Relative Dimensional Stabilities of a Selected Series of Stainless-Steel Decimeter Bars

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The results of intercomparisons of the lengths of a series of stainless-steel decimeter bars for the past twenty-five years are reported. Data on the degree of dimensional stability of these bars were obtained and the results were analyzed. Some of the bars were found suitable for use as line standards of length.

1. Introduction

It is known that the length of most linear standards at any specified temperature cannot be assumed to remain the same over a period of years, nor can it be assumed that the secular changes in the bars are uniform over any definite period of time. It is, therefore, essential that a stable material be obtained for use as a line standard of length so that frequent calibrations are not required. Other requirements are that the surface to be graduated must take a good polish, be free from pits and inclusions, have graduations that are straight, and have clean sharp edges in order that the standards can be fully relied upon to give the requisite degree of accuracy in their use.

Several of the National Bureau of Standards decimeter standards that have been frequently calibrated in terms of the International Prototype Meter were used in conjunction with the 13 stainless-steel decimeter bars selected for the present investigation. To calibrate these bars intercomparisons by pairs were made using different combinations in order to obtain the most probable corrections to their lengths.

Because the intercomparison of 15 bars (2 NBS standards and the 13 bars in this group) consumed a great deal of time in the earlier measurements, a method[1] that substantially diminished the time without significant loss of accuracy was used in the 1955 intercomparisons.

This group of 13 decimeter bars was first intercompared in 1930 and in subsequent years up to 1955. The trends in length changes of the bars over the 25-year period are discussed.

In making the present measurements on the decimeter bars, every reasonable precaution was taken to have the results reflect the accuracy and precision that is obtainable with the present NBS equipment.

The investigation was initiated by C. G. Peters, formerly of the Bureau. He prepared the decimeter blanks, determined some properties of the bars, and graduated them by the use of wavelengths of light.

[1] W. J. Youden and W. S. Connor, New experimental design for paired observations, J. Research NBS **53**, 191 (1954) RP2532; Willard H. Clatworthy, Partially balanced incomplete block designs with two associate classes and two treatments per block, J. Research NBS **54**, 177 (1955) RP2579.

2. Description of the Decimeter Bars

The 13 decimeter bars are made of stainless steel (Fe-Cr) 130 mm in length and have a cross section of 15 by 15 mm. The upper rectangular surface of each bar has a 3-mm bevel in order to facilitate the optical polishing of the surface to be graduated, and in each case is remarkably free from any pits, inclusions and scratches, and closely approximates a true plane.

Each decimeter bar has only a single 1-dm interval, which, as is the custom, is defined as the distance from the center of the central line of a group of three lines near one end to the center of the central line of a similar group of lines near the other end. The lines have a width of approximately 3 μ.

On each bar two parallel lines approximately 0.2 mm apart are ruled at right angles to the graduations. Only that portion of a graduation lying between these two lines is considered when measurements are made.

The coefficients of linear thermal expansion of the decimeter bars was determined as 10.25×10^{-6}/deg C at room temperature.

Table 1 gives available data on the chemical composition, heat treatment, and scleroscope hardness of the bars.

3. Description of Longitudinal Comparator

The longitudinal comparator described by Page[2] was used for the intercomparisons of the decimeter bars during the investigation of their dimensional stabilities. In this comparator there is a beam carrying two microscopes, which moves in a longitudinal direction. The axes of the scales being compared were collinear.

4. Calibration of the Decimeter Bars

The latest intercomparisons of the decimeter bars were made in 1955. The results of the previous measurements of the total lengths of these bars

[2] Benjamin L. Page, Calibration of meter line standards of length at the National Bureau of Standards, J. Research NBS **54**, 1 (1955) RP2559.

