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Assembled Polygon for the Calibration of Angle Blocks

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A method is described for constructing and calibrating an assembled, multiple-sided, angular standard of exceptional accuracy. Although designed as a master for the 30- and 45-degree angle blocks of a series made in this country, the polygon is equally suitable for the test or calibration of circular dividing equipment. Simpler forms may be easily and economically constructed as masters for a variety of applications in the mechanical and optical industries.

1. Introduction

The advent of World War II produced an acute shortage of precise angular dividing equipment in England and led to the development by the National Physical Laboratory of a series of solid angular standards.[1] These are hardened, ground, and lapped steel blocks about ⅝ in. wide, 3 in. long, and having specified angles between the two contact surfaces. Like gage blocks, they can be combined with little error by wringing, but since they may be combined to form the sum or difference of a pair, a series of as few as 12 blocks will form any angle up to a little over 80 deg. in increments of 3 sec. By mounting these blocks and a solid square (a rectangular prism having four finished faces), in various positions, any subdivision of a circle to the nearest 3 sec. may be obtained.

The angles of the individual blocks in the NPL set are, in general, based on the mathmatical relationship whereby each successively larger block is twice the sum of the preceding blocks plus 1, respectively, in seconds, minutes, and degrees. Exceptions to the general relationship are the omission of a 1-sec. block, and the use of a 41-deg. block instead of an 81-deg. block. A 1-sec. block is unwarranted in a set having permissible errors of 1 sec., and the 41-deg. block provides a more convenient size than the 81-deg. block. The series is thus comprised of blocks having angles of 3, 9, and 27 sec., 1, 3, 9, and 27 min., and 1, 3, 9, 27, and 41 deg.

At about the same time that the English series of blocks was developed, a similar series was patented in this country.[2] The American series differs from the English in that it is made up of more commonly used sizes and thus requires a few more blocks for the same angular coverage. The series is 1, 3, 5, 20, and 30 sec.; 1, 3, 5, 20, and 30 min.; 1, 3, 5, 15, 30, and 45 deg.

Because the generation of nearly optically plane surfaces and the control of pyramidal error within the required limits are not excessively difficult, it is possible to produce blocks that are susceptible to very precise calibration.

One of the most satisfactory methods of calibrating angle blocks is by comparison with a master set, provided that the angular values of the masters can be determined with adequate accuracy. Knoyle[3] has described a primary determination of the angles of all the blocks in a set without reference to an external standard. Using an interferometric method for the required comparisons, we have found his procedure entirely satisfactory except for measurements on master blocks or combinations of master blocks having angles of 45 deg or larger and particularly on the 90-deg combination, which is used in the primary determination for the entire series. Alinement of the sides of the blocks in a combination becomes increasingly critical as the angle increases, and some difficulty has been experienced in obtaining satisfactory agreement on 90-deg combinations.

Similar types of alinement difficulties arise in the comparison of 45-deg blocks by interferometric methods wherein two blocks are wrung side by side on an optical flat and the difference in angle established by the difference in fringe count over a specific length on the unwrung surfaces of the blocks. If, for example, the adjacent sides of the two 45-deg blocks are more than 0.00005 in./in. out of parallel along the length of the blocks, interference bands on the two blocks form too large an angle with each other to permit an accurate fringe count. Considering the errors that may accrue in the determination of the angle of 45-deg blocks, and possibly 30-deg blocks, an alternative method of determining their angles is clearly desirable.

The success of the National Physical Laboratory in the calibration of angular-dividing equipment by means of multiple-sided solid angular standards[4] suggested the possibility of using a 24-sided solid polygon having 15-deg exterior angles between adjacent faces. Such a polygon would permit direct calibration of both 30- and 45-deg blocks. The mechanical requirements for a polygon of the requisite accuracy are quite severe; the defining surfaces must be flat over practically their entire area to one or two millionths of an inch, the surface roughness of the faces must be less than 0.5 μin. rms with practically no visible scratches perpendicular to the side of the polygon, and the variations in the angle between the defining faces and the axis of the polygon not in excess of 15 sec. Convenience dictates a maximum departure of the angular intervals from nominal value of not more than 10 sec. The con-

[1] British patent 569,002.
[2] U. S. Patent 2,134,062.

