Journal of Research of the National Bureau of Standards Vol. 54, No. 3, March 1955 Research Paper 2574

Variation in Distortion with Magnification

Arthur A. Magill

The distortion introduced in the image by a lens for a given axial inclination of the chief ray is a linear function of the magnification. Specifically, if D_0 represents the distortion with parallel light incident on one side of the lens (zero magnification) and D_∞ the distortion with parallel light incident on the other, then the distortion D_m at any magnification m is given by $D_m = D_0 - {}^m D_\infty$. This equation has been experimentally verified for examples of three types of symmetrical lenses.

1. Introduction

The growth of photogrammetry with its high-precision imaging systems has necessitated accurate determination of the distortion introduced by a lens. Because the distortion varies with the magnification, it has been customary heretofore to have the distortion measured for each object-to-image ratio employed. To avoid making such a series of measurements, a simple linear equation for a lens with a single effective stop has been developed, from which the distortion at any magnification may be computed from the measured values obtained at two magnifications. The two distortions that would normally be used are those obtained with parallel light incident, in turn, on the front and on the back of the lens. These not only require the simplest experimental setup, but also have special significance as the limits of real image formation. Virtual images (i. e., negative magnifications) will be excluded as having little practical value.

2. Theoretical Development

In 1907 E. Wandersleb [1] developed a hyperbolic relation between magnification and the ratio of the lateral displacement of the image from its distortionless position to the height of the distortionless image. By defining the distortion as the later displacement of the image, the relationship may be reduced to a linear function, fully expressible by the introduction of the two values obtained with parallel light.

In figure 1, a lens of focal length f with surface 1 and final surface k has the paraxial foci F and F'. It is equipped with a single effective stop. The paraxial pupils are located at E_0 and E_0' at distances p_0 and p_0' from the foci. The spherical aberration of the lens elements preceding and following the stop changes the positions of the pupils for nonparaxial rays. For the chief ray with a slope angle of u to the optic axis in the object space and u' in the image space, the pupils are shifted by amounts δ and δ' to E and E' at distances p and p' from the foci.

Now consider the object point O with this same chief ray located at a height y from the optic axis, as shown in figure 2. The sign convention is that of taking figure 2 as an all-positive diagram. The intersection of the emergent chief ray with the Gaussian image plane at O′, a height y' from the axis, will be taken as the image of O. For low-aperture lenses with negligible zonal aberrations, this is certainly a legitimate definition of the image position. Because most precision projection is done

[1] E. Wandersleb, Uber die Verzeichnungsfehler Photographischer Objectiv, Z. Instrumentenk. **27**, 33–37, 75–85 (1907). The equation is also used in many optical texts.

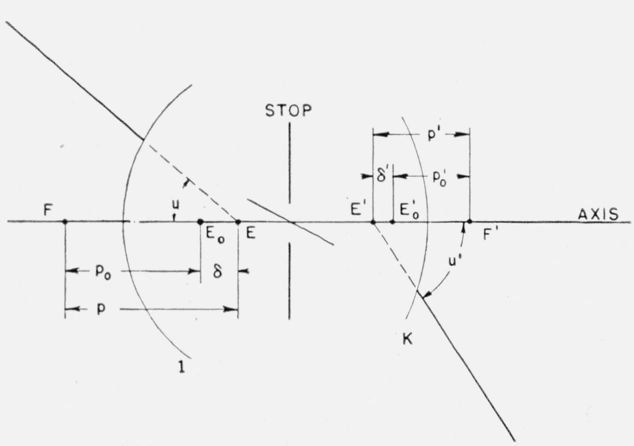

FIGURE 1. *Diagram of a lens and a given chief ray.*

The lens elements preceding and following the stop image is at E_0 and E'_0, respectively. For the indicated nonparaxial chief ray the spherical aberration of the elements relocates these images at E and E'.

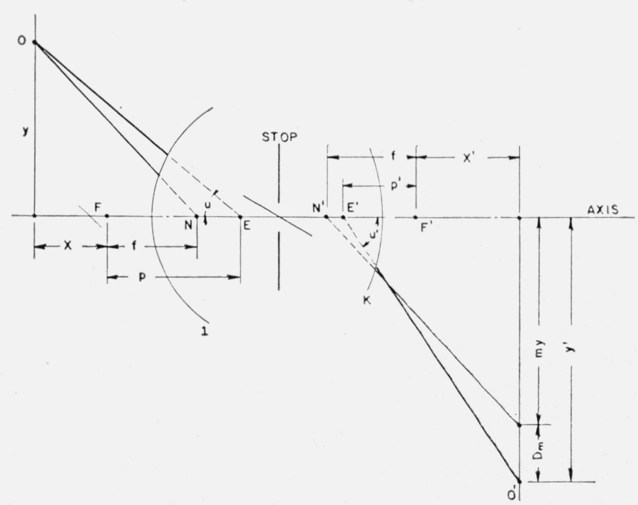

FIGURE 2. *Same lens as figure 1 with object and image planes included.*

The object O is imaged at O′ in the Gaussian image plane. The distortion D_m is the lateral displacement of O′ from its Gaussian position, $D_m = y' - my$. All distances and angles are to be considered positive, as shown.

at f-numbers of 22 or larger, the definition is also in accord with photogrammetric practice. Thus, the magnification m, can be defined in terms of Gaussian optics as

$$m = \lim_{y \to 0} \frac{y'}{y} = \frac{x'}{f} = \frac{f}{x}, \tag{1}$$

where x and x' are the distances of the object and image planes from their respective foci. A ray in the object space directed toward the first nodal point N, will emerge at the same slope angle in the image space as though emanating from the second nodal point, N'. This ray intersects the Gaussian image plane at the distortionless image position, a height my from the optic axis.

