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The Kösters Interferometer

J. B. Saunders

Results are given on an investigation of the Kösters double-image prism. Some of these results are not in harmony with those given by other investigators. A modification of the Kösters prism is described that forms a simple interferometer that is easy to apply to the testing of lenses, mirrors, and combinations of these elements. A practical test is given for determining the maximum size of the source that is usable in any interferometer.

1. Introduction

In 1953 the Kösters double-image prism [1] was investigated for possible use in testing lenses. It was found that the prism, when used with symmetrical systems that are assembled symmetrically with respect to the dividing plane of the prism, produced a striking polychromatic interference phenomenon. It was found that this prism could be used to test the asymmetry of mirrors and lenses, but that symmetrical (or even-order) aberrations disappeared because of this symmetry. When a lens or mirror is arranged unsymmetrically with respect to the dividing plane of the prism, the equation for optical-path difference is too complex for practical application, except for a particular position of the light source relative to the optics of the system.

A study of the large change in the interference patterns with corresponding small changes in position of source, and the limitation of source size that could be used to get good fringes, led to the discovery that for one particular position of the source many of the above-mentioned objections disappeared. This discovery led immediately to a modification of the prism, resulting in a simple arrangement that yielded an optical-path-difference equation that is simple, practical, and easy to apply. A report on this work, dated June 30, 1953, is not now available for distribution; its essentials are included here.

2. The Kösters Prism and Mirror Interferometer

A Kösters prism was mounted just inside the center of curvature of a concave mirror, M, shown in figure 1a. A small source of light was placed at S_1, where the light, after reflection from the mirror, forms one image of the source on itself and another image at S_1'. An observer's eye placed at this point sees an oval-shaped field of interference fringes. A ray of light from the source is divided into two components at the beam-dividing plane AB. After total internal reflections from faces AC and AD of the prism, the two components diverge from two separated coherent images, S_2 and S_2', of the source. After reflection from the mirror, the two component rays recombine at the dividing plane and proceed to S_1' where they are received by the eye of an observer. Refraction occurs at surfaces CB and BD of the prism but, to a first approximation, the two beams are affected similarly and compensation almost nullifies this effect.

The returning wave fronts are afflicted with off-axis aberrations, but these also are of the same magnitudes, and, when they are recombined, compensation is again effected to a first approximation. Thus, a concave mirror will produce approximately straight fringes if it has axial symmetry and the axis lies in the dividing plane of the prism. Zonal irregularities do not become apparent because of symmetry. When the mirror is rotated about an axis through its center of curvature and normal to the plane of figure 1 so that the dividing plane intersects it off center, zonal irregularities then become apparent. A similar interferometer has been described by Gates [2], using a different type of prism.

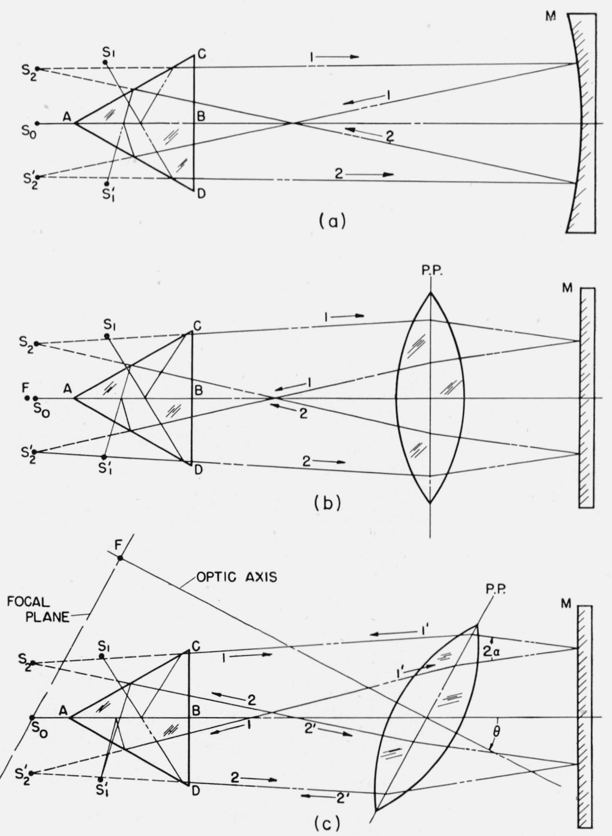

FIGURE 1. *The Kösters prism and lens interferometer.*

3. The Kösters Prism and Lens Interferometer

When the concave mirror of figure 1,a, is replaced by a lens and plane mirror (fig. 1,b), interference is again obtained. This arrangement has been described also by Gates [3], but his conclusions do not agree with the findings of this author.