[3] C. H. Knoyle, NPL combination angle gauges, Machinery (London) **77**, 1975 (1950).
[4] C. O. Taylerson, Testing circular division by means of precision polygons, Machinery (London) **71**, 1821 (1947).

struction of a conventional, solid 24-sided polygon to these limits constitutes a challenge to the best mechanical skill and equipment.

2. Construction of the Polygon

As a substitute for such a formidable undertaking, the possibility of assembling a polygon from more easily fabricated units was considered. The availability of satisfactory defining faces in the form of gage blocks suggested the assembly of modified blocks on a suitable base, using one finished surface of each block to define the polygon. Conventional gage blocks could readily be modified by lapping the side faces plane and parallel, with, of course, due control of the angle between adjacent lapped faces and by providing holes for fastening the blocks to the base. With this type of construction, control of any of the required features of the polygon could be readily accomplished, usually with little effect on the other dimensions.

In order to determine the merits of this method of construction, an experimental polygon was made. The base of the polygon was ¾ in. thick and 8 in. in diameter. It was made of oil hardened steel and heat treated for maximum dimensional stability. A 1¾-in.-diameter center hole and appropriate tapped holes in the base provided means for attaching a bushing so that the polygon could be used on centers if desired. In order to keep the diameter of the plate within reasonable limits, the gage blocks were placed in two layers of twelve each.

Twenty-four tapped holes were equally spaced on a circle of a radius such that the chord of a 30-deg sector corresponded to the spacing of the holes in the gage blocks. This permitted the use of the same screws for fastening both the upper and lower layers of blocks. The tapped holes in the base were countersunk and then counterbored to a depth of ¼ in. to minimize distortion of the base due to tension of the clamping screws.

After hardening, the base was ground on both sides and then hand-lapped until the error in parallelism of the two surfaces was less than 0.0001 in. and the maximum error in flatness of each surface less than 0.00006 in. A relatively high finish was imparted to one surface in the final lapping to permit wringing the special gage blocks in position.

Modified 0.750-in. rectangular gage blocks were obtained from a gage-block manufacturer for use as the defining faces. All four lapped sides were plane to 0.000005 in. or better, and adjacent sides were square to 2 sec of arc. The surface roughness of each of the sides was less than 1 μin. rms. The thickness (⅜-in. dimension) of the 24 blocks in the group varied less than 0.00003 in. Each block was measured and the bottom layer of blocks so selected that adjacent blocks did not differ in thickness by more than 0.00001 in. This permitted at least partial wringing of the blocks in the upper layer to their supporting blocks and reduced distortion arising from the pressure of the clamping screws.

Commercial gage blocks are frequently used as mirrors for an autocollimator, and as such form reasonably sharp images of the reticle. It was found, however, that the crispness of the image could be further enhanced by a limited amount of polishing on a cloth lap, and each of the defining faces was so finished. Figure 1, illustrating the finish before and after polishing, was made with the Zeiss micro-interferometer.

