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An Improved Method of Measuring Efficiencies of Ultra-High-Frequency and Microwave Bolometer Mounts

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A method is presented for measuring efficiencies of bolometer mounts used for ultra-high-frequency and microwave power measurement. It is based upon the impedance method of Kerns, but avoids the direct measurement of impedance. Pertinent theory is developed, and the errors in measuring efficiency by this method are analyzed and discussed. Experimental results are given.

1. Introduction

The efficiency, η, of a bolometer mount may be defined as the ratio of the power dissipated in the bolometer element to the power input to the bolometer mount. If the power dissipated in the bolometer element, P_b, can be accurately determined, the power input, P_I, to the bolometer mount is

$$P_I = \frac{P_b}{\eta}. \qquad (1)$$

P_b is usually measured by substitution techniques, in which it is customary to reduce the audio or d-c bolometer bias power (after the r-f power is applied) until the bolometer resistance returns to its original operating value. It is assumed that the change in bolometer resistance caused by the r-f power is identical to the change in resistance caused by an equal amount of a-f or d-c power P_d. The validity of this assumption has been analytically treated [1] for Wollaston wire bolometers cooled by convection. Based upon this analysis, Carlin and Sucher concluded that "Wollaston wire bolometers, when properly designed and mounted, afford a means of measuring cw power over a frequency range extending to the millimeter wavelength region, with an accuracy approaching that of low-frequency measurements." It should be noted, however, that under less favorable conditions the substitution error for convection-cooled Wollaston wire bolometers may be appreciable (let us say greater than 0.5 percent) at frequencies above the estimated limit of 3,000 Mc, depending upon the length and mounting of the bolometer element.

If the ratio of P_b to P_d is K_S,

$$P_I = \frac{K_S}{\eta} P_d. \qquad (2)$$

It is possible to estimate the limits of K_S for a specific Wollaston wire bolometer from the calculated curves of Carlin and Sucher (see footnote 1).

An impedance method of determining bolometer mount efficiency has been described [2] by Kerns. Unfortunately, relatively small errors in the required impedance measurements can lead to a large error in the efficiency as determined by this method.

A modification of Kerns' method will be described, in which the direct measurement of impedance is avoided, permitting the efficiencies of tunable bolometer mounts to be obtained with increased accuracy. Efficiencies of untuned bolometer mounts can then be obtained with very little loss in accuracy from comparative power measurements.

2. Impedance Method

In the impedance method of determining efficiency the bolometer mount is thought of as replaced by an equivalent two-terminal-pair network terminated in the bolometer resistance. As shown in figure 1, the input impedance (of the equivalent network) corresponding to each of three different bolometer resistances is obtained.

The normal operating resistance of the bolometer is designated as R_2. The efficiency for this condition may be calculated from an expression (see footnote 2) involving the three terminating resistances and the three corresponding input impedances.

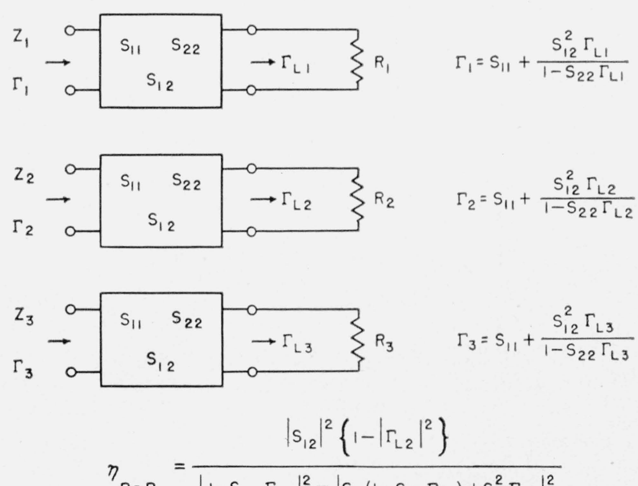

FIGURE 1. *Efficiency of a two-terminal-pair network (terminated in a resistance R_2) determined from three measurements of input impedance or reflection coefficient.*

[1] H. J. Carlin and Max Sucher, Accuracy of bolometric power measurements, Proc. Inst. Radio Engrs. **40**, 1042 (Sept. 1952).

[2] D. M. Kerns, Determination of efficiency of microwave bolometer mounts from impedance data, J. Research NBS **42**, 579 (1949) RP1995.

