ABSORPTION MEASUREMENTS OF THE X RAY GENERAL RADIATION

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ABSTRACT

Studied as a means for indicating the quality of the radiation, copper absorption curves for heterogeneous X rays were obtained, using a standard and two thimble ionization chambers. With the standard air chamber, logarithmic absorption curves were not found to become straight for filters up to 3 mm in thickness—indicating a gradual elimination of the longer wave lengths. With thimble chambers, owing presumably to errors from the chamber "wall effect," the curves were nonlinear only up to 1 mm thickness of filter.

Half-value layers and effective wave lengths derived from the three sets of results were compared. The "effective wave length" as derived from the slope of the absorption curve and the "half-value layer" are about equally sensitive to changes in quality at least up to 3 mm thickness of filter. As should be expected, effective wave lengths thus obtained are always longer than those obtained by Duane's finite filter method and, being less dependent upon the particular instrument used in the measurements, are, therefore, more consistent.

On the basis of the effective wave-length determinations it is found that, contrary to the accepted views, the greater part of the highly filtered energy is not concentrated in the tungsten K lines.

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I. INTRODUCTION

In connection with the proper control of X radiation in its various applications, reliable and relatively convenient means of measuring the radiant energy is the first necessity. This part of the problem seems to be satisfactorily solved by observing proper care in the design and use of the ionization chamber. The bureau has, therefore, announced its completion of a standard ionization chamber.[1] The next

[1] To be published in April issue of Radiology and March issue of American Journal of Roentgenology.

project on this program is the study of possible methods of determining "quality" of radiation. The accuracy with which it is necessary to describe the quality depends, of course, upon the particular use to which the radiation is put. In radiological practice an accuracy of 3 to 4 per cent may be sufficient, whereas in precise physical experiments the accuracy should be better as indicated, for example, in the recent papers by Compton,[2] and Crowther and Bond.[3]

In the radiological field recent work by Pohle and Barnes,[4] Quimby,[5] and Burby and Barry [6] has shown large discrepancies in the qualities of the different X-ray beams as measured by different methods. Moreover, Quimby showed that three beams having the same "quality" according to some method A, had radically different "qualities" according to another method B. It thus appeared desirable to make a fairly comprehensive study of available methods and the devices used in applying them.

Inasmuch as a beam of X rays may cover a range of wave lengths which are unequally effective in producing the results sought, next in importance to measuring the total energy is that of determining the distribution of the energy among the wave lengths present. A direct measurement of spectral energy distribution is the most reliable and satisfactory way of describing the quality of the radiation, but this is so difficult and cumbersome that the method eliminates itself from the field of practical applications. Fairly sound substitute methods of greater utility have been based on the degree of absorption of the total energy by filters of standard materials (copper, silver, aluminum, etc.), knowledge of the incident radiation being inferred from the absorption measurements of various filter thicknesses.

Even if the ionization spectrometer method of mapping the spectral radiation were not so difficult and cumbersome under present conditions, it would still be far from an ideal method owing to the many conditions involved, some of which are not well understood. The more common forms of the X-ray spectrograph as substitute for the ionization spectrometer are inadequate, for, due to such factors as X-ray scattering, photographic halation and spectral sensitivity of the emulsion, a simple density curve of the photographic plate may be very misleading. In using such data to determine minimum wave length,[7] it is necessary to extrapolate the curves to zero energy by starting at some point well up on the established curve, which, of course, makes for very uncertain conclusions.

With these facts in mind the best method, for practical purposes, for arriving at the quality of any given X-ray beam seems to be that involving direct absorption measurements.

In deep therapy wave-length ranges, Duane has shown that absorption measurements in copper are well suited for describing the quality.[8] In the superficial therapy range, either copper or aluminum absorption measurements [9] may be used; and, in the "Grenz-

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[2] A. H. Compton, Phil. Mag., **8**, p. 961; 1929.
[3] J. C. Crowther and W. N. Bond, Phil. Mag., **6**, p. 401; 1928.
[4] E. A. Pohle and J. M. Barnes, Radiology, **10**, p. 300; 1928.
[5] E. H. Quimby, Am. J. Roent., **21**, p. 275; 1929.
[6] J. J. Burby and M. W. Barry, Radiology, **12**, p. 275; 1929.
[7] A. Mutscheller, Radiology, **12**, p. 283; 1929.
[8] Wm. Duane, Am. J. Roent., **9**, p. 167; 1922.
[9] Wm. Duane, Am. J. Roent., **20**, p. 241; 1928.

therapie" range, most observers have used very thin aluminum absorbers.[10, 11, 12]

In their technical production, various types of high voltage generators are used, many having a characteristic wave form and, hence, furnishing X rays of periodically varying quality. If the peak voltage of such a generator remains constant, however, the radiant energy at all wave lengths integrated over any practical interval of time, will be proportional to that interval; and the resulting spectral energy distribution may be treated as constant. This is not merely an obvious assumption, but is supported by experience in that it has been found that essentially the same quality of radiation may be obtained from mechanical rectifiers as from constant potential sources if the rectifier be operated at a slightly higher peak voltage than the constant voltage source.

The present study has been confined throughout to a practically constant potential source,[13] inasmuch as this makes for more ready definition of the spectral energy distribution.

II. THEORY OF ABSORPTION MEASUREMENTS

1. ABSORPTION OF HETEROGENEOUS RADIATION

Representing by E_λ the energy between the wave lengths λ and $\lambda+d\lambda$ of the beam, the total energy (E) of a continuous X-ray spectrum, void of characteristic lines, may be expressed as an integral between the short wave length limit $\lambda=\lambda_o$, and $\lambda=\infty$; that is,

$$(E)=\int_{\lambda_0}^{\infty} E_\lambda \, d\lambda \tag{1}$$

If the form of E_λ is known the total energy (E) may be calculated. The magnitude of E_λ as it varies with λ has been derived theoretically by Behnken [14] who found fair agreement between his theoretical and Ulrey's [15] experimental curves for constant potential X rays. A more recent combined experimental and theoretical study by Kulenkampff [16] has revealed even better agreement between theory and experiment. In the present work we are concerned, however, with the problem of deducing some knowledge of the spectral distribution E_λ from measurements of the total energy (E) after passage through properly chosen filters.

If a filter of thickness x intercept a beam of radiation, the incident energy E_λ comprised between the wave lengths λ and $\lambda+d\lambda$ will be reduced by the filter to a quantity of energy E'_λ, emerging from the filter, of a magnitude given by the well-known equation

$$E'_\lambda=E_\lambda \, e^{-\mu_\lambda x} \tag{2}$$

where μ_λ is the "absorption coefficient" of the material of the filter for the radiation of wave length λ. Here, of course, scattering is neglected.

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[10] O. Glasser and U. V. Portmann, Am. J. Roent., **19**, p. 442; 1928.
[11] H. Kustner, Strahlen., **27**, p. 124; 1927.
[12] L. E. Jacobson, Am. J. Roent., **22**, p. 544; 1929.
[13] L. S. Taylor, B. S. Jour. Research, **2** (RP56), p. 771; 1929.
[14] H. Behnken, Zeit. f. Phys., **4**, p. 241; 1921.
[15] C. T. Ulrey, Phys. Rev., **9**, p. 548; 1917.
[16] H. Kulenkampff, Ann. der Phys., **69**, 548; 1922.

The value of μ_λ for copper or other material may be obtained from tables or computed from Richtmyer's [17] formula for the "mass absorption coefficient" $\frac{\mu_\lambda}{\rho}$, in which

$$\mu_\lambda/\rho = A\lambda^3 + K \tag{3}$$

A and K being constants for the given material and ρ the density of the material.