TABLE 1. *Chemical composition, heat treatment, and hardness of the stainless-steel bars*

Bar [a]	Chemical composition								Treatment		Scleroscope hardness
	Cr	C	Mn	P	S	Si	Cu	Ni	Quenched from 930° C	Tempered at—	
	%	%	%	%	%	%	%	%		°C	
2	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	In oil	400	66
7	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	do	360	67
8	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	do	340	70
9	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	do	320	67
10	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	do	300	73
11	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	In H_2O	400	70
12	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	do	310	72
13	13.9	0.32	0.28	0.02	0.03	0.35	0.03	0.07	do	Not tempered	67
14 [b]	14.0	0.3 to 0.4	0.4	------	------	0.05	------	------	do	do	72
15 [b]	14.0	0.03 to 0.4	0.4	------	------	0.05	------	------	do	do	do
16 [c]	12 to 14	0.10	------	------	------	------	------	------	do	do	48
17 [d]	16.5 to 18	1.05	------	------	------	0.05	------	------	(e)	------	33
20	11.0	0.5 to 1.0	------	------	------	------	------	0.9	In oil	480	70

[a] All bars conform to American Iron and Steel Institute specifications, AISI 420, with the exception of bars 16 and 17.
[b] Slight longitudinal crack developed when quenched.
[c] AISI 410.
[d] AISI 440.
[e] Annealed.

made in 1930, 1932, 1938, and 1941, together with the results of the latest measurements, are included here.

4.1. Determination of Total Length

All measurements on the bars were made with the NBS comparator described above, with the temperature of the room controlled at 20° C. The bars were allowed to remain in the comparator for a sufficient length of time to assure thermal equilibrium at a temperature very near 20° C before any measurements were begun. All measured differences between any two bars were corrected to 20° C by using the respective coefficients of linear thermal expansion.

For convenience, the corrections to the bars are given instead of actual lengths. The actual length is equal to the nominal length plus the correction; hence, a plus correction indicates that a bar is longer than the nominal length.

The 13 selected decimeter bars and 2 NBS standard decimeter bars were placed in the comparator in a random manner with respect to their relative positions, either to the right or to the left. Several bars were placed at either end of the comparator at any one time so that more than two bars could be intercompared in any one day without danger of a serious change in thermal equilibrium of the bars caused by their being handled and by the removal of the covers on the comparator tank in which the bars were mounted. All measurements were made with the bars in air while supported on a horizontal flat surface.

To obtain the differences in the length of two bars, two microscopes, separated by any convenient distance, were focused, one on the 0 graduation of the bar on the left and the other on the 0 graduation of the bar at the right. Micrometer observations were then made on the 0 graduation of each bar. The carriage was then moved so that the microscopes were over the 1-dm graduations before the observations were made. For convenience in computing, the bar at the right was always designated as A and the other as B. The comparisons to determine the differences in length between each pair of bars, A and B, were repeated six times, and the temperature of each bar was recorded. The mean of these differences was corrected to 20° C, and the differences in the length of the bars, $L=A-B$, was thus determined.

The results obtained in the 1955 series of comparisons are shown in table 2. The values in brackets were computed from the directly observed differences.

As the work required to observe and determine values for the $N(N-1)/2=105$ differences between 15 bars, consumes an excessive amount of time if all differences are directly determined, it was decided to divide the bars into 4 groups; 3 of 6 bars each, and 1 of 4 bars. The grouping of the 15 bars was as follows:

Group 1: 57, 24,2,7,8, and 9.
Group 2: 57, 24, 10, 11, 12, and 13.
Group 3: 57, 24, 14, 15, 16, and 17.
Group 4: 57, 24, 8, and 20.

This procedure reduces the number of observed and determined values between 15 bars to 51.

In order further, to reduce the number of directly determined differences between the bars, a method (see footnote 1) was used, whereby only 9 of the 15 differences were directly observed in each of the first 3 groups, with the remaining 6 differences computed from the directly observed values. All of the differences in group 4 were observed directly. This plan reduced the number of directly observed differences to a total of 33. As the value for dm 57—dm 24 is used 4 times, and the value for dm 57—dm 8 is used twice in the computations, the number of directly observed differences was finally reduced to 29.

It has been previously determined (see footnotes 1 and 2) that in spite of this reduction in the number of observations, the desired accuracy for the comparisons is obtained.

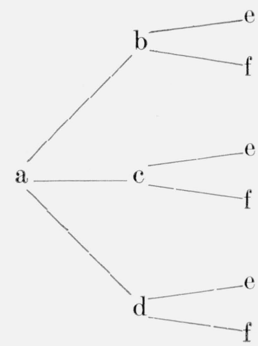
TABLE 2. *Differences in length of decimeter bars at 20° C*

(1955 series of comparisons)

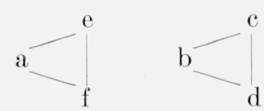
Observations made at temperature, *t*, and reduced to 20° C, using the respective values of the coefficients of linear thermal expansion. Directly observed differences (*A*−*B*) are the values not in brackets. Values in brackets have been computed from the directly observed differences.