The linear distortion for any magnification D_m is defined as the difference between the actual image position and this distortionless image position, that is,

$$D_m = y' - my. \tag{2}$$

From figure 2 it is apparent that the values of y and y' referred to the chief ray are

$$y = (x+p)\tan u, \tag{3}$$

$$y' = (x'+p')\tan u'. \tag{4}$$

By substituting the values of x and x' from eq (1) and multiplying y through by m, the following equations are obtained:

$$my = (f+mp)\tan u, \tag{5}$$

$$y' = (mf+p')\tan u'. \tag{6}$$

Substitution of eq (5) and (6) in eq (2) yields

$$D_m = (p' \tan u' - f \tan u) - (p \tan u - f \tan u')m. \tag{7}$$

For any given chief ray, all values in eq (7) are constants with the exception of the magnification, and D_m is a linear function of m.

The magnification for an object located an infinite distance to the left is zero and eq (7) reduces to

$$D_0 = p' \tan u' - f \tan u. \tag{8}$$

This is geometrically illustrated in figure 3 where parallel light incident on the left at the slope angle u forms an image in the paraxial focal plane on the right at a height of $p' \tan u'$. The distortionless image position is given as the limiting value of my as m approaches zero and y approaches infinity, which from eq (5) is $f \tan u$.

For an object in the focal plane at the left, the magnification and the linear distortion are both infinite. However, if an object point is considered to be located at the position of the infinite image, it would in turn be imaged at the intersection of the chief ray with the focal plane on the left. The linear distortion D_∞, is here defined as that obtained

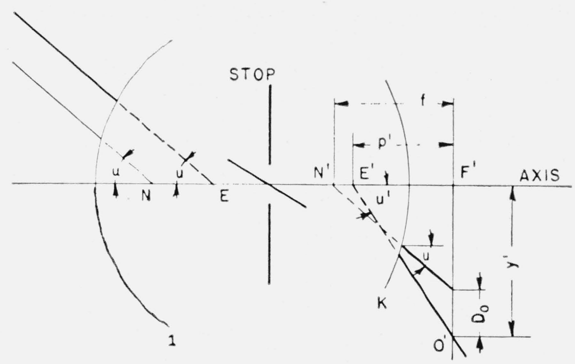

FIGURE 3. *Same lens as figure 1 at a magnification of zero.*

An infinite point object on the left is imaged in the focal plane at the right at O'. For this magnification of zero the distortion is given by $D_0 = y' - f \tan u$.

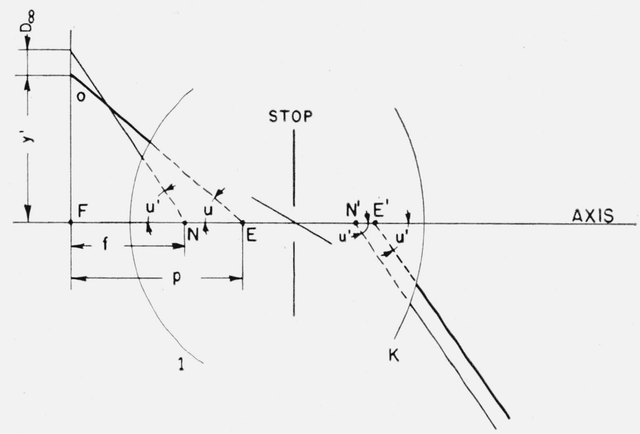

FIGURE 4. *Same lens as figure 1 at a magnification of infinity.*

An object O in the focal plane at the left is imaged at infinity on the right. If the infinite image were replaced by an object, it would in turn be imaged at O. The resulting distortion is designated D_∞ and in this case is negative.

when parallel light is incident on the right at a slope angle of u'. Referring to figure 4, it is clear that

$$D_\infty = p \tan u - f \tan u'. \tag{9}$$

By dividing eq (7) by m, it is also true that

$$D_\infty = -\lim_{m \to \infty} \frac{D_m}{m} = p \tan u - f \tan u', \tag{10}$$

where the negative sign indicates that D_∞ is measured with the light going backward through the lens. From eq (7), (8), and (9),

$$D_m = D_0 - mD_\infty. \tag{11}$$

This equation gives the linear distortion at any magnification in terms of the two distortions obtained with parallel light for any given chief ray. The only restrictions are that there be a single effective stop and a requirement that the chief ray adequately determine the center of gravity of the image or that, if it does not, the discrepancy be a linear function of the magnification.

