

Measurement of Multimegohm Resistors

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The method by which multimegohm resistors are measured at the National Bureau of Standards is a null method using an electrometer as the null detector. The charge flowing through the resistor during the time of measurement is obtained from a variable air capacitor maintained at a fixed potential. The potentials across the variable air capacitor and thus across the specimen are maintained constant as indicated by the null reading of the electrometer by decreasing the capacitance of the air capacitor at just the right rate. The capacitance of the capacitor is varied by a small direct-current motor geared to the shaft of the capacitor and whose speed can be controlled. Several multimegohm resistors of two different makes have been studied over a period of about three years. Although these are the most stable multimegohm resistors available, it was found that they had erratic fluctuations of 0.5 to 1 percent and were generally voltage sensitive. With the impressed voltage varied from 1.5 to 180 volts, various resistors showed resistance changes ranging from 0.4 to 26.9 percent.

1. Introduction

This paper has been prepared in response to numerous requests for a description of the methods used in the National Bureau of Standards for calibrating resistors having values above those conveniently and economically available in wire-wound form. A practical limit for wire-wound resistors may be taken as 10^7 ohms.

Bridges equipped with suitable detectors and capable of being balanced with a precision of a few tenths of a percent are commercially available for measurements in the range 10^7 to 10^{13} ohms. All of these equipments are comparison devices and involve the use of standard resistors either directly as components in the bridge circuit or as auxiliary calibrating units to be used in place of the unknown resistor. It is an unfortunate fact that even the best multimegohm resistors produced today do not possess sufficient stability to serve as reliable standards in the range mentioned above. A later section of this paper will present the results of a 3-year study in this Bureau on the best obtainable multimegohm resistors.

In the absence of suitable standards in this range, resistors must be evaluated by the application of other principles of measurement. These usually involve a variation of either the charge accumulation or the loss-of-charge method. For either method the null detector is usually an electrometer. In the accumulation method^{1 2 3} the charge flowing through the resistor is accumulated on a capacitor having good insulation. The potential from one side of the capacitor to ground is continuously increased at a rate such that the potential across the resistor remains the same throughout the measurement as indicated by the electrometer. The total charge that flows through the resistor in a known time is obtained from a knowledge of the capacitance of the capacitor system and the potential to which the

capacitor becomes charged as indicated by the potentiometer. The disadvantages of this method are the difficulty of accurately determining the effective capacitance and the difficulty of procuring a continuously variable potentiometer having the range and precision of setting desired.

A method for measuring small currents described by P. J. Higgs⁴ uses a variable capacitor rather than a potentiometer to measure the charge that flows through the resistor. In this case the loss-of-charge method is used. The charge that flows through the resistor is supplied by the variable air capacitor. Constant potential across the resistor as indicated by the electrometer is obtained by continuously decreasing the capacitance at the proper rate. It is not necessary to know the total capacitance of the system, only the amount by which the capacitance has been changed during measurement.

2. Method of Measurement

A slightly modified version of the Higgs' method has been developed by the present author. A diagram of the circuit is shown in figure 1. A potential divider is used to provide the potentials for the specimen and variable air capacitor rather than direct battery connections. This permits the ratio of the voltages to be more easily determined with the desired accuracy than when the voltages are measured. A small d-c motor is geared to the shaft of the variable air capacitor so that the capacitance can be changed more smoothly than by hand.

The basic principle of this method may be understood from the following considerations: Before the start of a measurement the switch, s , across the detector is closed; the capacitor, C_s , is charged to the full voltage of the battery, V ; and a steady current of magnitude $i = (r_x/r_s)(V/X)$ is flowing through the unknown resistor, X , the latter being very large compared with r_x . At time $t=0$ switch s is opened, and, if no other adjustments were made, the capacitor discharges exponentially to some lower voltage of

¹ John S. Townsend, The genesis of ions by the motion of positive ions in a gas, and a theory of the sparking potential, *Phil. Mag.* 6, 598 (1903).

² L. S. Taylor, Accurate measurement of small electric charges by a null method, *BS J. Research* 6, 807 (1931) RP306.