Bars compared		Observed difference, *L*	Observed temperature, *t*	
			Bar *A*	Bar *B*
A minus	*B*	μ	° *C*	° *C*
57	24	−3.24	20.00	20.00
2	24	−13.76	19.88	19.86
7	24	−14.15	19.88	19.86
8	24	[−13.76]	---------	---------
9	24	[−14.37]	---------	---------
2	57	[−10.37]	---------	---------
7	57	[−10.72]	---------	---------
8	57	−10.31	19.96	19.95
9	57	−11.00	19.96	19.95
7	2	[−0.35]	---------	---------
8	2	−0.07	20.00	19.96
9	2	−0.66	20.27	20.24
8	7	+0.24	19.99	19.95
9	7	−0.31	19.99	19.95
9	8	[−0.61]	---------	---------
57	24	−3.24	20.00	20.00
10	24	−13.72	19.98	19.94
11	24	−13.44	20.01	19.97
12	24	[−13.61]	---------	---------
13	24	[−16.20]	---------	---------
10	57	[−10.56]	---------	---------
11	57	[−10.20]	---------	---------
12	57	−10.53	20.02	19.98
13	57	−12.88	20.24	20.22
11	10	[+0.36]	---------	---------
12	10	+0.27	20.52	20.50
13	10	−2.48	20.62	20.57
12	11	−0.17	20.58	20.53
13	11	−2.83	20.60	20.55
13	12	[−2.59]	---------	---------
57	24	−3.24	20.00	20.00
14	24	−16.86	20.07	20.03
15	24	−16.16	19.97	19.94
16	24	[−14.11]	---------	---------
17	24	[−13.23]	---------	---------
14	57	[−13.60]	---------	---------
15	57	[−13.05]	---------	---------
16	57	−10.83	19.94	19.91
17	57	−10.14	20.03	20.00
15	14	[+0.54]	---------	---------
16	14	+2.64	20.10	20.06
17	14	+3.56	20.03	20.05
16	15	+2.13	20.06	20.04
17	15	+3.14	20.05	20.03
17	16	[+0.87]	---------	---------
57	24	−3.24	20.00	20.00
8	24	−13.58	20.20	20.17
20	24	−13.65	20.03	20.01
8	57	−10.31	19.96	19.95
20	57	−10.23	20.01	19.99
20	8	+0.22	19.99	19.96
Mean of all observed temperatures----------			20.08	

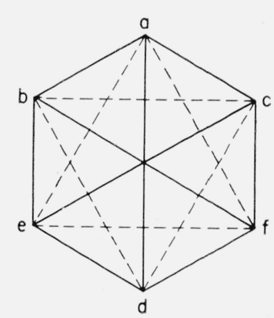
In order to select the pairs of bars, in a group of 6, that are to be compared directly and those to be computed, a procedure, called a partially balanced incomplete block design, can be developed for the intercomparison, in which 9 of the comparisons are made directly and 6 are computed from the direct comparisons. To illustrate this design, the bars can be represented by the letters a to f. The direct comparisons will be as follows: a with b, c, and d; and b, c, and d each with e and f. This can be illustrated as follows:



The remaining computed differences will be: a with e; a with f; e with f; b with c; c with d, and b with d. Illustrated graphically this will be



Another way to illustrate this design graphically is:



The letters at the ends of the solid lines represent the bars that will be compared directly, and those at the ends of the dotted lines represent the bars whose differences will be computed from the directly observed differences.

To compute these unobserved differences in length between the bars the following method, illustrated by 8–24, was used. First, the mean of the differences between 8 and each of the other bars with which it was directly compared, and for which there is a corresponding directly observed comparison of that other bar with 24, was found. Next, the mean of the differences between 24 and each of the others with which it was compared, and for which there is a corresponding directly observed comparison of that other bar with 8, was found. Finally, the mean values were added algebraically. This is the computed value for the difference, 8−24, or −13.76 μ, as is shown in brackets in table 3. The other bracketed figures in table 3 were computed in a similar manner.

