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Mixed-Path Ground-Wave Propagation: 2. Larger Distances

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The theoretical results given in part 1 (NBS Research Paper 2687) for ground wave propagation over a mixed path on a flat earth are generalized to a spherical earth. The problem is formulated in terms of the mutual impedance between two vertical dipoles which are located on either side of the boundary of separation. Extensive numerical results are given in graphical form for a mixed land-sea path at frequencies of 10, 20, 50, 100, and 200 kilocycles per second.

1. Introduction

In a previous paper [1] [1] (designated hereafter as part 1), calculations were presented for the propagation of vertically polarized ground waves across a boundary separating two contrasting homogeneous media. The earth was assumed to be flat so that the results were restricted to short distances. It is the purpose of this sequel to extend the analysis and computations to larger distances, where the curvature of the earth must be considered.

The mutual impedance, Z', between two vertical electric dipoles located at A and B, depicted in figure 1, is considered. The earth medium to the left of the boundary line has a

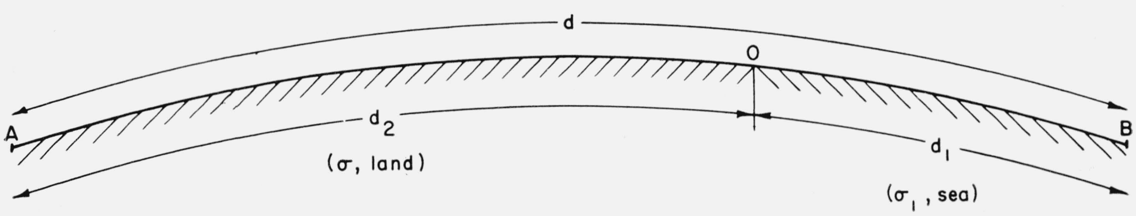

FIGURE 1. *Model for propagation between* A *and* B *straddling a land-sea boundary on a smooth spherical earth.*

conductivity, σ, and dielectric constant, ϵ, and the corresponding constants for the medium to the right of the boundary line are σ_1 and ϵ_1. Using the same approach as in part I, it follows that

$$Z'=Z+\frac{1}{I^2}\iint_S \left[\overrightarrow{}_{\mathrm{A}}\times\overrightarrow{H'_{\mathrm{B}}}-\overrightarrow{E'_{\mathrm{B}}}\times\overrightarrow{H_{\mathrm{A}}}\right] ds, \tag{1}$$

where Z is the mutual impedance between the dipoles over a homogeneous curved surface of conductivity σ and dielectric constant ϵ. E_{A}, E'_{B}, H_{A}, H'_{B} are the electric and magnetic fields tangential to the surface S which is the portion of the curved surface to the right of the boundary line. The unprimed quantities refer to the case where the dipoles are in the presence of a corresponding homogeneous surface of constants σ and ϵ throughout. The primed quantities refer to the case where A and B are on the modified surface. The subscripts A or B on E_{A}, E'_{B}, etc., specify that the source of the respective field is a dipole carrying a current, I, at A or B, respectively.

[1] Figures in brackets indicate the literature references at the end of this paper.

Invoking the principle of stationary phase, the surface integral in eq (1) can be reduced to a line integral between the coast line at O and the dipole B. The reasoning here is almost identical to that used in part 1. AOB, rather than being a straight line on a flat earth, is now the path of stationary phase between A and B. In the case of a spherical earth, AOB is on a great circle.[2] In the present problem, the concept of surface impedance is introduced as it was in part 1, the tangential electric and magnetic fields are assumed to be related by a complex constant of proportionality. For the medium to the left of the boundary, this constant is η and the corresponding constant for the medium to the right of the boundary is η_1, where, in ohms,

$$\eta = 120\pi \frac{i\beta}{\alpha}\left(1+\frac{\beta^2}{\alpha^2}\right)^{1/2} \tag{2a}$$

and

$$\eta_1 = 120\,\pi \frac{i\beta}{\alpha_1}\left(1+\frac{\beta^2}{\alpha_1^2}\right)^{1/2} \tag{2b}$$

with $\beta=2\pi/\text{wavelength}$, $\alpha=(i\sigma\mu\omega-\epsilon\mu\omega^2)^{1/2}$, $\alpha_1=(i\sigma_1\mu\omega-\epsilon_1\mu\omega)^{1/2}$ and $\mu=4\pi\times 10^{-7}$ h/m.

