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Applications of Dimensional Analysis to Spray-Nozzle Performance Data

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Some possible applications of dimensional analysis in studies of the performance of continuous fuel-spray nozzles of the centrifugal type are presented. Equations are developed showing the relations among nozzle capacity, mean drop diameter, spray angle, nozzle size, the density, viscosity, surface tension, and pressure of the fuel. Using experimental data available at the National Bureau of Standards and in the literature, good correlation is shown in considerations of nozzle capacity, and fair correlation in those involving mean drop diameter and spray angle.

1. Introduction

In the course of research sponsored by the Bureau of Aeronautics, Department of the Navy, on various fuel-handling and metering devices for aircraft engines, one of the accessories of interest has been the spray nozzle, which is widely used for the atomization of liquid fuels in turbine-type engines. The performance of such engines is influenced by the characteristics of the nozzles, including the quantity of fuel delivered, the size of the droplets, and the angle of the spray. The latter, in turn, are known to be influenced by the configuration of the nozzle, the nozzle-pressure drop, and by certain physical properties of the fuel, including density, viscosity, and surface tension.

The well-known advantages of dimensional analysis, namely, a reduction in the number of variables, the possibility that the effect of one quantity may be studied by varying another, and the ease with which it can be shown that certain variables are unimportant, have not been utilized heretofore in published analyses of the performance of fuel nozzles. These advantages are demonstrated here by applying this method of analysis to the limited data available from research at the National Bureau of Standards and elsewhere on fixed, continuous-spray nozzles of the centrifugal type.

Relations among several dimensionless variables are developed. The particular variables suggested herein are not new, and the reader will recognize in them the Reynolds and Weber numbers and products thereof. They seem convenient for showing both the individual and combined effects of nozzle size, fuel pressure, and the properties of the fuel upon the capacity of the nozzle, upon the angle of spray, and upon the mean drop diameter.

The nozzles under consideration are of fixed configuration and deliver a continuous fuel spray. Atomization in such nozzles is produced by the conversion of potential energy of the high-pressured liquid to kinetic energy in the high-velocity liquid discharged from the nozzle orifice. Both the simplex type, having a single internal flow passage from the pressurized fuel supply, and the duplex type, having two separate fuel inlets and internal flow passages, are considered. Although only centrifugal spray nozzles are treated specifically, it is well known that the analysis holds for nozzles of other types and for restrictions through which a liquid discharges, subject only to the limitation that the device have no moving parts.

2. Nomenclature

	Dimensions (m, l, t systems)
D = a length characterizing the size of the nozzle.	(l).
d = mean drop diameter of the spray	(l).
M = mass rate of flow	$((mt^{-1})$.
ΔP = pressure drop through the nozzle	$(ml^{-1}t^{-2})$.
ΔP_s = pressure drop through small passage of duplex nozzle.	$(ml^{-1}t^{-2})$.
r_i = length ratios designating shape of nozzle.	Dimensionless.
ρ = mass density	(ml^{-3}).
σ = surface tension	(mt^{-2}).
θ = spray angle	Dimensionless.
μ = absolute viscosity	$(ml^{-1}t^{-1})$.

3. Theory

A highly simplified description of the process of atomization from a simplex fuel nozzle, such as that shown schematically in figure 1, is adequate for present purposes. Pressurized liquid supplied to the nozzle inlet flows at high velocity through small tangential slots into the swirl chamber where a high angular velocity is attained. It then passes through the discharge orifice and emerges into the surrounding atmosphere in the form of a hollow, conical sheet, the apex of which is at the nozzle orifice. The liquid sheet becomes progressively thinner as it moves away from the orifice. Eventually the sheet becomes unstable, rupturing occurs, and small droplets are formed.

The effects of the liquid pressure drop, density, viscosity, and surface tension upon the rate of discharge, the mean drop size, and the angle of the spray are to be considered in this analysis. It is desired to develop relations by which experimantal data may be correlated to form characteristic curves and charts showing the relative influence of the various independent quantities upon nozzle performance.