For copper between $\lambda = 0.12$ and 0.30 angstroms, Richtmyer found [18]

$$\mu_\lambda/\rho = 153\lambda^3 + 0.2 \tag{3a}$$

λ being expressed in angstroms and μ in cm $^{-1}$.

From an inspection of equation (3) it is obvious that any filter inserted in a heterogeneous beam will absorb the longer wave lengths more than the shorter and, hence, "harden" the radiation. This means is used in practice. Furthermore the degree of hardness of the radiation transmitted by any given filter may be indicated by the extent to which a second additional filter inserted in the beam absorbs the radiation transmitted by the first. It is this application with which we are here concerned.

2. APPLICATION OF ABSORPTION MEASUREMENTS IN DESCRIBING X-RAY QUALITY

(a) EFFECTIVE WAVE LENGTH

One method, which makes use of absorption measurements for obtaining an indication of the quality of an unknown heterogeneous radiation of energy (E), is that which determines the wave length of the homogeneous radiation which would be reduced in intensity by a given filter in the same degree as the heterogeneous radiation is reduced by it. The heterogeneous radiation is then designated as having an *effective wave length* equal to that of its corresponding homogeneous radiation.

For example, relation (2) relates the transmitted to the incident radiation of wave length λ through the thickness of the filter x and the absorption coefficient μ_λ.[19] Taking now the incident and transmitted heterogeneous energies (E) and (E') we may use a *composite absorption coefficient* [20] μ, such that

$$(E') = (E)e^{-\mu x} \tag{4}$$

Having the values of μ_λ, either from tables or equation (3a), either (E) or (E') may be said to have that effective wave length λe for which

$$\mu = \mu_\lambda \tag{4a}$$

or, if x be taken sufficiently small, both may be assigned the effective wave length in question.

Duane uses a filter of given material and of finite thickness x for obtaining the effective wave length of the radiation incident thereon.[21]

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[17] F. K. Richtmyer, Phys. Rev., 18, p. 13; 1921.
[18] F. K. Richtmyer and F. W. Warburton, Phys. Rev., 6, p. 539; 1923.
[19] L. S. Taylor, Radiology, 12, p. 289; 1929.
[20] A. H. Compton, X Rays and Electrons.
[21] W. M. Duane, Proc. Nat. Acad., 13, p. 668; 1927.

The results of a comparative experimental study of this, and that of an infinitesimal thickness of filter with different available devices for measuring the energy of the beam, was the partial object of the present investigation.

Illustrations of the two methods of determining effective wave lengths are to be found in Figures 1 and 4 where, with increasing thickness of filter, the logarithm of the transmitted energy is plotted against thickness of a given filter material. Let in equation (4),

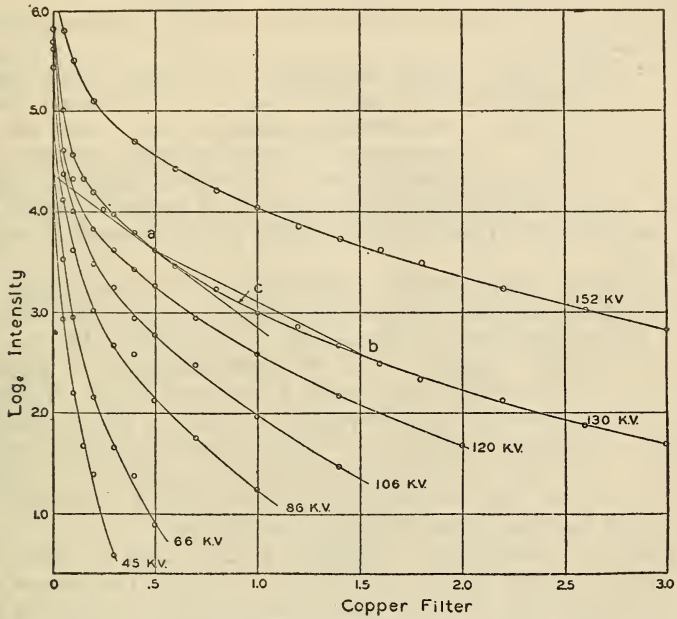

FIGURE 1.—*Copper absorption curves made with standard air ionization chamber for practically constant voltage X rays*

(E') be the energy at any thickness x', and (E'') that at a thickness $x''=x'+x$; then Duane's method applied gives

$$(E')=(E)e^{-\mu' x'} \tag{5}$$

$$(E'')=(E)e^{-\mu'' x''} \tag{6}$$

From this

$$\log\frac{(E')}{(E)}-\log\frac{(E'')}{(E)}=\mu''\ x''-\mu'\ x' \tag{7}$$

Noting that μ' in equation (5) depends on the entire thickness x', and μ'' in equation (6) depends on the same thickness of filter plus the increase $x=x''-x'$, it appears at least approximately correct within limited ranges to use an intermediate value of μ such that

$$\mu=\frac{\log\frac{(E')}{(E)}-\log\frac{(E'')}{(E)}}{x} \tag{8}$$

Here μ is dependent upon x as well as upon (E') and (E''). If, however, x be taken small enough, equal to Δx, so that the filter does not appreciably change the quality of the radiation passing through it, and yet produces a measurable change in the energy of the beam, then

$$\mu = \frac{\Delta \log \frac{(E')}{(E)}}{\Delta x} \tag{9}$$

is represented by the slope of the curve $\log \frac{(E')}{(E)}$ plotted against the thickness of filter x.

The effective wave length as defined by Duane [22] is "the wave length of monochromatic radiation that would produce the same effect (readings of the instruments employed, etc.) that the actual radiation produces." His method of measurement consists in determining the percentage transmission of the given filter for the given radiation, then determining λ_e from previously constructed calibration curves, as obtained from the absorption law indicated in equations (2) and (3).

The so-called "true effective wave length" is independent of any given filter thickness; and, in that one other factor is thus eliminated from its determination, it is physically a more desirable way to express the quantity sought. For its determination, a logarithmic absorption curve is established with a series of filters of increasing thickness and the slope at the point in question gives directly the composite absorption coefficient μ. From monochromatic wave-length absorption curves or Richtmyer's formula (equation (3a)) may thus be determined, as the true effective wave length λ_e, the single wave length which has the same absorption coefficient.

A "bracket" method for obtaining λ_e may also be used wherein two intensity measurements are made for filter thicknesses equally above and below the filter for which the radiation quality is desired. The absorption coefficient and corresponding λ_e is then obtained from the difference of the logarithms of these two intensity measurements, it being assumed that this absorption coefficient is the same as that corresponding to the intermediate filter. Such a method, while nearer the tangent method (true λ_e) than Duane's, does not make full allowance for the curvature of the logarithmic absorption curve between the points taken.

The bracket method is exactly the same as Duane's method as far as the operational procedure is concerned, except that the radiation for which λ_e is determined corresponds more nearly to that of the intermediate filtration instead of the smaller filtration. By Duane's method the ratio of the two intensities $(E')/(E'')$ for filtrations x' and x'' is said to give λ_e for the filtration x' whereas the bracket method gives λ_e for a filtration close to $(X'+X'')/2$. As shown later, the difference between the two varies according to the range in which the measurements are applied, and may be neglected for many practical purposes.