³ H. H. Race, Electrical conduction in hard rubber, Pyrex, quartz, and crystalline quartz, *Trans. AIEE* 47, 1044 (1928).

⁴ P. J. Higgs, A method of measuring high insulation resistance, *J. Sci. Instr.* 10 169 (June 1933).

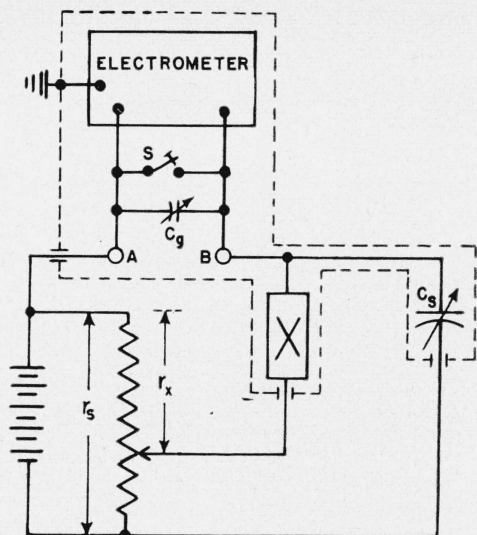


FIGURE 1. Circuit used to measure multimegohm resistors.

magnitude dependent upon the setting of the contact P . The procedure is to decrease the value of C_s continuously and smoothly in such a manner that the potential difference across it remains constant at its initial value as its charge diminishes. The rate at which C_s is decreased is such that the potential difference across open switch s remains zero as indicated by the detector. It will be obvious, therefore, that current through X is being maintained constant at its initial value i , with the difference that the energy dissipated in X is now furnished by the capacitor rather than the battery. Under these conditions the pertinent equation is:

$$R = \frac{V_x}{V_s} \frac{\Delta t}{\Delta C}, \quad (1)$$

where R is the resistance of the specimen, ΔC is the change in capacitance during time interval Δt and V_x and V_s are the potentials across the specimen and variable air capacitor, respectively. Since the current through the specimen is negligibly small compared to the current in the potential divider, the ratio V_x/V_s may be taken equal to r_x/r_s and eq (1) becomes:

$$R = \frac{r_x}{r_s} \frac{\Delta t}{\Delta C}. \quad (2)$$

3. Description of Equipment

Any good electrometer having the required sensitivity may be used as the null instrument to indicate when points A and B of figure 1 are at the same potential. The insulation between the terminals of the electrometer must be of such quality that the leakage current will be negligible compared to the current flowing through the specimen. Two types of electrometers have been used, a Compton quad-

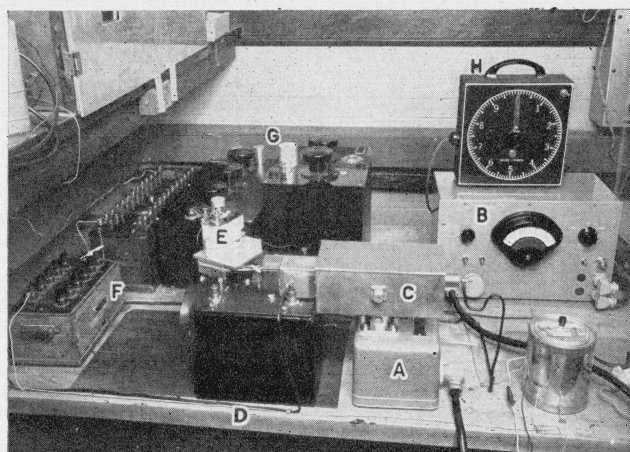


FIGURE 2. Equipment used to measure multimegohm resistances when the vibrating-reed electrometer is the null indicating instrument.

A, Electrometer head; B, electrometer controls; C, specimen shield; D, variable air capacitor, C_s ; E, motor drive for variable air capacitor; F, voltage divider; G, variable capacitor, C_s .

rant electrometer and a vibrating-reed electrometer.⁵ The equipment used in the latter case is shown in figure 2.