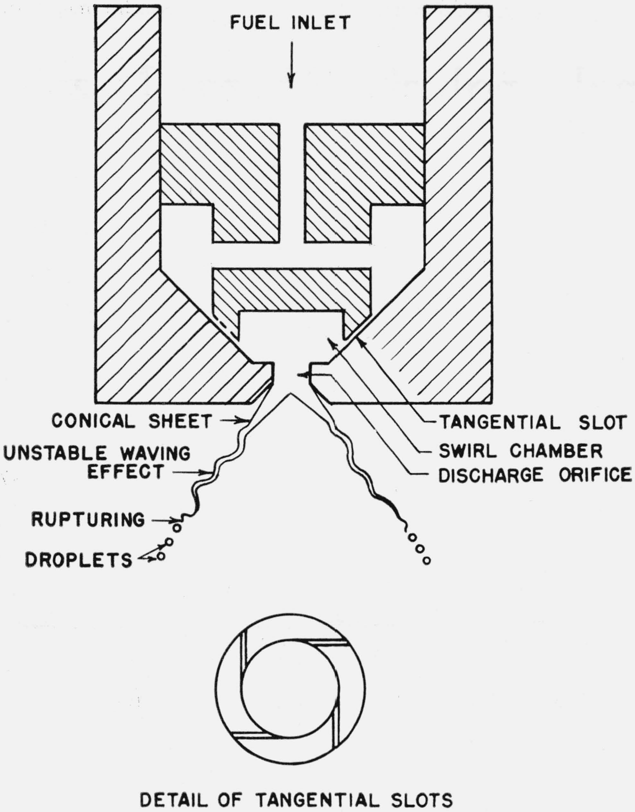

FIGURE 1. *Schematic diagram of a simplex spray nozzle.*

3.1. Flow Rate of the Simplex Nozzle

Again referring to figure 1, the flow rate of the liquid through the nozzle could be influenced by the pressure drop, i. e., the difference in pressure between the nozzle inlet and the atmosphere into which the spray discharges, by the density, viscosity, and perhaps surface tension of the liquid, and by those linear dimensions defining the configuration and the orientation of the flow passages and swirl chamber. Any one of these nozzle dimensions, such as the diameter of the orifice, may be designated as a characteristic length, D, and all others may then be expressed as dimensionless ratios ($r_1, r_2, \ldots, r_i$) of this characteristic length.

Assuming that all quantities that have an appreciable effect upon the flow rate have been enumerated their interrelation can be expressed by the general equation

$$F(M,\ \Delta P,\ \rho,\mu,\sigma,D,r_1,r_2,\ \ldots,r_i)=0, \tag{1}$$

in which F merely denotes "function of."

By applying Buckingham's pi theorem [1, 2],[1] eq (1) reduces to

$$\Pi_1=\Phi(\Pi_2,\ \Pi_3,r_1,r_2,\ \ldots,r_i), \tag{2}$$

in which each Π symbol represents a dimensionless variable or product formed through different combinations of not more than 4 of the 6 dimensional quantities in eq (1). Also, each Π is completely independent of the others, in that each contains one quantity not in either of the other two.

The three dimensionless Π products may be formed by selecting any 3 of the 6 dimensional quantities of eq (1) which are independent, and by associating these in turn with each of the remaining three dimensional quantities. Since primary interest is in the effects of viscosity and surface tension upon flow rate, it is desirable to derive each of the three Π products in such a way that one and only one will contain the quantities M, μ, and σ. This leaves ΔP, ρ, and D as the three quantities that may be involved in all three products. On this basis, the quantities can be arranged in dimensionless groups such as:

$$\Pi_1=M/D^2\sqrt{\Delta P\rho};\quad \Pi_2=D\sqrt{\Delta P\rho}/\mu;\quad \text{and}\quad \Pi_3=D\Delta P/\sigma.$$

Substitution of these values in eq (2) gives

$$M/D^2\sqrt{\Delta P\rho}=\Phi(D\sqrt{\Delta P\rho}/\mu,\ \ D\Delta P/\sigma,\ \ r_1,\ \ r^2,\ \ldots,\ r_i). \tag{3}$$

The function Φ is as yet unknown, and in fact it could have an infinite number of forms depending upon the shapes of the nozzles. However, if the consideration be now restricted to geometrically similar nozzles of fixed configuration but of different absolute sizes, i. e., to nozzles having the same r_i ratios, eq (3) reduces to

$$M/D^2\sqrt{\Delta P\rho}=\Phi(D\sqrt{\Delta P\rho}/\mu,\quad D\Delta P/\sigma), \tag{4}$$

in which the form of the function Φ will differ for various nozzle configurations.

As each of the quantities appearing in eq (4) can be measured, the explicit form of the function Φ can be determined by experiment for any particular nozzle. The experimental data might be plotted in terms of $M/D^2\sqrt{\Delta P\rho}$ as a function of $D\sqrt{\Delta P\rho}/\mu$ for different values of $D\Delta P/\sigma$. If such a plot shows that, regardless of the value of the latter parameter, a single curve is defined, then it has been demonstrated that surface tension has little or no effect upon the flow rate, since σ appears in eq (4) only in the product $D\Delta P/\sigma$. Thus, to investigate the effects of changes in surface tension, it is necessary only to change the variable $D\Delta P/\sigma$. This can be accomplished by varying either σ, ΔP, or D, although it is not simple to change D by fabricating geometrically similar spray nozzles of different absolute sizes.

