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Precise Comparison Method of Testing Alternating-Current Watthour Meters

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A brief description of the basic method of testing alternating current watthour meters at the National Bureau of Standards is given, followed by a description of equipment for a faster and less laborious method.

Equipment with several novel features has been assembled for making precise tests of alternating-current watthour meters by a comparison method employing a group of carefully selected alternating-current watthour meters, which serve as a secondary standard group. One of this group, designated the "Standard Watthour Meter", is used with multirange instrument transformers as a reference standard to test other watthour meters with good precision. The testing procedure is explained, and the formulas used in computing the results of the tests are derived.

An analysis of the possible errors of measurement and data from numerous tests indicate that the measurement of energy applied to a watthour meter under test can be relied upon to better than 0.06 percent.

1. Introduction

For many years the National Bureau of Standards has employed a method of testing alternating-current watthour meters, which, although very precise and accurate, is tedious and time consuming. By this method, the energy indicated by the watthour meter under test is compared with the true value of energy as measured in terms of a constant and accurately known power and an accurately determined time interval. The power is held constant with the aid of a standard electrodynamic wattmeter [1] which is calibrated on reversed direct current before and after each run. For this test, a very stable, manually controlled alternating-current source is required to insure a steady value of power.

In the midthirties it was observed that repeated tests on particular watthour meters showed remarkably small deviations from the initial test values. This suggested the possibility of using a group of watthour meters of demonstrated good repeatability as secondary standards. The register and gear train could be removed from each meter to reduce friction. Disk revolutions would be counted photoelectrically.

Four such meters were obtained, but unfortunately, work on this project was interrupted, and construction of a permanent setup and the apparatus for intercomparing the meters was delayed. In 1940 the paper of Goss and Hansen [2] describing the excellent performance of a group of watthour meters caused a revival of interest in this project. Further experimental work led to detailed plans, but actual shop construction was interrupted by duties imposed by World War II.

Work was resumed after the war, and the equipment, except for minor modifications, was constructed as originally planned. This apparatus has been under observation for several years, and its reliable performance has been verified. The new equipment is now used at the National Bureau of Standards in preference to the older equipment for practically all of the testing of a-c watthour meters. This paper describes the equipment, the procedures followed in its calibration and use, and the results obtained.

2. Equipment

Basically, the complete equipment consists of a group of modified commercially available portable watthour meters maintained at a constant temperature, and the means for their calibration and intercomparison with a high degree of precision. In order to eliminate friction except at the top and bottom bearings, the meters are not equipped with registers. Instead, phototubes serve for counting revolutions of these meters, which are operated continuously during a test. For any direct intercomparison, some means of reading the registration of at least one meter of the group to a fraction of a revolution is extremely desirable. This is accomplished by providing a light-beam pointer and a special circular phosphorescent scale for one of the meters. At the beginning and again at the end of each run the light source for the pointer (a mercury-vapor lamp) is flashed, leaving spots on the phosphorescent scale that persist for several seconds. The standard meters are always operated at the same voltage and current. Other ranges are provided by special voltage and current transformers.

Originally it was intended to use four house-type meters, one from each of four American manufacturers. Each of the manufacturers was consulted, and asked for advice in the selection of the meters. Two of the manufacturers recommended their portable standards rather than selected house-type meters. As a result, the original setup was designed to accommodate two portable standards and two (or more) house-type meters. More recently, however, a different type of watthour meter has proved to be so much more stable than the older house-type meters originally procured, that the latter have been replaced by two of the newer type.

[1] John H. Park and Arthur B. Lewis, Standard electrodynamic wattmeter and ac-dc transfer instrument, J. Research NBS **25**, 545 (1940) RP1344.

[2] J. H. Goss and A. Hansen, Jr., A precision rotating standard for the measurement of kilowatt hours, Trans. AIEE **59**, 412 (1940).

The two original portable standard meters were individually housed in separate temperature-controlled enclosures, each enclosure consisting essentially of a Bakelite housing completely surrounding the aluminum case of the meter, a mercury thermoregulator to control the temperature, and four small lamps distributed inside the aluminum case to serve as heaters. The temperature is regulated at approximately 35° C to better than ±0.2 deg C. The upper end of the meter shaft projects through the top of the Bakelite enclosure. The mirror for the light-beam pointer is attached to this end of the shaft at an angle of 45° to the axis of the shaft. Each meter enclosure is assembled on a brass base plate to make a complete unit for interchangeable mounting in the housing that holds the phosphorescent scale. The particular portable watthour meter that is used with the circular scale is referred to in this discussion as the Standard Watthour Meter, as distinguished from the Comparison Standard Watthour Meters.

The cast-aluminum base of the circular-scale housing is equipped with leveling screws. The circular scale, approximately 26 inches in diameter, is assembled on brackets attached to the base of the housing. The meter enclosure rests on screws for leveling the meter and adjusting its height for proper scale alinement. These screws, in turn, are mounted on an adjustable centering plate, which permits centering the axis of the meter shaft with respect to the scale.

The mercury-vapor lamp is mounted on a hinged arm above the meter. A diaphragm below the lamp has a ½-mm aperture that serves as a point source of light. Adjustments are provided for centering the light beam. The light path is vertically downward from the aperture through a lens to the mirror on the end of the meter shaft where it is reflected horizontally to the circular scale, a curved brass strip coated with phosphorescent paint. The dark-blue component of light from the aperture is focused by the lens to provide a well-defined image of 1-mm diameter when a flash occurs. Flush with this strip and directly below it is a circular strip of Lucite engraved with 1,000 small divisions (about 2 mm per division). Every tenth division is numbered. Effective use of the phosphorescent scale requires that the room be semidark; therefore, the Lucite scale is provided with edge illumination by 12 small panel lamps spaced at regular intervals. The zero mark on the scale, which is also the 1,000 division mark, is centered on the bracket supporting the mercury lamp. There is a narrow slit cut through the scale and bracket at this point. A phototube, which is used with auxiliary apparatus for counting revolutions, is mounted directly behind this slit. Figure 1, a photograph of the Standard Watthour Meter, shows most of these features.