Because of the separation of the two virtual sources, S_2 and S_2', the two beams of collimated light (one from each source) are not parallel between the lens and mirror. They are incident at angles that are of equal magnitude but of opposite signs. The resultant wave fronts are again afflicted with off-axis aberrations but, because of the symmetry, compensation is complete to a first approximation and straight fringes are again produced.

If the lens is rotated about an axis through its optical center and normal to the plane of figure 1 we have the arrangement of figure 1,c, which is similar to that described by Gates [3]. The interference fringes become curved. The condition of symmetry has been destroyed. If the light that returns into the source forms an image of the source

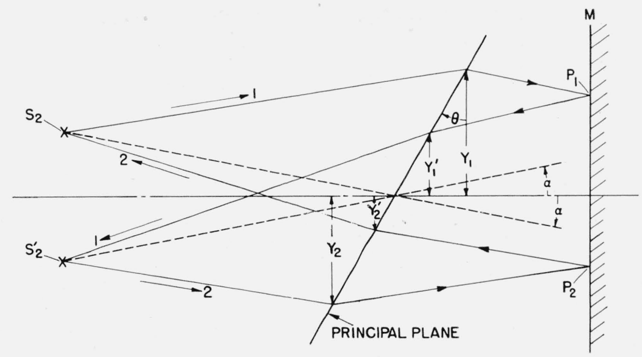

FIGURE 2. *Ray trace through the interferometer.*

on itself (as in autocollimation), the focal surface of the lens will pass through S_0, where S_0 is the bisecting point on the straight line joining S_2 and S_2'. Consequently, in general, S_2 and S_2' will lie on opposite sides of this surface. One beam becomes convergent at its first passage through the lens, whereas the other one remains divergent until its second passage through it. Neither beam becomes collimated

