$0 \leq \theta \leq \theta_0 = \cos^{-1} \dfrac{h}{\sqrt{h^2+\epsilon^2}}$	1
$\theta_0 < \theta \leq \theta_1 = \cos^{-1} \dfrac{h}{\sqrt{h^2+1}}$	$\dfrac{1}{2\pi} \sin^{-1}\left(\dfrac{\epsilon}{h \tan \theta}\right)$
$\theta_1 < \theta \leq \theta_2 = \cos^{-1} \dfrac{h}{\sqrt{h^2+\epsilon^2+1}}$	$\dfrac{1}{2\pi} \sin^{-1}\left(\dfrac{\epsilon}{h \tan \theta}\right) - \dfrac{1}{2\pi} \cos^{-1}\left(\dfrac{1}{h \tan \theta}\right)$
$\theta_2 < \theta \leq \pi$	0

(10)

The response function (10), evaluated with the parameters $\epsilon = 0.5$ and $h = 0.5$, is shown in figure 5.

5.3. Combination Rule

Expression (10) can readily be generalized so as to provide the response function for an arbitrary detector location. In the first place, it is to be noted that—within our schematization—the vertical distance of the detector from the shelter ceiling (or top of the open hole) must be assumed as the effective height H. Now consider a detector located at the bottom of the shelter, in a horizontal position P with coordinates $(\alpha, \beta\epsilon)$, as illustrated in figure 6. We now imagine that the shelter is partitioned by two vertical walls (perpendicular to each other), which pass through the point P, and are very thin, but perfect, radiation shields. The detector can then

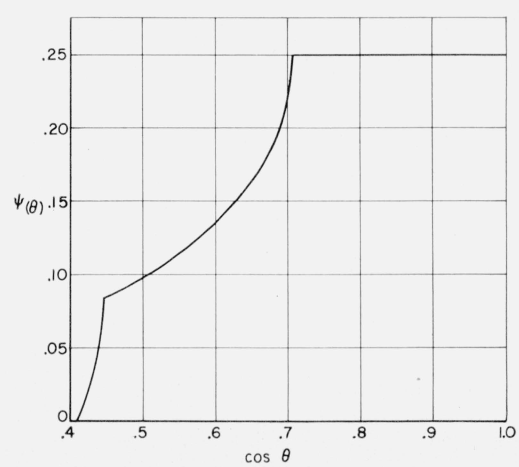

FIGURE 5. *Response function for a detector at a bottom corner of a box-shaped shelter with scaled width (horizontal eccentricity) $\epsilon = 0.5$ and height $h = 0.5$.*

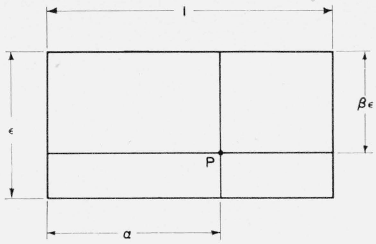

FIGURE 6. *Diagram illustrating the application of the combination rule by which response functions for off-corner positions are calculated.*

be thought of as being located at the corner of each of four adjoining box-shaped shelters with the dimensions listed below.

j	ϵ_j	h_j
1	$\frac{\min(\alpha, \beta\epsilon)}{\max(\alpha, \beta\epsilon)}$	$\frac{h}{\max(\alpha, \beta\epsilon)}$
2	$\frac{\min(1-\alpha, \beta\epsilon)}{\max(1-\alpha, \beta\epsilon)}$	$\frac{h}{\max(1-\alpha, \beta\epsilon)}$
3	$\frac{\min(\alpha, \epsilon-\beta\epsilon)}{\max(\alpha, \epsilon-\beta\epsilon)}$	$\frac{h}{\max(\alpha, \epsilon-\beta\epsilon)}$
4	$\frac{\min(1-\alpha, \epsilon-\beta\epsilon)}{\max(1-\alpha, \epsilon-\beta\epsilon)}$	$\frac{h}{\max(1-\alpha, \epsilon-\beta\epsilon)}$

$0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$.
min (x, y) = smaller of x and y.
max (x, y) = larger of x and y.