The temperature-controlled cabinet originally designed for the house-type meters is now being used to house the newer type watthour meters, which serve as comparison standards. Its dimensions are approximately 112 cm long, 37.5 cm wide, and 30 cm high. It consists of a wooden cabinet with a removable glass top, a thermoregulator, electric

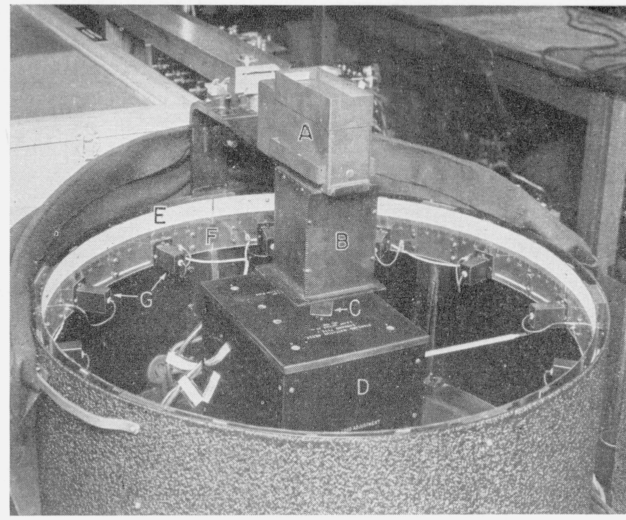

Figure 1. *Standard Watthour Meter with circular phosphorescent scale used as a reference standard for the testing of watthour meters by a comparison method.*

The light beam "pointer" is produced by a mercury vapor lamp and lamp combination located above the Standard Watthour Meter.
a, Mercury vapor lamp housing; b, lens box; c, mirror; d, Standard Watthour Meter; e, phosphorescent scale; f, engraved plastic scale; g, lamps.

heating coil, and a small blower for circulating the air within the cabinet. Temperature is maintained at approximately 40° C to within ±1 deg C.

The accessory electronic equipment performs three functions. The first function, that of counting, is accomplished by a relay pulse-counting circuit by means of which either a predetermined length of time or a predetermined number of revolutions of a watthour meter may be used to make or break electric circuits. The counting circuit may be preset to count seconds or revoltuions from 1 to 999. When calibrating the Standard Watthour Meter, using the Standard Wattmeter to aid in maintaining constant power, the counting circuit is operated by pulses accurately spaced 1 second apart. These pulses, or seconds signals, are derived from the quartz-crystal-controlled frequency standards maintained at the National Bureau of Standards, and, as a basis for timing, are reliable to better than a part in a million. With the aid of phototubes the pulse counter can be used for counting the revolutions of the disk of either a comparison standard, the reference standard, or a watthour meter under test.

The second function of the accessory electronic equipment is that of flashing the mercury-vapor lamp. The pulse counter completes a circuit for the starting pulse to trigger a thyratron whose plate circuit discharges a capacitor through the mercury-vapor lamp. The capacitor is charged to approximately 1,000 v by means of a conventional high-voltage power supply. The lamp is normally supplied with a current of 0.7 amp from a 240-v d-c power circuit. Normal voltage provides sufficient light intensity in the beam to trigger the phototube each time it passes over the slit in the scale. A large choke prevents the capacitor discharge from entering the power circuit. Just before the end of the preset time interval (or preset number of revolutions of a meter) the pulse counter again completes the circuit, so that the last or stopping pulse fires the thyratron. Thus the flashing is entirely automatic; the timing of the flash depends on the signal pulse, which in the case of the seconds signals is accurate to better than a part in a million.

The third function of the accessory electronic equipment is that of counting the disk revolutions of the Standard Watthour Meter. This, too, is automatic. During a run the constant light actuates the phototube behind the slit in the scale. This operates an electromagnetic counter by means of an amplifier and a small thyratron. An electronic switch opens the cathode circuit of the thyratron tube to prevent counting except during the run. The pulse that flashes the mercury lamp is also fed to the grids of the electronic-switch tubes. The first, or starting, pulse that flashes the lamp also enables conduction in the counter thyratron and allows counting to start. The second, or stopping, pulse that flashes the lamp again at the end of the preset timing or counting interval, disables the thyratron and stops the counting.

3. Testing

The accuracy of a test of a watthour meter, using the apparatus described is directly dependent upon the latest previous calibration of the reference standard, the stability of which is substantiated by more recent intercomparison tests. Phantom loading is employed in all tests. Correct circuit connections are made rapidly by means of links on an otherwise permanently wired circuit board.

3.1. Calibration of the Standard Watthour Meter

The word "calibration" usually implies not only a carefully made test, but also an initial careful adjustment or marking of the apparatus under test. For the purpose of this discussion, however, a calibration of the Standard Watthour Meter implies a test to determine accurately its percentage registration and possibly, but not necessarily, an adjustment of its mechanism in order to bring the percentage registration within certain limits.

The Standard Watthour Meter is periodically calibrated with 120 v, 5 amp applied, at unity power factor, and at 0.5 power factor with the current lagging the voltage. During calibration, a measured value of a–c power is maintained constant for an accurately determined time interval.

The testing circuit is shown in figure 2. In the potential circuit it is important that the effective terminals of the Standard Wattmeter be located precisely at the Standard Watthour Meter terminals on alternating current, and at the volt-box terminals on direct current. Hence the lead resistance from the a–c—d–c switch to the Standard Watthour Meter potential terminals and to the volt-box terminals must be equal to prevent errors that would otherwise be caused by a difference in IR drops in