When the Compton quadrant electrometer is used, the point A is directly connected to ground. The quadrant electrometer has a sensitivity such that a potential of 1 mv produces a deflection of about 1.5 mm at a scale distance of 2 m. Its period is about 11 sec, and this relatively long interval is a serious disadvantage when trying to observe potential changes and to compensate for them.

Of the two, the vibrating-reed electrometer is both more sensitive and more rapid in its response. Its sensitivity is such that a potential of 1 mv produces a meter deflection of 50 scale divisions. The response is almost instantaneous. A disadvantage is that neither of the measuring terminals may be connected to ground. This requires that all equipment be insulated from ground. Another disadvantage is that on the most sensitive range it has a zero drift that must be determined and for which a correction must be applied. The vibrating-reed electrometer is, however, generally more satisfactory than the quadrant electrometer.

The capacitor, C_s , must have negligible leakage between its plates. A three-terminal variable air capacitor is used. Each set of plates is insulated from the frame and case. There is no dielectric directly between the plates. Conductance to ground on the high-potential side causes no error because the leakage current returns to the battery without affecting the electrometer. There is no difficulty in keeping the leakage to ground on the low side sufficiently low because this part of the circuit is maintained at, or near, zero potential at all times, and the leakage current is therefore negligible.

⁵ This electrometer converts a d-c potential into an a-c potential by means of a reed vibrating near an anvil on which the d-c potential appears. The a-c potential is then amplified and rectified. A negative feedback stabilizes the circuit. A full description of this type of electrometer is given by H. Palevsky, R. K. Swank, and R. Grenchik, Design of dynamic condenser electrometers, Rev. Sci. Instr. 18, 298-314 (May 1947).

It has been found that when the relative humidity rises above about 40 percent appreciable conduction does occur directly between the plates of the capacitor. This is probably due to very fine fibers that are generally floating in the air and which settle onto the plates, occasionally bridging them. When a drying agent is placed in the capacitor, this conductance disappears.

Any one of three capacitors having ranges of 9.5 to 20.5, 15 to 125, and 100 to 1,100 picofarads (micro-microfarads) are available for use in the measurements of resistance. With these capacitors, and with proper potentials applied to them, measurements can be made involving currents from 10^{-9} to 10^{-13} amp. For instance, if a 10^9 -ohm resistor is being measured at 1.5 v, it will be seen from eq (1) that a potential of 500 v will be required on the variable air capacitor for a capacitance change of 900 picofarads in 300 sec. If a 10^{13} -ohm resistor is being measured at 1.5 v, a potential of 15 v will be required on the variable air capacitor for a capacitance change of 9 picofarads in 900 sec.

It is necessary to decrease the capacitance of the capacitor, C_s , at a specific uniform rate. For this purpose, a small d-c motor is geared to the shaft of the capacitor, as shown in figure 3. When using the vibrating-reed electrometer, the motor must be insulated from the shaft and frame of the capacitor, so a fiber gear and insulating supports are used. The speed of the motor is controlled by a series resistance. This resistance is adjusted so that the speed of the motor is just below the proper speed, and small changes in the speed are made by shorting part of the resistance by means of a key. By closing this key momentarily with varying regularity, it is possible to keep the potential between points *A* and *B* within plus or minus 1 mv as indicated by the electrometer.

The potential divider is a plug box having a total resistance of 1 megohm. This resistance is low compared to the resistances being measured, and yet is high enough to prevent undue drain on the battery used for potential source. The potential source must be very steady. The voltage on the variable air capacitor must not change sufficiently during the period of measurement to cause a change of charge on the capacitor, which is appreciable compared with the total charge that flows through the specimen. The charge Q_x that flows through the specimen during measurement is given by $Q_x = V_s \cdot \Delta C$, and the charge change due to voltage change is given by $Q_s = C \cdot \Delta V$. If the ratio Q_s/Q_x is to be less than the limit of error L , then ΔV must be less than $L(\Delta C/C)V_s$. If the limit of desired error is 0.1 percent, ΔC is 9 picofarads, and C is 20 picofarads, then ΔV must not be more than 0.045 percent of V_s . It has been found that heavy-duty radio B batteries will satisfy this requirement when connected to a potential divider that causes a current drain not exceeding 0.5 ma.