An equivalent expression for the efficiency can be obtained[3] in terms of the voltage reflection coefficients corresponding to the above terminating resistances and input impedances.

$$\underset{R=R_2}{\text{Efficiency }(\eta)}=\left|\frac{(\Gamma_2-\Gamma_1)(\Gamma_3-\Gamma_2)(\Gamma_{L3}-\Gamma_{L1})}{(\Gamma_3-\Gamma_1)(\Gamma_{L2}-\Gamma_{L1})(\Gamma_{L3}-\Gamma_{L2})}\right|\frac{1-|\Gamma_{L2}|^2}{1-|\Gamma_2|^2}, \tag{3}$$

where Γ denotes an input, and Γ_L a terminating reflection coefficient.

If the bolometer forms one arm of a Wheatstone bridge, it is convenient to adjust the bolometer resistances R_1, R_2, and R_3 to predetermined values. If the factor containing the real parameters Γ_{L1}, Γ_{L2}, and Γ_{L3} is denoted by C, eq (3) becomes

$$\eta=\frac{C}{1-|\Gamma_2|^2}\left|\frac{(\Gamma_2-\Gamma_1)(\Gamma_3-\Gamma_2)}{\Gamma_3-\Gamma_1}\right|, \tag{4}$$

where

$$C=\{1-|\Gamma_{L2}|^2\}\left|\frac{\Gamma_{L3}-\Gamma_{L1}}{(\Gamma_{L2}-\Gamma_{L1})(\Gamma_{L3}-\Gamma_{L2})}\right|=\frac{2R_2(R_3-R_1)}{(R_2-R_1)(R_3-R_2)},$$

and

$$\Gamma_L=\frac{R_L-Z_0}{R_L+Z_0},$$

Where Z_0 is an arbitrary real impedance.

It is generally true that the factor C can be more accurately determined than the other factors in eq (4), because C is a function of resistances determined by dc measurement.

The reflection coefficients Γ_1, Γ_2, and Γ_3 occur in difference terms of eq (4), with the unfortunate result that a given error in measuring individual reflection coefficients may produce a much larger error in the calculated efficiency.

For example, if $C=19.92$, $\Gamma_1=0.0676$, $\Gamma_2=0$, and $\Gamma_3=0.174\ e^{j61\pi/60}$, the efficiency is approximately 97 percent. An error of only ± 1 percent in measuring the voltage standing-wave ratios

$$\left(\text{VSWR}=\frac{1+|\Gamma|}{1-|\Gamma|}\right) \text{ corresponding to } |\Gamma_1| \text{ and } |\Gamma_3|$$

can produce an error of approximately ± 6 percent in the calculated efficiency.

In order to reduce this error in efficiency to the more useful value of ± 1 percent, it would be necessary in the example to make VSWR measurements to an accuracy better than approximately ± 0.2 percent. It is apparent that the determination of efficiency by this method places rather severe requirements on the accuracy of UHF or microwave impedance measurements.

[3] Equation (3) can be obtained by simultaneous solution of the equations appearing in figure 1, which are based upon the scattering equations of a two-terminal-pair network. See R. W. Beatty and A. C. MacPherson, Mismatch errors in microwave power measurements, Proc. Inst. Radio Engrs. **41**, 1112 (Sept. 1953).

3. Improved Method

It is possible to avoid the direct measurement of impedance of tunable bolometer mounts having a high efficiency (above approximately 90%) and thereby increase the accuracy of the efficiency determination.

Assuming that the bolometer mount can be made reflection-free ($\Gamma_2=0$) by an appropriate tuning adjustment when the bolometer is operating at its normal rated resistance, R_2, eq (4) becomes

$$\eta=C\left|\frac{\Gamma_1\Gamma_3}{\Gamma_1-\Gamma_3}\right|. \tag{5}$$

If, in addition, the bolometer mount has a high efficiency, it can be assumed with small error (as discussed later) that the vectors representing Γ_1 and Γ_3 terminate on a straight line[4] passing through the origin. The efficiency is

$$\eta=C\left|\frac{\Gamma_1\Gamma_3}{|\Gamma_1|\pm|\Gamma_3|}\right|. \tag{6}$$

The plus sign is used if the vectors representing Γ_1 and Γ_3 terminate on opposite sides of the origin, and the negative sign is used if they terminate on the same side.

Bolometer resistances R_1 and R_3 should be chosen above and below R_2 in order to obtain the greatest possible spread. In this case the vectors representing Γ_1 and Γ_3 terminate on opposite sides of the origin, and

$$\eta=C\,\frac{|\Gamma_1\Gamma_3|}{|\Gamma_3|+|\Gamma_1|}=\frac{C}{2}\,\frac{(\sigma_1-1)(\sigma_3-1)}{\sigma_1\sigma_3-1}, \tag{7}$$

where σ represents the VSWR corresponding to $|\Gamma|$.