The mutual impedance, Z, between the short antennas A and B for the homogeneous ground of electrical constants σ and ϵ is given by

$$Z=\frac{h_a h_b i\mu\omega}{2\pi d}\, e^{-i\beta d} W(d,\eta,a)\left[1+\frac{1}{i\beta d}-\frac{1}{\beta^2 d^2}\right], \tag{3}$$

where d is the great circle distance AOB, h_a and h_b are the effective heights of the dipole antennas at A and B, and $W(d,\eta,a)$ is a slowly varying function that approaches unity for a flat, perfectly conducting earth. The function W is obtainable from the Van der Pol-Bremmer theory [2] for the field of a dipole over a homogeneous sphere. It is given, in convenient form, by

$$W(d,\eta,a)=\left(\frac{2\pi d}{a}\right)^{1/2} e^{-i\pi/4}(\beta a)^{1/6} \sum_{s=0}^{\infty} \frac{\exp[-i\tau_s(\beta\alpha)^{1/3}d/a]}{(2\tau_s-1/\delta^2)} \tag{4a}$$

with $\delta=-i(\beta a)^{-1/3}(120\pi/\eta)$ and (4b)

$$\tau_s=\nu_s-\frac{1}{2\nu_s\delta}-\frac{1}{8\nu_s^3\delta^2}-\frac{1+\frac{3}{4\nu_s^3}}{12\nu_s^2\delta^3}\cdots, \tag{4c}$$

and

$\nu_0=0.808e^{-i\pi/3}$
$\nu_1=2.577e^{-i\pi/3}$
$\nu_2=3.824e^{-i\pi/3}$
$\nu_s\simeq 1/2[3\pi(s+1/4)]^{2/3}e^{-i\pi/3}$ for $s>2$.

An expansion for the coefficients τ_s in a power series in δ is also available [2]. In the preceding formula, a is the effective radius of curvature of the surface in the principal plane containing AOB.

The mutual impedance for the inhomogeneous path is now written in the form

$$Z'=\frac{h_a h_b i\mu\omega}{2\pi d}\, e^{-i\beta d} W'(d,\eta,\eta_1,a)\left[1+\frac{1}{i\beta d}-\frac{1}{\beta^2 d^2}\right], \tag{5}$$

where W' is some function of d, η, η_1, and a and can be expected to be slowly varying compared to $e^{-i\beta d}$ and must necessarily approach unity as d tends to zero or if η and η_1 tend to zero and a approaches infinity.

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[2] The coastal refraction effect is neglected here because for most cases it is very small, as shown in part 1.

The essential modification here to the flat-earth theory in part 1 is to replace the functions $F(d,\eta)$ and $F'(d,\eta,\eta_1)$ by $W(d,\eta,a)$ and $W'(d,\eta,\eta_1,a)$. It then follows that

$$W'(d,\eta,\eta_1,a) \simeq W(d,\eta,a) - \left(\frac{i\beta d}{2\pi}\right)^{1/2} \left(\frac{\eta_1-\eta}{120\pi}\right) \int_0^{d_1} \frac{W(d-\alpha,\eta,a)W(\alpha,\eta_1,a)}{[\alpha(d-\alpha)]^{1/2}} d\alpha, \tag{6}$$

which is the spherical-earth counterpart of eq (13) of part 1. In the above, α, the integration variable, ranges from B to O (i. e., d_1=OB). This result is an approximation that is not valid near the boundary line because it utilizes a stationary phase principle, as in part 1. Furthermore, the wave reflected from the boundary is neglected. Actually, this effect is very small at distances greater than a wavelength from the boundary line.