The time is measured by means of an electric timer that indicates to tenths of a second. This timer is started by a switching mechanism that opens switch

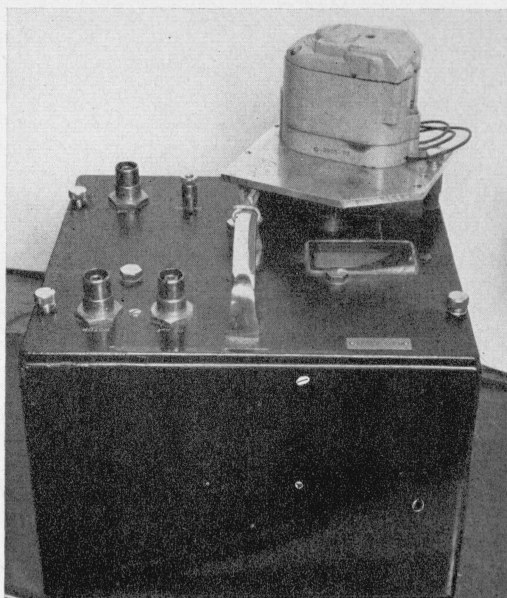


FIGURE 3. Motor drive for variable air capacitor

S and starts the motor driving the capacitor C_s so that all three events occur simultaneously.

For attainment of the desired accuracy (limit of error of 0.1 percent in the present case), the following requirements must be met. (1) The capacitance change, ΔC , during measurement must be large enough so that it can be determined with the required accuracy. This depends upon the range of the capacitor being used. (2) The time of measurement must be long enough so that Δt can be determined with the required accuracy. The timer used in this work can be read to 0.1 sec, so a time of measurement of about 300 sec is generally used. (3) The sensitivity of the electrometer to voltage must be such that in combination with proper regulation, the deviations of the potential between points *A* and *B* from zero are negligible in comparison to the voltage applied to the specimen.

4. Measurement Procedure

The measurement procedure can be best described by an example such as the following. Suppose that a resistor having a nominal value of 10^{10} ohms is to be measured at 1.5 v. A time of measurement of 300 sec is chosen. If the capacitor having a range of 1,000 picofarads is used, a ΔC of 900 picofarads is chosen. As seen from eq (1), choice of the above three values will require that the voltage on the capacitor be 50 v. As this is also the value of the voltage across the voltage divider, the battery connections are adjusted to give that value as nearly as possible. The battery pack is so constructed that its voltage output can be adjusted in steps of 1.5 v. As 0.5 ma is the current chosen for the potential divider, the resistance of the potential divider is adjusted to 100,000 ohms. The tap for the specimen is placed at 3,000 ohms to give 1.5 v. The value of the voltage across the potential divider is watched by means of a potentiometer connected across 300

ohms. To keep the sensitivity of the circuit within a practical working range, the value of C_g is adjusted until the potential difference across the open switch, s , changes at the rate of about 20 mv/sec when the capacitance, C , remains fixed. An appreciably higher rate than this makes it difficult to adjust the speed of the driving motor on the variable air capacitor quickly enough to keep the potential difference across the open switch, S , from fluctuating more than 1 mv. The equipment is now adjusted for measurement.

To make a measurement, switch s is closed, the capacitor is set at its highest value, and the battery is connected to the potential divider. At time $t=0$, switch s is opened at the same time that a switch is closed, which starts both the motor drive on the capacitor and the timer. The potential difference across the open switch, s , is kept less than 1 mv (with practice it can be kept less than 0.4 mv) by slightly changing the speed of the driving motor. At the end of about 300 sec the switch is opened, which stops the motor and timer. Switch s is then closed. The capacitance difference is determined from its original and final readings, and the time interval is read from the timer. The resistance of the specimen is computed by eq (2). The capacitor is then reset to its highest value, the timer reset to zero, the battery to the potential divider is reversed, and another measurement is made, as described above. An average of this pair of measurements is considered a single determination. Two determinations are made on each specimen.