In Duane's earlier work he specified the use of 1 mm of copper for obtaining the effective wave length. The use of such a large filter-difference will give reasonably consistent agreement with the true λ_e

[22] Wm. Duane, Am. J. Roent., 20, p. 241; 1928.

only for very high filtrations as shown in Table 2 and when used according to the bracket method (equation (8)). For lower filtrations the differences between the two methods increases. In later work he specified the use of 0.25 mm filter separation, thus increasing considerably the range over which λ_e might be determined with fair agreement with the other method (true λ_e).

Duane also indicated at the time that small errors in the measurement of filter thickness may cause an appreciable error in the determination of λ_e by his method. The effect on λ_e of errors in thickness will, of course, be greater for the method using 0.25 mm filter separation than for the 1 mm method. In the case of obtaining λ_e from a complete absorption curve such errors are averaged out.

(b) HALF-VALUE LAYER

One of the earliest methods of describing the quality of a heterogeneous X-ray beam was to measure the absorption by a series of filters of some material and then determine from the absorption curve the thickness of that material which would reduce the intensity of the given beam to one-half its initial value. If, for example, a beam has an intensity of E_{F_1} after passing through a filter of thickness x_1, and E_{F_2} is the half-intensity ($E_{F_1}/2$) after passing through a filter x_1+x_2, then x_2 is the half-value layer of the particular material for that radiation which has passed through x_1. While then the quality is expressed by the effective wave length as determined by a given thickness of filter of given material, the half-value method expresses the quality in terms of the thickness of filter (of given material) required to reduce the intensity by a given amount (one-half).

Obviously, the shorter the effective wave length of the given radiation, the thicker this half-value layer of filtering material must be. Inasmuch, therefore, as the composite absorption coefficient μ of equation (4) varies with the thickness x of the added filter, there is no simple direct quantitative relation connecting effective wave length with half-value layer.

Comparing the half-value layer and effective wave-length method, the latter has the advantage in that it presents a clearer physical picture of the radiation quality.

In Roentgenological practice, the voltage wave forms are usually of two general types, being either approximately constant or approximating a sine wave; and the target material is always tungsten. Here the "half-value" method has served as a useful approximation for expressing the radiation quality. However, as a particular example, it is unsafe to rely on this method for comparing qualities of two radiations, one excited by constant and the other by a fluctuating voltage. Quimby [23] made a detailed study of the qualities of various radiations, all having the same half-value layer of copper, and found large differences between them.

The inset (A) in Figure 5 shows a plot of the half-value layer taken from the absorption curve in Figure 5, for filtrations as high as 1.6 mm of copper. Investigations cited [24] have indicated that such a curve would be different for each voltage wave form applied to the tube.

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[23] E. Quimby, Am. J. Roent., 21, p. 64; 1929.
[24] See footnotes 4, 5, 6, and 29.

(c) AVERAGE WAVE LENGTH

This method was proposed by Mutscheller [25] several years ago. It follows the true effective wave-length method in that from the tangent to a heterogeneous radiation absorption curve a coefficient of absorption is determined from Richtmyer's tables with which the corresponding monochromatic wave length is identified. He definitely divides his measurements into two classes, those for the portion of the absorption curve from zero to the "homogeneity filter" [26] and those beyond this point.

According to the results presented in this paper there can be no "homogeneity filter," strictly speaking. Practically a filtration may be reached beyond which the radiation quality changes comparatively little—the absorption curve approaching linearity.

In most of his published absorption curves he shows such linearity above about 0.5 mm copper filtration, indicating the attainment of effectively homogeneous radiation. There is no evidence that such a condition should be rigidly true and it is not reached within the range of filtration (0 to 3 mm Cu) used in this paper. It appears, therefore, that the average wave lengths so obtained in the region beyond the "homogeneity filter" are subject to errors larger than herein contained; that is, Mutscheller's method is that of the true effective wave length above but applied in the range where the errors are such as to render the results questionable. Applied in the region below the "homogeneity filter"—that is, for small filtrations—his method is identical with the so-called true effective wave-length method.

III. EXPERIMENTAL RESULTS

1. COMPARISON OF DIFFERENT IONIZATION CHAMBERS

In the development thus far we have dealt with ideal conditions wherein it has been considered possible to make direct energy measurements. However in applying absorption measurements as an indication of the quality of a given X-ray beam, the ionization current produced by the beam is the quantity measured. The current per unit energy does not change rapidly with wave length, and for simplicity we will speak of this as energy, understanding, of course, that the measured ionization is proportional to the energy absorbed from the beam by the air in the ionization chamber.

The results of absorption measurements made by a number of observers are not in close agreement; consequently the technique of making such measurements has been studied in the greater detail given here.

In some recent work at this laboratory a study was made of the calibration of the thimble ionization chambers against a standard air ionization chamber [27] in which it was brought out that there was a marked difference in the copper absorption curves for the same radiation when measured with several different thimble ionization chambers and the standard chamber.

In Figure 1 is shown a series of semilogarithmic copper absorption curves obtained by measuring the incident and transmitted intensities

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[25] A. Mutscheller, Radiology, May, 1924; October, 1924; April, 1929.
[26] E. A. Pohle, Radiology, 10, p. 300; 1928.
[27] L. S. Taylor and G. Singer, B. S. Jour. Research, 4, (RP169) p. 631, May; 1930.

with the standard air ionization chamber. The tungsten X-ray tube was operated at practically constant potential as recently described. The diaphragming system[28] to limit the beam was in accordance with the necessary conditions to make an accurate determination of the effective intensity of the beam in Roentgens.[29] When a thimble ionization chamber was used, it also was in accordance with previously described technique.[30, 31] The configuration of the system in Figure 2 shows the arrangement diagrammatically, and is self-explanatory.

For the curves shown in Figure 1 the diaphragms M and N each had a radius of 12 mm and the interdiaphragm distance B was 65 cm. The filters were 15 cm from M, thus making the distance C between filter and chamber diaphragm 50 cm. In taking this data the precision of observation for a single filter was about ± 0.2 per cent. The copper filters had a tested purity of 0.99998 and their thicknesses were uniform to within ± 0.002 mm. It is clearly evident that for the curves shown there is no straight portion up to a copper filtration of 3 mm, although it is apparent that they are asymptotically

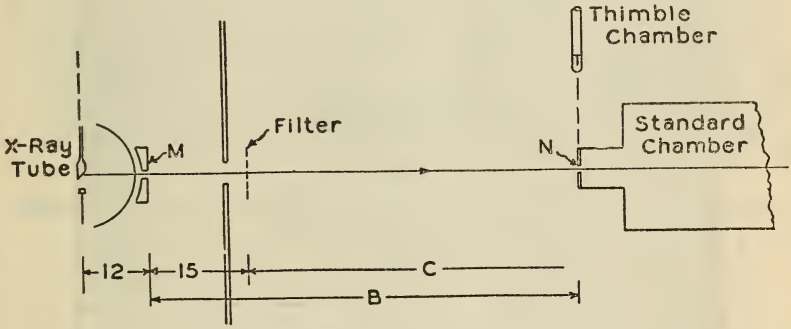

FIGURE 2.—*Diagram of X-ray tube-filter-ionization chamber system*

approaching a straight line. A large number of other such curves were obtained under varying conditions of the distance B and the diaphragms M and N, and all were exactly similar to those shown.

In order to be certain that no peculiarity of this system could be responsible for the form of these curves, I have obtained through the courtesy of Dr. Failla and of Dr. Quimby, of Memorial Hospital, New York, N. Y., the set of absorption curves for tube potentials up to 200 kv, shown in Figure 3. These were made some months previously with a thimble air wall ionization chamber differing slightly in construction from that used at the bureau. Here also it is clearly evident that the absorption curves do not become straight.