The assembly of the components of the polygon

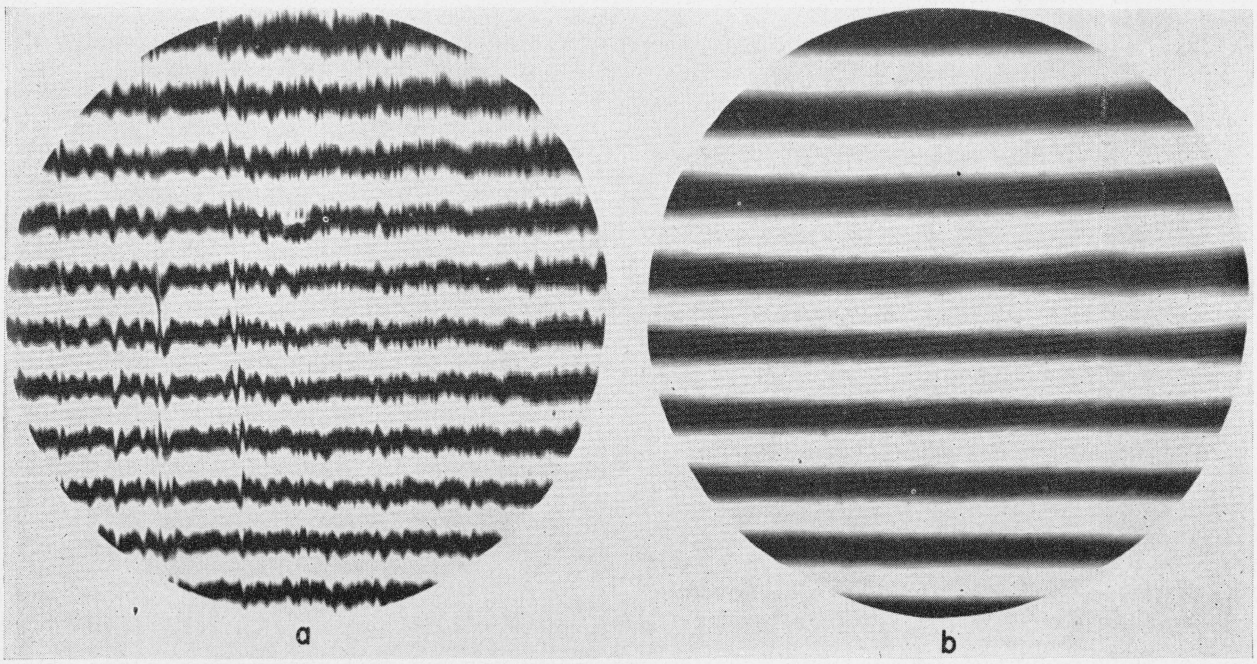

FIGURE 1. *Interference micrographs of a reflecting face (a) before and (b) after cloth polishing,* $\times$ *400.*

The interference fringes may be considered as contour lines with the distance between adjacent fringes equal to 0.000011 inch. Thus the excursion of a fringe one-half way toward an adjacent fringe indicates a hill or valley 0.0000055 inch high or low with the nature of the irregularity determined by the direction of the fringe displacement.

was made on a rotary table by the use of 15-, 30-, and 45-deg angle blocks, a solid square, and an autocollimator. The rotary table had previously been adjusted so that its surface was closely perpendicular to its axis of rotation and, for convenience in alining the autocollimator, parallel to the surface plate on which it rested. The autocollimator was mounted on a rigid base with its axis parallel to the surface plate at an appropriate height to cover both the polygon faces and the square or angle block.

The 0-, 90-, 180-, and 270-deg faces were positioned by means of the square and the intermediate faces by adding or subtracting 15, 30, and 45 deg from these positions. For example, the 0-deg block was wrung to the base with its two holes approximately concentric with two of the tapped holes; one face of a 30-deg angle block was adjusted parallel to the 0-deg block by means of the autocollimator; the base and the angle block were rotated through 30 deg by means of the rotary table, and the 30-deg polygon block was positioned parallel to the second face of the angle block. Because the 0- and 30-deg blocks could not be fastened to the base until the 15-deg block was in place, this block was positioned next, using the same method as for the 30-deg block. All the other blocks of the polygon were similarly located, working alternately in clockwise and counterclockwise directions from the 0-deg block.

The use of the same screws for holding both layers of blocks to the base made the assembling somewhat tedious, as the top block frequently moved when the screws were tightened, even though well lubricated, cupped, brass washers were used between the screw heads and the blocks. At least a part of this difficulty arose from the fact that some of the tapped holes were not accurately square with the base. The angular position of the top block was observed as the screws were tightened and, when necessary, the positioning repeated until the block was located within the required limits. The blocks in the lower layer had little tendency to move as they wrung quite firmly to the base, but their position was checked, as a precautionary measure, after the fastening screws were tightened.

It was found that quite accurate settings could be made by gently tapping a block to its final position, and, if only a single layer of blocks had been involved, it is believed that the angular intervals could easily have been set to an accuracy of 2 sec.

On this particular polygon no further adjustment was attempted when a block was once located to an accuracy of ± 10 sec, as moderate random departures from nominal angle may, in some applications, be used to good advantage to confirm the direction of corrections.

3. Calibration of Equipment

The autocollimators used in the calibration have a total range of 10 min and a least reading of 0.5 sec.