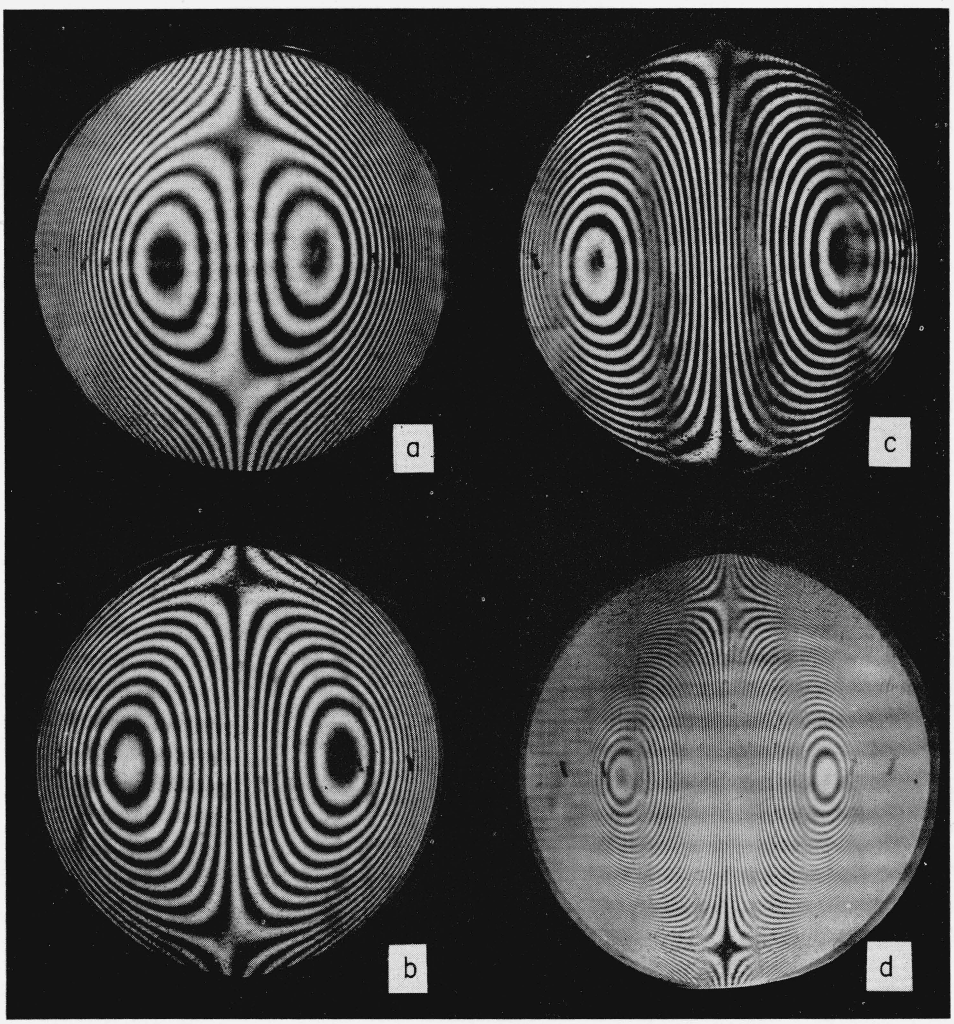

FIGURE 3. *Interferograms with the Kösters double-image prism.*

a, b, and c, Change resulting from change in position of the source alone; c and d, effect of two small sources located close together.

outside the lens. Consequently, the method described above does not permit off-axis testing of lenses with one conjugate at infinity.

A characteristic ray of light emanating from the source, S_1 of figure 1,c, divides at the beam-dividing plane, AB, into two ocherent component rays, 1 and 2. These two rays are incident at distances y_1 and y_2 from the dividing plane (fig. 2) at their first incidence and at distances y_1' and y_2' at their second incidences. In general, the magnitudes of these y-values are all different, and each reference point located on the lens appears in quadruplicate. To illustrate this, a mark was purposely placed on the lens used to produce figure 3. The relative separation of these four images depends upon the angle θ, the distance from lens to mirror, and the position of the source. The separation of the images in figure 3,c, corresponding to y_1 and y_1' of figure 2, is three times as large as that for figure 3,a. This change is due to positions of source alone. The differences in the absolute values of the four y's will be relatively large if θ is appreciable. Therefore, the equation for optical-path difference must either include the separation of mirror from lens and position of source, or include four different y's. In either case the resultant equation for the fringe pattern is too complex for practical application.

The two virtual sources, S_2 and S_2', must be separated by an appreciable distance if each beam is to fill completely the aperture of the lens. Figure 4 shows the triangular areas, $G'H'K'$ and $G''H''K''$, in which the two virtual sources must lie. If a $1\frac{1}{2}$-in.-aperture prism is used to fill a 20-in. focal-length lens of f/6.3 aperture, the lowest practical value for the separation of the two virtual sources is approximately $\frac{1}{2}$ in. Consequently, the principal rays of the two beams are at an appreciable angle, 2α, to each other (see fig. 2). A ray that is undeviated on its first transmission through the lens will suffer deviation at its second passage through it.

The ray trace shown in figure 2 ignores refraction at the surfaces of the prism and also assumes the focal plane of the lens to pass through S_2 and S_2'. Under these assumptions the two beams are collimated to the right of the lens and the equation for the optical-path difference is simplified accordingly. However, the angle of incidence on the mirror is $+\alpha$ for ray 1 and $-\alpha$ for ray 2, where α is the angle between a principal ray and the dividing plane. The angle between these collimated rays and normals to the principal plane of the lens are: For ray 1, $(\theta-\alpha)$ before and $(\theta+\alpha)$ after reflection from M; and for ray 2, $(\theta+\alpha)$ before and $(\theta-\alpha)$ after reflection from M, where θ is the angle between the optic axis and the dividing plane. Thus, the image height (distance from image to optic axis) is different for the two component beams, and the resultant equation relating optical path difference to the aberration constants of the lens must include these two image heights or their equivalent.