Attention may also be called to a paper by Rump,[32] in which is given a similar series of absorption curves for potentials up to 150 kv (fig. 5 in his article). Again, it is found that the curves are nonrectilinear. In view of the excellent agreement of these different workers there seems no question of the validity of the results.

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[28] L. S. Taylor, B. S. Jour. Research (RP56), 2, p. 771; 1929.
[29] L. S. Taylor, B. S. Jour. Research (RP119), p. 807; 1929.
[30] See footnote 28, p. 524.
[31] L. S. Taylor, Radiology, 15, 227.
[32] W. Rump, Zeit. f. Phys., 43, p. 254; 1927; 44, p. 396; 1927.

The same type of copper-absorption measurement was next made with several thimble ionization chambers placed at various distances from the filter, but all for the same beam of radiation. It must be pointed out that the precision of observation with such chambers is not as great as with a standard air chamber, and consequently we can not place the same reliance on the measurements made with them.

In order to be certain that the position of the ionization chamber with respect to the filter did not influence the measurements, a series of intensity readings were made keeping the chamber fixed and changing the filter position. For the standard ionization chamber no measurable effect was produced by moving the filter from 0.5 to 30 cm from the entrant diaphragm. However, shifting the filter over the same range for a thimble chamber, the measured intensity fell off

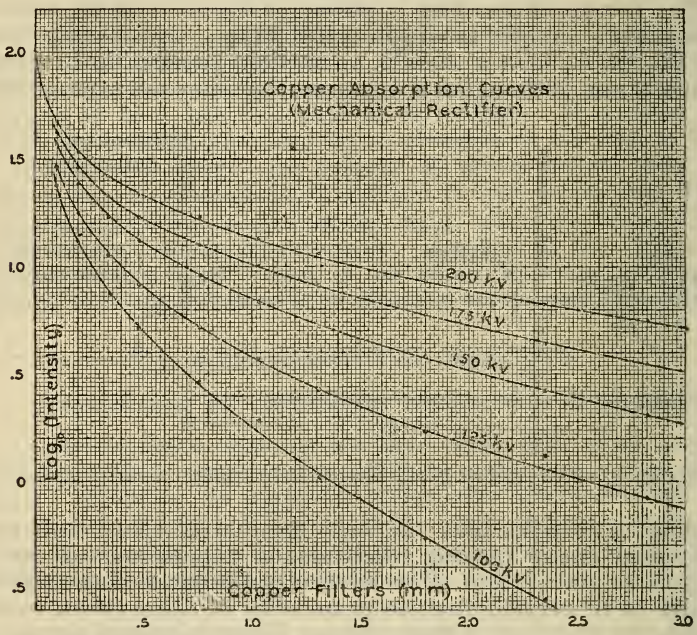

FIGURE 3.—*Copper absorption curves made for mechanically rectified voltage*

steadily up to a distance of about 10 cm where it reached a steady value. Consequently, all measurements with a thimble chamber, were made at least 15 cm from the filter.

In Figure 4 are shown three semilogarithmic absorption curves: *S*, obtained with a standard; *G*, with a Glasser thimble chamber, and *F* with a Friedrich thimble chamber. The X-ray tube was operated at 133 kv, d. c., for all. The standard chamber was set 45 cm from the filter and the Glasser chamber and Friedrich chamber at about 15 cm.

The original data for these curves are given in Table 1. The intensities I_F and I_G, as measured with the Friedrich and Glasser chambers, respectively, have each been corrected for a very small natural leakage. There was no measurable leakage in the standard chamber system. Half-value layers obtained from this data were in

good agreement with Holthusen's curves for the voltage used,[33] both measurements being made with a standard chamber.

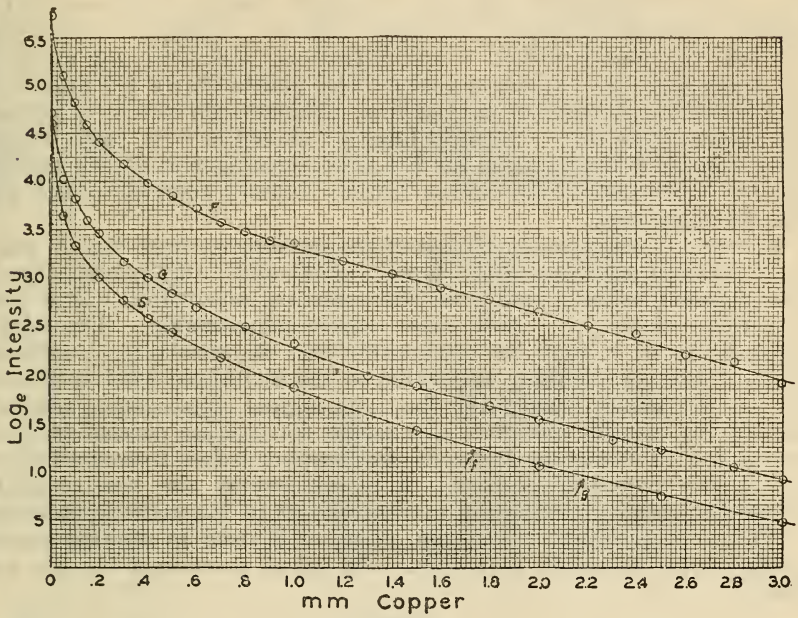

FIGURE 4.—*Copper absorption curves made for 133 kv radiation, using three different chambers*

TABLE 1.

Filter	Standard		Friedrich		Glasser	
	I_S	$\log_e I_S$	I_F	$\log_e I_F$	I_G	$\log_e I_G$
1	2	3	4	5	6	7
0	100.0	4.60	312.0	5.74	111.0	4.71
.05	38.4	3.65	164.0	5.10	55.7	4.01
.1	28.6	3.355	123.0	4.81	45.4	3.82
.15	----------	----------	99.0	4.58	35.7	3.58
.2	20.0	3.00	81.6	4.40	30.4	3.46
.25	(17.8)	----------	72.6	4.285	(26.8)	----------
.3	16.1	2.78	66.3	4.194	23.8	3.16
.4	13.0	2.565	53.6	3.98	20.0	3.00
.5	11.5	2.44	48.1	3.87	17.2	2.84
.6	----------	----------	41.2	3.72	14.8	2.69
.7	8.70	2.16	35.6	3.57	(13.4)	----------
.8	----------	----------	32.5	3.48	12.1	2.495
.9	----------	----------	29.8	3.39	----------	----------
1.0	6.49	1.87	28.8	3.36	10.2	2.315
1.2	(5.4)	----------	23.6	3.16	(8.4)	----------
1.3	----------	----------	----------	----------	7.35	1.99
1.4	----------	----------	20.8	3.035	----------	----------
1.5	4.11	1.43	(19.4)	----------	6.62	1.89
1.6	----------	----------	18.0	2.89	----------	----------
1.8	----------	----------	15.8	2.76	5.26	1.66
2.0	2.86	1.05	14.2	2.65	4.58	1.52
2.2	----------	----------	12.2	2.50	----------	----------
2.3	----------	----------	----------	----------	3.72	1.31
2.4	----------	----------	11.2	2.42	----------	----------
2.5	2.08	.733	----------	----------	3.36	1.21
2.6	----------	----------	9.0	2.20	----------	----------
2.8	----------	----------	8.5	2.14	2.84	1.04
3.0	1.60	.469	6.7	1.90	2.56	.714

[33] H. Holthusen, Radiology, 10, p. 292; 1928.