In order to reduce autocollimator errors to a minimum, a number of precautions were observed. Rotation of the reticle from its true position in a plane perpendicular to the optical axis introduces an error. The true position of the reticle is parallel to the setting hairlines of the micrometer eyepiece and perpendicular to their direction of travel. When the autocollimator is properly alined, the setting hairlines are parallel to the axis of rotation of the work. The relationship can be expressed by the formula $\theta = \beta \tan \alpha$, where θ is the error in seconds of the reading, β the departure in seconds of the reflecting surface from the plane of measurement of the micrometer eyepiece, and α the deviation of the reticle line from its true position. For a 30-sec change in vertical angle of the polygon reflecting surface and for a 2-deg rotation of the reticle line, the error is 1.05 sec. Because visual alinement of a reticle line is usually limited to an accuracy of about 2 deg, a more accurate method of alinement was required. Accordingly, we removed all the lenses of the autocollimator, projected an image of the reticle on a screen, and, by adjustment of the autocollimator rotational stops, brought the image into coincidence with the image of a suitably disposed machinist square. By this method the reticle line could be set perpendicular to the base of the instrument to within less than 15 min.

After adjustment of the reticle line, the autocollimator microscope was carefully focused on the reticle and the autocollimator objective adjusted to form a crisp image with a representative polygon face.

The two lines of the micrometer eyepiece so closely bracketed the reticle image that traces of color along each edge of the image affected the precision of setting. After some experimentation, a blue-green Wratten filter (B2–58) was found that transmitted an adequate amount of light but largely suppressed the objectionable colors. The remaining secondary chromatic aberration appeared to the operator to be equal on both sides of the reflected line.

Both autocollimators were calibrated over the central 9 min of range in 3-min intervals and over the central 5 min in 1-min intervals by means of master angle blocks. No error in excess of 0.3 sec/min was found in the central part of the range on either autocollimator. Because the differences in the angles to be measured were not expected to exceed 10 sec, it was considered that the progressive errors of the autocollimators could be neglected. As a check on the possibility of periodic errors in the micrometer eyepiece screws, comparisons were made of the angular value of the micrometer eyepiece screws of the two autocollimators in 5-sec intervals over a 1-min range by directing the autocollimators at separate faces of an angle block and rotating it in 5-sec increments between readings. The angular values of the screws on both instruments were found to be uniform within 0.3 sec, which is about the limit to which two settings can be repeated.

4. Calibration of Polygon

All the equipment for the calibration of the polygon was then assembled on a large surface plate. The polygon was mounted on a rotary table and each autocollimator on a substantial iron base with its optical axis in line with the junction of the upper and lower layers of blocks. Each autocollimator was centered on the appropriate polygon face by viewing with a low-power microscope the real image of the face that is formed in the exit pupil of the autocollimator eyepiece. At the same time, the position of the autocollimator light source was adjusted to provide uniform illumination over the entire polygon face.

Preliminary observations soon indicated that, after handling of the equipment, the entire setup required a fairly long stabilizing period in a constant-temperature room and that temperature gradients in the surface plate due to the presence of observers changed the angle between the autocollimators enough to introduce intolerable errors when the observations extended over a period of 1 or 2 hr. These effects were largely overcome by erecting an insulating barrier completely around and over the top of the setup and scheduling the observations so that at least 4 hr intervened between the readjustment of units and a series of measurements. Small openings in the insulating wall at each autocollimator position permitted reading of the autocollimator. As a check on angular change in autocollimator positions, the initial interval was remeasured after six (or fewer) consecutive intervals were observed.

Measurements were made by directing each autocollimator toward the center of the appropriate polygon face at such an angle that the reflected image of the reticle came within the selected range of the autocollimator. Figure 2 shows the disposition of polygon and autocollimators for the measurement of the 30-deg intervals. After recording the reading of the two autocollimators, the rotary table on which the polygon was mounted was then turned to present the faces of the adjacent angular interval. This process was repeated until every interval of the same nominal size had been measured and the circuit closed. Because some of the intervals are aliquot parts of 360 deg and others are aliquots of a multiple of 360 deg, a closure consisted of from 2 to 24 readings. Where a closure consisted of less than 24 readings, additional closures were required to cover all the intervals. For example, 12 closures of two readings each were required to cover the 180-deg intervals, whereas one closure consisting of 24 readings over 3,960-deg (11 revolutions) was required for the 165-deg intervals.