5. Accuracy and Precautions

A limit of error of 0.1 percent can be obtained with this equipment if the following precautions are observed. The capacitance difference, ΔC , should be large enough so that it can be determined with the required accuracy. The variable air capacitors used are calibrated to 0.01 percent of their maximum range. Because only about nine-tenths of the maximum range is used in a measurement and inaccuracies due to stopping the motor are involved, ΔC cannot be determined with an accuracy greater than about 0.04 percent.

The time interval, Δt , must be of such duration that it can be determined with sufficient accuracy. Because the timer reads to 0.1 sec, 300 sec is chosen. Care must be taken to assure that the timer is started at the same instant switch s is opened. Under present procedure, the error due to lack of the synchronization is not greater than 0.1 sec. Thus Δt can be determined with an accuracy of 0.03 percent.

The resistances in the potential divider can be readily determined to 0.01 percent, so these present no problem regarding limit of error.

It is not possible to keep the potential difference across the open switch, s , always at zero. The appearance of this potential causes an error in two ways. It changes the potential across the specimen and across the capacitor. Because a voltage of 1.5 v is applied to the specimen, a voltage of 0.5 mv

across switch, s , would change the voltage on the specimen and hence the current by about 0.03 percent. The potential that appears across the switch, s , is random in nature, being both positive and negative, so the average effect is less than 0.03 percent.

The appearance of a potential across switch s also changes the potentials across the capacitors, C_s and C_g . The appearance and disappearance of these random potentials do not affect the measurement except as they produce a leakage of charge over the insulation from the low-potential side of the capacitor, C_s , to the shield or through the insulation of the capacitor, C_g . If the insulation is good (10^{10} ohms or greater), this leakage will be negligible. However, if the measurement is terminated at a time when the potential across the switch s is a maximum of 0.5 mv, then the resistance of the specimen as computed by eq (2) will be in error because part of the charge that flowed through the specimen came from a change in potential of the capacitors, C_s and C_g , rather than from a change in capacitance of capacitor C_s . The error due to this can be estimated by recalling that C_g is adjusted so that the rate of change of potential across switch s is about 20 mv/sec when the capacitance, C_s , remains fixed. This is equivalent to about 6,000 mv in the 300 sec during which the measurement is made, so an error of 0.5 mv at the end of the measurement would cause an error of about 0.01 percent.

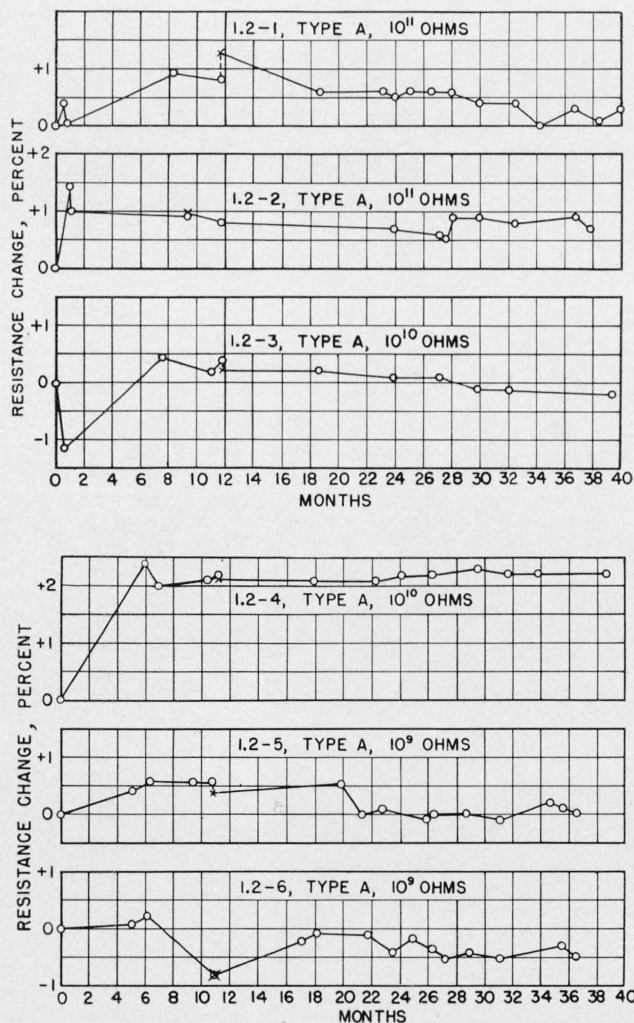
In those cases where the current flow is less than 10^{-12} amp (resistance greater than 10^{12} ohms at 1.5 v), the capacitance desired for C_g becomes less than the minimum capacitance of the system and cannot be realized. Therefore, the measurement time must be increased correspondingly if the same accuracy is to be obtained.