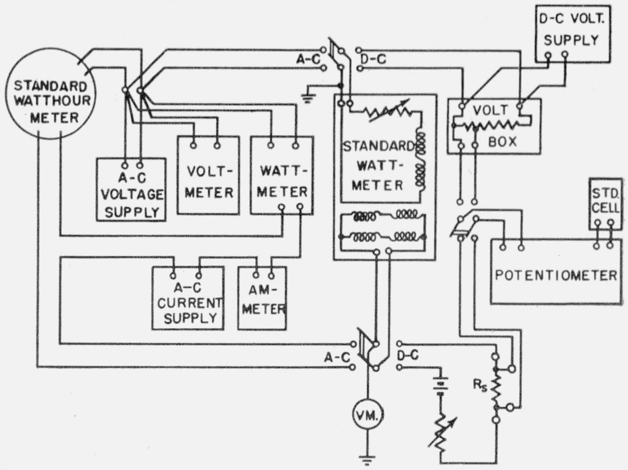

FIGURE 2. *Calibration of Standard Watthour Meter.*

the leads. It is also important that the voltage supplies be connected to these same terminals, otherwise a phase-angle error in the watthour-meter-potential circuit might result from an IR drop in the leads. Lead resistances are checked and adjusted, if necessary, just before each calibration. Note also in figure 2 that the moving-coil side of the Standard Wattmeter potential circuit is grounded, and that the current circuit is connected to ground through a rectifier-type voltmeter, a 300-v instrument having a total resistance of 300,000 ohms. This serves as an electrostatic tie to ground, which minimizes electrostatic forces between fixed and moving coils of the wattmeter, and provides an indication of the presence of leakage current and its magnitude.

In calibrating the Standard Watthour Meter the procedure is as follows. First, temperature equilibrium of the entire equipment is established by applying a–c power adjusted (in the case of the wattmeter and watthour-meter circuits) to approximately the values required for the test and then waiting for an hour or two before proceeding. After temperature equilibrium has been attained, the Standard Wattmeter is calibrated on reversed direct current to indicate the desired value of power. Alternating-current power is restored to the Standard Wattmeter and Standard Watthour Meter circuits and adjusted to give the same deflection of the Standard Wattmeter as that obtained on direct current. The a–c power is held substantially constant throughout the run.

The pulse-counting circuit is then energized. The next seconds signal flashes the mercury-vapor lamp and starts the run. The position of the spot on the phosphorescent scale is recorded to the nearest division. At the end of the preset interval (usually 100 seconds) the final seconds pulse again flashes the mercury-vapor lamp, thereby ending the run. The position of this last spot is recorded as well as the reading of the revolution counter, which indicates the number of complete revolutions. Thus, the number of revolutions of the Standard Watthour Meter's disk equals the reading of the revolution counter plus the scale reading at the end of the run minus the scale reading at the beginning of the run. For example, if the reading at the start of the run were 872 divisions, and at the end of the run, 493 divisions, and the revolution counter read 10, the number of revolutions would be $10+0.493-0.872=9.621$. After the a-c run, two additional readings of the Standard Wattmeter on reversed d-c power are obtained in the same manner as at the start, giving a total of four d-c readings. Care is taken to insure a minimum of delay between the d-c readings and the run with alternating current. If this is done, any small linear drift that may be present in the deflection of the Standard Wattmeter causes no significant error in the result. The d-c readings are averaged and a correction applied for the difference between the a-c setting and the average d-c reading. Corrections are also applied for known errors in the resistance standard, volt box, wattmeter (phase-defect angle), and for deviations from normal of the average frequency of the source, if significant.

The formula used in this calibration for computing percentage registration is

$$\mathscr{R}=100\left[\frac{KR_s n3600}{E_p N_{vb} E_c t}+\frac{D_{dc}-D_{ac}}{D_{dc}}+c_s-c_{vb}+c_f-T_w \tan\theta\right] \tag{1}$$

where

K = disk constant of the watthour meter, in watthours per revolution.
R_s = nominal resistance of resistance standard, in ohms.
n = number of disk revolutions of watthour meter.
E_p = potentiometer setting, in volts, when adjusting the d-c voltage.
N_{vb} = nominal volt box ratio.
E_c = potentiometer setting, in volts, when adjusting the d-c current.
t = time of run in seconds.
D_{dc} = average of the four deflections of the Standard Wattmeter, in centimeters, observed during the d-c calibrations before and after each run.
D_{ac} = average deflection of Standard Wattmeter, in centimeters, maintained during the a-c run.
c_s = correction to the resistance standard, in parts per unit.
c_{vb} = correction to the volt box, in parts per unit.
c_f = frequency correction to the Standard Watthour Meter, determined from the reading of the synchronous timer, the time duration of the run, and a frequency correction factor obtained in a separate test of the Standard Watthour Meter.
T_w = phase defect angle of the Standard Wattmeter, in radians, positive if the moving coil current leads the supply voltage.
θ = power factor angle, which is the angular phase difference between the voltage, E, applied to the Standard Wattmeter, and the current, I, in the Standard Wattmeter current coils, and is positive if the current lags the voltage.

A complete derivation of this equation appears in the appendix.

3.2. Tests of the Comparison Standard Watthour Meters

The watthour meters used as Comparison Standards are tested with 120 v and 5 amp applied, at unity power factor, and at 0.5 power factor, with the current lagging the voltage. The method of testing a Comparison Standard Watthour Meter involves a comparison with the Standard Watthour Meter immediately following or during its calibration and periodically between calibrations. Figure 3 is a diagram of the circuit involved in the intercomparison of the standard watthour meters.

The similarity between the circuit shown in figure 3 and the alternating-current portion of figure 2 should be noticed. When a calibration of the Standard Watthour Meter is in progress the Comparison Standard Watthour Meters shown in figure 3 are incorporated in the circuit of figure 2, although in the interest of simplicity, this modification is not shown in the figure. With the circuit revised in this manner, the Standard Watthour Meter, the Comparison Standard Watthour Meters, and the Standard Wattmeter are energized simultaneously, and the calibration runs on the reference standard can be performed alternately with intercomparison test runs, thereby insuring nearly identical conditions for both tests. Truly simultaneous testing of the reference standard (having the circular phosphorescent scale) and the comparison standards is possible with the addition of more equipment, but it is believed that any improvement that might accrue from simultaneous testing would be too small to detect with certainty.

There is a phototube-lamp combination associated with each Comparison Standard, but only one of these combinations is energized at a time. The phototube produces a pulse for each revolution of the disk of the Comparison Standard under test. This pulse is amplified and used in conjunction with the pulse-counting circuit to start and stop the run.