The response function ψ_P at point P is

$$\psi_P(\theta) = \sum_{j=1}^{4} \psi(\theta, \epsilon_j, h_j). \quad (11)$$

5.4. Legendre Polynomial Expansion

In order to obtain the geometric protection factor $G(z)$, it is necessary to evaluate the integral in (4) for each distribution $g(z,\theta)$ of interest. When many distributions must be included, this procedure tends to become laborious. It is then convenient to perform a set of standard integrations based on an expansion of the response function into a series of Legendre polynomials:

$$\psi(\theta) = \sum_{l=0}^{\infty} \frac{2l+1}{2} a_l P_l(\cos\theta). \quad (12)$$

The theory of radiation diffusion provides as the most direct result, a similar Legendre polynomial expansion for the angular distribution:

$$g(z,\theta) = \sum_{l=0}^{L} \frac{2l+1}{2} g_l(z) P_l(\cos\theta). \quad (13)$$

As a rule, only the first few harmonics ($l=0, 1, \ldots, L \leq 7$) are at one's disposal, but they suffice to specify $g(z,\theta)$. When (12) and (13) are substituted into (4), it follows from the orthogonality of the Legendre polynomials that

$$G(z) = \sum_{l=0}^{\infty} \sum_{n=0}^{L} \frac{2l+1}{2} a_l \frac{2n+1}{2} g_n(z) \int_0^{\pi} P_l(\cos\theta) P_n(\cos\theta) \sin\theta d\theta \quad (14)$$

$$= \sum_{l=0}^{L} \frac{2l+1}{2} a_l g_l(z).$$

Thus, one can limit oneself to the calculation of the first few response function expansion coefficients

$$a_l = \int_0^{\pi} \psi(\theta) P_l(\cos\theta) \sin\theta d\theta, \; l=0, 1, \ldots, L. \quad (15)$$

With these coefficients, one is in a position to evaluate $G(z)$ for any angular distribution $g(z,\theta)$ through simple superposition according to (14).

Numerical evaluations of G have been carried out for detectors in various positions in box-shaped shelters, on the basis of the angular distribution functions shown in figure 2. These results apply as follows:

Problem	Angular distribution	Infinite medium dose-rate $\frac{\text{rads/hour}}{\text{curies/foot}^2}$
Underground shelter or house with roof covered by fallout, roof thickness $\mu_0 z = 4$(0.896 lb of concrete per square inch).	(a)	$f(z) = 0.983$
Isotropic comparison distribution.	(b)	-----------
Sky shine (scattered radiation) into open hole.	(c)	$f_s(z) = 33.5$

The results are presented as functions of the following dimensionless parameters:

$$\text{Horizontal eccentricity} = \frac{\text{width}}{\text{length}} = \epsilon \quad (16)$$

$$\text{Vertical eccentricity} = \frac{\sqrt{\text{roof-area}}}{\text{height}} = \frac{\sqrt{\epsilon}}{h} = \tau. \quad (17)$$

The range of parameters considered includes $0 \leq \epsilon \leq 1$ and $0.2 \leq \tau \leq 20$. This appears sufficient for all practical applications.

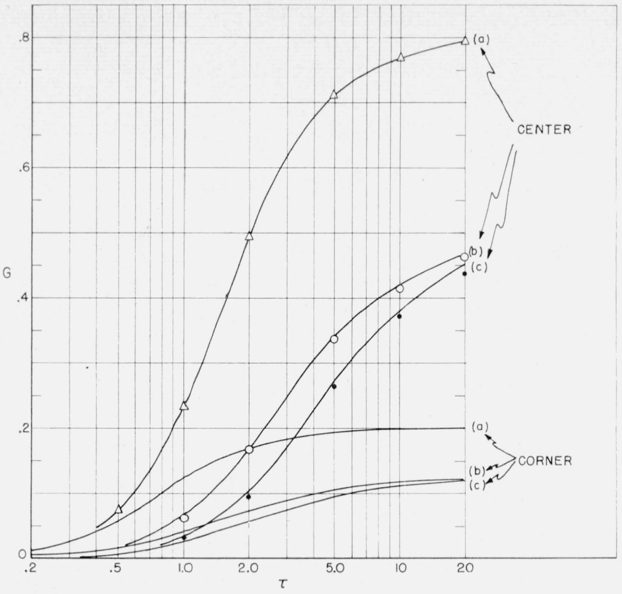

FIGURE 7. *Geometric protection factor versus vertical eccentricity, for fixed horizontal eccentricity $\epsilon=1$.*

The curves labeled (a), (b), and (c) are based on the use of the corresponding angular distributions in figure 2. The triangles, solid circles, and hollow circles represent results obtained with the response function (7) for a cylindrical shelter.