The prime task now is the evaluation of the integral in eq (6). Inserting the Van der Pol-Bremmer formula for the W's, the integrations can be carried out to yield

$$W'(d,\eta,\eta_1,a) = W(d,\eta,a) + (2\pi i\beta d)^{1/2}\left(\frac{\eta_1-\eta}{120\pi}\right)\sum_{s=0}^{\infty}\sum_{q=0}^{\infty}\frac{\exp[-i\tau_s(\beta a)^{1/3}d/a]\left[\exp\left[i(\tau_s-\tau_q^1)\frac{d_1}{a}(ka)^{1/3}\right]-1\right]}{(2\tau_s-1/\delta^2)(2\tau_q^1-1/\delta_1^2)(\tau_s-\tau_q^1)}, \tag{7}$$

where the coefficients τ_s are given the eq (4c) et seq. and τ_q^1 are identical in form to τ_s, except that σ_1 and ϵ_1 replace σ and ϵ. This double summation is very cumbersome and converges poorly. An alternative formula may be derived by employing the following series formula [3, 4] for the W's:

$$W(d-\alpha,\eta,a) = \sum_{n=0}^{\infty} A_n\left(\frac{d-\alpha}{T}\right)^{n/2}, \tag{8}$$

where $1/T = -(i\beta/2)(\eta/120\pi)^2$. The coefficients up to order 9 are given by

$$\begin{aligned}
&A_0=1,\ A_1=-i\sqrt{\pi},\ A_2=-2,\ A_3=i\sqrt{\pi}(1-\delta^3/2)\\
&A_4=\frac{4}{3}(1-\delta^3),\ A_5=-\frac{i\sqrt{\pi}}{2}(1-3\delta^3/2),\ A_6=-\frac{8}{15}\left(1-2\delta^3+\frac{7}{8}\delta^6\right),\\
&A_7=\frac{i\sqrt{\pi}}{6}\left(1-\frac{5}{2}\delta^3+2\delta^6\right),\ A_8=\frac{16}{105}\left(1-3\delta^3+\frac{27}{8}\delta^6\right),\\
&A_9=\frac{-i\sqrt{\pi}}{24}\left(1-\frac{7}{2}\delta^3+5\delta^6-\frac{21}{8}\delta^9\right),
\end{aligned} \tag{9}$$

where $\delta=-i(\beta a)^{1/3}(120\pi/\eta)$. The corresponding expression for $W(\alpha,\eta_1,a)$, for example, is identical to the above with $d-\alpha$, T, and δ being replaced by α, T_1 and δ_1, respectively, where $1/T_1=-(i\beta/2)(\eta_1/120\pi)^2$ and $\delta_1=-i(\beta a)^{-\frac{1}{3}}(120\pi/\eta_1)$. Therefore,

$$W(\alpha,\ \eta_1\ a) = \sum_{m=0}^{\infty} A_m^1\left(\frac{\alpha}{T_1}\right)^{m/2}, \tag{10}$$

where the A_m^1 coefficients are functions of δ_1. After carrying out the integration indicated in eq (6), the formula for W' takes the form

$$W'(d,\eta,\eta_1,a) = W(d,\eta,a) - \left(\frac{i\beta d}{2\pi}\right)^{1/2}\left(\frac{\eta_1-\eta}{120\pi}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A_n A_m^1 I_{m,\,n}, \tag{11}$$

where

$$I_{m,\,n} = \int_0^{d_1} \alpha^{\frac{m-1}{2}}(d-\alpha)^{\frac{n-1}{2}}\,d\alpha. \tag{12}$$

The integral $I_{m,n}$ can be reduced by the formulas

$$
\begin{aligned}
I_{m,n}&=\frac{2}{m+n}(d_1)^{\frac{m+1}{2}}(d-d_1)^{\frac{n-1}{2}}+\frac{(n-1)d}{m+n}I_{m,n-2},\\
I_{m,n}&=\frac{1}{m+n}\left[(m-1)dI_{m-2,n}-2(d-d_1)^{\frac{n+1}{2}}(d_1)^{\frac{m-1}{2}}\right],\\
I_{m,1}&=\left(\frac{2}{m+1}\right)(d_1)^{\frac{m+1}{2}},\\
I_{1,n}&=\left(\frac{2}{n+1}\right)\left(d^{\frac{n-1}{2}}-(d-d_1)^{\frac{n-1}{2}}\right),
\end{aligned}
\tag{13}
$$

and

$$I_{0,0}=2\tan^{-1}\left(\frac{d_1}{d-d_1}\right)^{1/2}.$$

The preceding expression for W' is useful when distance d is small compared to T and T_1.