Instead of measuring σ_1 and σ_3, more accurate results may be obtained by measuring the relative voltage output of a loosely coupled, properly positioned fixed probe.

A simplified representation of a slotted section and probe is shown in figure 2. It is seen that the voltage, E_P, (in wave guide of rectangular cross section operating in the dominant mode, E_P corresponds to the strength of the transverse electric field) is a function of the reflection coefficients of the generator, probe, and load referred to the probe position. From inspection of the equivalent circuit

$$E_P=E_G\,\frac{(1-\Gamma_G)(1+\Gamma_P)(1+\Gamma_L)}{(1+\Gamma_G)(1-\Gamma_P)(1+\Gamma_L)+2(1+\Gamma_P)(1-\Gamma_G\Gamma_L)}, \tag{8}$$

in which the subscripts G, P, and L, refer to the generator, probe, and load, respectively.

By means of a matching transformer following the generator, it is possible to make Γ_G vanish. In this case,

$$E_P=E_G\,\frac{1}{y_P+\dfrac{2}{1+\Gamma_L}}, \tag{9}$$

[4] See appendix, section 6.

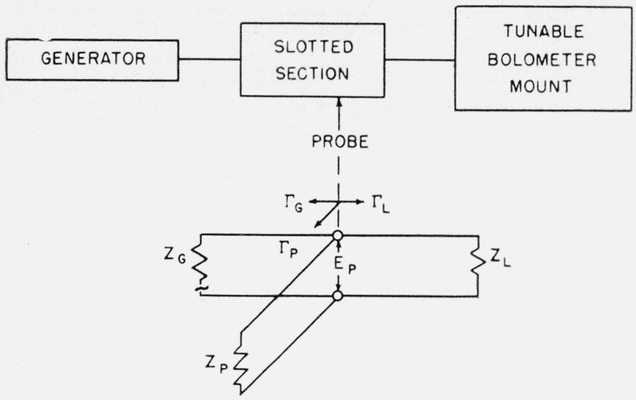

FIGURE 2. *Simplified block diagram of measuring apparatus and an equivalent circuit representation.*

where $y_P=(1-\Gamma_P)/(1+\Gamma_P)$.

If in addition, the probe is loosely coupled ($y_P\approx 0$),

$$E_P\approx\frac{E_G}{2}(1+\Gamma_L). \tag{10}$$

If the probe is located at a position where its response is maximum when the bolometer resistance is R_1, E_{P1} is proportional to $(1+|\Gamma_1|)$. With the probe fixed in that position, the bolometer resistance is changed to R_2 and then R_3, observing the probe response,

$$\begin{aligned} E_{P1}&=k(1+|\Gamma_1|)\\ E_{P2}&=k \\ E_{P3}&\approx k(1-|\Gamma_3|). \end{aligned} \tag{11}$$

Defining the ratios K_1 and K_3 as follows,

$$\begin{aligned} K_1&=\frac{E_{P1}}{E_{P2}}=1+|\Gamma_1|\\ K_3&=\frac{E_{P3}}{E_{P2}}\approx 1-|\Gamma_3|, \end{aligned} \tag{12}$$

the efficiency may be written

$$\eta=C\frac{(E_{P1}-E_{P2})(E_{P2}-E_{P3})}{E_{P2}(E_{P1}-E_{P3})}=C\frac{(K_1-1)(1-K_3)}{K_1-K_3}. \tag{13}$$

A correction to eq (13), to compensate for failure of the assumption that the vectors representing Γ_1 and Γ_3 are colinear, can be made if the other sources of error are neglected for the moment. Let the ratio of η given by eq (5) to that given by eq (13) be

$$\zeta_1=\left|\frac{\Gamma_1\Gamma_3}{\Gamma_1-\Gamma_3}\right|\frac{(K_1-K_3)}{(K_1-1)(1-K_3)}. \tag{14}$$

If the angular difference between Γ_1 and Γ_3, as shown in figure 3, is $\pi+\delta$, where $\delta \lesseqgtr 0.1$, it can be

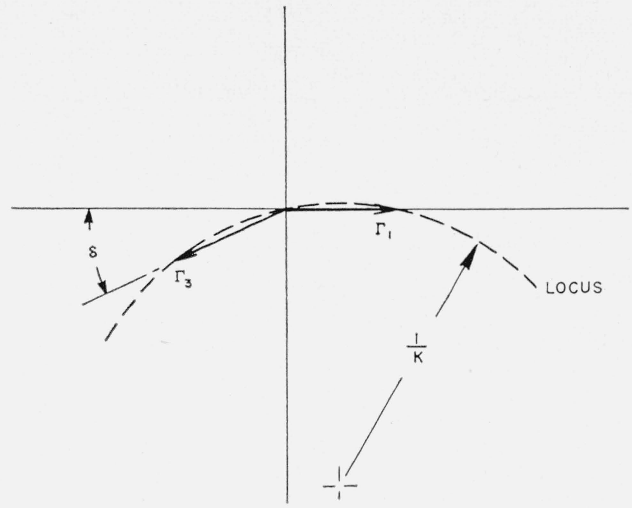

FIGURE 3. *Typical curvature of input reflection coefficient locus for resistive termination of bolometer mount.*