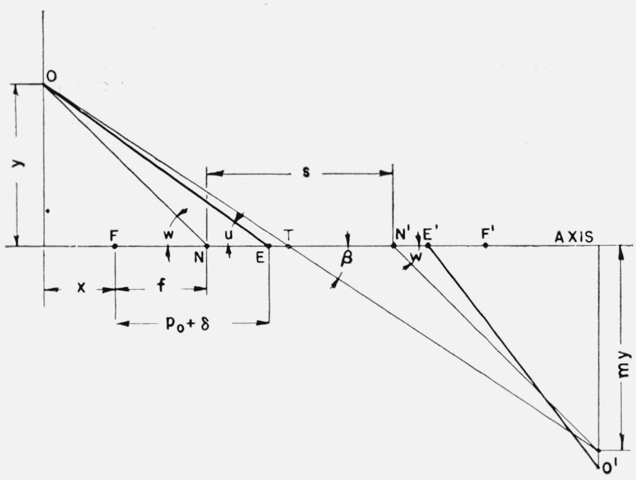

FIGURE 5. *Same lens as figure 1 illustrating other reference angles.*

It is often more convenient to refer the distortion to the slope angles w or β than it is to the slope angle, u. For a symmetrical lens, u, w, and β may be used interchangeably with little error.

3. Experimental Method

The equation has been verified for three types of commonly used symmetrical objectives at relative apertures of $f/22$. The experimental method is the same as that described by Bennett,[2] using a nodal slide and optical bench. The probable error of measurement varies with the slope angle, ranging from approximately 1 to 5 μ at 45° with parallel incident light. For finite object distances the probable error is greater by the nominal factor of $(m+1)$.

For the measurement of D_∞, the value of u' that corresponds to any given u may be found from eq (8), providing the distance $p'=(p_0'+\delta')$ is known. Furthermore, for the user of the lens to be able to refer distortion to the chief ray requires that he know either $(p_0+\delta)$ or $(p_0'+\delta')$. The measurement of the location of the paraxial pupils presents no particular problem. To determine the values of δ and δ', the stop could be equipped with a cross hair and the displacement of its image measured as the lens is rotated around each paraxial pupil in turn. The magnitude of δ could also be computed from eq (9) as a check on the measured values.

In practice it is often more convenient to refer the distortion to some other basis than the slope angle of the chief ray. In the nodal slide method the distortion is referred to the slope angle of the straight line joining the object and its Gaussian image, indicated as β in figure 5. The slope angle, w, of the ray in the object space directed toward the first nodal point might serve as a more convenient basis for the user of the lens, who normally computes the conjugate distances from the nodal points. The equations relating these two slope angles to that of the chief ray are

$$\tan w = \frac{p_0+\delta+x}{f+x} \tan u, \tag{12}$$

$$\tan \beta = \frac{p_0+\delta+x}{f+x+\dfrac{s}{m+1}} \tan u, \tag{13}$$

where s is the separation of the nodal points.

For the ideal symmetrical lens, the pupils coincide with the corresponding nodal points and $p_0=p_0'=f$, $u=u'$, and $\delta=\delta'$. Also from eq (8) and (9), $D_0=D_\infty=\delta \tan u$. Remembering that $x=f/m$, eq (12) and (13) may be rewritten

$$\tan w = \tan u + \frac{D_0}{f}\left(\frac{m}{m+1}\right), \tag{14}$$

$$\tan \beta = \frac{1}{1+\dfrac{s}{f}\dfrac{m}{(m+1)^2}} \tan w. \tag{15}$$

That u, w, and β may be used interchangeably with little error is superficially apparent because D_0 is normally a very small percentage of f and the discrepancy between β and w maximizes for $m=1$, for which magnification the distortion is zero. Although nominally symmetrical lenses are actually slightly asymmetric resulting in obvious differences between D_0 and D_∞, the effect on the reference slope angle has been ignored in obtaining the data for this paper; that is, the assumption has been made that u, u', and β are all equal. A detailed defense of this assumption for each lens tested would appear to be trivial in view of the good experimental agreement that was obtained. It should be pointed out, however, that if u' is not equal to u, the assumption of equality will in itself tend to minimize the errors introduced by employing a constant β for finite object distances.

4. Experimental Verification

The experimentally determined distortions are given for examples of the three types of symmetrical lenses in tables 1, 2, and 3. In all cases the value of the distortion at a slope angle of 5° is assumed to be zero. The errors in magnification are so small with respect to the other errors involved that all magnifications are considered exact. The measurements were made with a tungsten light source and a narrow-band filter having a dominant wavelength of 575 mμ.

The italicized digits represent the differences between the computed and observed values, and give a measure of the error involved in predicting distortion from D_0 and D_∞. In general these differences are what would be expected from the probable error of measurement. The greatest discrep-

[2] A. H. Bennett, The distortion of some typical photographic objectives, J. Opt. Soc. Am. and Rev. Sci. Instr. **14**, 235 (March 1927).