In view of the fact that extensive numerical data for the W functions are available [5, 6], it would seem to be preferable to evaluate the integral by a numerical method. Certainly this appears simpler than attempting to sum the doubly-infinite series formulas discussed above. To convert the integral to a form suitable for numerical integration, it is desirable to change the variable of integration to remove the singularity at $\alpha=0$. Then, letting $\alpha=x^2$, it follows that

$$W'=W+2\left(\frac{\rho_0}{\pi}\right)^{1/2}\left[\frac{1-k^{1/2}}{k^{1/2}}\right]e^{-i\frac{\pi}{2}}\int_0^{d_1^{1/2}}\frac{W(d-x^2,\eta,a)W(x^2,\eta_1,a)dx}{\sqrt{d-x^2}},\tag{14}$$

where

$$k=\frac{\sigma_1+i\omega\epsilon_1}{\sigma+i\omega\epsilon}\text{ and }\rho_0=-\frac{i\beta d}{2}\left(\frac{\eta}{120\pi}\right)^2.$$

At low radiofrequencies, displacement currents in the ground are small, and consequently k and ρ_0 are real and positive.

For purposes of computation, the following values of the parameters are employed:

$\sigma=10^{-2}$ mho/m corresponding to land,
$\sigma_1=4$ mhos/m corresponding to sea, and
$a=4/3$ times the earth's radius.

In figure 2 the amplitude of W' is shown plotted as a function of distance d for frequencies of 10, 20, 50, 100, and 200 kc. Two values of d_1/d are indicated on the curves, namely, 0 corresponding to an all land path, and 0.8, corresponding to an 80 percent sea path. For the lower frequencies, the curves are almost indistinguishable, and the shape of W' is determined primarily by diffraction by the earth's curvature. The phase lag ϕ'(i. e., $\phi'=-\arg W'$) is shown plotted in figures 3 to 7 as a function of d for the same frequencies. The values of d_1/d, which indicate the ratio of the length of the sea path to the length of the total path, are 0, 0.2, 0.4, 0.6, 0.8, and 1.0.

The preceding analysis has been discussed in relation to the mutual impedance between two dipole antennas at A or B. It is probably desirable to express the results in terms of the vertical electric field, E, at B (or A) for a standard source at A (or B). For example,

$$E=E_0W'=E_0|W'|e^{-i\phi'},\tag{15}$$

where

$$E_0=\frac{160.5}{d}\left[1-\frac{1}{(\beta d)^2}-\frac{i}{\beta d}\right]e^{-i\beta d}=|E_0|e^{-i\Phi_0}e^{-i\beta d}.\tag{16}$$