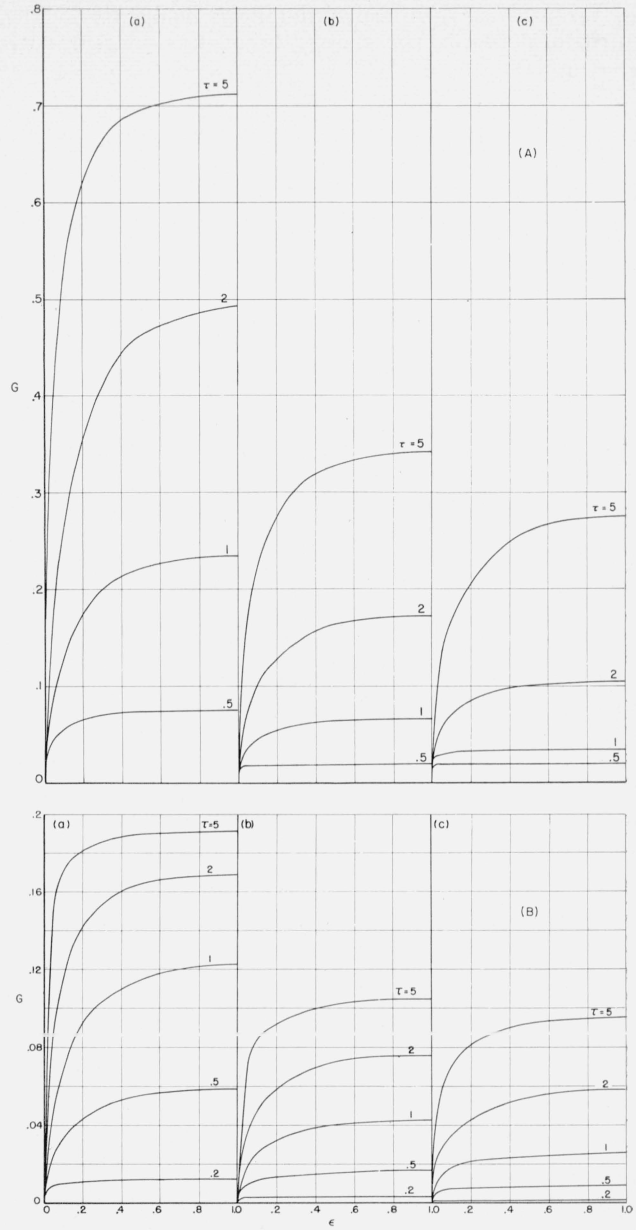

FIGURE 8. *Geometric protection factor versus horizontal eccentricity, for various constant values of the vertical eccentricity.*

The curves labeled (a), (b), and (c) pertain to the corresponding angular distributions of figure 2. (A) Detector in center; (B), detector in corner.

Figure 7 contains plots of G versus τ, for a shelter with a square cross section ($\epsilon=1$). Figure 8 contains plots of G versus ϵ, for different fixed values of τ. Results are shown for the detector in a corner, and in a center position ($\alpha=\beta=\frac{1}{2}$). The figures confirm quantitatively what one would expect on the basis of a commonsense estimate. The protection provided by the shelter increases rapidly as τ is increased, but is generally a very slowly varying function of ϵ for fixed τ, except when ϵ has a very small value. Dose rates in the corner are on the order of four times smaller than dose rates in the center. The angular distribution (a) with its strong forward peak gives rise to G-values 2 to 3 times larger than those obtained with the diffuse skyshine distribution (c), whereas there is little difference between cases (c) and (b).