Because of the small currents involved, it was found necessary to take special precautions in shielding to eliminate spurious effects. It was found that contact-potential differences of different metals in the shielding and conductor can cause a drift of the electrometer even though no potential is applied. It was not practical to make all parts of the equipment from the same metal. However, the effect was virtually eliminated by the proper selection by a cut-and-try process of metals for the various parts. Correction for the residual can be made by making measurements with the battery connected direct and reversed. An average of these two measurements gives a value from which the error is eliminated, provided the effect does not change during the time of measurement. The terminals on some of the resistors are especially bad in this respect. The trouble from specimen terminals may be eliminated by surrounding the terminal with a sheath of metal that does not produce a drift of the electrometer. A bare copper wire wound into a tight helix makes an excellent sheath.

There is a drift in the vibrating-reed electrometer that is equivalent to a current of about 10^{-16} amp. This is not significant when measuring resistances below 10^{12} ohms, but becomes appreciable for

resistances above this value. This drift is sufficiently constant over the period of measurement, so that an average of two measurements with reversed battery practically eliminates this effect.

To test the equipment for spurious currents, a measurement is made with the resistor removed from the circuit. When the circuit is set for measuring resistances in the range 10^9 up to 10^{11} ohms, and when it is operating properly, no detectable change in potential (equivalent to less than 0.02 percent of the resistance for which the circuit is set to measure) is observed across open switch *s* in 300 sec. When the circuit is set for measuring resistances of the order of 10^{13} ohms, the readings are appreciable and somewhat erratic. The potential appearing across open switch *s* always has the same polarity, regardless of the polarity of the battery across the potential divider and is great enough to produce an error of from 0.1 to 0.5 percent. Averaging a pair of measurements with the battery reversed tends to cancel this effect, but because this effect is somewhat erratic, the limit of error for measurements of 10^{13} ohms is 0.2 to 0.3 percent.



6. Aging and Voltage Coefficient of Certain Commercial Resistors

A number of multimegohm resistors have been studied over the past 3 years, during which time they have been kept under standard laboratory conditions (23°C and less than 50 percent relative humidity). Most of these resistors have been measured by the method described in this paper, but a few of these resistors had resistances too low to be measured in this manner. These latter resistors were measured either by the "comparison method, using a galvanometer" described in ASTM Tentative methods of test for electrical resistance of insulating materials D257-52T, or more lately by means of the Wheatstone bridge. The study included two types of resistors. One type (1.2-1 through 1.2-9) consisted of a small coated rod sealed in a glass envelope. The glass envelope had been treated to reduce surface conductance. The other type (1.2-10 through 1.2-14) consisted of molded composition material.

The values of resistance fluctuated with time, as shown in figure 4 and were a function of the voltage