TABLE 3. *Calculations of corrections to decimeter bars (group 1) at 20° C*

Directly observed differences ($A-B$) are taken from table 2, and are the values not in brackets or parentheses. Values in brackets have been computed from the directly observed differences. Calculated ("most probable") differences for $A-B$, given in parentheses, are derived by taking the differences of $\Sigma/6$; e. g., dm 57−dm 24=+6.53−(+9.88)=−3.35 μ, dm 2−dm 57=−3.84−(+6.53)=10.37 μ, and dm 7−dm 2=−4.19−(−3.84)=−0.35 μ. Correction to dm 24=+12.96 μ at 20°C. Correction dm 57=+9.72 μ. Correction to bar equals the mean of the corrections to dm 24+(bar−24) and dm 57+(bar−57). Length of bar equals 1 dm+correction. All values expressed in microns.

B	*A*						Residuals,
	24	57	2	7	8	9	*v*
24	0	−3.24	−13.76	−14.15	[−13.76]	[−14.37]	+0.11 −.04 −.08 .00 .00 .00 .00 +.10 +.02 .00 −.03 −.01 −.07 −.01 .00
		(−3.35)	(−13.72)	(−14.07)	(−13.76)	(−14.37)	
57	+3.24	0	[−10.37]	[−10.72]	−10.31	−11.00	
			(−10.37)	(−10.72)	(−10.41)	(−11.02)	
2	+13.76	[+10.37]	0	[−0.35]	−0.07	−0.66	
				(−0.35)	(−0.04)	(−0.65)	
7	+14.15	[+10.72]	[+0.35]	0	+0.24	−0.31	
					(+0.31)	(−0.30)	Σv^2=0.0365
8	[+13.76]	+10.31	+0.07	−0.24	0	[−0.61]	[a]P. E.=±0.025
						(−0.61)	
9	[+14.37]	+11.00	+0.66	+0.31	[+0.61]	0	
Σ	+59.28	+39.16	−23.05	−25.15	−23.29	−26.95	
Σ/6	+9.88	+6.53	−3.84	−4.19	−3.88	−4.49	
Bar−24	0	---------	−13.72	−14.07	−13.76	−14.37	
Correction to bar	+12.96	---------	−0.76	−1.11	−0.80	−1.41	
Bar−57	---------	0	−10.37	−10.72	−10.41	−11.02	
Correction to bar	---------	+9.72	−0.65	−1.00	−0.69	−1.30	
Mean correction to bar	---------	---------	−0.70	−1.06	−0.74	−1.36	

[a] Probable error of the calculated value of the differences in the corrections between any two bars that were directly compared.

The reduction of these observed and computed differences by the method of least squares was carried out for bars of group 1 by the use of the form [3] shown in table 3. Tables for the bars of groups 2 to 4 are not shown because they are similar in form.

The differences, v, between observed and calculated differences are shown in table 3. These 15 residuals range from 0.00 to 0.11 μ, and the average, neglecting sign, is 0.03 μ. The probable error of the calculated value of the differences in the corrections between any two bars that were directly compared, r, is given by the formula [4]

$$r=0.6745\sqrt{\frac{\Sigma v^2}{\frac{N}{N_i}(N-N_i)}},$$

where Σv^2 is the sum of the squares of the residuals; N, the number of observational equations; N_i, the number of unknowns. In the example considered, N_i is equal to 5 and N equal to 9. For the case given in table 3, r is calculated to be ± 0.025 μ. The probable error of the calculated value of the differences in the corrections between any two bars that were not directly compared is slightly different. The probable errors of the calculated values of the other 3 groups of bars in the 1955 series of comparisons are of the same order of magnitude.

In the comparison of the bars to determine their total corrections for the study of their stabilities, the smallness of the residuals indicates that the bars were in good thermal equilibrium while the observations were being made. It is believed that the corrections determined for the total lengths of the decimeter bars in each of the five series of calibrations are not in error by more than 0.1 μ.

During the period 1930 to 1956, inclusive, 5 series of determinations of the corrections to the group of 13 dm bars were made. The corrections to the total length of the bars at 20° C are given in table 4.