The question may be asked to what extent the results for the geometrical protection factor are affected by the choice of a rectangular shelter cross section. To obtain a partial answer, calculations were carried out for a detector at the center of a cylindrical shelter, with the use of the response function (7). As can be seen from these values plotted in figure 7, the results thus obtained are, for the same value of τ, practically identical with the results for a detector at the center of a box-shaped shelter with $\epsilon=1$. This means that the geometric protection factor depends very little on the shape of the cross section. Combined with the previously demonstrated insensitivity of G with respect to the horizontal eccentricity ϵ, this has the following implications: (1) The geometric protection factor at the center of a shelter of any shape is, in good approximation, only a function of the vertical eccentricity τ; (2) under these circumstances, a calculation using the very simple response function (7) will be adequate.

Center and corner values provide upper and lower limits of G. To indicate the transition between the two extremes, a systematic mapping of G is presented in table 1 for numerous detector positions in the horizontal plane, the shelter being box-shaped with $\epsilon=1$ and $\tau=2$.

TABLE 1. *Geometric protection factor G as a function of the position in the horizontal plane, for a detector in a shelter with horizontal eccentricity* $\epsilon=1$ *and vertical eccentricity* $\tau=2$, *calculated for the angular distribution (a) of figure 2*

The position indicators α and β are those defined in figure 6. Note that the ransformations $\alpha\rightarrow 1-\alpha$, $\beta\rightarrow 1-\beta$, or $(\alpha\rightarrow\beta,\ \beta\rightarrow\alpha)$ leave G unchanged.

β \ α	0. 0	0. 125	0. 25	0. 375	0. 5
0. 0	0. 17	0. 22	0. 26	0. 27	0. 29
. 125	. 22	. 31	. 34	. 37	. 38
. 25	. 26	. 34	. 40	. 43	. 44
. 375	. 27	. 37	. 43	. 47	. 48
. 5	. 29	. 38	. 44	. 48	. 49

6. Corrections, Accuracy

6.1. Penetration Through the Side Walls

This effect has, so far, been investigated only in a limited manner to obtain the order of magnitude of the necessary correction. To simplify the calculations, they were done for a detector at the center of the bottom of a cylindrical cavity. For the case of an underground shelter (the problem in section 2.1), the response function (7) could be amended as follows:

$$\left.\begin{aligned} &\theta_1<\theta\leq\frac{\pi}{2},\ \psi(\theta)=\frac{\varphi(z+z')}{\varphi(z)}\,\frac{g(z+z',\theta)}{g(z,\theta)} \\ &\frac{\pi}{2}<\theta\leq\pi,\ \psi(\theta)=0 \end{aligned}\right\} \qquad (7')$$

where

$$z'=H-R\cot\theta.$$

The meaning of (7′) is that the surface elements of the side wall contribute to the dose in proportion to the intensity and angular distribution of the radiation at the distance $z=z'$ of the surface elements from the source plane.

Sample calculations have been carried out for a shelter with roof thickness $\mu_0 z=4$, horizontal eccentricity $\epsilon=1$, and a depth of 8 ft. (The side-wall correction differs from the uncorrected response function in that it depends on the absolute size of the shelter.) It was found that for vertical eccentricities

$$\tau=1,\ 2,\ \text{and}\ 4,$$

the increase of the dose is 7, 3, and 1 percent, respectively.

6.2. Backscattering

Some radiation will be scattered toward the detector by reflection from the side walls, floor, and ceiling of the shelter. The order of magnitude of this effect can be estimated roughly on the basis of available Monte Carlo calculations of backscattering. The results of reference [6] suggest a dose increase by approximately 15 percent.

6.3. Over-all Correction and Error Estimate

In the final equation for the dose received by the partially shielded detector,

$$D(z)=Cf(z)G(z), \qquad (3)$$

the factor C, incorporating side-wall and backscattering effects, is estimated to be 1.2. It is further estimated that the factors in (3) have the relative errors $(\delta fG)/fG\approx\pm 0.2$ and $(\delta C)/C\approx\pm 0.2$, which leads to the following error estimate for D:

$$\frac{\delta D}{D}=\sqrt{\left(\frac{\delta fG}{fG}\right)^2+\left(\frac{\delta C}{C}\right)^2}\approx\pm 0.3. \qquad (18)$$

The authors are indebted to L. V. Spencer for valuable discussions, and to J. Hubbell for help with the calculations.

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