Herein lies one of the important advantages of dimensional analysis, in that the effect of variations in one quantity, surface tension in this illustration, may be investigated by changing some other quantity, in this case the pressure drop or the size. Another advantage of this method is seen by comparing eq (1) and (4). The former contains six independent dimensional variables, which have been combined into three independent products in eq (4). This reduction in the number of variables from 6 to 3 simplifies the presentation and application of results.

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[1] Figures in brackets indicate the literature reference at the end of this paper

3.2. Flow Rates of Duplex Nozzles

As is shown in figure 2, duplex nozzles are provided with two inlets and two separate internal flow passages. The inlet pressures may or may not be equal. From the viewpoint of dimensional analysis, the duplex nozzle of fixed configuration differs from the simplex nozzle in a single important respect, namely, that the pressure drops ΔP and ΔP_s for the main and small flow passages must both be included in the consideration. Thus eq (1) for the duplex nozzle becomes

$$F(M, \Delta P, \Delta P_s, \rho, \mu, \sigma, D, r_1, r_2, \ldots, r_i)=0. \quad (5)$$

By forming the dimensionless groups as before, and by limiting the consideration to geometrically similar nozzles, there results the relation

$$M/D^2\sqrt{\Delta P\rho}=\Phi(D\sqrt{\Delta P\rho}/\mu,\ \Delta P/\Delta P_s,\ D\Delta P/\sigma). \quad (6)$$

It is known that variations in the parameter $D\Delta P/\sigma$ have no appreciable influence upon the function Φ for either the simplex or the duplex nozzles investigated. Thus $D\Delta P/\sigma$, and consequently surface tension, can be omitted from both eq (4) and (6). Equation (6) then becomes

$$M/D^2\sqrt{\Delta P\rho}=\Phi(D\sqrt{\Delta P\rho}/\mu,\ \Delta P/\Delta P_s). \quad (7)$$

In applying this relation to the experimental data, it will be found convenient to form charts of $M/D^2\sqrt{\Delta P\rho}$ versus $D\sqrt{\Delta P\rho}/\mu$ for various constant values of the pressure ratio $\Delta P/\Delta P_s$.

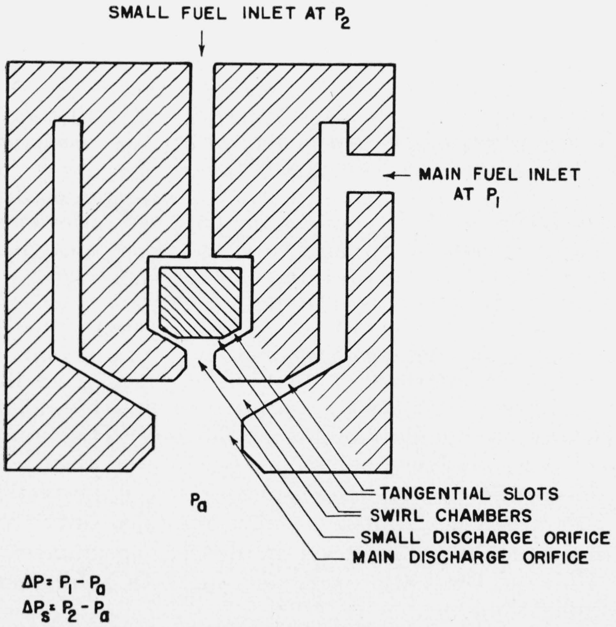

FIGURE 2. *Schematic diagram of a duplex spray nozzle.*

3.3. Mean Drop Size of the Spray

Consider next the mean diameter of the droplets formed during the rupture of the conical sheet emerging from the nozzle orifice. In an actual spray having a condensed volume, V, and consisting of N droplets of various sizes, the mean drop size, d, is taken [3] as the diameter of a sphere having a volume V/N.

Previous investigators [4, 5, 6, 7] have shown that the density, surface tension, and viscosity of the liquid, as well as the relative velocity between the liquid and the air just prior to atomization, may all influence drop size. Although the velocity of a liquid emerging from a centrifugal-type nozzle is not readily measurable, it depends upon the nozzle configuration and upon the flow rate, the latter being influenced by the pressure drop and by the density and viscosity of the liquid. The size of the drops also depends upon the shape of the spray chamber and the location of the drop-sampling apparatus within the spray. Thus, with the understanding that the procedure applies only to geometrically similar nozzles, spray chambers, and drop-determination methods, the general equation

$$F(\Delta P, \rho, \mu, \sigma, D, d)=0 \quad (8)$$

includes all the quantities that are expected to exert an appreciable influence when the spray discharges into still air.