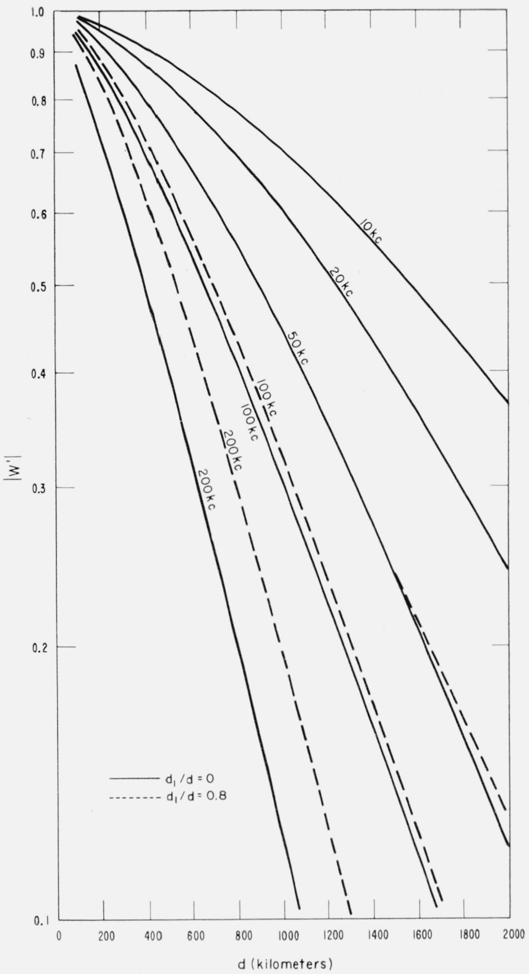

FIGURE 2. *Amplitude factor for mixed-path propagation.*

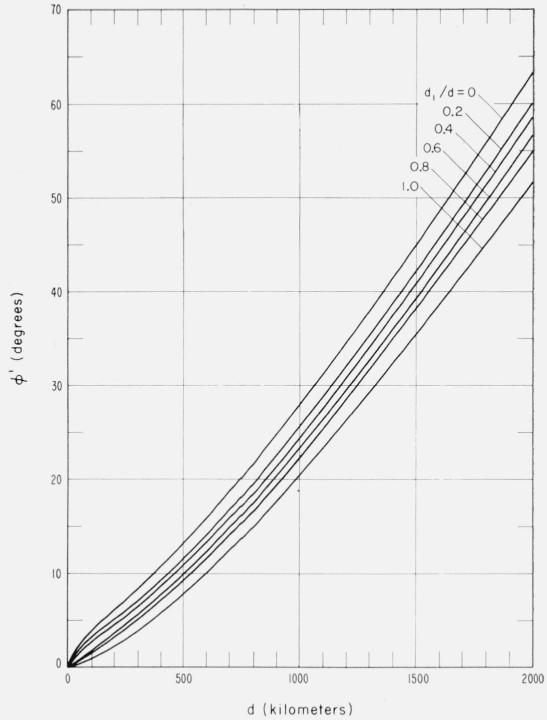

FIGURE 3. *Phase factor for mixed-path propagation.* f=10 kc.

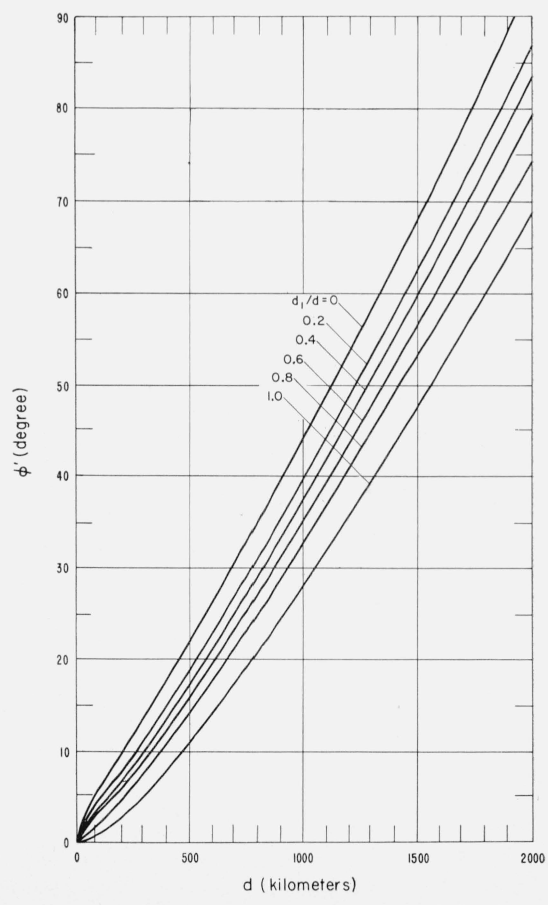

FIGURE 4. *Phase factor for mixed-path propagation.* f=20 kc.

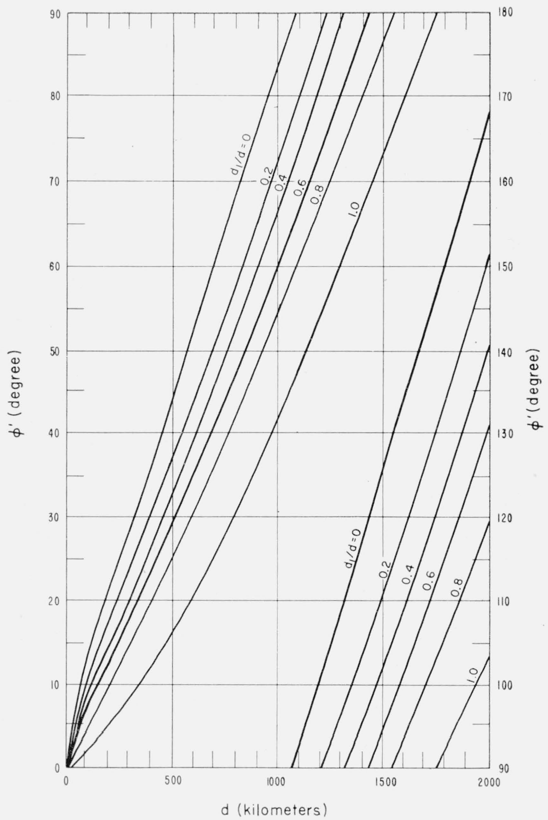

FIGURE 5. *Phase factor for mixed-path propagation.*
f=50 kc.