shown that

$$|\Gamma_3|\approx(1-K_3)(1+\delta^2/2K_3),$$

$$|\Gamma_3-\Gamma_1|\approx(K_1-K_3)\left[1+\delta^2\frac{K_1(1-K_3)^2}{2K_3(K_1-K_3)^2}\right],$$

and

$$\zeta_1\approx 1+\delta^2\frac{(K_1-1)(K_1-K_3^2)}{2K_3(K_1-K_3)^2}. \tag{15}$$

The angular difference, δ, is simply related to the curvature, K, of the locus of the reflection coefficient. This locus may be determined by measuring the input reflection coefficient (referred to the fixed position of the probe) as the bolometer resistance is varied. The expression relating K and δ is

$$\delta\approx\frac{K}{2}(K_1-K_3)\qquad(\delta\lesseqgtr 0.1). \tag{16}$$

Equation (15) may be written in terms of K:

$$\zeta_1\approx 1+K^2\frac{(K_1-1)(K_1-K_3^2)}{8K_3}. \tag{17}$$

A graph representing the percentage correction according to eq (17) is shown in figure 4.

Another correction to eq (13) is based upon the fact that there may be appreciable losses between the fixed probe position and the bolometer mount input. The efficiency of a length of line or waveguide having a known attenuation is shown in figure 5. If the line or guide section is not uniform, the efficiency must be determined by other means, such as measuring the bolometer mount efficiency with another identical slotted section inserted between the bolometer mount and the measuring slotted section.

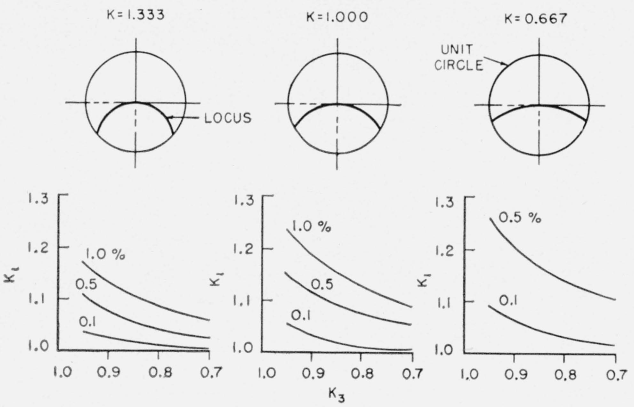

FIGURE 4. *Percentage correction to efficiency corresponding to curvatuve of reflection coefficient locus for three values of curvature.*

If the efficiency of that portion of the circuit between the fixed probe position and the bolometer mount input is η_{P-B}, the efficiency of the bolometer mount, applying the above corrections is

$$\eta=\frac{C}{\eta_{\text{P-B}}}\left|\frac{(K_1-1)(1-K_3)}{K_1-K_3}\right|\left\{1+K^2\frac{(K_1-1)(K_1-K_3^2)}{8K_3}\right\}. \tag{18}$$

It is seen that both corrections increase the efficiency over the value obtained in eq (11).

The method just described is applicable to tunable bolometer mounts, in which the bolometer element can be represented by a resistance terminating the bolometer mount. (Barretters are generally suitable, but there is evidence that thermistors do not fulfill this condition.)

The efficiency of tunable bolometer-mount assemblies, including matching transformers, can also be measured by this method. After the efficiency of a tunable bolometer-mount assembly has been determined, at a specified operating frequency, the efficiency of another tunable or untuned bolometer mount or assembly can be obtained by comparing the power readings of the two mounts when alternately connected to a stable, well-padded generator. Assuming that the power dissipated in the element can be accurately measured by d-c substitution techniques,[5] and letting the subscripts A and B refer to the two mounts,

$$\begin{aligned}\eta_A P_A &= P_{Ad}\\ \eta_B P_B &= P_{Bd},\end{aligned} \tag{19}$$

where η is the efficiency, P is the input power, and P_d is the power dissipated in the bolometer element.

If Γ_B is the input reflection coefficient of the bolometer mount whose efficiency is to be determined, the ratio of powers absorbed by the two

[5] See a text on microwave measurements, for example, C. G. Montgomery, Technique of microwave measurements (McGraw Hill Book Co., Inc., New York, N. Y., 1948).