To start the run, a pulse from the phototube associated with the Comparison Standard Watthour

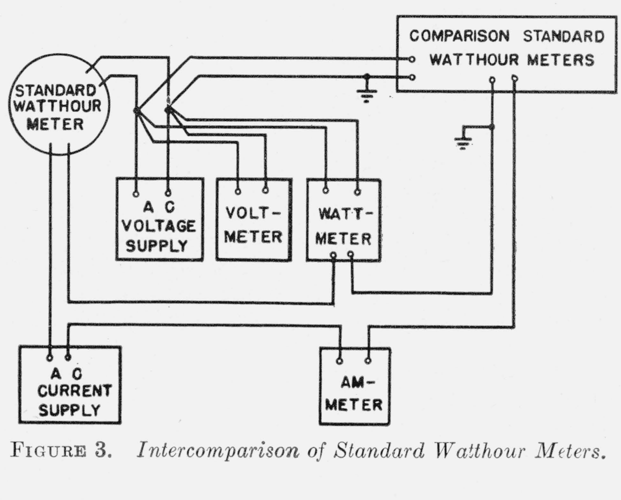

FIGURE 3. *Intercomparison of Standard Watthour Meters.*

Meter is used to flash the mercury-vapor lamp and a spot is left on the phosphorescent scale of the Standard Watthour Meter. The position of this spot is recorded on the data sheet. When the preset number of turns have been completed, the mercury-vapor lamp again flashes; another spot is left, and its position is recorded together with the reading of the revolution counter. From these readings the number of revolutions of the Standard Watthour Meter's disk is computed. The average percentage registration of the Comparison Standard obtained from several runs is taken as the most probable value. The other Comparison Standard Watthour Meters are tested in the same way.

The formula used in this test for computing the percentage registration of the Comparison Standards is

$$\mathscr{R}_c = \frac{n_c K_c}{K_s} \frac{\mathscr{R}_s}{n_s}, \tag{2}$$

where

n_c=number of disk revolutions of the Comparison Standard Watthour Meter.
n_s=number of disk revolutions of the Standard Watthour Meter.
K_s=disk constant of the Standard Watthour Meter, in watthours per revolution.
K_c=disk constant of the Comparison Standard Watthour Meter, in watthours per revolution.
$\mathscr{R}_s$=percentage registration of the Standard Watthour Meter.

A derivation of this equation appears in the appendix.

3.3. Tests of Other Watthour Meters, Using the Standard Watthour Meter

Alternating-current watthour meters that are sent to the National Bureau of Standards for test are compared with the Standard Watthour Meter in the following manner. The current and potential circuits of the Standard Watthour Meter are energized from the secondaries of a current transformer and a potential transformer having multiple ranges. The primary windings of these transformers and the watthour meter under test are excited by the a-c power supplies, as shown in figure 4.

The resistance of the leads that connect the potential circuit of the watthour meter under test to the primary of the voltage transformer is made small enough to render negligible the error in phase angle caused by the voltage drop in the leads. A voltmeter, ammeter, and a wattmeter of good quality are connected in the secondary circuit, and they enable an observer to set the secondary voltage, current, and power factor.

A phototube-lamp combination is arranged to produce a pulse at each revolution of the disk of the watthour meter under test if this is conveniently possible. The pulses are amplified, and, in conjunction with the pulse counter, are used to determine the duration of the run.

An alternative method of testing meters, particularly those that have no good means of operating a phototube, is to let the duration of the run be determined by the Standard Watthour Meter. This is accomplished by connecting the pulse counter to the output of the phototube located behind the slit in the scale of the Standard Watthour Meter. The potential circuit of the watthour meter under test is then closed at the beginning of the run and opened at the end by relays operated by the pulse counter. Contacts on one of the relays, when energized, close the potential circuit; while normally closed, contacts on the other relay, when energized, open the circuit. The time lags in the operation of these relays are carefully equalized to reduce timing errors to a negligible amount.

As the percentage registration of the Standard Watthour Meter is accurately known, the percentage registration of the watthour meter under test can be computed. Corrections are applied for the ratio and phase-angle errors of the current and potential transformers.

The procedure followed in making a test of this sort is practically identical to that followed in testing a Comparison Standard Watthour Meter.

The formula used in this test for computing the percentage registration of the test watthour meter is

$$\mathscr{R}_t = \frac{K_t n_t \mathscr{R}_s}{K_s N_{pt} N_{ct} n_s} + 100[c_\theta - c_{ct} - c_{pt}], \tag{3}$$

where

K_t=disk constant of the watthour meter under test, in watthours per revolution.
n_t=number of disk revolutions of the test watthour meter.
$\mathscr{R}_s$=percentage registration of the Standard Watthour Meter.
K_s=disk constant of the Standard Watthour Meter, in watthours per revolution.
N_{pt}=nominal potential transformer ratio.
N_{ct}=nominal current transformer ratio.
n_s=number of disk revolutions of Standard Watthour Meter.
c_{ct}=correction for the current transformer ratio (=ratio factor −1), in parts per unit.
c_ϕ =correction for the potential transformer ratio (=ratio factor −1), in parts per unit.
c_φ=correction for the phase angle, in parts per unit. It is equal to $(\beta - \gamma) \tan \theta_s$.
β=phase angle of the current transformer, in radians, considered positive when the reversed secondary-current vector leads the primary-current vector.
γ=phase angle of the potential transformer, in radians, considered positive when the reversed secondary-voltage vector leads the primary-voltage vector.
θ_s=power-factor angle at the Standard Watthour Meter, considered positive when the current vector lags the voltage vector.

A complete derivation of this equation appears in the appendix.