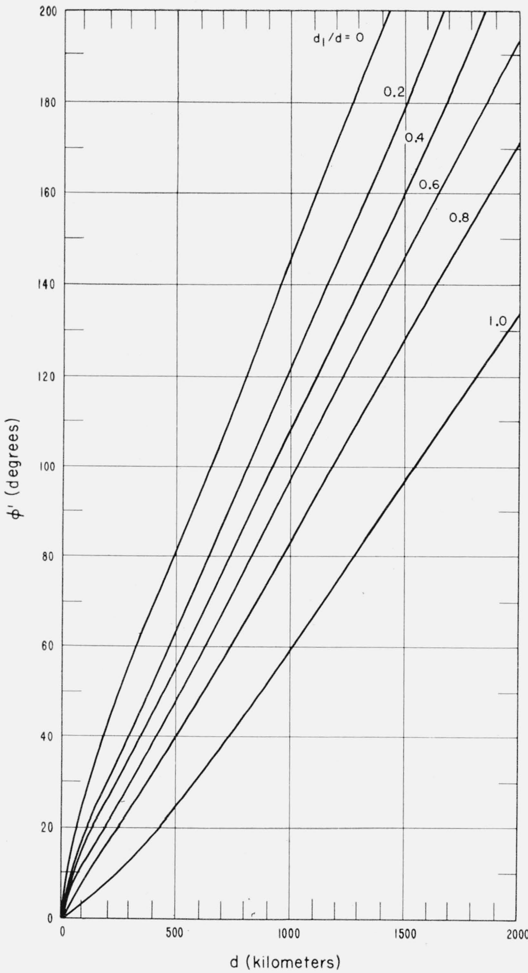

FIGURE 6. *Phase factor for mixed-path propagation.*
f=100 kc.

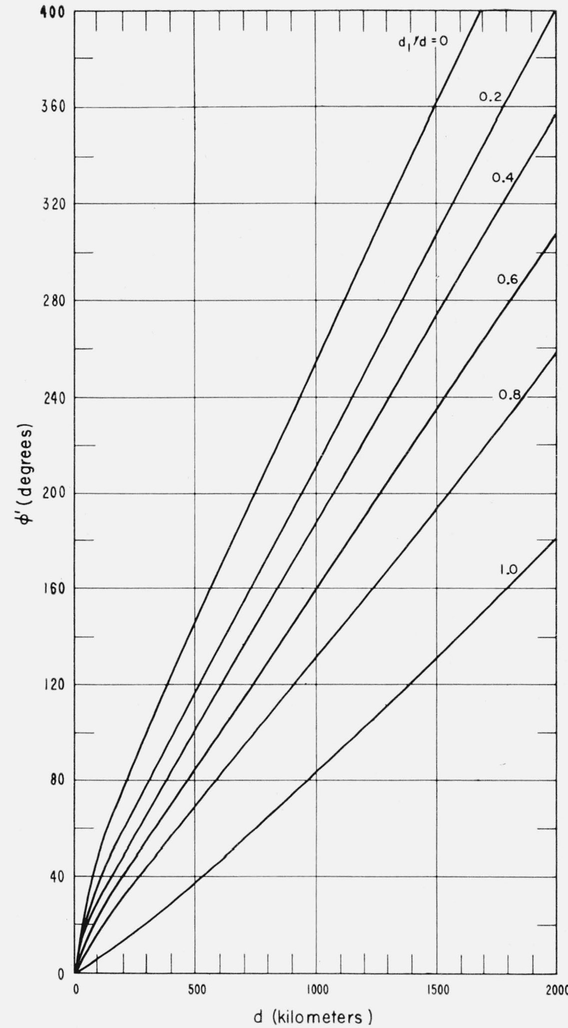

FIGURE 7. *Phase factor for mixed-path propagation.*
f=200 kc.