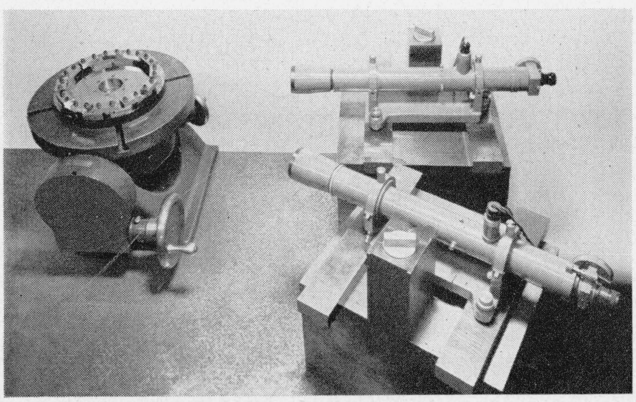

FIGURE 2. *Arrangement of polygon and autocollimators for the measurement of 30-degree intervals.*

One autocollimator is directed at a selected face and the other at a face 30 degrees away.

The difference between the readings of the two autocollimators is an estimate of the difference in angle defined by the two autocollimators and the angle between the two polygon faces. Because the angle between the collimators was only approximately equal to the nominal polygon interval, each difference, d_i, in any particular closure is an estimate of the error of the polygon interval, γ_i, plus a constant error, Δ, due to the deviation from nominal of the angle between the autocollimators. Thus $d_i = \gamma_i + \Delta$.

For a complete closure

$$\sum_{i=1}^{n} d_i = \sum_{i=1}^{n} \gamma_i + n\Delta,$$

where n is the number of readings required for a closure. But since $\sum_{i=1}^{n} \gamma_i$ for a circle or any multiple thereof equals 0,

$$\Delta = \frac{1}{n} \sum_{i=1}^{n} d_i.$$

Therefore,

$$\gamma_i = d_i - \frac{1}{n} \sum_{i=1}^{n} d_i,$$

from which values for each polygon interval of a closure can be obtained. However, each difference is subject to a random error e, so that we have an e_i associated with each d_i. It is reasonable to assume that all errors, e_i, arise independently and randomly from the same error distribution, and that the distribution is characterized by an average value of zero and a dispersion of the magnitude indicated by a standard deviation, σ. This standard deviation is a measure of the precision of an individual difference by this processs of measurement; an average of n individual values will have the precision indicated by a standard deviation of $\sigma/\sqrt{n}$ and the sum of k individual values will have a standard deviation of $\sigma\sqrt{k}$.

If only the closure for the 15-deg interval is made, the precision with which the corrections to each of the 24 basic angles is known would be the same for all angles, but that of any angle comprised of the sum of two or more 15-deg angles would vary with the number of 15-deg angles involved.

In order to provide corrections of equal precision for all intervals, 552 measurements involving 76 closures were made to obtain all possible sums of consecutive angles. From these measurements corrections were derived for each interval. For example, in the determination of the correction for the 0- to 15-deg interval, 48 values were obtained from the following two series of differences:

[(0- to 15-deg correction)—(15- to 15-deg correction)],
[(0- to 30-deg correction)—(15- to 30-deg correction)], . . .,
[(0- to 360-deg correction)—(15- to 360-deg correction)], and

[(360- to 15-deg correction)—(360- to 0-deg correction)],
[(345- to 15-deg correction)—(345- to 0-deg correction)], . . .,
[(15- to 15-deg correction)—(15- to 0-deg correction)].

The least-squares estimate for each angular interval involves the same number of measurements, and the precision with which the correction to any angle is known is $\sigma/\sqrt{24}$. An estimate of the standard deviation, σ, was determined from the least-squares solution. For any single measurement, the standard deviation, σ, was found to be 0.22 sec. Hence, the final value for each of the intervals had a standard deviation of $0.22/\sqrt{24}$, or 0.045 sec.

In addition to random errors, any calibration process commonly has systematic errors associated with it. In this case, however, no error existed in the primary standard, a circle, and any possible small errors in the autocollimator screws were so distributed over the entire series of measurements that their effect largely assumed a random characteristic. Accordingly, it is believed that the systematic errors may be considered negligible and, therefore, disregarded. Knowledge of the true corrections is, therefore, obscured solely by the random errors of measurement, and the accuracy of the process involves only some upper limit to the magnitude of the random errors.

A reasonable upper limit for the magnitude of the random errors can be set by selecting the value from the error distribution for which there is a chance of only one in one hundred of a greater discrepancy occurring. For this definition (assuming a normal distribution of errors), the accuracy of the corrections is $\pm 2.58(.045) = \pm .12$ sec.