It is seen that the curve obtained with the standard chamber does not become straight for filtration up to at least 3 mm of copper. For curve *F* obtained with the Friedrich chamber, however, it is equally possible to pass a straight line or a slightly curved line through the points above 1 mm filtration. Likewise for curve *G*, obtained with the Glasser chamber, the points above 1.5 mm could be joined by a straight or curved line, although the former is perhaps the better of the two. Straight lines are shown in both cases to illustrate a possible explanation for the absorption curves shown invariably by Mutscheller, his curves usually having been obtained with thimble chambers.

If now we measure the slopes of the curves, we find that *F* and *G* have straight portions with slopes −6.80 and −6.28, respectively. The points marked *f* and *g* on the standard-absorption curve have also these slopes of −6.80 and −6.28, respectively. Thus for a given beam of X rays we apparently find an absorption coefficient which varies considerably according to the technique used in measuring it. To attain uniformity the most logical consequence seems to discard the thimble chamber as a means for making precise measurements; for, inasmuch as the open-air chamber is free from wall effects and is the accepted standard, it is apparently the best available means for making absorption measurements of the general radiation.

Where, in practice, a thimble chamber is used in making precise quality measurements, care should be taken to correct for the wall effect.

TABLE 2

Filter	Slope μ	True λ_e	Bracket λ_e		Equivalent filter	Duane λ_e	H. V. L. Cu
			μ	λ_e			
1	2	3	4	5	6	7	8
mm	*cm*⁻¹	*Å*	*cm*⁻¹	*Å*	*mm*	*Å*	*mm*
0.1							0.238
.2	25.5	0.256					
.25	22.8	.243					
.3	19.9	.230					.475
.4	17.5	.224				0.183	.56
.5	15.4	.207	28.0	0.262	0.15	.178	.62
.6	14.4	.202	16.6	.214	.45	.172	.68
.7	12.8	.192	13.8	.198	.6	.168	.73
.8	11.4	.184	12.2	.189	.7	.164	.79
.9	10.5	.178	11.3	.183	.8	.161	
1.0	9.86	.174	10.5	.178	.9	.158	.86
1.1	9.25	.170	9.6	.172	1.05		
1.2	8.64	.165	9.0	.168	1.15	.153	1.00
1.3	8.40	.163	8.5	.164	1.25		
1.4	7.94	.160	8.1	.161	1.37	.148	1.04
1.5			7.8	.158	1.45	.144	
1.6	7.53	.156					1.18
1.7			7.2	.153	1.8		
1.8	7.15	.153					
1.9			6.7	.148	1.9		
2.0	6.42	.146					
2.1			6.2	.144	2.1		
2.2	5.98	.142					
2.4	5.80	.140					
2.6	5.40	.136					
2.8	5.25	.134					

2. COMPARISON OF QUALITY MEASUREMENTS

Having reviewed the methods of making quality measurements, we shall now compare the results obtained by applying each method to the same heterogeneous beam of X rays generated by a practically constant potential source of voltage.

In order to better coordinate this comparison we shall refer all measurements of quality to those of the so-called true effective wave length without meaning to imply thereby that this method is preferable or necessarily the most correct. It is, however, less arbitrary than other methods.

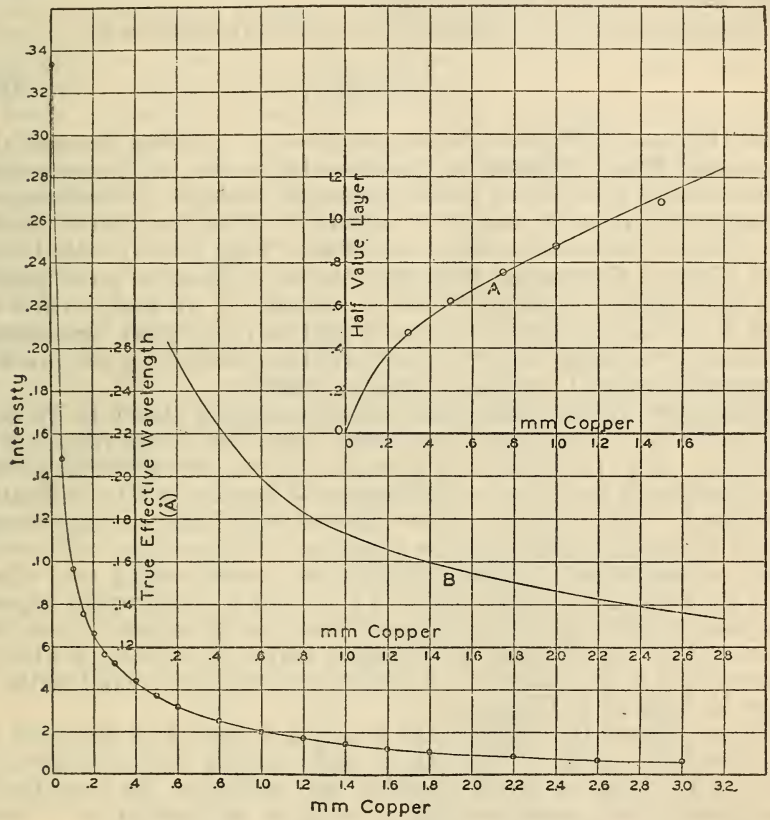

FIGURE 5.—*Curves showing half-value-layer and true effective wave length obtained from the same absorption curve*

Through the courtesy of Prof. H. Holthusen, of Hamburg, a family of curves was available, showing the relation between the half-value layer in copper and the tube voltage (constant) for a series of filtered radiations. Half-value layers were obtained from each of his curves at points corresponding to 130 kv and plotted as circles in the H. V. L. curve. (Inset A in fig. 5.) The excellent agreement of our experimental results with Holthusen's shows the reliability of the H. V. L. method for constant potential, and gives further proof of the correctness of our above experimental procedure.

We may now make a simple comparison of the effective wave length obtained by the slope and bracket methods. For example, if we were to take as the absorption coefficient the slope of the line joining the two points *a* and *b* on the absorption curve (fig. 1), corresponding to filters of 0. 5 and 1. 5 mm of copper, this will give the slope corresponding to a filtration of 0. 9 mm at the point *c* instead of at 1 mm. If the points 1 and 2 mm were chosen instead, the effective wave length would be much more nearly the correct value at 1. 5 mm. For small filtrations the error in the bracket type of measurement is considerable. Above a certain filtration the method may be sufficiently accurate for many purposes. It has the advantage of requiring but two measurements

From equation (8) the absorption coefficient μ is given by

$$\mu=\frac{1}{x}\log_e\frac{(E')}{(E'')} \tag{10}$$

where (E') and (E'') are the intensities after passing through the two copper filters differing in thickness by x cm. If, for example, we wish to find μ for 133 kv radiation filtered through 1 mm of copper, we have from Table 1, column 3, $\log_e (E')=2.44$ for filter 0. 5, and $\log_e (E'')=1.43$ and for filter, 1. 5 mm. This gives $\mu=10.1$ and $\lambda_e=0.174$ Å. Comparing this with the true effective wave length for 1 mm copper filtration (Table 4, column 3) we find a value of 0. 169 Å for the true effective wave length, a fairly good agreement.

A series of values of λ_e for 130 kv radiation obtained by the bracket method (where $x=1$ mm) are given in Table 2.