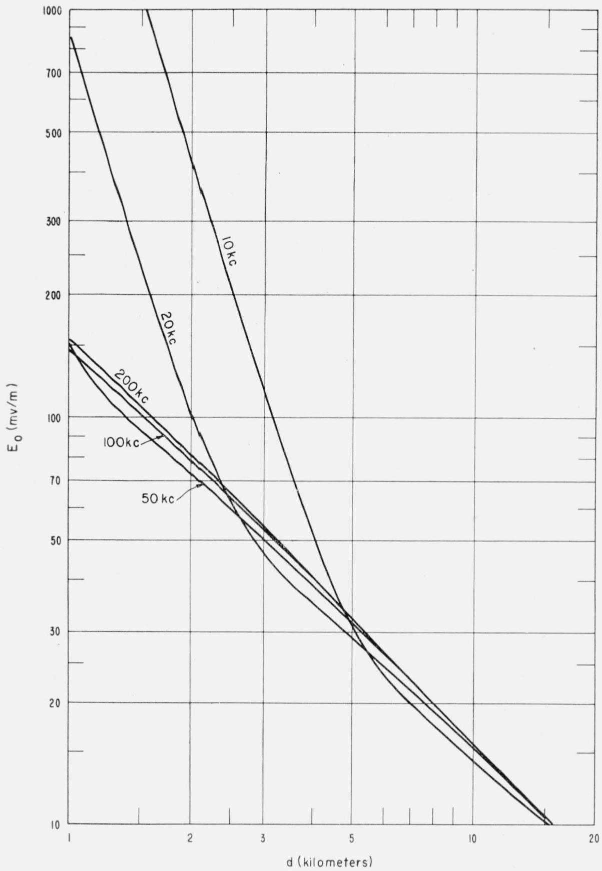

FIGURE 8. *Amplitude for near field.*

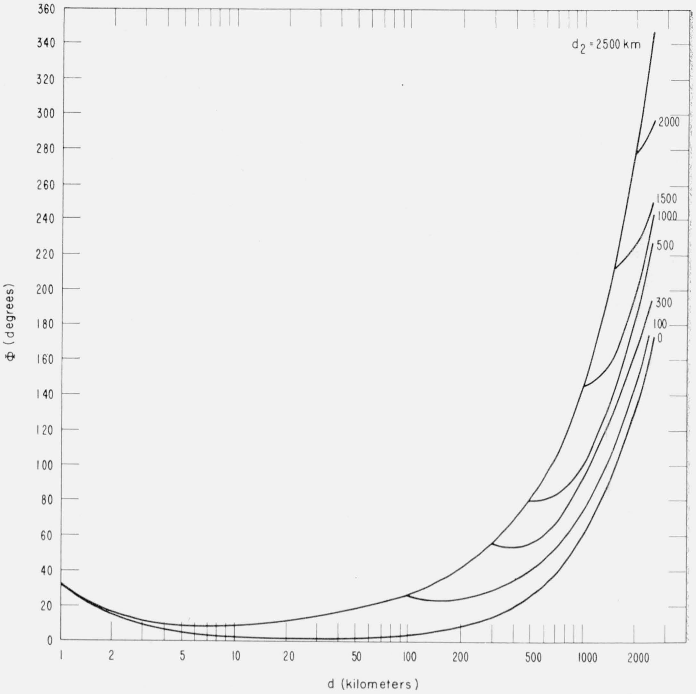

FIGURE 10. *Phase of the ground wave for propagation from land to sea.*
(Transmitter is distance d_2 from the coast) f=100 kc.