When the ray trace shown in figure 1, c, is analyzed it is found that the components of a given ray, after division, do not recombine at the dividing plane, but converge to S_1' from different directions. Interference does not result from the recombination of the components of an original ray but from the combination of two rays that leave the source from different directions. The Rayleigh refractometer [4] is a familiar example of this manner of combining rays to produce interference. An illustration of the course

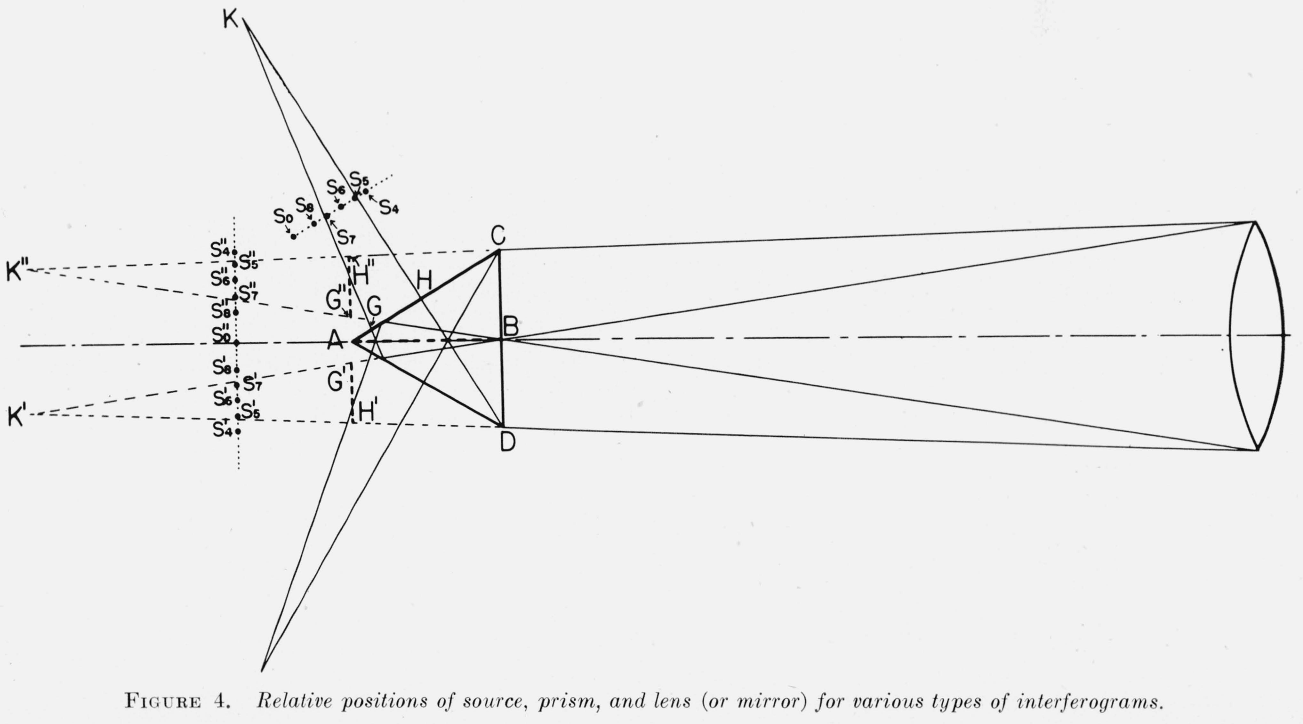

FIGURE 4. *Relative positions of source, prism, and lens (or mirror) for various types of interferograms.*

of two such rays may be obtained by placing the source at S_1' (in fig. 1, c) instead of S_1 as was previously the case, thus reversing the directions of the beams. The two rays 1′ and 2′ leave plane AB from different points, diverge from the two virtual sources S_2 and S_2', suffer differential refraction at the prism faces CB and BD, and traverse the lens at points that are at different distances from the meridional plane of the lens. After reflection from M, the two coherent beams return to points in the principal plane of the lens that appear coincident from S_1. These two points are at equal angular distances on opposite sides of the dividing plane of the prism. The rays suffer equal refraction when reentering the prism faces CB and BD (if the prism is perfectly symmetrical) and combine at a common point on plane AB, from which they proceed colinearly to point S_1. The resultant differential refraction of the two component beams is not negligible.