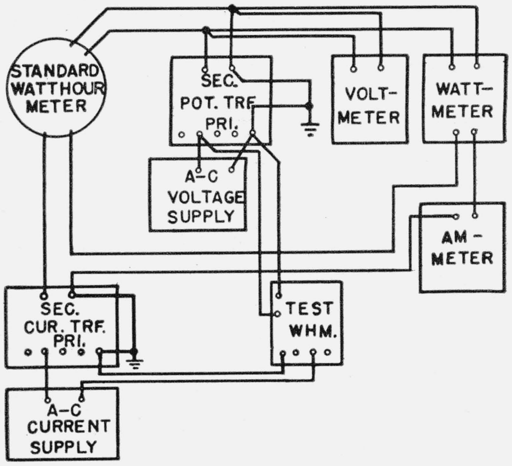

FIGURE 4. *Testing watthour meters, using the Standard Watthour Meter.*

4. Calibration and Testing Schedule

The periodic primary calibrations of the Standard Watthour Meter serve to accurately ascertain its percentage registration. Primary calibrations performed at monthly intervals for a period of time have indicated a long-time stability of the reference standard that will permit an increase in the interval between calibrations to 6 months or longer. A graphical record is kept of the intercomparison tests, which are performed at more frequent intervals, to detect any appreciable drifts or erratic changes in registration of any one of the group relative to the others, and possibly, to form a basis for a revision of the registration assigned to the Standard Watthour Meter if an observed change is not large enough to warrant a primary calibration. When a revision of the registration of reference standard is warranted on the basis of intercomparison tests and the evidence does not indicate gross instability in any one particular meter, the computation of registration is performed on the assumption that the mean registration of all meters remains unchanged between calibrations. With this assumption, the entire group of meters is useful in maintaining the unit of energy, although transfer of the unit to a watthour meter under test is accomplished through only one member of the group.

5. Precision and Accuracy Attainable

Both systematic and random errors limit the accuracy of test results obtained with the Standard Watthour Meter and its associated apparatus. Systematic errors, those which cannot be considered accidental, can be further classified into several types. First, there are the errors that have escaped the attention of the observers. After a careful analysis of all possible sources of error and the comparison of the results of tests made by several methods, it is believed that unknown systematic errors of magnitudes larger than about 0.01 percent are highly improbable.

Second, there are residual errors from the application of imperfect corrections. These errors are systematic in nature and have been minimized in the present work by carefully calibrating each piece of apparatus for which a correction must be applied in terms of the National Electrical Standards, which are readily available. The largest systematic errors of this type result from the uncertainty in the ratio and phase-angle corrections of the current and potential transformers. Other residual errors of this type, such as those associated with the standard cell, potentiometer, and volt box, occur in the primary calibrations of the Standard Watthour Meter, and are rather small. The history of the components used in the measurements circuit is well known. The maximum net change in value of the components between their periodic tests, for the last several years, is listed in table 1.

The third type of systematic error is, in a sense, similar to the second type, in that it arises from the imperfect application of corrections, but in this case the corrections are for the compensation of fluctuating errors. Such errors may vary with certain parameters in a definite though unknown manner. On the other hand, the fact that an error exists and that it is a function of a certain parameter may be known, but its effect may be negligibly small (and a correction impractical to apply) compared with that of the other errors. Varying systematic errors are evident in the Standard Watthour Meter as well as the Comparison Standard Watthour Meters and, in fact, sharply limit the accuracy of which any watthour meter is capable. Reference is here made to the effect on the percentage registration of a watthour meter of such factors as incorrect magnitude of applied voltage and current, incorrect phase angle, incorrect frequency, or the presence of harmonics in the applied voltage or current. Studies have been made of the effect of each of these variables on the meters used as standards. It has been found feasible to use a good quality voltmeter, ammeter, and wattmeter, to facilitate setting the voltage, current, and phase angle. The power supplies are electronically regulated to maintain the initial adjustment of voltage and current, and are operated and loaded in such a manner as to minimize harmonics.

TABLE 1. *Observed maximum net change in the value of the components necessary for the accurate standardization of the Standard Electrodynamic Wattmeter*

Component	Maximum net change in value
	ppm
Standard cell	10
Potentiometer	10
Volt box	11
Total	31
Standard cell	10
Potentiometer	10
Resistance standard	6
Total	26
Grand total	57

A third harmonic of as little as 1 percent of the fundamental in the current circuit of a watthour meter may cause a registration error of several hundredths of a percent if certain phase relationships exist. This error arises from the existence of a third harmonic flux in the air gap of the electromagnet even if a purely sinusoidal voltage is applied to the potential coil. It is important to realize that even if a watthour meter is tested using alternating current of practically sine wave form it will register inaccurately if used in a circuit in which the wave form is nonsinusoidal. Although it would be unfair to impute errors due to nonsinusoidal wave form to watthour meters, which are designed for use on a sinusoidal waveform, the possibility of an appreciable error arising from this cause should not be overlooked if accurate measurements are desired. Odd harmonics which are present in the wave form of the current and voltage supplied to the Standard Watthour Meter and associated apparatus at the time of this writing are, under most load conditions, considerably less than 1 percent, much less than usually found in commercial power sources.

During calibrations of the Standard Watthour Meter, average frequency of the applied power during a run is measured and a correction applied for any deviation from 60 c/s. In most other tests a measurement of average frequency, and a correction for errors therein, is not conveniently possible, and this probably contributes the greatest share of systematic errors that vary in a definite, but unknown, manner. However, it has been determined that errors arising from customarily observed deviations in frequency are of the order of a few thousandths of a percent. The effect of temperature changes on the watthour meter standards is reduced to a small value by keeping all meters in temperature-controlled enclosures. The standards are not removed from the position in which they are calibrated. Throughout all tests the Standard Watthour Meter is operated continuously, thus eliminating those errors associated with starting and stopping the disk.

Since the scale of the Standard Watthour Meter is marked with 1,000 small divisions, any small variations in the angular speed of the disk which would be undetectable in other meters may very likely be easily discernible. To determine the nature and the magnitude of errors due to angular speed variations, the standard was centered as accurately as feasible in its scale and carefully levelled. It was operated at a reduced speed, and seconds pulses were used to flash the mercury vapor lamp once every second. The applied power was held constant at a value sufficient to cause successive spots from the mercury-vapor lamp to appear on the phosphorescent scale at four points as the disk revolved. Scale readings corresponding to spot positions indicated a regular (nearly sinusoidal) variation of disk speed during each revolution. The variations were such as to cause a maximum shift in scale reading of about 0.005 revolution. A method was devised for aligning the watthour meter in its scale to reduce this error to less than 0.001 revolution, thereby partially compensating for the angular speed variations. An error of 0.001 revolution would cause a scale reading error of only 0.001 percent if the watthour meter is operated for 100 revolutions, as is usually done.