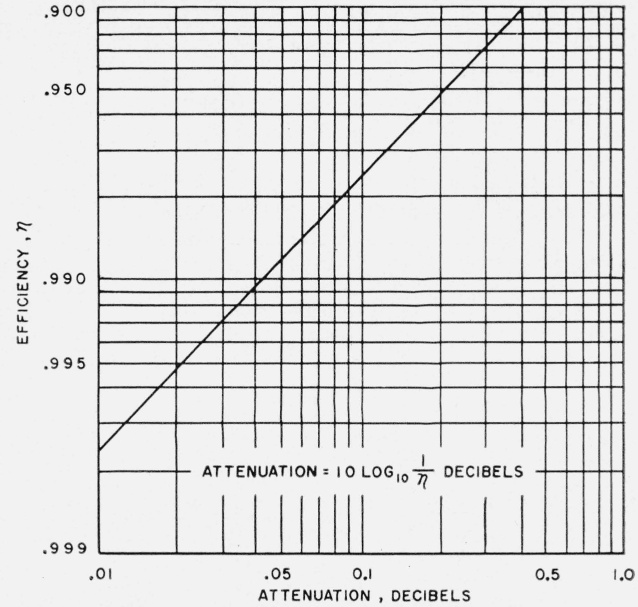

FIGURE 5. *Efficiency of a symmetrical, matched attenuator (or a length of uniform line) terminated in a reflection-free load.*

bolometer mounts or assemblies, (assuming a matched generator) is (see footnote 3)

$$\frac{P_A}{P_B}=\frac{1}{1-|\Gamma_B|^2}=\frac{(\sigma_B+1)^2}{4\sigma_B}. \tag{20}$$

The efficiency of the second bolometer mount is

$$\eta_B=\frac{P_A}{P_B}\frac{P_{Bd}}{P_{Ad}}\eta_A=\frac{(\sigma_B+1)^2}{4\sigma_B}\frac{P_{Bd}}{P_{Ad}}\eta_A, \tag{21}$$

where σ_B is the VSWR corresponding to $|\Gamma_B|$.

An error in measuring σ_B will cause an error in determining η_B, but fortunately the error is small in most practical cases. For example, if σ_B is determined to be 1.20 with an accuracy of ± 2 percent, the corresponding error in η_B is approximately ± 0.2 percent.

4. Discussion of Errors

An accurate knowledge of the efficiency of bolometer mounts used for microwave power measurement is essential to accurate power measurement. For this reason, it is felt that a detailed discussion of the error in measuring efficiency is desirable.

Certain sources of error seem to be common to most measurements at high frequency. Among these are instability of oscillators and amplifiers, unwanted frequency modulation (FM), spurious amplitude modulation (AM) and harmonics in the generator output, pulling of the oscillator by changes in loading, erratic or unknown detector characteristics, errors in measuring the detector output, impedance mismatches at junctions, and mechanical instability of the components. Error from these sources is minimized by careful instrumentation and the use of recognized good practice in measurement techniques. For example, the stability of electronic equipment is improved by using voltage-stabilized power supplies and by avoiding ambient temperature variations. Oscillator pulling is minimized by the use of nonreciprocal transmission-line elements or attenuator pads with at least 20-db attenuation. Unwanted FM is reduced by careful modulation practices or by the use of high-Q transmission cavities to attenuate undesired side bands. Parasitic oscillations, causing spurious AM, can be eliminated by usual procedures, e. g., damping, shielding; and minimizing feedback. Low-pass filters are used to reduce the harmonic output of generators. Detectors can be calibrated before use or the need for known detector characteristics may be avoided by use of calibrated attenuators. Matching transformers can be used to reduce impedance mismatches, and careful attention to reducing movement of the components will reduce mechanical instability.

After the above precautions are taken, observations should be made to verify the desired conditions. For example, the generator output can be observed with a spectrum analyzer to verify the reduction in unwanted FM and spurious AM. The oscillator-output amplitude and frequency can be monitored during load changes to observe pulling, and the detector output can be monitored with a continuous recorder to observe system stability.

Additional sources of error, which can be minimized by careful instrumentation and experimental procedure, are instability of the bolometer bias supply, inaccuracy of resistance measurement, mechanical irregularities in the slotted section and traveling probe, excessive coupling, and incorrect position of the probe. The use of heavy-duty, low-discharge storage batteries will generally provide a stable bias supply.