4. Size of Light Source

In order to show the effect which the position of the light source has on an interferogram a small pinhole source (diam=0.2 mm) of filtered yellow light of helium was used to produce the interferograms of figures 3, a, and 3, b. The faces of the prism were 1½ in., the focal distance and aperture of the cemented achromatic lens were 9 and 2 in., respectively. The position of the source, for the interferogram marked A in figure 3, was at point S_5 of figure 4. The next picture is marked B and the corresponding position of the source was at S_6. Points S_5 and S_6 are approximately 3 mm apart. Photograph C was obtained with two similar pinhole sources, 0.6 mm apart near point S_7 and in line with the other source positions. This results in a double set of fringes that produce a Moiré pattern. The value of θ for pictures A, B, and C in figure 3 was approximately 2 degrees. Photograph D shows a similarly obtained Moiré pattern with a larger value of θ. The Moiré fringes became more numerous and more curved with increasing values of θ.

Gates attributes the "limitation to the size of source which may be used with the double image interferometer" to imperfections in his prism. The quantity $(\epsilon_1-\epsilon_2)$, described in reference [3] as the factor that determines the maximum size of the source, was less than 1 sec of arc for the author's prism and, according to Gates, should permit the use of an extended source. The existence of Moiré fringes in figures 3, c, and 3, d, indicates that an extended source could not have been used even with a perfect prism. This use of two small sources is found to be quite practical for ascertaining the maximum size of the source that can be used with any interferometer.

5. The Inverting Interferometer

When the source is placed at S_4 or S_8 of figure 4 (producing pairs of image sources S_4', S_4'' and S_8', S_8'') the lens is completely filled but not by either beam separately. When placed at S_0 the lens is also completely filled, but no part of it is covered by both sources. The wave fronts for the two component beams have a common boundary that coincides with the dividing plane of the prism. These two wave fronts diverge from their common centers at S_0'' as parts of the same sphere. When they return through the prism one of them suffers two, and the other one, inversion. This results in the folding of one wave front onto the other about their common boundary. If the center of the lens or mirror is outside the dividing plane of the prism (fig. 5), the two returning wave fronts differ in area and shape. They form the two parts of a circular area that is divided by a chord of the circle. Interference fringes appear only in the overlapping area. If the dividing plane intersects the lens or mirror at its center, the interferogram is semicircular in shape.

Point S_0'' corresponds to the position of the source as it is used with the inverting [5] interferometer. As the source is moved from position S_4, of figure 4, to S_0 the two images of the source converge toward and become coincident at S_0''.

If there were no refraction at the surfaces of the prism, in figure 1, and if the images of the source were at S_0, which lies in the focal plane of the lens, the two beams would become collimated and parallel to each other. By making surface CBD of the prism spherical, with S_0'' as its center of curvature, no re-

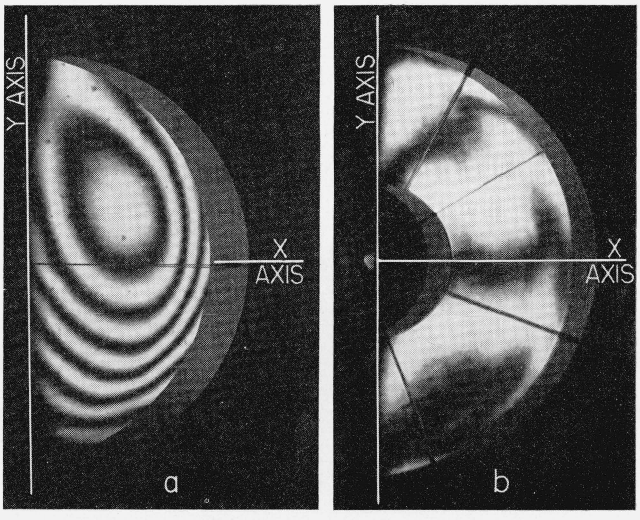

FIGURE 5. *Interferograms.*

a, Parabolic mirror tested at its center of curvature; b, Cassegrainian telescope tested at its focus.