[3] A. Pérard and C. Volet, Les mètres prototypes du Bureau International Travaux et Mémoires du Bureau International des Poids et Mesures, p. 21, (1945).

[4] T. W. Wright and J. F. Hayford, The adjustment of observations, p. 133, 137, 143, (1906).

Table 4. *Determination of corrections to decimeter bars at 20° C*

(1930 to 1955 series of comparisons)

Correction to bar=correction to standard used+(bar−standard used). All values expressed in microns.

Bars	Year of correction determination					Differences				
	1930	1932	1938	1941	1955	1932−1930	1938−1932	1941−1938	1955−1941	1955−1930
2	−0.95	−0.85	−1.15	−1.30	−0.70	+0.10	−0.30	−0.15	+0.60	+0.25
7	−0.60	−0.65	−0.80	−0.80	−1.05	−0.05	−0.15	0.00	−0.25	−0.45
8	−0.50	−0.35	−0.35	−0.60	−0.75	+0.15	0.00	−0.25	−0.15	−0.25
9	−0.60	−0.65	−0.75	−0.65	−1.35	−0.05	−0.10	+0.10	−0.70	−0.75
10	−0.55	−0.65	−0.55	−0.70	−0.85	−0.10	+0.10	−0.15	−0.15	−0.30
11	−0.45	−0.30	−0.20	−0.35	−0.45	+0.15	+0.10	−0.15	−0.10	0.00
12	−0.40	−0.35	−0.50	−0.50	−0.65	+0.05	−0.15	0.00	−0.15	−0.25
13	−0.60	−1.45	−2.10	−2.35	−3.25	−0.85	−0.65	−0.25	−0.90	−2.65
14	−0.70	−1.70	−2.80	−3.15	−3.85	−1.00	−1.10	−0.35	−0.70	−3.15
15	−0.40	−1.40	−2.10	−2.70	−3.30	−1.00	−0.70	−0.60	−0.60	−2.90
16	−0.65	−1.20	−1.10	−1.55	−1.15	−0.55	+0.10	−0.45	+0.40	−0.50
17	−0.25	−0.15	−0.35	−0.30	−0.30	+0.10	−0.20	+0.05	0.00	−0.05
20		−0.10	−0.15	−0.30	−0.60		−0.05	−0.15	−0.30	[a] −0.50

[a] 1955−1932.

It will be noted that all of the bars, with the exception of bars 2 and 11 shortened during the 25-year period. Bar 2 lengthened by 0.25 μ; bar 11 remained the same length. The changes in length range from $-3.15\ \mu$ to $+0.25\ \mu$, as shown in figures 1 and 2.

Table 5 shows the average annual changes for each of the decimeter bars over a period of 25 years. These changes in length per decimeter are rather small, with the exception of AISI 420 bars 13, 14, and 15, which were quenched but not tempered. The average annual change in length over a period of 25 years for these three bars was $-0.12\ \mu$/dm, or $-1.2\ \mu$/m. It is significant that the average annual change in length over the same period for the remaining 8 AISI 420 bars, which were both quenched and tempered, was only $-0.01\ \mu$/dm, or $-0.1\ \mu$/m. The annealed bar 17 was exceedingly stable, but it did not take a good polish on account of its low hardness and therefore was not suitable for a line standard.