Resistance R_1, R_2, and R_3 are measured at direct current and assumed to be the same at UHF or microwaves. It was pointed out by Kerns (see footnote 2), and can be seen from eq (4), that even if the d-c resistances are multiplied by a constant real factor, there will be no error in efficiency. The effect of random errors in resistance measurement upon the efficiency is the same as the random errors in VSWR measurements, discussed in section 2. It was seen that an error in VSWR between the limits ± 0.2 percent will produce an error in efficiency between the limits of approximately ± 1 percent.

Resistance measurements between 100 and 300 ohms can be made with an accuracy of approximately ± 0.05 percent with a good Wheatstone bridge. The corresponding error in efficiency would be approximately ± 0.25 percent.

The choice of a slotted section and traveling probe is important in adjusting Γ_G and Γ_2 for minimum value, and in approximating the assumed uniform, lossless line or waveguide.

The error caused by excessive probe coupling is difficult to evaluate analytically (see eq (8) and (9)). However, it is possible to determine experimentally when the probe is sufficiently decoupled by making a series of efficiency measurements, each with a diminshing value of probe coupling. When there is no further appreciable change in the measured efficiency, the probe has been sufficiently withdrawn. Another method of checking the effect of probe coupling consists in making two efficiency measurements, one with the probe set to the position for maximum response corresponding to a bolometer resistance R_1, and the other with the probe set to the position for maximum response when the bolometer resistance is R_3. If nothing else is changed, the two probe positions are separated by approximately $\lambda/4$, so that the phase of the reflection from the load as seen at the probe position differs by approximately 180 degrees in the two cases. An example of this method is given in table 1, where it is assumed that the average of the two efficiency measurements closely approximates the correct efficiency with the probe sufficiently decoupled. This assumption was found to be valid for small variations in efficiency.

It is possible to evaluate the effect of certain sources of error analytically. The error in measurement of the relative voltage output of the probe, the incorrect positioning of the probe, the generator and load mismatch, and the curvature of the input reflection coefficient locus can be taken into account if the resulting error in efficiency is small.

If ϵ_1, ϵ_2, and ϵ_3 are the errors made in measuring the probe relative voltages E_{P1}, E_{P2}, and E_{P3}, respectively, the error in efficiency from this source alone is approximately

$$\epsilon \approx \frac{-1}{K_1-K_3}\left[\frac{K_1(1-K_3)}{(K_1-1)}(\epsilon_1-\epsilon_2)+\frac{K_3(K_1-1)}{(1-K_3)}(\epsilon_2-\epsilon_3)\right]. \tag{22}$$

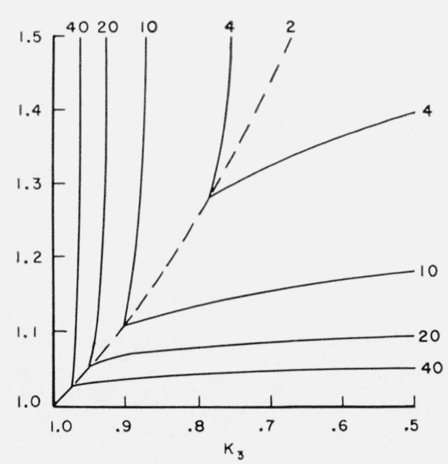

FIGURE 6. *Factor by which the random error in measuring the relative output voltage of the probe is multiplied in order to obtain the corresponding error in efficiency.*

If the individual errors lie within the range indicated by $|\epsilon_1|=|\epsilon_2|=|\epsilon_3|\leq\epsilon'$, the maximum error in efficiency would be less than

$$\epsilon=\pm\frac{2K_1(1-K_3)}{(K_1-1)(K_1-K_3)}\,\epsilon' \text{ or } \epsilon=\pm\frac{2K_3(K_1-1)}{(1-K_3)(K_1-K_3)}\,\epsilon', \tag{23}$$

whichever is largest. A graph of this relationship is shown in figure 6. Using a 200-ohm barretter, the limiting values of K_1 and K_3 were determined to be approximately 1.33 and 0.75. Referring to figure 6, with ϵ' assumed equal to ± 0.1 percent, the error in efficiency would be less than ± 0.4 percent. As this is a random error, improved accuracy can be obtained by averaging the results of a number of measurements.