The actual values of the intervals are of no particular interest to a reader of this paper, but it may be noted that the maximum deviation from nominal size of any 15-deg interval was 8.2 sec and for any iterval, 9.0 sec.

5. Application of the Polygon

The polygon was used to calibrate a master 45-deg. angle block belonging to the Bureau. The calibration could be made either by comparing the angle block with one or more intervals of the polygon or by comparing the angle block with a sufficient number of consecutive intervals to encompass an integral number of polygon rotations or circuits. In the case of a 45-deg block, whose external angle is 135 deg, eight consecutive readings covering 1,080 deg are required for a closure. Two additional series of eight readings each are required to provide comparisons with every 135-deg interval of the polygon.

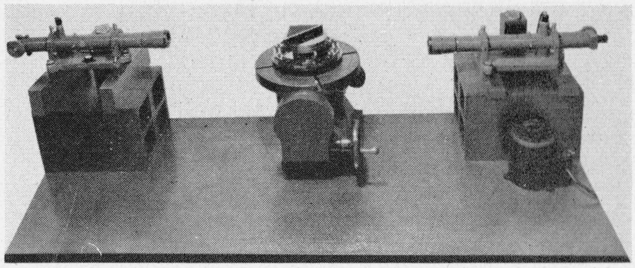

FIGURE 3. *Arrangement of polygon, angle block, and autocollimators for calibration of 45-degree angle block.*

One autocollimator is directed at the polygon and the other at the angle block.

The disposition of the equipment for the calibration of a 45-deg angle block is shown in figure 3. The polygon and angle block were mounted on a rotary table with the angle block elevated just above the polygon faces by means of a plane parallel block interposed between the polygon base and the block. Two autocollimators were positioned 180 deg apart, with one directed at a polygon face and the other at an angle-block face. The same care was observed in alining the equipment as for the calibration of the polygon, but temperature effects were largely eliminated by the 180-deg disposition of the autocollimators.

With the autocollimators appropriately alined, the readings of the two instruments were recorded. The assembly of polygon and angle block was then rotated 135 deg by means of the rotary table, and the autocollimator readings were again noted. The angle block was next rotated 135 deg with respect to the polygon, and comparison of the angle block with the adjacent 135-deg polygon interval was obtained by observing readings of both autocollimators at that position and after another 135-deg rotation of both polygon and angle block. Similar comparisons with the next six consecutive intervals of the polygon completed a closure and permitted computation of the angle of the angle block. In order to obtain a more accurate value, additional closures were made, relating the angle block to other 135-deg-polygon intervals.

Table 1 is a tabulation of two closures on a 45-deg angle block. Column 1 is the polygon interval; column 2, the difference in autocollimator readings on the polygon; column 3, the difference in autocollimator readings on the angle block; column 4, the difference between columns 2 and 3; column 5, the error in the polygon interval by the previous calibration; column 6, the error of the angle block by direct comparison with the polygon interval; and column 7, the error of the angle block by comparison with all the polygon intervals in each of the separate closures. The latter is wholly independent of any errors in the values for the polygon intervals and therefore somewhat more accurate than the average of eight values not constituting a closure.

The angular rotation of the block and polygon between readings was 135 deg$+e_p+d_1$ in terms of polygon values and 135 deg$+e_b+d_2$ in terms of angle block values, where e_p is the error in the polygon interval, d_1 is the difference in autocollimator readings on the polygon, e_b is the error in the angle of the block, and d_2 is the difference in autocollimator readings on the angle block.

Equating the two expressions, we have $e_b = d_1 - d_2 + e_p$. The values of e_b are given in column 6 of table 1. Since the sum of the e_p's for a closure is zero, e_b is estimated by $\frac{1}{8}\Sigma(d_1 - d_2)$, where 8 is the number of readings in a closure. This estimate is given in column 7 of table 1, and it is independent of the errors in the polygon.