fraction occurs, and the rays return upon themselves. After reflection from M they again traverse the lens along the same paths, suffer no refraction at faces CB and BD, reunite at the point of division, and proceed collinearly to the point of observation. The quantity α does not appear, having been reduced to zero by superposition of the two virtual sources. Consequently, the angle of incidence on M is zero, making $y_1=y_1'=-y_2=-y_2'$. The equation for optical-path difference (OPD), based on Conrady's equations and using Kingslake's [6] terminology, is

$$(OPD)=2\lambda(y-g)\{(4Ag+B)[3g^2+x^2+(y-g)^2] \\ +2g(3C+D-4Ag^2)+E\}, \quad (1)$$

where g is the distance from the center of the lens to the dividing plane of the prism. The quantity (OPD), used here, is the optical-path difference between rays 1 and 2 of figure 1,c, when the images of the source coincide at S_0. If the dividing plane of the prism is parallel to the x-axis and the coordinates of the intersection of ray 1 with the lens are (x,y), the corresponding coordinates for ray 2 are $(x,y-2g)$. When g is adjusted to zero, eq (1) becomes

$$(OPD)=2\lambda y[B(x^2+y^2)+E]. \quad (2)$$

The quantity E is the displacement of the chosen image point from the dividing plane of the prism. When one is calculating the interference patterns, or otherwise analyzing the data for a given lens, the values for λ, x, and y will be known and (OPD) is observed directly. The quantity E may be eliminated by adjustment of the prism. The quantity B is then directly computable and is a measure of coma. In order to evaluate the spherical and astigmatic coefficients of aberrations (A and C in formula 2) the quantity g is adjusted to a convenient magnitude. The procedures described by Kingslake or Gates may then be applied for these evaluations.

Equation (2) is quite similar to Gates' formulas if the term in $z^4=(x^2+y^2)^4$ of reference [4] is omitted.

Figure 5,a, is an interferogram of a 12-in.-aperture parabolic mirror, tested at its center of curvature. Figure 5,b, is an interferogram of a 12-in-aperture f/11 Cassegrainian telescope, tested at its focus. A plane mirror was used to return the collimated light to the focus of the telescope. A perfect telescope would have produced straight fringes. The shapes of the fringes indicate zonal aberration.

The difficulty (or ease) of applying the inverting interferometer is about equal to that of applying the Foucault knife-edge test.

6. The Prism

The prisms used for these photographs were experimental models, cemented together with Canada balsam. The edge, A, of the inverting interferometer prism should be relatively sharp to avoid obstructing the light from along the line of inversion, which coincides with the dividing plane of the prism. These prisms were adjusted to introduce a wedge between the two component wave fronts so that when perfect optical systems are tested, the fringes are straight and perpendicular to the dividing plane of prism. This permits the use of convenient fringe widths when testing nearly perfect systems. Without this wedge the photograph of figure 5,b, would have shown one broad, fluffed-out fringe that would have been difficult to measure.

The beam-dividing surfaces of these experimental prisms were too thin to produce equally intense component beams. This accounts for the low contrast in the Moiré fringes of figure 3. The ratio of transmission to reflection is not critical in prisms to be used as inverting interferometers because each beam suffers one transmission and one reflection and, after recombination, they will always be equally intense. However, when used as shown in figure 1, one beam suffers two transmissions and the other, two reflections. To obtain equal transmission and reflection, after the prisms are cemented with Canada balsam, the reflectivity should approximate three times the transmission when tested at normal incidence and with air-glass as the mediums.

7. Conclusions

The interferometer has proved itself quite practical for laboratory test of lenses, mirrors, and combinations of these during figuring operations. It has been used to test parabolic, elliptical, and spherical mirrors. These operations are performed with remarkable simplicity. As yet, no test has been made of a telescope when using a celestial star as source. This test, however, is believed to be quite simple.

8. References

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