An analysis of the error in efficiency caused by generator and load mismatch, curvature of the reflection coefficient locus, and incorrect probe position yields, after some manipulation, a correction factor to apply to eq (13). It is

$$\zeta_2\approx\left\{1+\left[\begin{array}{l}\left(\dfrac{1-K_3}{K_1-K_3}\right)\dfrac{\alpha^2}{2K_1}+\left(\dfrac{K_1-1}{K_1-K_3}\right)\dfrac{(\delta+\alpha)^2}{2K_3}+\dfrac{\delta^2}{2}\,\dfrac{(K_1-1)(1-K_3)}{(K_1-K_3)^2}+\dfrac{(1-K_1K_3)}{(K_1-1)(1-K_3)}\,|\Gamma_2|\cos\psi_2\\[2ex] -\dfrac{1}{(K_1-1)}\,|\Gamma_2|\cos(\psi_2-\alpha)+\dfrac{1}{(1-K_3)}\,|\Gamma_2|\cos(\psi_2-\delta-\alpha)-\dfrac{K_1(1-K_3)}{(K_1-K_3)}\,|\Gamma_G|\cos(\psi_G+\alpha)\\[2ex] -\dfrac{K_3(K_1-1)}{(K_1-K_3)}\,|\Gamma_G|\cos(\psi_G+\delta+\alpha)\end{array}\right]\right\} \tag{24}$$

where ψ and ψ_2 are the angular arguments of Γ_G and Γ_2, respectively, and α represents twice the angular error ($2\beta\Delta l$) in setting the probe to its correct position. (Δl is the distance the probe position is in error.)

In the derivation of eq (24) approximations were made (very small higher-order terms were neglected), assuming that $|\Gamma_G|<0.005$, $|\Gamma_2|<0.005$, $\delta<0.1$, and $\alpha<0.1$. The magnitude of the error represented by eq (24) can be illustrated by considering some of the sources of error separately. For example, if $\delta=\alpha=0$,

$$\zeta_2\approx 1-|\Gamma_G|\cos\psi_G+|\Gamma_2|\cos\psi_2. \tag{25}$$

If $|\Gamma_G|\doteq|\Gamma_2|=0.005$, the total mismatch error lies between the limits ± 1 percent.

If $\Gamma_G=\Gamma_2=0$,

$$\zeta_2\approx 1+\frac{1}{2(K_1-K_3)}\left[\left(\frac{1-K_3}{K_1}\right)\alpha^2+\left(\frac{K_1-1}{K_3}\right)(\delta+\alpha)^2+\frac{(K_1-1)(1-K_3)}{(K_1-K_3)}\,\delta^2\right]. \tag{26}$$

A graph of the effect of changing the probe position upon the calculated efficiency is shown in figure 7

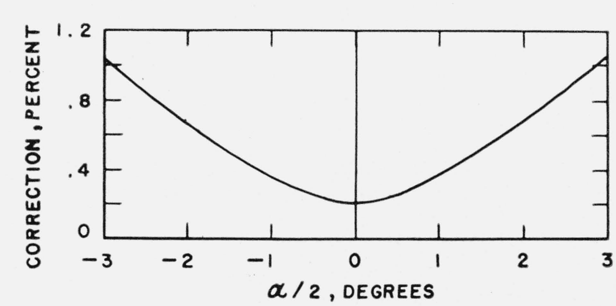

FIGURE 7. *Effect of varying probe position upon the efficiency correction according to equation (26).*

for $K_1=1.0676$, $K_3=0.826$, and $\delta=5°$.

If $\Gamma_G=\Gamma_2=\alpha=0$, eq (24) reduces to eq (15), as represented by figure 4.

5. Experimental Results

The efficiencies of two commercially available bolometer mounts were measured at 600, 1,000, 2,000, and 3,000 Mc. The efficiency of a commercially available tunable bolometer mount (A) was measured first, and then the efficiency of a commercially available bolometer mount (B) was determined from comparative power measurements. The data obtained in a typical measurement of the efficiency of a bolometer mount is shown in table 1. It was found that the efficiency of the tunable bolometer mount remained at approximately 96 percent over the above frequency range, while the efficiency of mount B decreased with rising frequency, as shown in figure 8. Because only one of each of the two types of mounts was investigated, the measured efficiencies are not necessarily representative of these types of bolometer mounts.

TABLE 1.—*Typical efficiency measurement at 1,000 Mc.*

K	R_1=150 ohms		R_1=250 ohms	
	R_n	E_{pn}	R_n	E_{pn}
1	150.0	1.270	250.0	1.277
2	200.0	1.119	200.0	1.155
3	250.0	1.000	150.0	1.000

In each case, the probe was in position for maximum response when $R=R_1$.
C=16.00 (calculated from eq (4))
η=0.951 (measured when R_1=150 ohm)
η=0.946 (measured when R_1=250 ohm)
η=0.948 (average of above two values)
$\eta_{P\text{-}B}$=0.988 (measured)
C_1=1.002 (correction for locus curvature)
η_A=0.962 ((mount A) calculated from eq (18))
P_{Ad}=0.807 mw,
P_{Bd}=0.823 mw,
σ_B=1.020 (measured)
η_B=0.981 ((mount B) calculated from eq (21)).