Through the same procedure used in reducing eq (1) to eq (3), eq (8) may be reduced to the form

$$d/D=\Phi(D\Delta P/\sigma,\ D\rho\sigma/\mu^2). \quad (9)$$

Equation (9) is convenient for the present purpose, as it separates the effect of fluid viscosity. If experimental results show that the effect of viscosity is appreciable, it would be convenient to form a chart of d/D versus $D\Delta P/\sigma$ for different constant values of $D\rho\sigma/\mu^2$. For any given value of D, the latter variable is constant for a given fluid at constant temperature, and can be varied conveniently by proper selection of the test fluids.

3.4. Angle of the Spray

Similar considerations may be applied to the spray angle. For this correlation, use is again made of the data of Rupe [3], who states arbitrarily that this angle shall be symmetrical about the axis of the spray and shall include 80 percent of the total-weight flow. It seems reasonable that this dimensionless angle, θ, could be influenced by the configuration and size of the nozzle, and by the pressure drop, density, viscosity, and surface tension of the liquid. It does not seem probable that any other quantity will have a significant effect when the nozzle is discharging into stagnant air, except for the currents created by the spray itself. Thus, as before, the general equation may be written

$$F(\Delta P, \rho, \mu, \sigma, D, \theta)=0, \quad (10)$$

which reduces to

$$\theta=\Phi(D\Delta P/\sigma,\ D\rho\sigma/\mu^2). \quad (11)$$

4. Experimental Results and Discussion

The validity of the relations developed above is demonstrated if plots of the experimental values of the dimensionless parameters give curves from which the deviations of the individual points do not exceed the experimental error. Such plots provide means of correlating the experimental data, and show the influence of the various quantities upon the flow rate, mean drop size, and spray angle. In the graphs that follow, the experimental data on the flow capacity of the nozzles were obtained at the Bureau. The data on drop size and spray angle are original with Rupe [3].

4.1. Nozzle Flow-Capacity Correlation

The nozzles selected for the flow-capacity tests were types currently in use in aircraft gas turbines. These included three simplex nozzles of one make, design, and nominal size, and two duplex nozzles of different designs. All were of the fixed configuration, continuous spray, centrifugal type, and each was an unmodified production unit.

Four different liquids were used in the flow-capacity measurements. The temperature of the liquid at the nozzle, as measured in each run by a thermocouple, varied from 77° to 90° F, depending upon the operating pressure and flow rate. The pertinent physical properties of these test liquids at the average operating temperature of 81.5° F are given in table 1.

Table 1. *Properties of the capacity test liquids at 81.5° F*

Description	Density, ρ	Viscosity, μ	Surface tension, σ
	g/cm³	*cp*	*Dynes/cm*
Commercial grade of heptane	0.691	0.412	22.8
Light petroleum solvent	.780	.863	27.6
45% of white mineral oil plus 55% of light petroleum solvent	.813	2.38	29.2
75% of white mineral oil plus 25% of light petroleum solvent	.837	7.02	31.3

The variations of the properties of the liquids with temperature over the operating range of 75° to 90° F were also known, and the actual values corresponding to the operating temperature observed in each run were used in computing the dimensionless variables.

With a given liquid in the test system, the flow rates (M) were observed at nozzle pressure drops of 20, 40, 70, 100, 175, and 275 lb/in^2. These flow rates were determined for each nozzle at each of the aforementioned pressure drops before changing the liquid. This procedure requires that each nozzle be installed and removed whenever the liquid is changed. Although the simplex nozzles were torqued to approximately the same value at each insertion, it is possible that this process caused small changes in configuration, which would influence the correlation among results with different fluids.