TABLE 1. *Distortion of two Hypergon-type lenses*

The measured values of the linear distortion D_m in millimeters for two Hypergon-type lenses are given for the indicated magnifications m and slope angles. The italicized digits beneath each distortion give the differences in microns between the values computed from D_0 and D_∞ and the measured values.

m	Slope angle							
	10°	15°	20°	25°	30°	35°	40°	45°
	Lens A1, f/22, 127 mm							
0	0.000	0.001	0.005	0.012	0.024	0.041	0.067	([a])
∞	.001	.002	.004	.003	.000	−.007	−.013	([a])
0.2	.000	.001	.004	.013	.021	.044	.074	([a])
	0	*0*	*0*	*−2*	*+3*	*−2*	*−4*	
0.5	−.001	−.001	.003	.010	.025	.044	.074	([a])
	+1	*+1*	*0*	*0*	*−1*	*0*	*0*	
1	−.001	.000	.003	.014	.029	.054	.087	([a])
	0	*−1*	*−2*	*−5*	*−5*	*−6*	*−7*	
2	−.002	−.005	−.006	.003	.022	.044	.085	([a])
	0	*+2*	*+3*	*+3*	*+2*	*+11*	*+8*	
5	−.001	−.014	−.013	.001	.025	.068	.138	([a])
	−4	*+5*	*−2*	*−4*	*−1*	*+8*	*−6*	
	Lens A2, f/22, 127 mm							
0	0.001	0.002	0.003	0.003	0.002	−0.001	−0.004	−0.008
∞	.001	.002	.007	.013	.031	.048	.064	.093
0.2	.000	.000	.000	.000	−.004	−.009	−.019	−.025
	+1	*+2*	*+2*	*0*	*0*	*−2*	*+2*	*−2*
0.5	.000	−.001	−.003	−.006	−.011	−.020	−.036	−.054
	0	*+2*	*+2*	*+2*	*−3*	*−5*	*0*	*0*
1	.000	.000	−.002	−.007	−.020	−.040	−.062	−.096
	0	*0*	*−2*	*−3*	*−9*	*−9*	*−6*	*−5*
2	−.002	−.004	−.012	−.028	−.049	−.090	−.146	−.209
	+1	*+2*	*+1*	*+5*	*−11*	*−7*	*+14*	*+15*
[b]5	−.002	−.003	−.017	−.046	−.092	−.160	−.262	−.375
	−2	*−5*	*−15*	*−16*	*−61*	*−81*	*−62*	*−98*

[a] The special barrel in which this lens was mounted limited the effective total field to about 84°.

[b] The differences obtained at this magnification are an example of the systematic error introduced by a poor setting of the transverse axis over the center of rotation of the nodal slide. The differences could be materially reduced by employing a calibrating procedure such as that illustrated in figure 10.

TABLE 2. *Distortion of three Dagor-type lenses*

The measured values of the linear distortion D_m in millimeters for three Dagor-type lenses are given for the indicated magnifications m and slope angles. The italicized digits beneath each distortion give the differences in microns between values computed from D_0 and D_∞ and the measured values.

m	Slope angle						
	10°	15°	20°	25°	30°	35°	40°
	Lens B, f/22, 6½ in.						
0	0.004	0.018	0.048	0.110	0.232	0.462	0.919
∞	.005	.017	.041	.101	.212	.413	.838
0.25	.002	.011	.032	.081	.173	.355	.702
	+1	*+3*	*+6*	*+4*	*+6*	*+4*	*+7*
1	.001	.001	.004	.010	.020	.052	.095
	−2	*0*	*+3*	*−1*	*0*	*−3*	*−14*
4	−.009	−.048	−.120	−.280	−.603	−1.194	−2.380
	−7	*−2*	*+4*	*−14*	*−13*	*+4*	*−53*

TABLE 2. *Distortion of three Dagor-type lenses*—Continued

The measured values of the linear distortion D_m in millimeters for three Dagor-type lenses are given for the indicated magnifications m and slope angles. The italicized digits beneath each distortion give the differences in microns between values computed from D_0 and D_{00} and the measured values.

m	Slope angle						
	10°	15°	20°	25°	30°	35°	40°
	Lens B3, f/22, 6½ in.						
0	0.004	0.017	0.049	0.116	0.244	0.488	0.967
∞	.002	.011	.032	.077	.171	.346	.706
0.25	.005	.013	.043	.100	.208	.412	.787
	−1	*+1*	*−2*	*−3*	*−7*	*−10*	*+3*
1	.001	.005	.016	.038	.073	.144	.265
	+1	*+1*	*+1*	*+1*	*0*	*−2*	*−4*
4	−.004	−.021	−.072	−.188	−.439	−.883	−1.884
	0	*−6*	*−7*	*−4*	*−1*	*−13*	*+27*
	Lens B4, f/22, 6½ in.						
0	0.004	0.018	0.049	0.111	0.232	0.464	0.914
∞	.002	.012	.038	.086	.185	.372	.745
0.25	.005	.015	.039	.089	.191	.379	.739
	−1	*0*	*0*	*0*	*−5*	*−8*	*−11*
1	.002	.005	.013	.026	.047	.089	.163
	0	*+1*	*−2*	*−1*	*0*	*+3*	*+6*
4	−.009	−.031	−.100	−.216	−.526	−1.042	−2.098
	+5	*+1*	*−3*	*−17*	*+18*	*+18*	*+32*