A summary of the absorption measurements is shown in Table 2 wherein all the calculations are taken from the absorption curve at 130 kv. (Fig. 1.) Column 2 gives the slope of the curve at points corresponding to the different thicknesses of the copper filter indicated in column 1. Since the curves were plotted with Naperian logarithms, column 2 thus gives directly the coefficient of absorption μ for each quality of radiation. Column 3 gives the corresponding true effective wave lengths when the values of μ are put in Richtmyer's copper absorption coefficient curves for monochromatic radiation. From this it is seen that the effective wave length varies appreciably for filtrations up to 2. 8 mm at least. A plot of the effective wave lengths is shown in inset *B* of Figure 5.

Column 4 gives the value of the absorption coefficient obtained by the "bracket" method (Sec. II, 2 (*a*)) wherein the coefficient is given by the slope of the line joining two points on the logarithmic absorption curve, corresponding to a filter separation of 1 mm. The intermediate filtration of the two is that shown in column 1. Column 5 gives the effective wave lengths corresponding to the absorption coefficients of column 4, from which it is seen by comparison with column 3 that, as should be expected, the values approach the same limit for higher filtrations. Column 6 gives the approximate filter thickness on the true absorption curve for the effective wave lengths of column 5, thus indicating the divergence in terms of filter thickness. For example, consider a beam having an initial filtration of 0.6 mm copper. Its true effective wave length is 0.202 Å, whereas that derived by the "bracket" method is 0.214 Å and would correspond to a filtration of about 0.45 mm on the logarithmic absorption curve. Such a discrepancy shows at

once that the bracket method, using a filter separation of 1 mm is not satisfactory for obtaining effective wave lengths in close agreement with the true effective wave length except for higher filtrations.

Column 7 gives the effective wave lengths obtained by Duane's method, using a 1 mm filter separation, from which it is seen that the difference between λ_e, as obtained by the slope of the curve and as obtained by Duane's method, is large over the whole range of filtrations used. Applying the same measurements, however, we obtain a fairly good measure of λ_e for the radiation corresponding to the average filtration. Thus for 0.7 mm initial filtration, the true effective wave length is 0.192 Å; by the 1 mm bracket method, 0.198 Å, whereas by Duane's (1 mm) method, 0.168 Å. When comparing Duane's effective wave length using 0.25 mm copper with the true effective wave length, the differences are not large except for unfiltered or lightly filtered radiation.

From the practical standpoint we must not ignore the advantages of Duane's method. It requires but two intensity measurements with given filters and reference to a predetermined calibration curve; it is sufficiently accurate for many of the present-day medical needs.

The half-value layers in copper for the same radiation are given in column 8 and it is seen that, throughout the entire length of the absorption curve, these are decreasing continuously at least up to an initial filtration of 1.6 mm. The data of column 8 are plotted in insert *A* of Figure 5. As pointed out in (II, 2 (*c*)) there can be no fixed relation between the half-value layer and effective wave length except for strictly constant-potential X rays. Curves showing the relation between the two have been given by Neeff and Reisner [34] and others, under different conditions, but none lead to any coordinated relationship.

TABLE 3

Chamber	Filter	E/E_o	λ_e (Duane)	Percentage difference E/E_o	Percentage difference λ_e	λ_e calculated
1	2	3	4	5	6	7
	mm	*Per cent*				
S	0.25 Cu	17.7	0.37	22.4	8.1	0.368
F	.25 Cu	23.3	.34			.343
S	1.0 Cu	6.5	.26	30.9	7.5	.263
F	1.0 Cu	9.4	.24			.234
S	2.0 Al	41.8	.47	11.6	4.2	---------
F	2.0 Al	47.3	.45			---------
S	4.0 Al	27.4	.41	15.1	2.5	---------
F	4.0 Al	32.3	.40			---------

Table 3 shows the results of determining the effective wave length of the unfiltered radiation by Duane's method, using both a standard (*S*) and a thimble (*F*) ionization chamber for the intensity measurements. Column 1 indicates the type of chamber used; column 2 is the filter thickness used in the determination of λ_e; column 3 gives the percentage transmission of the respective filters for which are given the corresponding values of λ_e in column 4, obtained from Duane's curves for effective wave lengths.

[34] T. C. Neeff and A. Reisner, Strahlen., 32, p. 190; 1929.

It is interesting to note for lightly filtered radiation, how dependent is the measurement of λ_e on the type of ionization chamber used. For example, in the first case λ_e differs by 8.1 per cent for the two chambers, while the respective percentage transmissions differ by 22.4 per cent (column 5). In order to check Duane's curves, λ_e was calculated using equations (2) and (3a), and the very good agreement is indicated in column 8.

If the quality of a beam of unfiltered radiation is desired, we are confronted by the question of which one of the eight values in Table 3 is to be used. Obviously no one of the methods will give a correct value of λ_e, nor can we readily obtain the slope of the absorption curve with any degree of accuracy at zero filter. We must thus choose an arbitrary techinque for making this measurement or else use some other method, such as the half-value layer in copper or aluminum. Here, again, it will be found that only a standard ionization chamber will give accurate and reproducible results.

TABLE 4

Chamber	Filter	True λ_e	Percentage difference, standard	Bracket λ_e	Percentage difference, bracket and same chamber	Percentage difference, standard	λ_e Duane	Percentage difference, Duane and same chamber	Percentage difference, standard
	1	2	3	4	5	6	7	8	9
Standard	0	-------	-------	-------	-------	-------	0.37	-------	-------
	.25	0.247	-------	0.248	+0.4	-------	.222	−11.3	-------
	.5	.207	-------	.204	−1.5	-------	.192	−7.8	-------
	.75	.185	-------	.184	−.5	-------	.177	−3.9	-------
	1.0	.169	-------	.169	0	-------	.167	−1.2	-------
Friedrich	0	-------	-------	-------	-------	-------	.34	-------	-------
	.25	.241	−2.4	.249	+3.2	+0.8	.222	−8.3	−11.3
	.5	.210	−1.4	.205	−2.4	−1.0	.190	−9.5	−8.9
	.75	.172	−7.0	.173	+.6	−6.5	.165	−4.2	−12.1
	1.0	.156	−7.7	.156	0	−7.6	.150	−3.9	−12.6
Glasser	0	-------	-------	-------	-------	-------	.34	-------	-------
	.25	.250	+1.2	.255	+2.0	+2.7	.224	−10.5	−10.3
	.3	.201	−4.5	.210	+4.5	+1.4	.193	−4.1	−7.3
	.75	.184	−.5	.185	+.5	0	.176	−4.3	−5.1
	1.0	.170	+.6	.170	0	+.6	.164	−3.5	−3.0

NOTE.—Groupings as follows: Columns 1, 2, and 3; columns 4, 5, and 6; columns 7, 8, and 9.

Table 4 gives a further comparison of the three methods of determining the effective wave length of 133 kv filtered radiation, using for each filter the standard Friedrich and Glasser ionization chambers. Column 1 gives the filtration of the beam; column 2, the "true effective wave length" obtained from the slope of the absorption curve; column 4, the effective wave length by the bracket method with a filter separation of 0.25 mm; and column 7, Duane's effective wave length, using a 0.25 mm copper filter.

Column 3 gives the percentage difference between the true effective wave lengths obtained with two thimble chambers and the values obtained with the standard chamber. It is evident that the disagreement is somewhat random, although the values obtained with the Glasser chamber appear to agree better with those of the standard.

Column 5 gives the percentage difference between the effective wave lengths obtained when using the slope and bracket methods with each of the three chambers. Here we see that when using the standard chamber the two methods agree very closely, whereas for the thimble chambers the agreement is considerably poorer. It is also seen that for high filtrations the two methods agree well within the experimental error.