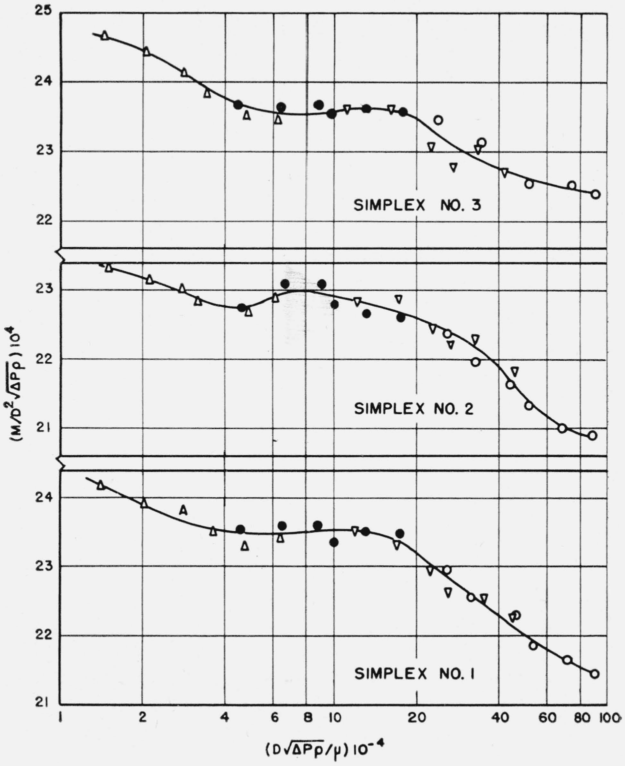

Figure 3. *Capacity curves for three simplex nozzles.*

The capacity of the duplex nozzles was determined in the same way as that of the simplex nozzles, except that the pressure ratio $\Delta P/\Delta P_s$ was controlled and measured. Ratios of infinity, 1.0, 0.6, and 0.0 were selected arbitrarily for these tests. The 0.6 pressure ratio had to be omitted in the case of nozzle 2, because its flow rate proved so sensitive in this region that accurate measurements could not be made.

In dealing with fuel nozzles it is common practice to express capacity in pounds per hour, pressure drop in pounds per square inch, density of the liquid in grams per cubic centimeter, its absolute viscosity in centipoises, and its surface tension in dynes per centimeter. Although these mixed units are used in this report for speciying experimental conditions and characteristics of the liquid, all measured quantities were converted to cgs units before computing the dimensionless variables presented herein. Thus the dimensionless variables are based on flow rates in grams per second, pressure drops in dynes per square centimeter, densities of the fluid in grams per cubic centimeter, its absolute viscosity in poises, and its surface tension in dynes per centimeter. The dimensionless equations are independent of the system of units employed, and the user may select the system with which he is most familiar, provided only that the same units are used in developing numerical values of the parameters and in their subsequent applications.