TABLE 3. *Distortion of two Artar-type lenses*

The measured values of the linear distortion D_m in millimeters for two Artar-type lenses are given for the indicated magnifications m and slope angles. The italicized digits beneath each distortion give the differences in microns between the values computed from D_0 and D_∞ and the measured values.

m	Slope angle				
	10°	15°	20°	25°	30°
	Lens, C1, f/22, 12 in.				
0	0.010	0.033	0.088	0.196	0.392
∞	.005	.024	.051	.121	.222
0.25	.008	.026	.073	.163	.331
	+1	*+1*	*+2*	*+3*	*+5*
0.5	.008	.023	.064	.137	.287
	0	*−2*	*−2*	*−1*	*−6*
1	.004	.008	.034	.073	.154
	+1	*+1*	*+3*	*+2*	*+16*
2	.000	−.006	−.016	−.054	−.061
	0	*−9*	*−2*	*+8*	*+2*
4	−.008	−.056	−.120	−.281	−.509
	−2	*−7*	*+4*	*−7*	*+13*
	Lens C2, f/22, 16½ in.				
0	0.014	0.043	0.106	0.214	0.354
∞	.011	.040	.098	.215	.386
.5	.008	.022	.054	.115	[a]
	0	*+1*	*+3*	*−9*	
1	.001	.000	−.001	−.004	−.036
	+2	*+3*	*+9*	*+3*	*+4*
2	−.006	−.031	−.090	−.232	[a]
	−2	*−6*	*0*	*+16*	

[a] The conjugate distances were too great for the range of the optical bench.

Table 4. *Distortion of twenty Hypergon-type lenses*

The measured values of the linear distortion D_m in microns for 20 Hypergon-type lenses are given for parallel light and the indicated slope angles. These lenses have a nominal focal length of 127 mm and a nominal aperture of $f/22$.

Lens	m	Slope angle							
		10°	15°	20°	25°	30°	35°	40°	45°
A_3	0	2	3	3	7	12	19	29	54
	∞	2	4	6	7	12	17	21	27
A_4	0	−1	−2	−6	−11	−19	−24	−42	−63
	∞	1	5	10	18	32	52	82	124
A_5	0	2	3	8	13	20	29	41	64
	∞	0	1	3	3	2	0	−2	1
A_6	0	1	1	0	1	3	4	6	12
	∞	2	5	9	15	23	34	47	64
A_7	0	0	1	−5	−7	−6	−5	−4	6
	∞	0	4	6	15	23	33	49	67
A_8	0	3	7	12	19	28	42	65	91
	∞	−2	−4	−7	−9	−12	−16	−19	−19
A_9	0	0	2	3	6	11	14	20	26
	∞	0	2	4	6	11	16	27	45
A_{10}	0	0	2	6	10	16	25	38	56
	∞	1	3	5	5	2	1	1	1
A_{11}	0	0	1	3	6	8	12	12	12
	∞	0	1	2	6	11	17	30	52
A_{12}	0	0	0	3	6	13	20	32	54
	∞	0	1	3	7	10	14	16	22
A_{13}	0	0	0	1	3	5	7	11	22
	∞	1	3	5	9	17	27	39	56
A_{14}	0	0	2	3	6	11	19	30	53
	∞	0	0	0	2	6	12	18	23
A_{15}	0	1	3	5	6	9	12	14	22
	∞	0	−2	−2	4	11	22	39	55
A_{16}	0	−2	−4	−5	−6	−6	−3	1	14
	∞	1	3	7	12	25	32	40	58
A_{17}	0	0	1	2	5	9	13	18	25
	∞	0	0	1	5	10	17	26	42
A_{18}	0	1	3	4	6	7	7	9	10
	∞	−1	−2	−1	6	13	24	40	65
A_{19}	0	0	0	0	5	8	13	18	29
	∞	1	2	3	8	17	24	36	60
A_{20}	0	1	0	1	5	9	17	23	42
	∞	1	4	6	11	15	21	35	49
A_{21}	0	3	7	9	14	17	28	32	48
	∞	1	1	1	4	9	18	26	47
A_{22}	0	0	1	3	5	8	13	22	38
	∞	2	6	9	16	22	34	44	57

ancy occurs for lens A_2 at a magnification of 5, representing about three times the nominal probable error. The good agreement at other magnifications for this lens indicates that the greatest part of the discrepancy is a result of errors in the observed values. That the differences have the same algebraic sign is evidence of a systematic error, which can be attributed to the method of measurement. This consisted of making a run at 5° intervals of the slope angle without resorting to the time-consuming process of resetting the lens on the nodal slide for each angle measured. Thus, any longitudinal displacement of the transverse axis (located at T in fig. 5) from the center of rotation of the nodal slide for a given run results in a systematic error in the distortion. The magnitude of this error is very closely proportional to the tangent of the slope angle. With the exception of B_2 and B_5, all of the lenses were submitted to the Bureau for routine tests and were not available for sufficient time to allow more than 2, and in some cases 3, such runs to be made at each magnification.