Some of the residual errors thus far discussed will be positive, others will be negative, and the magnitude of many of these systematic errors will appear to vary in a random manner as more and more tests are performed. To this must be added the truly random errors, the distribution of which is entirely by chance, and to which the theory of errors may be applied. The values listed in table 2 include an estimate of the standard deviation of random errors based upon numerous test data, and half the maximum systematic error that it is believed may possibly be present in any one measurement of energy. It is recognized that the errors listed do not represent truly random variations, yet their individual contribution (some being positive and some negative) to the final result partakes of randomness and probably justifies the computation of the total propagated error from the square root of the sum of the squares of the component errors. There is one exception. The error in the standard cell and the error in balancing the potentiometer against the electromotive force of the standard cell occur twice in a single measurement of d-c power; hence, these component errors are not independent, and the propagation of these errors is by simple addition. The potentiometer error also occurs twice, but these are independent, since different resistance sections of the potentiometer are used for the measurement of voltage and current. The total propagated error provides an estimate of expected accuracy with which the applied energy is known, when the Standard Watthour Meter and its associated apparatus is used to test other meters. It is not to be inferred that the other meters are capable of calibration or maintaining their calibration to the degree of accuracy indicated at the bottom of the table.

A test was made to determine the repeatability of results in the comparison testing of the Comparison Standard Watthour Meters against the Standard Watthour Meter. Individual runs of about 100 seconds duration were made about 10 minutes apart until a group of 10 runs had been made. This was repeated the next day, and again 4 days later. The relative percentage registrations of the two Comparison Standards are shown in table 3. The values listed are the means of the groups of 10 runs, and the measure of precision is the standard deviation of a single run of the group.

The precision indices listed in table 3 result from a combination of random errors inherent to (1) the Comparison Standard Watthour Meter being tested, and (2) the Standard Watthour Meter, including observational errors in reading its scale. If it is assumed that each meter contributes equally to the total error, but that the contributions are random, then an index of precision for each meter may be computed. If the total error is considered as 0.012 percent, the individual errors which produce this error are each $0.012/\sqrt{2}=0.008$ percent. From these data it is estimated that the precision of each of the watthour meters in the setup over a period of a few hours is about 0.008 percent.

TABLE 2. *Effect of residual errors on the test of a watthour meter*

Type of measurement and components	Estimated residual error	
Measurement of d-c power with a potentiometer, volt box, and resistance standard:	%	
Standard cell	0.001	
Error in standardizing potentiometer	.002	
Twice the square root of the sum of the squares from the above sources		0.005
Potentiometer:		
a. Measurement of voltage		.002
b. Measurement of current		.002
Volt box		.001
Resistance standard		.001
Reading deflection of wattmeter on direct current		.002
Calibration of the Standard Watthour Meter:		
Setting and holding deflection of Standard Wattmeter on alternating current		002
Timing errors		.001
Standard Watthour Meter scale errors		.001
Errors in maintenance and use of the standard meters:		
Maintenance of the unit of electric energy by the Standard Watthour Meter and the Comparison Standard Watthour Meters (long-time stability, see table 4)		.012
Standard Watthour Meter registration variations (short-time stability)		.010
Test of a watthour meter at unity power factor in a comparison test with the Standard Watthour Meter:		
Transformer errors:		
a. Current transformer ratio		.010
b. Potential transformer ratio		.010
Square root of sum of squared residual errors at unity power factor. Total propagated error equals		0.022
Test of a watthour meter at 0.5 power factor in a comparison test with the Standard Watthour Meter:		
Transformer errors (additional errors due to uncertainty of transformer phase angles:		
a. Current transformer		0.010
b. Potential transformer		.010
Square root of sum of squared residual errors at 0.5 power factor. Total propagated error equals		0.026

TABLE 3. *Short-time repeatability of intercomparison tests on the Comparison Standard Watthour Meters against the reference Standard Watthour Meter*

At 0.5 power factor, the current lagged the voltage.

Group	Comparison Standard No. 1				Comparison Standard No. 2			
	1 power factor		0.5 power factor		1 power factor		0.5 power factor	
	Registration	Standard deviation	Registration	Standard deviation	Registration	Standard deviation	Registration	Standard deviation
	%	%	%	%	%	%	%	%
A	100.067	0.014	99.923	0.012	100.140	0.011	100.068	0.007
B	100.085	.001	99.932	.016	100.160	.010	100.064	.010
C	100.056	.010	99.936	.016	100.135	.015	100.082	.018

TABLE 4. *Precision of intercomparison test results over a period of several months (from same data as figure 5)*

At 0.5 power factor, the current lagged the voltage.

Watthour meter	Standard deviation	
	1.0 power factor	0.5 power factor
	Percent	*Percent*
Standard Watthour Meter	0.012	0.014
Comparison Standard Watthour Meter No. 1	.008	.007
Comparison Standard Watthour Meter No. 2	.007	.012

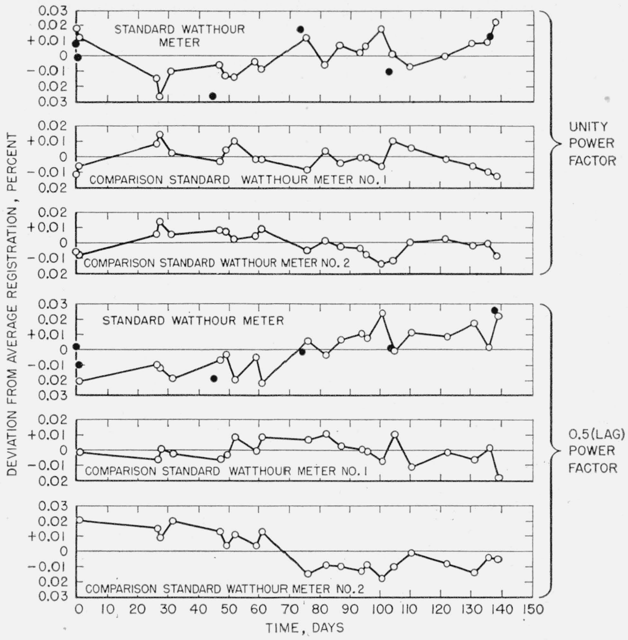

FIGURE 5. *Percentage registration deviations of the Standard Watthour Meter and Comparison Standard Watthour Meters over a period of several months.*

Connected points are results of intercomparison tests; isolated points are results of calibrations of the Standard Watthour Meter.