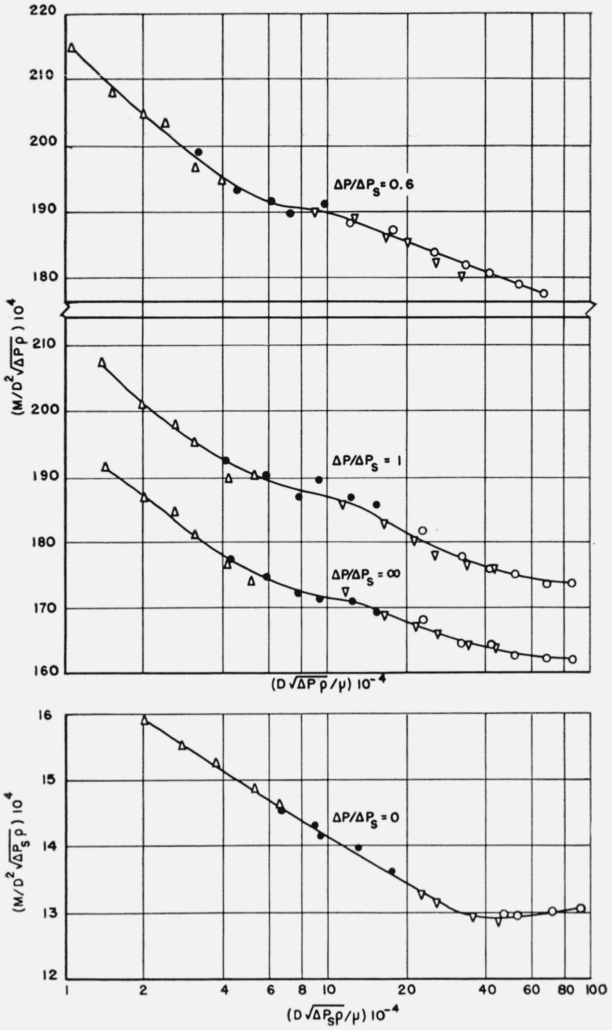

FIGURE 4. *Capacity curves for duplex nozzle 1.*

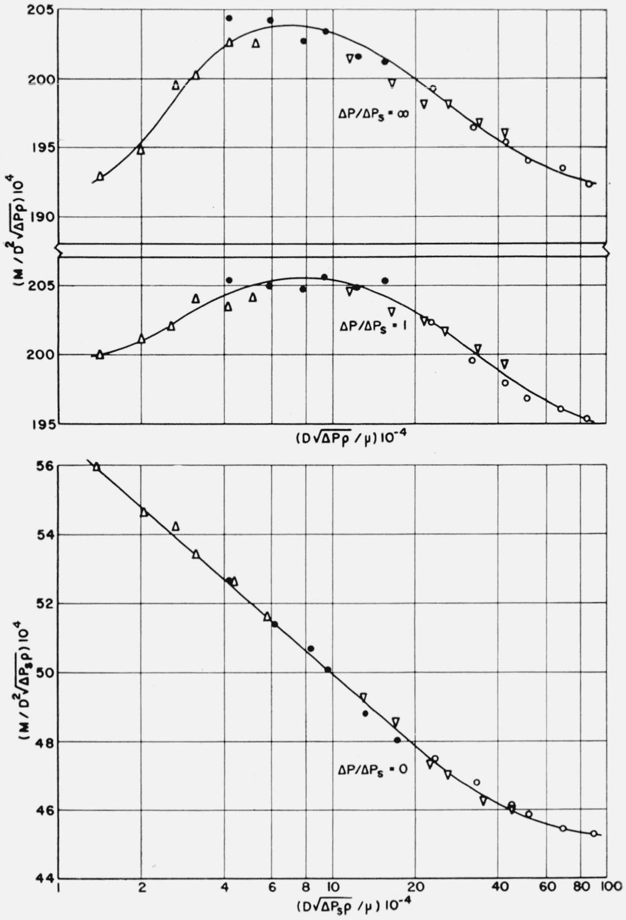

FIGURE 5. *Capacity curves for duplex nozzle 2.*

In the present state of the manufacturing art it is not practicable to make the small and intricate flow passages of production nozzles geometrically similar. It was therefore anticipated that correlation of the data for the three simplex nozzles by a single curve would not be possible. Hence a separate characteristic curve is presented for each nozzle, and since D for a given nozzle is constant, its value need not be determined. For simplicity the value of one centimeter is arbitrarily assigned to D.

With this assigned value of D, the parameters $M/D^2\sqrt{\Delta P\rho}$ and $D\sqrt{\Delta P\rho}/\mu$ can be evaluated from the experimental data for each nozzle and for each observed value of pressure drop, density and viscosity. The results for the three simplex nozzles are shown in figure 3, and those for duplex nozzles 1 and 2 are shown in figures 4 and 5, respectively. The same symbol is used in all these figures for a given test fluid. It will be seen that the results for each nozzle define a single curve or chart, from which the individual points deviate by less than ± 1 percent on the average. Such scattering is commensurate with the experimental error, thus confirming eq (4) and (7).

Referring to figures 4 and 5, the capacity curves for the duplex nozzles show that the treatment of the two inlet pressures, through the formation of the dimensionless pressure ratio $\Delta P/\Delta P_s$, is valid. Thus, through the relation expressed in eq (7), it is possible to obtain characteristic capacity curves at selected, constant values of pressure ratio for the duplex nozzles.

In general, the capacity curves show slightly increasing flow rates with increasing viscosity, for constant values of D, ΔP, and ρ. They also show a noticeable difference between nozzles that have the same nominal size and shape. After a curve of this type has been established for a given nozzle, it may be used for estimating the capacity of that nozzle to deliver other fluids.

As an illustration of the utility of such capacity curves, the possible effects of changes in surface tension upon flow rate will be considered. In performing the capacity tests, the product $D\Delta P/\sigma$ was varied between 4.5×10^4 and 83×10^4. Despite the fact that the parameter $D\Delta P/\sigma$ appears in eq (4), all values of $M/D^2\sqrt{\Delta P\rho}$ and of $D\sqrt{\Delta P\rho}/\mu$ for a given nozzle lie on a single curve. As the surface tension does not appear in the plotted parameters and as the experimental values of the latter lie on a smooth curve, regardless of the value of $D\Delta P/\sigma$, the results indicate that the flow rate is not affected significantly by the surface tension of the liquid that is flowing. This conclusion is limited to variations in σ that produce changes in $D\Delta P/\sigma$ within the range specified above.

4.2. Mean Drop-Size Correlation

Although no measurements of drop size or spray angle have been made at the Bureau, the applicability of eq (9) and (11) can be examined by referring to data obtained elsewhere. Rupe [3] presents extensive data on a few nozzles with different liquids, and his results will now be inserted in the equations. The pertinent properties of the liquids used by Rupe are listed in table 2.