The data obtained on four other lenses are presented graphically in figures 6, 7, 8, and 9. For convenience in plotting, the equation has been defined in two regions around unit magnification. D_m is plotted against m for $0 \leq m \leq 1$ and D_m/m against $1/m$ for $1 \leq m \leq \infty$. This allows the point $-D_\infty$ to be included as the $\lim_{m \to \infty} D_m/m$. The points give the experimental distortions and the straight lines the computed values from D_0 and D_∞. For the perfectly symmetrical lens, all the lines would intersect at zero distortion and unit magnification; only in this case would there be no discontinuity. For the lenses measured, minimum distortion occurs at some other magnification than unity, indicating slight asymmetries introduced in manufacture. For very low distortion lenses, these slight asymmetries can result in pronounced differences in distortion between lenses of the same type. This is clearly illustrated in table 4, where the measured values of D_0 and D_∞ in microns are given for 20 Hypergon-type lenses of the same nominal focal length submitted to the Bureau for test.

Equation (11) may also be applied to minimized distortions that have been calibrated by a given cri-

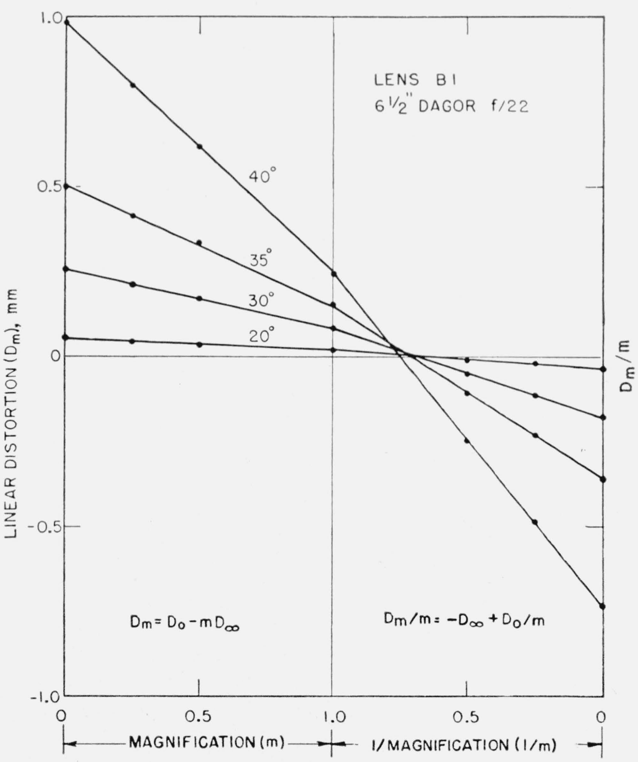

Figure 6. *Distortion plotted against magnification for the indicated slope angles.*

In order to include infinite magnifications, the graph has been split as shown into two regions at unit magnification. The straight lines represent the values computed from the two equations shown in their respective regions. The points indicate the experimentally determined values. The same graphical form has been employed in figures 7, 8, 9, 10, and 11.

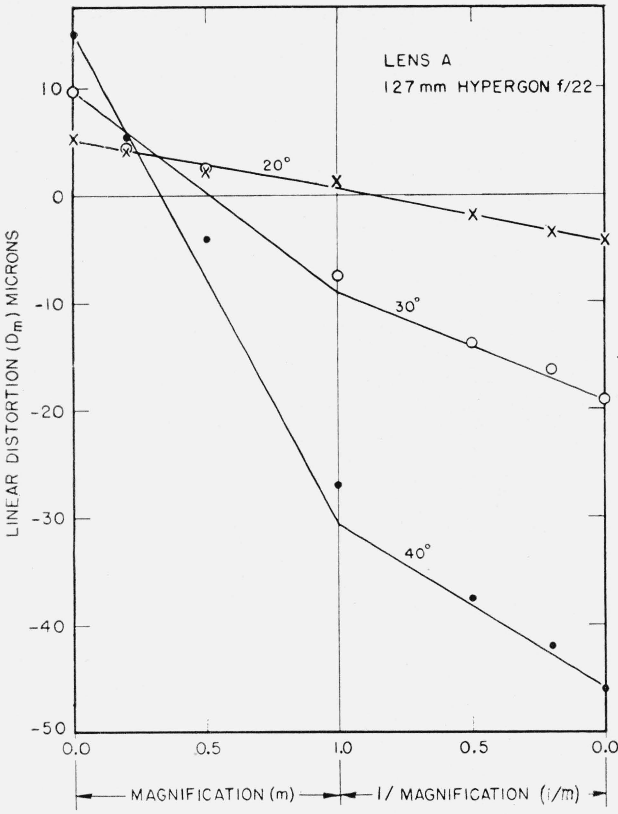

Figure 7. *Distortion plotted against magnification for the indicated slope angles.*