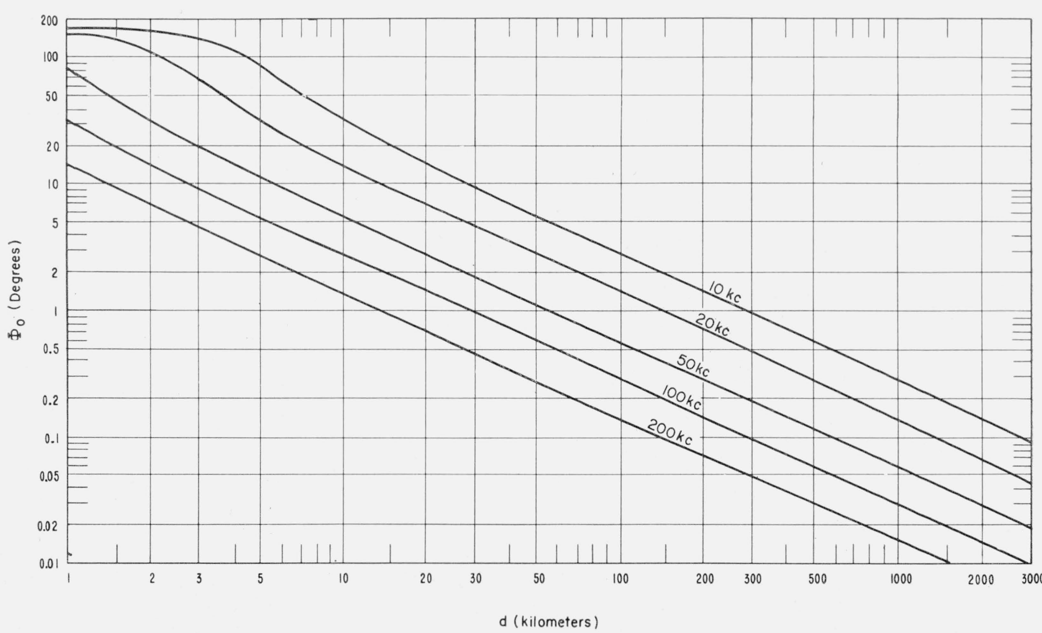

FIGURE 9. *Phase correction for near field.*

The "near field", E_0, is chosen such that the radiation component is 0.10 v/m at $d=1$ mile $=1{,}605$ m. The amplitude, E_0, and the phase, Φ_0, are shown plotted in figures 8 and 9, respectively, for frequencies of 10, 20, 50, 100, and 200 kc. It should be noted that $|E_0|\simeq 160.5/d$ and $\Phi_0\simeq 1/\beta d$ when $\beta d>>1$. Actually, E_0 is the total field of the source on a flat, perfectly conducting earth. To a good first approximation the total field E over a mixed path on a spherical earth is obtained by multiplying the near field E_0 by the attenuation function, W', as indicated above [1].

To provide an illustration of the applicability of the numerical results, the total phase correction $\Phi(=\Phi_0+\phi')$ at 100 kc, is shown in figure 10 for the case of a transmitter on land, a distance d_2 from the coast. The plotted curves show how Φ varies with $d(=d_1+d_2)$ for various values of d_2. As the receiver moves away from the transmitter, the rate of increase of the phase is characteristic of propagation over land. As the coastline is crossed at $d=d_2$, the phase lag is reduced somewhat and eventually continues to increase at a much slower rate as the receiver moves out to sea. This abrupt reduction of the rate of increase of phase lag is known as a "recovery effect" and has been verified experimentally [7].

2. Concluding Remarks

The results presented here should be useful in predictions of ground-wave propagation at low radiofrequencies over a two-part mixed land-sea path. It should be emphasized that the sky waves have not been considered. At ranges greater than 200 km or so, the ionospherically reflected waves must be separately accounted for. In any case, the ground wave is omnipresent even out to 2,000 km and is a substantial contribution to the field.

It is of interest to note that a formula almost identical to eq (13) of part 1 has been reported by Godzinski [8] in Poland at the recent International Radio Consultive Committee meeting in Warsaw. His results, however, are not valid for short distances or very low frequencies, where the induction and static fields are significant. He has also developed a formula essentially equivalent to eq (6) of the present paper, when $(d/a)^3<<1$. Furutsu [9] has also recently investigated the problem of ground-wave propagation over an inhomogeneously conducting spherical earth. He has indicated that the basic integral equation can be solved by an iterative procedure. The first term of his rather elaborate analysis agrees with eq (7) of the present paper. Neither Godzinski or Furutsu present any explicit numerical result for the mixed-path attenuation function.

The results in this paper, as well as in part 1, are not valid for points close to the boundary because a first-order stationary-phase evaluation of the integrals has been employed. Furthermore, the wave reflected from the boundary has been neglected. A somewhat different analytical approach has been used [10] to study the field very close to the boundary, which indicates the stationary-phase approximation is valid if A and B are at least two wavelengths from the boundary or coastline.

3. References

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