As these meters are used to maintain the unit over much longer periods of time than were used in the test described, it might be expected that the long-time stability of the meters should not be as good as that for short intervals because of varying systematic errors. On the other hand, the use of a multiplicity of meters improves the over-all precision. A measurement of the precision with which the unit is maintained is obtained from the records of calibration and intercomparison tests. Figure 5, for example, shows the test results over a period of several months. The points connected by straight lines represent the results of intercomparisons, and are based on an assumed constant mean registration of all three meters. The validity of this assumption is proved by the proximity of the intercomparison test results to the isolated points representing calibrations of the Standard Watthour Meter by the wattmeter method. The standard deviation for each of the meters (assuming all deviations are normally distributed and random) computed from the data shown in figure 5 is given in table 4. From data such as these it is estimated that the unit of the watthour can be maintained over a period of several months with precision represented by a standard deviation of 0.012 percent or better. Many tests that have been run in rapid succession, a few minutes apart, indicate drifts in registration in one direction for a period, then a reversal or rapid change of some sort. Such drifts or changes have been very small (few thousandths of a percent). However, an abnormal distribution of this sort reveals the presence of those varying systematic errors, discussed previously, which depend upon other parameters. A comparison of tables 3 and 4 shows that the stability over several months is very nearly the same magnitude as that over a period of an hour or two. This implies that any varying systematic errors that may exist vary rapidly and in such a manner that their effect on the mean of a number of observations of percentage registration taken several hours or more apart is negligibly small. Accordingly, when calibrations of the Standard Watthour Meter are performed, the individual runs are taken at time intervals sufficient to extend the duration of the test to several hours.

When the apparatus is used for testing other watthour meters it is believed that the measurement of the energy applied to the test watthour meter is valid and can be relied upon to better than 0.06 percent.

6. Conclusion

At the National Bureau of Standards a set of several carefully selected and prepared watthour meters has been arranged as a standard group for testing alternating-current watthour meters by a comparison method.

Although the initial cost of designing, constructing, and assembling this apparatus would ordinarily deter the general adoption of such an elaborate system of testing watthour meters, the use of the apparatus described has yielded a definite saving in time and labor, without significant loss of accuracy when compared with the primary method used in the past.

Accurate initial calibration of the apparatus is accomplished by averaging the results of a number of primary calibrations of the Standard Watthour Meter, using (1) a transfer wattmeter and potentiometer for measurements in terms of the fundamental electrical units of electromotive force and resistance, and (2) standard seconds signals for time measurements. Assurance of the continued accuracy of the reference standard watthour meter is obtained by intercomparison tests with other members of the standard group. The reference standard, supplemented by calibrated instrument transformers, accurately measures the energy supplied to a watthour meter under test.

J. B. Dempsey initiated the early work in setting up selected watthour meters for secondary standards. F. B. Silsbee proposed the light-beam pointer and phosphorescent scale. F. J. Gross carried out the developmental work and the original design of the comparison apparatus. Most of the mechanical construction of this apparatus was done by C. H. Hochgesang of the Bureau's Instrument Shop. Robert W. Balcom assembled most of the electronic equipment and provided many valuable suggestions. Thomas W. Cushing carried the project forward for a time and designed and supervised the construction of the constant-temperature cabinet for the comparison standards.

7. Appendix

The derivations of eq (1), (2), and (3) proceed most easily from the definition of percentage registration, which is the ratio of the indicated energy as obtained from the reading of the watthour meter to the true energy, expressed in percent.

$$\mathscr{R}=100\,\frac{\text{Indicated energy}}{\text{True energy}}. \tag{4}$$

In a test of the Standard Watthour Meter, using the Standard Wattmeter, the indicated energy in watthours is Kn. The true energy is determined by the readings and settings of the Standard Wattmeter. Because this instrument is very nearly astatic, its sensitivity, s, will be the same for both directions of the direct current, and the average deflection is

$$D_{dc}=sW_{dc}, \tag{5}$$

where W_{dc} is the true power impressed on the Standard Wattmeter.[4] The true power in terms of the potentiometer settings and circuit constants is

$$W_{dc}=\frac{E_pN_{vb}E_c}{R_s}+c \tag{6}$$

where c represents the correction terms for the circuit components. The average a-c deflection of the Standard Wattmeter is

$$D_{ac}=sEI\cos(\theta+T_w), \tag{7}$$

where E is the a-c voltage between the Standard Wattmeter potential terminals causing a potential coil current, I_p, see figure 6. It is the magnetic field from this current that reacts with the magnetic field from the current, I, in the fixed coils of the Standard Wattmeter to produce the deflection, D_{ac}.

The wattmeter sensitivity, s, will be the same on alternating current as on direct current provided all readings are grouped closely on the Standard Wattmeter's scale.

Dividing (5) by (7)

$$\frac{D_{dc}}{D_{ac}}=\frac{sW_{dc}}{sEI\cos(\theta+T_w)}. \tag{8}$$

Solving for EI

$$EI=W_{dc}\,\frac{D_{ac}}{D_{dc}\cos(\theta+T_w)}. \tag{9}$$

The a-c power impressed on the Standard Watthour Meter is

$$W_{ac}=EI\cos\theta \tag{10}$$

and substituting for EI from eq (9)

$$W_{ac}=W_{dc}\,\frac{D_{ac}\cos\theta}{D_{dc}\cos(\theta+T_w)}. \tag{11}$$

[4] The symbols not specifically defined in the appendix have been previously defined in the text.

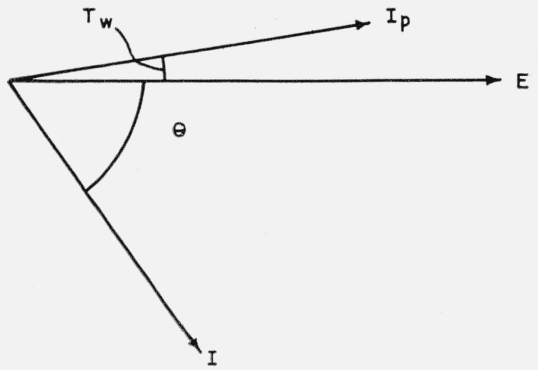

FIGURE 6. *Vector relationships at the Standard Wattmeter terminals during a calibration of the Standard Watthour Meter.*

No scale is intended and the angles indicated are grossly exaggerated for clarity.