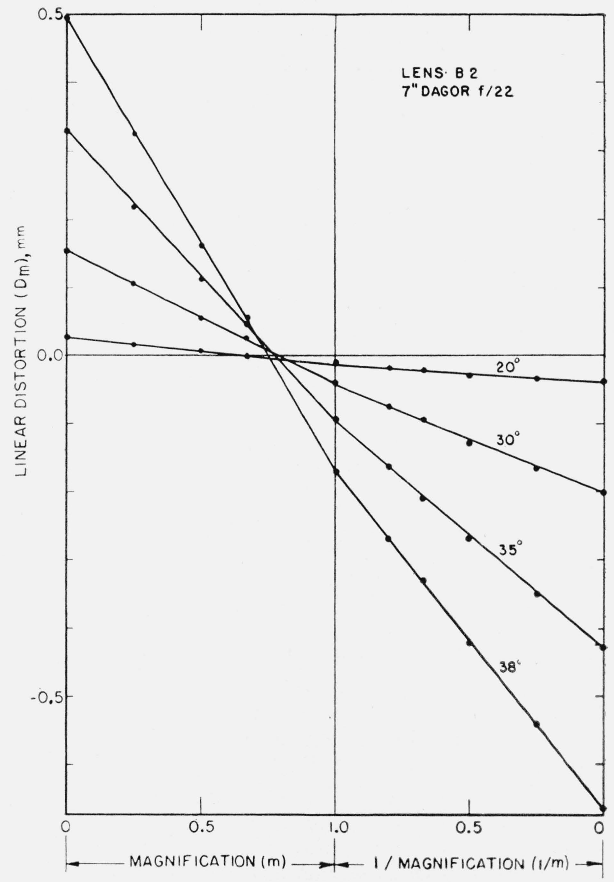

Figure 9. *Distortion plotted against magnification for the indicated slope angles.*

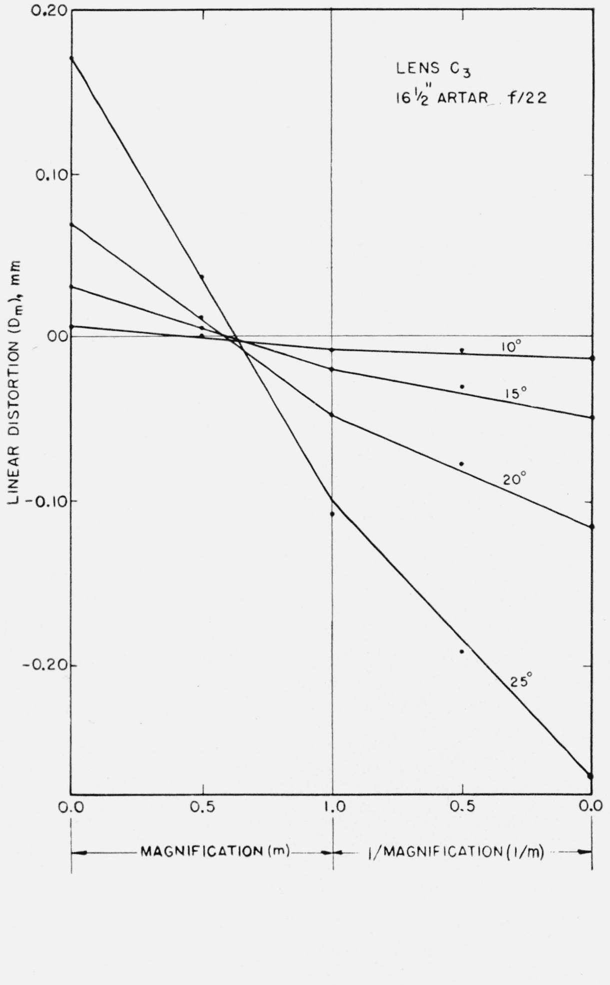

Figure 8. *Distortion plotted against magnification for the indicated slope angles.*

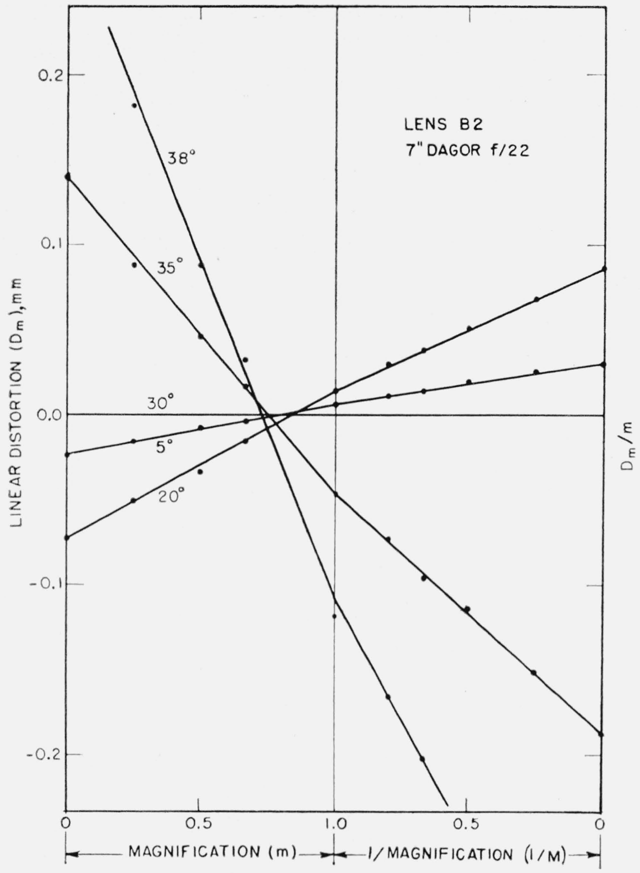

FIGURE 10. *Calibrated distortion plotted against magnification for the indicated slope angles.*

The data for the same lens as presented in figure 8 has been calibrated to give zero distortion at a slope angle of 30°. The magnitude of the change in distortion for any given magnification is proportional to the tangent of the slope angle.