Rupe's method of determining drop size involved collecting the spray droplets for a predetermined time interval in cells having non-wetting glass bottoms. These contained an immersion liquid having a lower density than that of the sprayed fluid. The droplets collected on the bottom were photographed under magnification and their images were counted with an automatic electronic scanner-counter in 14 size groups down to 5 microns. The results of interest here, as presented in figures 12, 13, and 15 of reference [3], apply to a single spray nozzle with droplets collected from the same relative position in successive runs.

TABLE 2. *Properties of the drop-size and spray-angle test liquids*

Fluid No.	Components			Properties at 20° C			
	Water	Nigro-cin	Addi-tive [a]	Surface tension	Specific gravity 20°/20° C	Vis-cosity	$\frac{D\rho\sigma}{\mu^2}\times$ [b]10^{-4}
	wt %	*wt %*	*wt %*	*Dynes/cm*		*cp*	
1	96.63	3.37	0.00	67.0	1.003	1.158	49.7
2	86.93	3.37	9.70	63.3	1.029	1.532	27.8
3	79.92	3.37	16.71	61.6	1.048	1.936	17.2
4	73.00	3.37	23.63	59.4	1.068	2.531	9.9
5	95.13	3.37	1.50	46.0	1.008	1.283	28.2
6	93.63	3.37	3.00	38.0	1.001	1.284	23.1
7	90.36	3.34	6.30	28.0	0.996	1.429	13.6
8	100-octane gasoline			19.0	[c].712	[c]0.497	[c]54.6

[a] Additive to fluids 2, 3, and 4 was glycerol; to fluids 5, 6, and 7 was butyl alcohol.

[b] Value of D taken as 1 cm in the parameter $D\rho\sigma/\mu^2$.

[c] Specific gravity and viscosity are estimated values.

Rupe's figure 12 applies for fluid 1, and shows mean drop diameter as a function of pressure drop. In his figure 13 the ratio of the mean drop diameters of fluids 5, 6, and 7 to that of fluid 1 are plotted as functions of surface tension at pressure drops of 25, 30, and 100 lb/in^2. In his figure 15 the ratio of the mean drop diameters of fluids 2, 3, and 4 to that of fluid 1 are shown as functions of viscosity at pressure drops of 50, 100, and 150 lb/in^2. His method of analyzing the results required data on two groups of fluids, one (fluids 1, 5, 6, and 7) having equal viscosities but different surface tensions and one (fluids 1, 2, 3, and 4) having equal surface tensions but different viscosities. Three separate graphs were required for presenting the results, and these concealed rather than revealed possible unevaluated influences. Had he planned to treat the results by dimensional analysis, there would have been much greater leeway in the choice of test fluids and in the selection of experimental pressure drops. The results could also have been presented on a single chart.

In the present treatment the data of reference [3] have been substituted in eq (9), using the arbitrary value of 1 cm for D. The resulting values of d/D and $D\Delta P/\sigma$ are plotted in figure 6 for the seven test fluids. It will be noted that the points define a single curve, from which the deviations are usually within ±10 percent. For values of $D\Delta P/\sigma$ between 2.8×10^4 and 10×10^4, even better correlation is obtained. Considering the difficulties encountered in collecting, photographing and counting the numerous droplets ranging in diameter from 5 to 200 microns, such correlation is surprisingly good. Nevertheless, an examination of the probable causes of the scattering is of interest, as it indicates the presence of some unevaluated influence in the experiments.

It seems improbable that the effects of viscosity are a primary cause of the scattering. Equation (9) indicates that a plot of d/D versus $D\Delta P/\sigma$ should yield a chart with each curve representing a constant value of the product $D\rho\sigma/\mu^2$. Thus all fluids having the same value of this parameter should define the same curve, even though their separate values of ρ, σ, and μ may differ considerably. Furthermore, if the plot of d/D versus $D\Delta P/\sigma$ yields a single curve for any particular range of the product $D\rho\sigma/\mu^2$, then it is known that changes in ρ or μ, which cause the product to vary within this range will not affect the d/D ratio, and consequently the drop size.