TABLE 1. *Measurements of a 45-deg angle block by comparison with 135-deg intervals of polygon*

1	2	3	4	5	6	7
Polygon interval	Difference in polygon readings, d_1	Difference in block readings, d_2	$d_1 - d_2$	Error of polygon intervals	Error of angle block	$\frac{\Sigma(d_1-d_2)}{8}$
deg	*sec*	*sec*	*sec*	*sec*	*sec*	*sec*
0 to 135	12.13	10.10	2.03	—1.76	0.27	------
135 to 270	1.77	0.63	1.14	—1.19	—.05	------
270 to 45	—5.57	—3.20	—2.37	2.34	.03	------
45 to 180	12.86	12.03	0.83	—0.45	.38	------
180 to 315	13.60	14.70	—1.10	0.87	—.23	------
315 to 90	4.13	1.47	2.66	—2.41	.25	------
90 to 225	6.53	7.43	—0.90	0.48	—.42	------
225 to 0	—5.14	—2.76	—2.38	2.12	—.26	—0.01
15 to 150	—7.34	—8.60	1.26	—1.17	.09	------
150 to 285	6.97	2.06	4.97	—4.96	.01	------
285 to 60	—3.17	—1.20	—1.97	1.82	—.15	------
60 to 195	—11.13	—5.93	—5.20	4.81	—.39	------
195 to 330	5.44	4.07	1.37	—0.76	.61	------
330 to 105	1.47	—1.57	3.04	—3.02	.02	------
105 to 240	12.70	10.94	1.76	—1.43	.33	------
240 to 15	1.46	6.67	—5.21	4.71	—.50	0.00

Four additional measurements gave values of $+0.01$, -0.01, -0.12, and $+0.03$ sec for $\Sigma(d_1 - d_2)/8$.

The standard deviation, σ, for the six values of $\Sigma(d_1 - d_2)/8$ is 0.02 sec. If we use a 99 percent confidence limit, as in the case of the polygon, the average value of -0.01 sec may be considered accurate to ± 0.09 sec.

6. Other Possible Uses

Although designed primarily as a master for angle blocks, the polygon is equally suitable for the test or calibration of angular dividing equipment. Used alone, it permits the calibration of 15-deg. intervals and, in conjunction with other polygons, of intervals as small as 1 deg by the method described by Taylerson (see footnote [3]).

Simple arrangements of a few blocks can be easily assembled to serve as masters or gages in the optical or mechanical industry.

A further extension of the application of assembled polygons to the calibration of circular dividing equipment is being considered.

Taylerson's method for calibrating 1-deg intervals requires the use of a number of polygons, each having a different number of sides. The complete calibration, using one suggested group of polygons, requires 49 circuits of three polygons, with a total of 408 readings.

The ease and precision with which defining faces can be positioned on an assembled polygon suggests the possibility of using a single polygon having in addition to the usual large intervals, one or more blocks offset at such angles as to subdivide the principal intervals. If, for example, a 12-sided polygon having 30-deg intervals is used, the addition of a single block offset from any one of the faces by 1 deg permits the calibration of test equipment in 1-deg increments. The 1-deg interval would fix the relationship between successive series of comparisons with the equal intervals of the polygon and necessarily enter the complete calibration as many as 15 times. If, however, two additional blocks are offset 10 and 20 deg, respectively, no more than five 1-deg intervals need be added or subtracted to establish the relationship between circuits. The addition of three more blocks offset 5, 15, and 25 deg, respectively, further reduce the number of 1-deg intervals entering a calibration and also provides very convenient method of calibrating equipment in 5, 10, or 15-deg increments.

The procedure for calibrating a rotary table in increments of 1 deg with a 12-sided polygon having three additional blocks offset 1, 10, and 20 deg, respectively, is as follows: Every 30-deg interval of the polygon is compared with rotary-table intervals starting at zero, then at 1, 2, 3, etc. up to 29 deg, inclusive. Additional comparisons are made with the 1-, 10-, and 20-deg intervals starting at zero and with the 1-deg interval on succeeding circuits. From the values of the polygon intervals, the errors of the rotary table can be derived in 1-deg increments.

For simplicity of explanation, the additional 1-, 10-, and 20-deg blocks have been indicated as offset from the zero polygon face, whereas space limitations would require that they be distributed around the polygon and offset from other polygon faces. The only effect of such a distribution is to change the computational procedure.

The use of a single polygon having faces offset in fractions of the main intervals has two advantages over Taylerson's method:

1. Thirty circuits with 392 readings are required, as compared with 49 circuits with 408 readings. The minor reduction in the number of readings is of little importance, but the large reduction in the number of circuits and the concomitant manipulation of the polygon should effect an appreciable saving in time.
2. A major reduction in the cost of equipment is achieved by the substitution of a single polygon for a group of three.

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