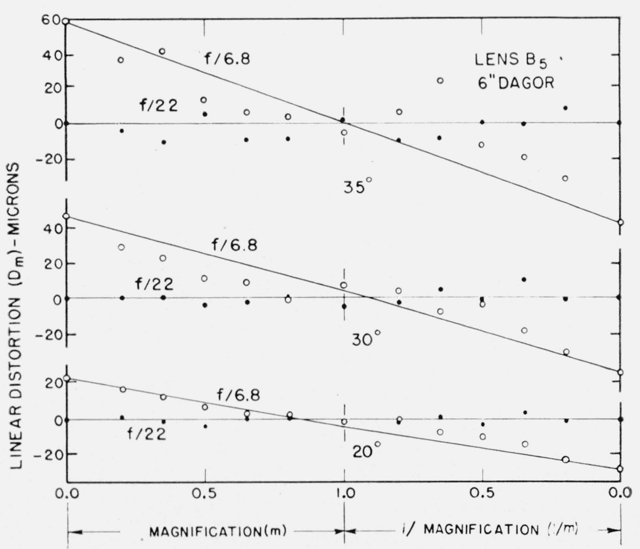

FIGURE 11. *Difference between the experimentally determined distortions for the indicated slope angles and those computed from the measured values of D_0 and D_∞ at f/22.*

The open circles represent data obtained at an aperture of $f/6.8$, and the solid circles at an aperture of $f/22$.

terion. At any given magnification, the magnitude of the change in distortion introduced by this calibration is proportional to the tangent of the slope angle, representing an effective shift in the position of the nodal points. For parallel incident light this results in calibrated focal lengths, which are equal only for a perfectly symmetrical lens. At finite object distances this results in a calibrated image distance or, more conveniently, a calibrated magnification. The calibrated focal lengths and magnifications are applied solely as scale factors and are not used in the determination of conjugate distances. Figure 10 shows the distortion presented in figure 8 calibrated to zero at a slope angle of 30°.

A detailed discussion of the effects on distortion of zonal aberrations introduced at apertures larger than $f/22$ is beyond the scope of this paper. However, their net effect can only result in a shift of the center of gravity of the nonparaxial image and a change in the plane of best focus. If the center of gravity shift is a linear function of the magnification and if the Gaussian identities expressed in eq (1) adequately determine the image plane of best focus when the higher aperture focal length is used, then eq (11) will still be valid.

In figure 11 the experimental distortions of a symmetrical-type lens at $f/22$ and the maximum aperture of $f/6.8$ have been plotted as the difference from the values computed from D_0 and D_∞ at $f/22$. The focal lengths and the values of D_0 and D_∞ for both apertures are given in table 5. No attempt was made to determine whether the image planes computed from the measured focal lengths were actually the planes of best over-all definition. At maximum aperture the effective stops were the outer lens retaining rings.

For this lens the experimental points for $f/6.8$ seem to lie along an S-shaped curve rather than being distributed about the straight line given by eq (11). The probable error of measurement is certainly higher at $f/6.8$ than at $f/22$, particularly for the larger slope angles. More data would have to be taken to determine if figure 11 is typical of all lenses of the same type, so at the present time the results must be considered as applying only to this particular lens. It is of interest that the difference between the distortion at $f/22$ and $f/6.8$ minimizes in the region about unit magnification, and elsewhere is generally greater than the deviation of the experimental distortions at $f/22$ from the computed values.

TABLE 5. *Change in focal length and distortion with aperature*

The change in focal length and distortion with aperature for two apertures of the Dagor-type lens B_5, with a nominal focal length of 6 in. The distortion is given in millimeters at the indicated slope angles.

Aperture	Focal length	m	Distortion 20°	Distortion 30°	Distortion 35°
	mm				
f/22	150.52	0	0.028	0.152	0.302
		∞	.047	.224	.435
f/6.8	150.12	0	.052	.199	.360
		∞	.075	.267	.492

5. Conclusion

The findings reported show that for low-aperture symmetrical lenses, the distortion at finite object distances can be simply computed from the two values obtained with parallel incident light with about the same accuracy as the direct experimental measurement. The necessary modifications required in carrying out this computation on other types of lenses have been indicated, and it is hoped that experimental verification of these procedures will soon be available.

Acknowledgment is made to F. E. Washer, who suggested the project and to W. R. Darling, who made many of the calculations and experimental measurements.

Washington, October 5, 1953.