Referring to figure 6 and table 2, the points representing fluids 2, 3, and 4 all define the same curve, even though their respective values of $(D\rho\sigma/\mu^2)\times10^{-4}$ are 27.8, 17.2, and 9.9. Thus, con-

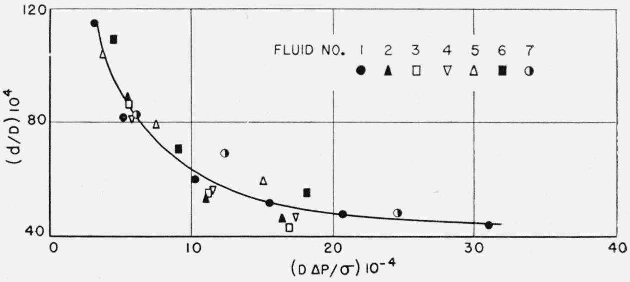

FIGURE 6. *Mean drop-size curve (Rupe's nozzle 4).*

sidering only these three fluids, it could be concluded that variations in viscosity that cause the product $(D\rho\sigma/\mu^2)\times 10^{-4}$ to vary within the range 10 to 28 will not affect the drop size. Considering fluids 5, 6, and 7, for which the respective values of $(D\rho\sigma/\mu^2)\times 10^{-4}$ are 28.2, 23.1, and 13.6, it appears that these fluids define a different single curve. As these two groups of fluids embrace the same range of $D\rho\sigma/\mu^2$, the separation between the curves defined by the two groups cannot be attributed to either viscosity or density effects.

Some possible explanations of the scattering in figure 6 are that some influential quantity has been omitted from the theoretical consideration; that the nozzle configuration may have changed, either from dirt or rust, during the experiments; or that properties of the test fluids at the instant of drop formation may have been different from the tabulated static values. Rupe mentions the latter possibility in connection with the surface tension of the water-alcohol mixtures. The method of collecting and counting the smaller droplets may also have been inadequate, thus resulting in appreciable errors in the total number of drops and consequently in their mean diameter.

The curve of figure 6 may be used to predict with fair accuracy the mean drop diameter from geometrically similar nozzles of different sizes, with any liquid and pressure drop, provided only that the values of $D\Delta P/\sigma$ and $D\rho\sigma/\mu^2$ are within the ranges investigated. As the ratio d/D is nearly constant at the higher values of $D\Delta P/\sigma$, the mean drop size would be expected to increase in direct proportion to the size of the nozzle. This is not true, however, in the region of lower values of $D\Delta P/\sigma$.

4.3. Spray-Angle Correlation

Spray angle data for a single nozzle at different pressure drops within the range from 5 to 100 lb/in^2 are given in figure 24 of reference [3]. These data, obtained with liquids 1, 5, 6, 7, and 8 of table 2, were presented in terms of θ as a function of ΔP, with a separate curve for each liquid.

From these data, the dimensionless variables of eq (11) were computed, with the result shown in figure 7. Each symbol represents a single test liquid, and hence also a single value of the product $D\rho\sigma/\mu^2$. As the individual points do not deviate from the curve by more than $\pm 5°$, the correlation is considered reasonable. However, for $D\Delta P/\sigma$ from 3.5×10^4 and 10×10^4, fluid 1 appears to define a different curve from that of fluids 5 through 8. As in the case of the mean drop-size correlation, viscosity effects are not believed responsible, because table 2 shows that fluid 1 has a value of $D\Delta P\sigma/\mu^2$ within the range encompassed by fluids 5 through 8. Therefore, it appears that either the nozzle configuration changed or some unknown influence has been omitted from the consideration.

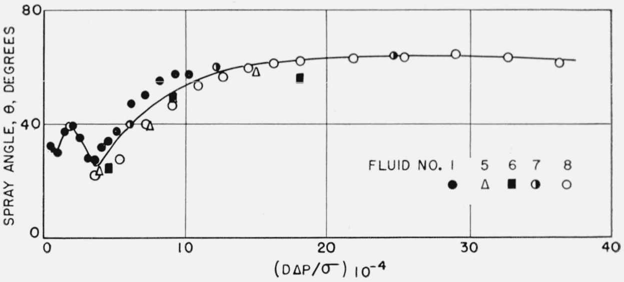

Figure 7. *Spray-angle curve (Rupe's nozzle 4).*

5. Conclusion

Dimensional analysis provides a useful and convenient method for correlating and presenting experimental data on the performance of fuel spray nozzles. It is shown that experimental values of flow capacity with liquids of various physical properties define a single curve for each nozzle, when plotted in terms of logical dimensionless variables. Correlations of available data on mean drop size and on spray angle are less exact than those on capacity. This lack of correlation may be ascribed, at least in part, to experimental error. However, the possibility that some important quantity has been omitted in formulating the dimensionless relations is not excluded.

The curves presented to indicate nozzle performance apply specifically only to the particular nozzles for which they were determined. They are used solely to illustrate possible applications of dimensional analysis to the study of spray nozzles for such purposes as determining effects of individual variables that cannot be changed readily and independently; developing test programs leading to a desired end result from a limited number of experiments; analyzing, correlating, and interpreting experimental data; and predicting differences in performance due to the use of liquids of different physical properties.

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