Column 8 gives the percentage difference between Duane's and the true effective wave length as measured with the standard and thimble chambers. As expected from Section II the differences are large for all filtrations, although they become less for increasing filtration.

It must be emphasized again that, for much of the medical work done, the differences in effective wave lengths due to the use of thimble chambers is of no great importance. The fundamental difference between the three methods described is, however, of sufficient magnitude to be taken into consideration for all types of work.

Table 5 summarizes a series of absorption measurements made by Failla for the same radiation, but using six different types of ionization chamber. Columns 2 to 7 give the true effective wave lengths for the initial filtrations given in column 1.

From the discussion above we see that, while Duane's effective wave lengths are not the closest to the true effective wave length, they are the simplest to determine, requiring but one intensity measurement besides that of the original beam. The bracket method is considerably closer, but requires two extra intensity measurements. In both of these methods the accuracy of the measurement is very dependent upon the filter thickness, as stressed by Duane, who recommends the use of the mass per square centimeter of the filter instead of its thickness, the former being much easier to measure accurately. To obtain the "true effective wave length," of course, requires a series of intensity measurements, and consequently is not so applicable in practice as the other two methods.

TABLE 5

Filter	Standard λ_e	Drum λ_e	Bakelite λ_e	Graphite sphere λ_e	Aluminum mesh λ_e	Aluminum sphere λ_e
(1)	(2)	(3)	(4)	(5)	(6)	(7)
mm	Å	Å	Å	Å	Å	Å
0.25	0.256	0.244	0.261	0.246	0.247	0.264
.50	.192	.193	.187	.201	.185	.212
.75	.177	.177	.171	.178	.181	.192
1.00	.164	.165	.160	.166	.170	.179

For reference, curves are given showing λ_e as a function of the percentage transmission of the filter. (Fig. 6.) These are identical with Duane's curves and may be used in determining λ_e by his method. In applying these curves to the bracket method it must be clearly understood that the intensity measurements are made for filter thicknesses equally above and below the filter used in practice.

IV. RECOMMENDATIONS ON THE PROCEDURE FOR QUALITY EXPRESSION

In the foregoing discussion it has seen shown that a large number of methods of quality expression have been proposed and used. For example, in the work of Burby and Barry, and Pohle and Barnes,[35] we find some half-dozen methods of determining the effective wave length, although the authors have carefully explained the use of each method. Likewise, Quimby [36] used several kinds of λ_e in addition to the absorption coefficient of the radiation to express quality—with

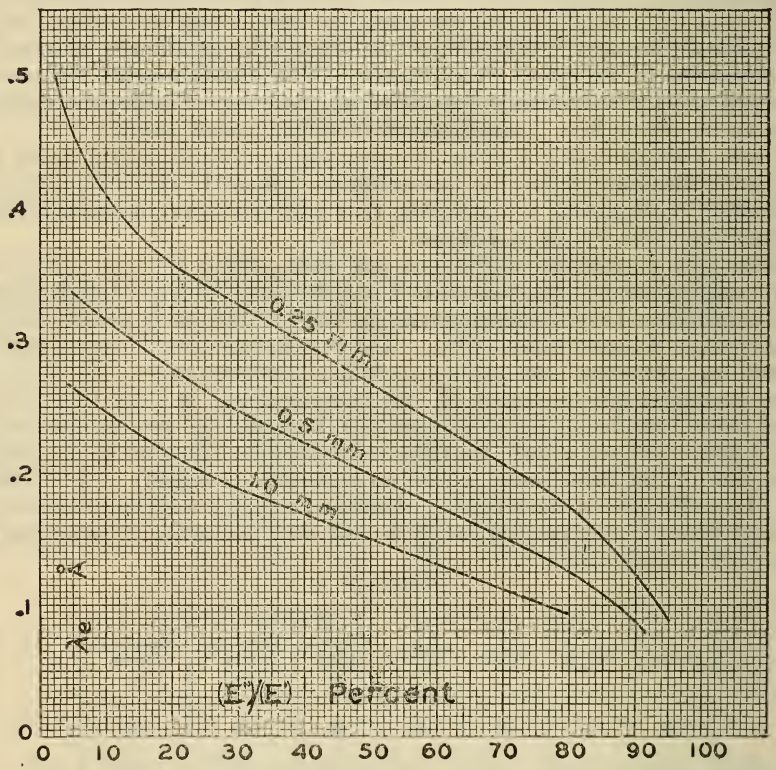

FIGURE 6.—*Effective wave length as a function of the percentage transmission through given filters of finite thickness*

proper explanation in each case. As shown in (III, 3, (b)), it is impossible to readily correlate effective wave lengths when obtained with different filter thicknesses.

In all the methods for obtaining λ_e, two steps are involved—determining an absorption coefficient μ of the radiation by some means and from this the wave length of monochromatic radiation having the same μ. For certain ranges Pohle and Barnes found that Duane's effective wave-length curves were inadequate, and consequently expressed λ_e in terms of the percentage transmission of a given filter—a physically undesirable procedure. The fact remains that at present

[35] See footnotes 4 and 6, p. 518.

[36] See footnote 5, p. 518.

the monochromatic wave length copper absorption coefficients have not been determined over the full range needed to determine λ_e for all the radiations used in X-ray therapy.

It seems reasonable, therefore, to recommend that the absorption coefficient in copper or aluminum of a heterogeneous beam of X rays be used alone to express the quality, stating the methods used in its determination. If desired at any future time, it is then a simple matter to express from available data the effective wave length in the ranges where possible. Such a method would, for example, eliminate any confusion in Pohle and Barnes' tables.

As shown in II, 1, the most accurate value of the composite absorption coefficient is obtained when using an infinitesimal filter thickness. The degree of agreement between this and one of the finite filter methods has been shown in Table 2.

In practice, μ may be determined (1) directly from the slope of the copper absorption curve, giving the true absorption coefficient; (2) by Duane's method, giving an approximate absorption coefficient; and (3) by the bracket method, giving a value in closer agreement with the true absorption coefficient. The first requires a series of measurements. The second and third require but two measurements and μ is calculated from equation (10).

For reference, Figure 7 gives curves for the absorption coefficient as a function of the percentage transmission of several thicknesses of filter which may be used in applying methods 2 and 3 above. These apply for a filter of any material but care must be taken that the material and thickness are always specified.

V. SPECTRAL ENERGY DISTRIBUTION AS DERIVED FROM COPPER ABSORPTION MEASUREMENTS

We may now turn to one other application of the foregoing copper absorption measurements, namely, the determination of a qualitative picture of the energy distribution in a beam of filtered heterogeneous radiation without the use of a spectrometer.

The effect on an X-ray beam of adding successive layers of a filtering material (copper for example) is to reduce the intensity of all incident wave lengths according to the absorption law expressed in equations (3) and (3a). As seen from this equation, the greatest absorption occurs in the longer wave lengths so that by increasing the filter thickness the energy becomes relatively more and more concentrated in the shorter, more penetrating wave lengths.

If, to take a particular case as an example, a tungsten target X-ray tube be operated below the critical voltage (70 kv) at which the characteristic K series of lines appear, the effect of increased filtration is such that the maximum of the spectral energy distribution curve of the filtered radiation shifts steadily toward shorter wave lengths. It would be expected that for such conditions a logarithmic plot would approach asymptotically a straight line, the slope of which is the μ for the short wave length limit. Since this would require that the remaining energy be practically zero, we would not expect the absorption curve to become straight for filtrations ordinarily used (0 to 3 mm copper).