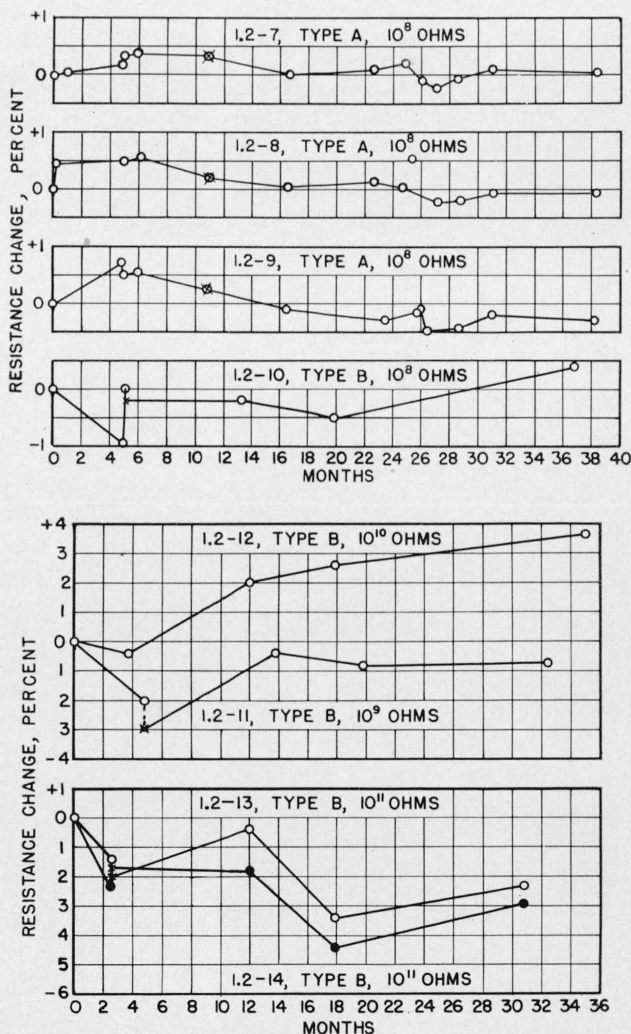


FIGURE 4. Change of resistance of multimegohm resistors with time. The crosses are values obtained after the resistors had been subject to a voltage of 180 volts.

at which they were measured, as shown in table 1. The values of the molded composition resistors (1.2-10 to 1.2-14) fluctuate more with time than do the glass-sealed resistors (1.2-1 to 1.2-9), which may be due to humidity effects. One of the molded composition resistors (1.2-12) now has over twice the resistance that is stamped on it. When originally received about 12 years ago, it was measured by a moderately accurate method (limit of error probably about 2 percent) and found to have approximately the value of resistance stamped on it. This means that in the intervening years, the resistance had doubled. This is the resistor that shows the greatest voltage coefficient.

Three other types of resistors have been tested, but their stability was so much poorer than the two mentioned above that no extended study has been made of them.

There is some indication that the fluctuations in resistance of some of the sealed resistors are decreasing with age. But even the best of these resistors have erratic fluctuations of resistance of the order of 0.5 percent.

The effect of change in voltage is often very different for different resistors, even when they are the same make (see table 1). The percentage changes between 1.5- and 180-v range from 0.4 to 26.9 percent. There appears to be no significant difference between the two types of resistors as regards effect of voltage.

Although the null-electrometer method described above is capable of measuring resistances up to 10^{12} ohms at 1.5 v, with a limit of error of 0.1 percent, no multimegohm resistors (10^9 ohms or above) have

TABLE 1. *Change of resistance of multimegohm resistors with voltages above 1.5 volts*

Resistor	Volts		Nominal value
	1.5 to 45	1.5 to 180	
	<i>%</i>	<i>%</i>	<i>Ohms</i>
1.2-1	-2.0	-8.3	10^{11}
1.2-2	-----	-3.4	10^{11}
1.2-3	-0.4	-2.5	10^{10}
1.2-4	-4	-2.5	10^{10}
1.2-5	-4	-2.7	10^9
1.2-6	+7	-1.5	10^9
1.2-7	0	-1.9	10^8
1.2-8	-5	-3.0	10^8
1.2-9	-3.9	-12.4	10^8
1.2-10	-0.1	-1.1	10^8
1.2-11	+6	-0.4	10^8
1.2-13	-----	-0.4	10^{11}
1.2-14	-0.3	+2.0	10^{11}
1.2-12	-10.4	-26.9	2×10^{10}

been noted that are stable with time to that accuracy. They all have voltage coefficients. All these things considered, the presently available resistors may not be relied upon to maintain their values closer than 0.5 to 1.0 percent. There is great need for a more stable resistor for use as a standard in the multimegohm range.

The author is indebted to J. F. Richardson for his aid in making many of the measurements and to the late Dr. Charles Moon for his suggestions and advice.

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