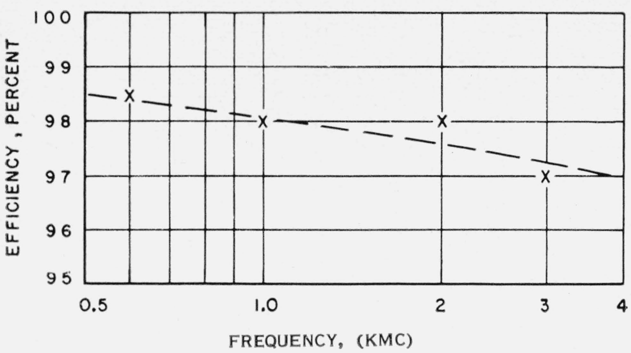

FIGURE 8. *Measured efficiency of a coaxial bolometer mount B.*

TABLE 2.—*Estimate of limits of error in single efficiency measurement*

Principal sources of error	Approximate limits of error in efficiency
	%
Measurement of $\eta_{P\text{-}B}$	±0.5
Measurement of probe voltage	±.4
Measurement of resistances	±.2
Generator mismatch	±.1
Load mismatch	±.1
Estimated limits of error in single efficiency measurement of tunable bolometer mount	±1.3
Measurements of power	±0.2
Measurement of VSWR	±.1
Estimated limits of error in single efficiency measurement of untuned bolometer mount	±1.6

An approximate evaluation of the error in measuring efficiency is given in table 2. It represents an estimate of the limits of error in a single measurement of efficiency. The actual error can be considerably less than this, if the effect of random errors is reduced by averaging the results of a number of measurements. A further reduction of error could be obtained by use of better equipment and improved measuring techniques.

6. Appendix

It will be shown that the locus of the input voltage reflection coefficient of a lossless, tuned, linear, passive, two-terminal-pair network terminated in loads having real reflection coefficients is a straight line passing through the origin.

The lossless condition requires that [6]

$$S^*S=1, \tag{27}$$

where S represents the scattering matrix of the network, S^* is its complex conjugate, and their product equals the unit matrix. Solution of this equation for a two-terminal-pair network yields the relationships

$$\begin{aligned} |S_{12}|^2&=1-|S_{11}|^2=1-|S_{22}|^2\equiv 1-S^2 \\ 2\psi_{12}&=\psi_{11}+\psi_{22}\pm\pi, \end{aligned} \tag{28}$$

where ψ represents the angular argument of a scattering coefficient and $S=|S_{11}|=|S_{22}|$.

The input reflection coefficient of a two-terminal-pair network terminated in a load having a real reflection coefficient $|\Gamma_L|$, is

$$\Gamma=S_{11}+\frac{S_{12}^2|\Gamma_L|}{1-S_{22}|\Gamma_L|}. \tag{29}$$

Because the network is matched ($\Gamma=0$) when terminated in a load having a reflection coefficient Γ_{L2},

$$S_{11}=-\frac{S_{12}^2|\Gamma_{L2}|}{1-S_{22}|\Gamma_{L2}|}. \tag{30}$$

Combining eq (28) and eq (30),

$$Se^{j\psi_{11}}=S^2|\Gamma_{L2}|e^{j(\psi_{11}+\psi_{22})}-(1-S^2)|\Gamma_{L2}|e^{j2\psi_{12}} \tag{31}$$

or

$$Se^{j\psi_{11}}=|\Gamma_{L2}|e^{j(\psi_{11}+\psi_{22})}.$$

It is evident that $S=|\Gamma_{L2}|$ and $\psi_{22}=0$ for the above lossless, tuned, two-terminal-pair network. Substituting the results of eq (28) and eq (31) into eq (29), the input reflection coefficient is

$$\Gamma=\frac{|\Gamma_{L2}|-|\Gamma_L|}{1-|\Gamma_{L2}\Gamma_L|}\,e^{j\psi_{11}} \tag{32}$$

As $|\Gamma_L|$ varies, the locus of Γ is a straight line passing through the origin.

It should be noted that the above conditions imposed upon the network (lossless, matched input when terminated in a load having the real reflection coefficient Γ_{L2}) are sufficient to produce a linear input reflection coefficient locus passing through the origin, but are not necessary. The amount of locus curvature is not necessarily an indication of the amount of loss, because it is possible to obtain a straight line locus with a lossy network having $\psi_{22}=0$.

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C. G Montgomery, R. H. Dicke, and E. M. Purcell, Principles of microwave circuits, p. 149 (McGraw-Hill Book Co., Inc., New York, N. Y., 1948).