However, in the case of operating the same tube well above its critical voltage; for example, at 150 to 200 kv, the K lines will be

present and conditions altered. The effect now of increased filtration is again to reduce the energy at all wave lengths, the resulting energy distribution being dependent upon the voltage and filtration; but, in general, the energy transmitted through thick copper filters will consist of several practically monochromatic lines—the several K lines and the "end" radiation at the short wave length limit.

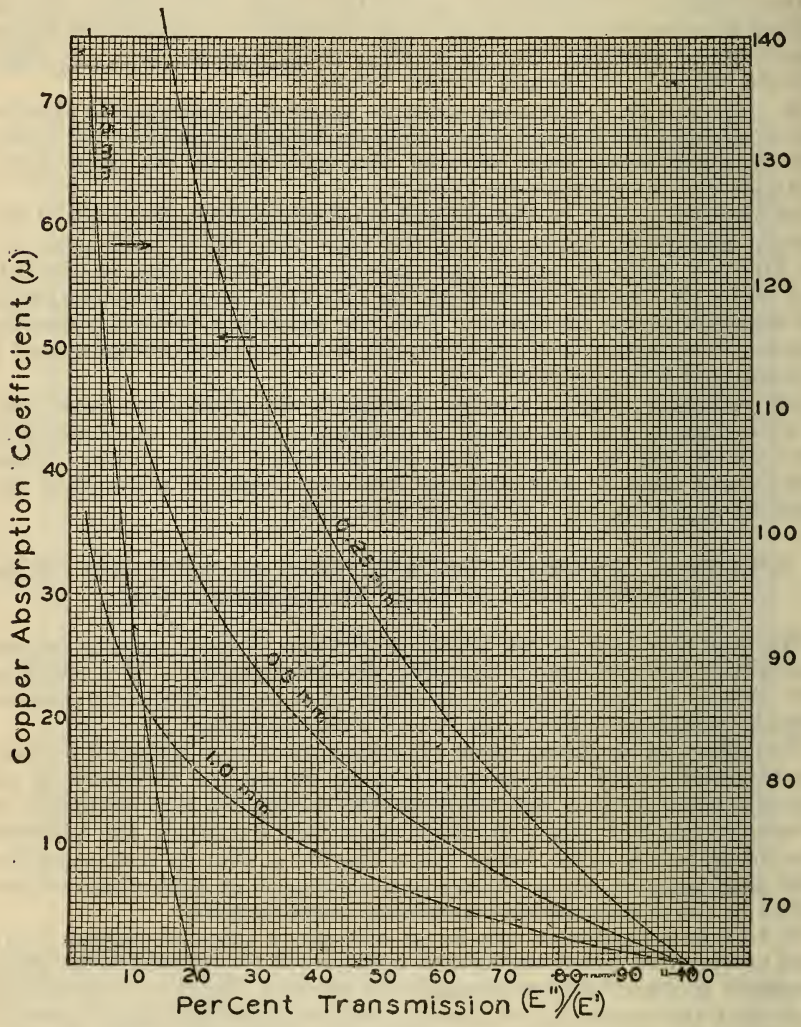

FIGURE 7.—*Copper absorption coefficients as a function of the percentage transmission through given filters of finite thickness*

Note that the curve for 0.25 mm filter is in two parts.

At present we have no very accurate data concerning the relative intensity of the K_α lines in comparison with the continuous spectrum background. There will always be present a certain amount of this radiation of shorter and less easily absorbed wave lengths, and it is thus impossible to state in advance the expected form of the logarithmic copper absorption curves for such radiation after high filtration.

Only if the amount of continuous radiation were negligible, compared with that of the line radiation, could it be expected that the absorption curve would become straight, thus indicating a constant absorption coefficient μ. At tube potentials of the order of 150 kv it has been tacitly assumed by some workers that a large part of the spectral energy comes from the K_α lines of tungsten which appear at wave lengths of about 0.2 Ångstroms, thus accounting for a straight end to the absorption curve. However, such a distribution of energy, even in highly filtered radiation, is contrary to experience.

Having obtained values of λ_e for copper filtrations up to 3 mm we see that the effective wave length of 130 kv radiation does not approach the wave lengths of the tungsten K_α lines which occur at about 0.208, 0.213, 0.184, and 0.179, Å, respectively. The nearest approach of λ_e to these wave lengths occurs for about 0.6 to 0.8 mm of copper filter; whereas, for the maximum filtration used, λ_e was about 0.134 Å. This is a reasonable value of λ_e, since for the voltage used the minimum wave length λ_o of the spectrum is 0.095 Å. The fact that $\lambda_e = 0.134$ Å is nearer $\lambda_o = 0.095$ Å than the wave length of the K lines gives further indication that the larger part of the energy passing through the filter is not that of the K lines, but that of the continuous spectrum background instead.

As a qualitative check of the above reasoning, we may refer to some approximate spectral energy distribution curves obtained by Duane [37] with an ionization spectrometer. The curve shown was made at 161 kv (constant potential) and 1 mm of copper, and it is shown thereby that the greater part of the remaining energy apparently lies at shorter wave lengths than the first-order tungsten lines.

VI. SUMMARY

1. A qualitative theory of copper absorption measurements shows that, for a known spectral energy distribution, an effective wave length may be determined which is definitely related through known absorption laws to that distribution.

2. The logarithmic copper absorption curves for heterogeneous high voltage X-radiation do not become straight for filtrations up to 3 mm copper when the measurements are made with a standard ionization chamber.

3. Absorption measurements, applied to the problem of describing the quality of a heterogeneous beam of X rays, show (*a*) that measurements of the so-called "average wave length" of the radiation have no meaning as long as they are based strictly on the logarithmic absorption curve becoming straight for filtrations greater than the "homogeneity filter"; (*b*) the "half-value layer" method of expressing quality is sensitive at least up to 1.6 mm copper filtration but is readily reproducible only for strictly constant voltage applied to the X-ray tube; (*c*) the most logical expression of the "effective wave length" is obtained when using the absorption coefficient given by the slope of the logarithmic absorption curve. This is called the true effective wave length.

[37] W. Duane, Am. J. Roent., 9, p. 167; 1922.

4. Duane's method for obtaining the effective wave length gives values consistently lower than the true effective wave length, although the difference is not great enough at higher filtrations to have any significance for most medical work. If this method is used the thickness of copper used in the measurement should be stated.

5. The bracket method for determining the effective wave length is more accurate than any other approximate method, but requires one more observation than the Duane method.

6. Absorption measurements made with the more common form of thimble ionization chamber are subject to error due to the chamber "wall effect." Such error is minimized in the true effective wave-length method.

7. The absorption coefficient obtained with a standard thickness of copper (or aluminum) is recommended as the simplest general method of describing the quality of a heterogeneous beam of X rays for therapeutic uses. Due to a lack of monochromatic wave-length absorption data over a sufficient range, it is impossible to extend the effective wave-length method over the whole range of radiations encountered in X-ray therapy. Thus, for a perfectly general physical definition of quality, the "true" absorption coefficient obtained from the slope of the absorption curve is recommended.

8. The effect on heterogeneous tungsten radiation of increasing the copper filtration is the concentration of the major part of the transmitted energy in wave lengths shorter than the K series of lines.

Recognition is due G. Singer, of this laboratory, for having carried out the experimental part of this work.

WASHINGTON, December 1, 1929.