This can be rearranged in the form

$$W_{ac}=W_{dc}\left[1+\frac{D_{ac}}{D_{dc}}-\frac{D_{dc}}{D_{dc}}\right]\left[1+\frac{\cos\theta}{\cos(\theta+T_w)}-\frac{\cos(\theta+T_w)}{\cos(\theta+T_w)}\right]$$

$$W_{ac}=W_{dc}\left[1+\frac{D_{ac}-D_{dc}}{D_{dc}}\right]\left[1+\frac{\cos\theta-\cos\theta\cos T_w+\sin\theta\sin T_w}{\cos\theta\cos T_w-\sin\theta\sin T_w}\right] \quad (12)$$

$$W_{ac}=W_{dc}\left[1+\frac{D_{ac}-D_{dc}}{D_{dc}}\right]\left[1+\frac{\cos\theta\,(1-\cos T_w)+\sin\theta\sin T_w}{\cos\theta\cos T_w-\sin\theta\sin T_w}\right]$$

and since T_w is a very small angle (116 microradians at 60 c/s), sin T_w very nearly equals T_w in radians and cos T_w is very nearly 1; therefore, the equation can be simplified by neglecting the extremely small terms.

$$W_{ac}=W_{dc}\left[1+\frac{D_{ac}-D_{dc}}{D_{dc}}\right][1+T_w\tan\theta] \quad (13)$$

and since the second terms in both pairs of brackets are much smaller than 1, the equation may be further simplified by multiplying and then neglecting the smaller second-order terms.

$$W_{ac}=W_{dc}\left[1+\frac{D_{ac}-D_{dc}}{D_{dc}}+T_w\tan\theta\right]. \quad (14)$$

When W_{dc} from eq (6) is substituted and the result is multiplied by $t/3600$ to obtain energy in watthours, the true energy thus obtained may be used in eq (4), and the results, showing all corrections as correction terms, is eq (1) shown in section 3.

The derivation of eq (2), which is used in intercomparison tests, follows logically from eq (4). The indicated energy equals $K_c n_c$. Hence the percentage registration of the Comparison Standard Watthour Meter is

$$\mathscr{R}_c=100\,\frac{K_c n_s}{\text{True energy}} \quad (15)$$

and that of the Standard Watthour Meter is

$$\mathscr{R}_s\,\frac{K_s n_s}{\text{True energy}}. \quad (16)$$

Since, in this test, the true energy is identical for both watthour meters, eq (15) and (16) can be combined to yield eq (2), as shown in section 3.2.

In the derivation of eq (3) the following additional symbols will be used:

θ=power factor angle at the test watthour meter, considered positive when the current vector lags the voltage vector. It is equal to $\theta_s+\beta-\gamma$.

E_p=primary voltage applied to the test watthour meter.

I_p=primary current applied to the test watthour meter.

E_s=secondary voltage applied to the Standard Watthour meter.

I_s=secondary current applied to the Standard Watthour meter.

t=time of run, in seconds.

N'_{pt}=true potential transformer ratio= $N_{pt}(1+c_{pt})$.

N'_{ct}=true current transformer ratio= $N_{ct}(1+c_{ct})$.

The indicated energy of the watthour meter under test is $K_t n_t$. The true energy that should be measured by the test watthour meter is $E_p I_p t \cos\theta$, or, if E_p and I_p are expressed in terms of secondary voltage and secondary current, the true energy is $E_s N'_{pt} I_s N'_{ct} t \cos\theta$. Substituting these factors in eq (4), the percentage registration of the test watthour meter is

$$\mathscr{R}_t=100\,\frac{K_t n_t}{E_s N'_{pt} I_s N'_{ct} t\cos\theta}. \quad (17)$$

Now consider the energy measured by the Standard Watthour Meter. Since the percentage registration of the Standard Watthour Meter is known, and the indicated energy is $K_s n_s$, then the true energy measured by the Standard Watthour Meter is

$$\frac{100\,K_s n_s}{\mathscr{R}_s}=E_s I_s t\cos\theta_s \quad (18)$$

or

$$\frac{1}{E_s I_s t}=\frac{\mathscr{R}_s\cos\theta_s}{100\,K_s n_s} \quad (19)$$

and substituting eq (19) in eq (17)

$$\mathscr{R}_t=\frac{K_t n_t \mathscr{R}_s}{K_s N'_{pt} N'_{ct} n_s}\,\frac{\cos\theta_s}{\cos\theta}. \quad (20)$$

In this equation $\cos \theta_s/\cos \theta$ is a correction factor for the phase angles of the current and voltage transformers. To simplify this correction the denominator may be expanded

$$\frac{\cos \theta_s}{\cos (\theta_s+\beta-\gamma)}=\frac{\cos \theta_s}{\cos \theta_s \cos (\beta-\gamma)-\sin \theta_s \sin (\beta-\gamma)}. \tag{21}$$

Dividing both numerator and denominator by $\cos \theta_s$ the correction factor becomes

$$\frac{1}{\cos (\beta-\gamma)-\sin (\beta-\gamma) \tan \theta_s}. \tag{22}$$

Since $(\beta-\gamma)$ is a small angle, $\cos (\beta-\gamma)$ is very nearly 1 and $\sin (\beta-\gamma)$ is very nearly $(\beta-\gamma)$, hence the correction factor is

$$\frac{1}{1-(\beta-\gamma) \tan \theta_s} \tag{23}$$

and when the indicated division is performed, neglecting the extremely small second-order terms, the result is

$$1+(\beta-\gamma) \tan \theta_s. \tag{24}$$

Thus the correction term for the phase angle in parts per unit is $c_\varphi=(\beta-\gamma) \tan \theta_s$, and when the correction terms for the current transformer ratio, potential transformer ratio, and phase angles are shown, the final form of eq (20) is that of eq (3) in part 3.3.

WASHINGTON, March 5, 1954.