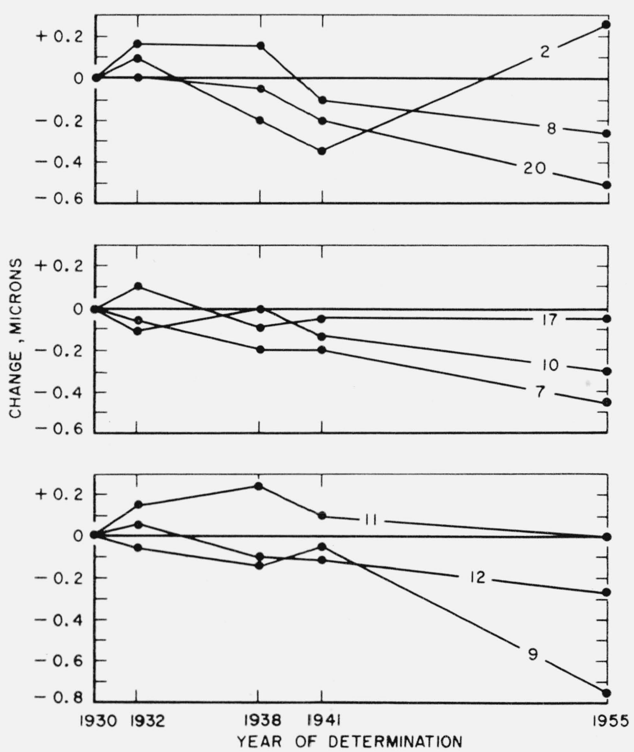


Figure 1. *Change in length of decimeter bars.*

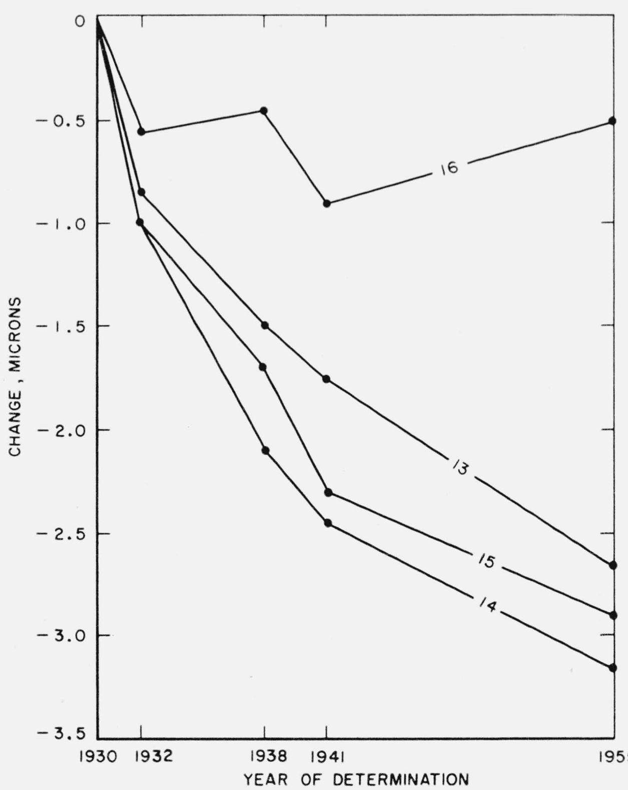


Figure 2. *Change in length of decimeter bars.*

TABLE 5. *Length changes of decimeter bars*

Bars	Average changes per year				
	1930 to 1932	1932 to 1938	1938 to 1941	1941 to 1955	1930 to 1955
	μ	μ	μ	μ	μ
2	+0.05	−0.05	−0.05	+0.04	+0.01
7	−0.03	−0.03	0.00	−0.02	−0.02
8	+0.08	0.00	−0.08	−0.01	−0.01
9	−0.03	−0.02	+0.03	−0.05	−0.03
10	−0.05	+0.02	−0.05	−0.01	−0.01
11	+0.08	+0.02	−0.05	−0.01	0.00
12	+0.03	−0.03	0.00	−0.01	−0.01
13	−0.43	−0.11	−0.08	−0.06	−0.11
14	−0.50	−0.18	−0.12	−0.05	−0.13
15	−0.50	−0.12	−0.20	−0.04	−0.12
16	−0.28	+0.02	−0.15	+0.03	−0.02
17	+0.05	−0.03	+0.02	0.00	0.00
20	------	−0.01	−0.05	−0.02	[a]−0.02

[a] 1932 to 1955.

5. Conclusion

The average annual change in length of 8 dm bars of quenched and tempered AISI 420 stainless steel was −0.1 μ/m during a period of 25 years. This small change was approximately one-half the average annual changes of a group of four meter bars made of invar, 42 percent nickel-steel, and "Fix Invar", over a similar period of time. This degree of stability indicates that AISI 420 stainless steel would be suitable for line standards of length, for which a permanence of 0.1 μ/m over many years is required.

The author expresses his appreciation to W. B. Emerson, P. Hidnert, L. V. Judson, and W. J. Youden for valuable discussions and assistance during the preparation of this paper.

WASHINGTON, October 25, 1956.